Dynamic Mechanism Design: Robustness and Endogenous Types

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Mechanism Design

- Mechanism Design: auctions, regulation, taxation, political economy, etc...

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- Many settings

  - **information arrives over time** (serially correlated, possibly endogenous)

  - **sequence of decisions**

Long-Term Contracting

- Long-term contracting

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- Long-term contracting
    - Trade

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  - Trade
  - Employment

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- Value of relationship changes over time
- "Shocks" to:
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  - productivity
  - outside options
  - etc.
- Changes often anticipated albeit not necessarily commonly observed

Questions

- Structure of optimal LT contract in changing environments

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- Dynamics of distortions — convergence to FB?

Dynamic Mechanism Design

- Applications:

  - **revenue management** (Courty and Li, 2000, Battaglini 2005, Boleslavsky and Said, 2013, Ely, Garrett and Hinnosaar, 2014, Board and Skrzypacz, 2015, Akan, Ata, and Dana, 2015,...)

  - **disclosure in auctions** (Eso and Szentes, 2007, Bergemann and Wambach (2015), Li and Shi (2015)...)

  - **experimentation** (Bergemann and Välimäki, 2010, Pavan, Segal, and Toikka, 2014, Fershtman and Pavan, 2015...)

  - **taxation** (Farhi and Werning, 2012, Kapicka, 2013, Stantcheva, 2014, Makris and Pavan,2015,...)

  - **managerial compensation** (Garrett and Pavan, 2012, 2014,...)

  - **insurance** (Hendel and Lizzeri, 2003, Handel, Hendel, Whinston, 2015,...)

Dynamic Mechanism Design

- Most work:

  - Myersonian/first-order approach ("global" IC slack)

  - exogenous types

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- **Robust Predictions** (to binding global IC constraints)**?**

- **Novel effects due to endogeneity of types?**

Plan

1. Robust Predictions

Plan

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2. Endogenous Types

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1. Robust Predictions
2. Endogenous Types
3. Conclusions

Static example

- Price discrimination (Mussa-Rosen, Maskin & Riley, Myerson)

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with θ drawn from F (density f), privately observed by Buyer

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- Envelope Th.

$$V^A(\theta) = V^A(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} q(s)ds \quad \text{with} \quad q(\cdot) \text{ nondecreasing}$$

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$$\mathbb{E}\left[\left(\theta - \frac{1-F(\theta)}{f(\theta)}\right) q(\theta) - c(q(\theta))\right] \quad \text{s.t. } q(\cdot) \text{ nondecreasing (M)}$$

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- **Robust predictions** (e.g., Hellwig, 2010):

1. participation constraint binds only for lowest type: $V^A(\underline{\theta}) = 0$
2. no distortion at the top: $q(\bar{\theta}) = q^{FB}(\bar{\theta})$
3. downward distortions elsewhere: $q(\theta) < q^{FB}(\theta)$

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- Binding (M): "ironing" (just more pooling)

Dynamic Environment

- $t = 1, \ldots, T$ (possibly infinite)

- Intertemporal payoffs

$$U^P = \sum_t \delta^{t-1}(p_t - c(q_t)) \quad \text{and} \quad U^A = \sum_t \delta^{t-1}(\theta_t q_t - p_t)$$

- θ_t privately observed by agent at beginning of period t

Type process

- type θ_t drawn from (exogenous) Markov chain on $\Theta = [\underline{\theta}, \overline{\theta}] \subseteq \mathbb{R}_+$
- transition probability kernels $F \equiv (F_t)$
- $F_t(\cdot \mid \theta)$: cdf of θ_t, given $\theta_{t-1} = \theta$
- F_1 : cdf of initial distribution; density f_1
- *stochastic monotonicity (FOSD):* $\theta' > \theta \Rightarrow F_t(\cdot \mid \theta') \succsim_{FOSD} F_t(\cdot \mid \theta)$
- *ergodicity:* $\exists!$ invariant distribution π s.t., for all $\theta \in \Theta$

$$\sup_{A \in \mathscr{B}(\Theta)} |F^n(A, \theta) - \pi(A)| \to 0 \text{ as } n \to \infty$$

- *stationarity:* $F_1 = \pi$ and $F_t = F_s$ all $t, s > 1$.

Principal's problem

- Principal designs $\chi = \langle \mathbf{q}, \mathbf{p} \rangle$ to maximize

$$\mathbb{E}\left[\sum_t \delta^{t-1}(p_t(\theta^t) - c(q_t(\theta^t)))\right]$$

subject to IR-1 and IC-t, all $t \geq 1$

- Stronger (periodic) IR

- Complexity:

- different types have different beliefs about future

- multi-period deviations

IC-IR-extended

State representation and impulse responses

Eso-Szentes (2007), Pavan, Segal, Toikka (2014)

- Auxiliary shocks, orthogonal to initial private information
- $\theta_t = Z_t(\theta_1, \varepsilon)$ where $\varepsilon \equiv (\varepsilon_t)$ are iid r.v.s
- Integral-transform-probability theorem (F_t^{-1} inductively)
- **Impulse responses**:

$$\boxed{I_t(\theta_1, \varepsilon) = \frac{\partial}{\partial \theta_1} \theta_t = \frac{\partial Z_t(\theta_1, \varepsilon)}{\partial \theta_1}}$$

Examples

- AR(1):

$$\begin{aligned} \theta_t &= \gamma\theta_{t-1} + \varepsilon_t \\ &= Z_t(\theta_1, \varepsilon) = \gamma^{t-1}\theta_1 + \gamma^{t-2}\varepsilon_2 + \cdots + \gamma\varepsilon_{t-1} + \varepsilon_t \\ &\to I_t(\theta_1, \varepsilon) = \gamma^{t-1} \end{aligned}$$

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- More generally,

$$I_t = \prod_{s \le t} \frac{\partial}{\partial \theta} F_s^{-1}(\varepsilon_s \mid \theta_{s-1})$$

Local IC – heuristics

- Assume $T = 2$
- Fix period-1 report, $\hat{\theta}_1$, and priod-2 reporting strategy, $\sigma(\varepsilon)$
- Agent's payoff

$$U^A(\theta_1, \hat{\theta}_1; \sigma) = \theta_1 q_1(\hat{\theta}_1) - p_1(\hat{\theta}_1) + \delta \mathbb{E}\left[Z_2(\theta_1, \varepsilon) q_2(\hat{\theta}_1, \sigma(\varepsilon)) - p_2(\hat{\theta}_1, \sigma(\varepsilon))\right]$$

- If $\chi = \langle \mathbf{q}, \mathbf{p} \rangle$ is IC, then

$$V_1^A(\theta_1) = \sup_{\hat{\theta}_1; \sigma} U^A(\theta_1, \hat{\theta}_1; \sigma)$$

- Envelope theorem

$$\frac{\partial V_1^A}{\partial \theta_1} = \frac{\partial}{\partial \theta_1} U^A(\theta_1, \theta_1; \sigma^{truth}) = q_1(\theta_1) + \delta \mathbb{E}\left[\frac{\partial Z_2(\theta_1, \varepsilon)}{\partial \theta_1} q_2(\theta_1, \varepsilon))\right]$$

$$= \mathbb{E}\left[\sum_{s \geq 1} \delta^{s-1} I_s q_s \mid \theta_1\right]$$

Local IC – general case

- More generally,

Theorem (Pavan, Segal, Toikka, 2014)

If $\chi = \langle \mathbf{q}, \mathbf{p} \rangle$ is IC, then, for every truthful history θ^{t-1}, $t \geq 0$, V_t^A is equi-Lipschitz-continuous in θ_t and

$$\frac{\partial V_t^A}{\partial \theta_t} = \mathbb{E}\left[\sum_{s \geq t} \delta^{s-1} I_{t \to s} q_s \mid \theta^t\right] \quad a.e., \qquad \text{(ICFOC)}$$

where $I_{t \to s} = \frac{d}{d\theta_t}\theta_s$ (with $I_t \equiv I_{1 \to t}$)

▸ ICFOC-proof

Sufficiency and Integral Monotonicity

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Theorem (PST, 2014)

Mechanism $\chi = \langle \mathbf{q}, \mathbf{p} \rangle$ is IC iff, for all $t \geq 0$,

$$\frac{\partial V_t^A}{\partial \theta_t} = \mathbb{E}\left[\sum_{s \geq t} \delta^{s-1} I_{t \to s} q_s \mid \theta^t\right] \text{ a.e.,} \qquad \text{(ICFOC)}$$

and, for all θ^t and $\hat{\theta}_t$,

$$\int_{\hat{\theta}_t}^{\theta_t} [D_t((\theta^{t-1}, x); x) - D((\theta^{t-1}, x); \hat{\theta}_t)] dx \geq 0 \qquad \text{(INT-M)}$$

where

$$D_t(\theta^t; \hat{\theta}_t) \equiv \mathbb{E}\left[\sum_{s \geq t} \delta^{s-1} I_{t \to s} q_s(\theta^s_{-t}, \hat{\theta}_t) \mid \theta^t\right]$$

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- Int-M $\to$ one-stage deviations suboptimal

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- Int-M $\longrightarrow$ one-stage deviations suboptimal
- Int-M + Markov + OSDP $\longrightarrow$ all deviations suboptimal

Int-M-Proof

Stronger sufficient conditions

- Int-M holds if expected future output, *discounted by impulse responses*

$$D_t(\theta^t;\hat{\theta}_t) = \mathbb{E}\left[\sum_{s\geq t}\delta^{s-1}I_{t\to s}q_s(\theta^s_{-t},\hat{\theta}_t) \mid \theta^t\right]$$

is *nondecreasing* in current report $\hat{\theta}_t$.

- Output need not be monotone history by history, enough to have monotonicity **on average** over time and states.

- Literature typically checks "strong monotonicity" (i.e., $q_t(\theta^t)$ nondecreasing in θ^t), but that's stronger than necessary.

Full program

- Principal's **full program**

$$\max_{\chi=\langle \mathbf{q},\mathbf{p}\rangle} \mathbb{E}\left[\sum_t \delta^{t-1}(p_t - c(q_t))\right]$$

subject to

IR: $V_1^A(\theta_1) \geq 0$ all θ_1

ICFOC-(t): $\dfrac{\partial V_t^A(\theta^t)}{\partial \theta_t} = D_t(\theta^t;\theta_t)$

Int-M: $\displaystyle\int_{\hat{\theta}_t}^{\theta_t} [D_t((\theta^{t-1},x);x) - D_t((\theta^{t-1},x);\hat{\theta}_t)]dx \geq 0.$

Relax program – Myersonian/First-Order Approach

- Principal's **relaxed program**

$$\max_{\chi=\langle \mathbf{q},\mathbf{p}\rangle} \mathbb{E}\left[\sum_t \delta^{t-1}(p_t - c(q_t))\right]$$

subject to

IR: ~~$V_1^A(\theta_1) \geq 0$ all θ_1~~ $\rightarrow$ $V_1^A(\underline{\theta}) \geq 0$

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Relax program – Myersonian/First-Order Approach

- Principal's objective as "**Dynamic Virtual Surplus**"

$$\max_{\mathbf{q}} \mathbb{E}\left[\sum_t \delta^{t-1}\left(\theta_t - \frac{1-F_1(\theta_1)}{f_1(\theta_1)} I_t\right) q_t - c(q_t))\right]$$

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- Pointwise maximization:

$$\text{period-}t \text{ virtual value } = \theta_t - \frac{1-F_1(\theta_1)}{f_1(\theta_1)} I_t = c'(q_t) = \text{ marginal cost}$$

$\Rightarrow$ **distortions** driven by **impulse responses** I_t

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- Suppose $c(q) = \frac{1}{2}q^2$. Solution to relaxed program

$$q_t = \theta_t - \frac{1-F_1(\theta_1)}{f_1(\theta_1)} I_t$$

Monotone enough?

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Example (AR-1)

$q_t = \theta_t - \frac{1-F_1(\theta_1)}{f_1(\theta_1)} \phi^{t-1} \Rightarrow$ suffices that F_1 log-concave

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- **Variational approach** $\longrightarrow$ robust predictions for **average distortions**

▸ Existence

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- Simple perturbation: add constant $a \in \mathbb{R}$ to period-t allocation (equivalently, Gateux derivative in direction $(0, \ldots, 0, 1, 0, \ldots)$)
- FOC for optimum at $a = 0$:

$$\mathbb{E}\left[\theta_t - \frac{1-F_1(\theta_1)}{f_1(\theta_1)} I_t\right] = \mathbb{E}\left[c'(q_t)\right]$$

$\Rightarrow$ *average* virtual value equals *average* marginal cost

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- Assume IR binds only at $\theta_1 = \underline{\theta}$ (always under FOSD and $\mathbf{q} \geq 0$) and interior solutions.
- Simple perturbation: add constant $a \in \mathbb{R}$ to period-t allocation (equivalently, Gateux derivative in direction $(0, \ldots, 0, 1, 0, \ldots)$)
- FOC for optimum at $a = 0$:

$$\mathbb{E}\left[\theta_t - \frac{1-F_1(\theta_1)}{f_1(\theta_1)} I_t\right] = \mathbb{E}\left[c'(q_t)\right]$$

$\Rightarrow$ *average* virtual value equals *average* marginal cost

- Same prediction as under FOA, **but only in expectation**!

$$\begin{aligned}\mathbb{E}[\text{period-}t\text{ distortion}] &\equiv \mathbb{E}[\theta_t - c'(q_t)] \\ &= \mathbb{E}\left[\frac{1-F_1(\theta_1)}{f_1(\theta_1)} I_t\right]\end{aligned}$$

Handicap Dynamics

Theorem (Garrett, Pavan, Toikka, 2015)

Assume F is ergodic. Then

$$\mathbb{E}\left[\frac{1-F_1(\theta_1)}{f_1(\theta_1)}I_t\right] \to 0.$$

Moreover, if F satisfies FOSD, then convergence is from above.

If, in addition, F is stationary, then convergence is monotone in t.

Handicap-proof

More general bounds

- When IR binds only at bottom and $\mathbf{q}$ interior

$$\mathbb{E}[\text{distortion}] = \mathbb{E}[\text{handicap}] = \mathbb{E}\left[\frac{1-F_1(\theta_1)}{f_1(\theta_1)} I_t\right]$$

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Theorem (Garrett-Pavan-Toikka, 2015)

If F is ergodic, then

$$\limsup_{t\to\infty} \mathbb{E}[\theta_t - c'(q_t)] \leq 0 \qquad \textit{(limit upward distortions)}$$

If, in addition, q eventually strictly interior, then

$$\lim_{t\to\infty} \mathbb{E}[\theta_t - c'(q_t)] = 0$$

Finally, if distortions are eventually downward, then

$$q_t \xrightarrow{p} q_t^{FB}$$

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Corollary

Failure to converge → over-consumption and exclusion eventually infinitely often.

Dynamic Managerial Compensation: A Variational Approach

Garrett and Pavan, JET 2015

- Suppose now

$$U^P \equiv \sum_{t=1}^{T} \delta^{t-1} [y_t - c_t]$$

$$U^A \equiv \sum_{t=1}^{T} \delta^{t-1} \left[v^A(c_t) - \psi(y_t, \theta_t) \right]$$

with

$$\psi(y_t, \theta_t) = \frac{1}{2} (y_t - \theta_t)^2 \text{ and } \theta_t = \rho \theta_{t-1} + \varepsilon_t$$

- Non quasi-linear environment

Theorem (Garrett and Pavan, JET 2015)

When the agent's risk aversion and productivity persistence are low, distortions decrease, on average, over time. Opposite is true when they are sufficiently high.

- distortions increasing over time reduce compensation risk

Managerial Turnover in a Changing World

Garrett and Pavan (JPE, 2013)

- Under risk neutrality and declining impulse responses

  $\longrightarrow$ Effort gradually converges to FB

- Suppose firm can replace its managers and that each new employment relationship is affected by same information frictions as with incumbent

Theorem (Garrett and Pavan, JPE 2013)

Firm's optimal contract

(i) either induces excessive retention (inefficiently low turnover) at all tenure levels

(ii) or excessive firing early on followed by excessive retention in the long run.

Plan

1. Robust Predictions
2. **Endogenous Types**
3. Conclusions

Motivation/Perspective

- Most of DMD literature: exogenous types/information

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- Many problems of interest: information/types **endogenous**
  - bandit auctions for sale of experience goods – Pavan, Segal, Toikka, 2014
  - dynamic matching w. unknown values – Fershtman and Pavan, 2015
  - habit formation/addiction
  - **taxation under learning-by-doing** (Makris and Pavan, 2015)

Incentives for Endogenous Types: Makris and Pavan (2015)

- Design of incentive schemes in Markovian setting with

  - endogenous types

  - non-quasilinear payoffs

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- Theory + quantitative analysis

Questions

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- (Mirrleesian) taxation:

  - Effect of LBD on **progressivity** and **dynamics of marginal taxes**

  - Welfare gains of tax reforms

  - Loss from neglecting LBD

Related literature (incomplete)

- **Dynamic Mechanism Design**

  ....

- **New Dynamic Public Finance**

  Farhi and Werning (2012) ⟵

  Kapicka (Restud 2013)

  Golosov, Troshkin, Tsyvinski (2014)

  ...

- **Taxation w. Human Capital Accumulation / LBD**

  Kapicka (2014)

  Kapicka and Neira (2014) ⟵

  Stantcheva (2014) ⟵

  Krause (2009)

  Best and Kleven (2013) ⟵

  Heathcote, Storesletten and Violante (2014)

  ...

Design problem

- Agent's productivity

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  with z_t increasing

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- Principal's (dual) problem: choose $\chi = \langle \mathbf{y}, \mathbf{c} \rangle$ to maximize

$$\mathbb{E}\left[\sum_{t=1}^{T} \delta^t (y_t - c_t)\right]$$

  s.t. χ be IC and

$$\int_{\underline{\theta}_1}^{\overline{\theta}_1} q\left(V_1^A(\theta_1)\right) dF_1(\theta_1) \geq \kappa$$

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- Rawlsian: $q\left(V_1^A(\theta_1)\right) = 0$ all $\theta_1 > \underline{\theta}$

Incentive Compatibility

- Analysis similar to exogenous case BUT

$$I_{t \to s}(\theta^s, y^{s-1})$$

now **depend on past income**

Wedges

- Period-t labor wedge (equivalently, marginal tax rate)

$$\left[1 + LD_t^{FB;\chi}\right][1 - W_t] = \frac{\psi_y(y_t, \theta_t)}{v^{A\prime}(c_t)}$$

where $LD_t^{FB;\chi}(\theta^t)$ are FB effects of **learning-by-doing**

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where $LD_t^{FB;\chi}(\theta^t)$ are FB effects of **learning-by-doing**

- Interested in how LBD affects progressivity and dynamics of W_t

- We study

$$\hat{W}_t(\theta^t)\equiv\frac{W_t(\theta^t)}{1-W_t(\theta^t)}$$

Wedges: Two-period Rawlsian

- Period-2 wedge

$$\hat{W}_2(\theta) = RA_2(\theta)\left[\hat{W}_2^{RN}(\theta)\right]$$

where

- $\hat{W}_2^{RN}(\theta)$ is wedge under risk neutrality and no LBD

- $RA_2(\theta) > 0$ is correction due to risk aversion

Wedges: Two-period Rawlsian

- Period-1 wedge under LBD

$$\boxed{\hat{W}_1(\theta_1) = RA_1(\theta_1)\left[\hat{W}_1^{RN}(\theta_1) + \Omega_1(\theta_1)\right]}$$

where $\Omega_1(\theta_1)$ is effect of LBD on period-1 wedge

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where $\Omega_1(\theta_1)$ is effect of LBD on period-1 wedge

- $\Omega_1(\theta_1)$ takes into account how variation in y_1 affects **cost of future incentives** through

  - **distribution of future productivity** $F_2(\theta_2|\theta_1, y_1)$

  - **impulse responses** (handicaps): $I_2(\theta, y_1)$

Wedges: General case

Theorem (Makris and Pavan, 2015)

At any history θ^t, optimal wedge given by

$$\boxed{\hat{W}_t(\theta^t) = RA_t(\theta^t)\left[\hat{W}_t^{RN}(\theta^t) + \Omega_t(\theta^t)\right]}$$

Literature: special cases

- Static RN (Mirrlees-Diamond-Saez)

$$\hat{W}_1(\theta_1) = \hat{W}_1^{RN}(\theta_1) = \frac{1 - F_1(\theta_1)}{f_1(\theta_1)} \frac{1}{\theta_1} \varepsilon_\theta^{\psi_y}(y_1(\theta_1), \theta_1)$$

where $\varepsilon_\theta^{\psi_y}(y_1, \theta_1)$ is elasticity of marginal disutility of effort.

Literature: special cases

- Static RA (Mirrlees-Saez):

$$\hat{W}_1(\theta_1) = RA_1(\theta_1)\hat{W}_1^{RN}(\theta_1)$$

$$= \left[\int_{\theta_1}^{\overline{\theta}_1} \frac{1}{v'(c_1(s))} \frac{dF_1(s)}{1-F_1(\theta_1)} v'(c_1(\theta_1))\right] \frac{1-F_1(\theta_1)}{f_1(\theta_1)} \frac{1}{\theta_1} \varepsilon_\theta^{\psi_y}(y_1(\theta_1),\theta_1)$$

Literature: special cases

- Dynamic RN (exogenous types):

$$\hat{W}_t(\theta^t) = \hat{W}_t^{RN}(\theta^t) = \frac{1 - F_1(\theta_1)}{f_1(\theta_1)} \frac{I_t(\theta^t)}{\theta_t} \varepsilon_\theta^{\psi_y}(y_t(\theta^t), \theta_t)$$

Literature: special cases

- Dynamic RA (exogenous types; Farhi and Werning, 2013):

$$\hat{W}_t(\theta^t) = RA_t(\theta^t)\left[\hat{W}_t^{RN}(\theta^t)\right]$$

Literature: special cases

- Dynamic RA (exogenous types with training; Stantcheva, 2014):

$$\hat{W}_t(\theta^t) = RA_t(\theta^t)\left[\hat{W}_t^{RN}(\theta^t, b^t(\theta^{t-1}))\right]$$

where

$$\hat{W}_t^{RN}(\theta^t, b^t(\theta^{t-1})) = \frac{1-F_1(\theta_1)}{f_1(\theta_1)}\frac{I_t(\theta^t)}{\theta_t}\varepsilon_\theta^{\psi_y}(y_t(\theta^t), \theta_t, b^t(\theta^{t-1}))$$

General case

- More generally:

$$\boxed{\hat{W}_t(\theta^t) = RA_t(\theta^t)\left[\hat{W}_t^{RN}(\theta^t) + \Omega_t(\theta^t)\right]}$$

- Key: **endogeneity of type process**

Example of effects of endogenous types on progressivity/dynamics

- $\psi(y,\theta) = \frac{1}{1+\phi}\left(\frac{y}{\theta}\right)^{1+\phi}$ where $1/\phi > 0$ is Frisch elasticity

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- Pareto F_1 : $\frac{f_1(\theta_1)}{1-F_1(\theta_1)}\theta_1 = \lambda$

- Ben-Porath specification:

$$\theta_2 = z_2(\theta_1, y_1, \varepsilon_1) = \theta_1^{\rho} \cdot y_1^{\zeta} \cdot \varepsilon_2$$

where

- ρ controls for exogenous type persistence
- ζ controls for **intensity of LBD**

Example (cont'd)

- Consider economy described above, and assume agent is risk-neutral and principal Rawlsian

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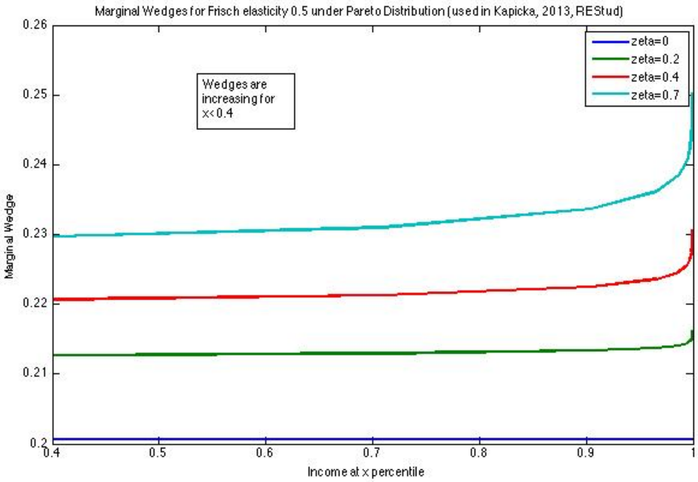
- With LBD

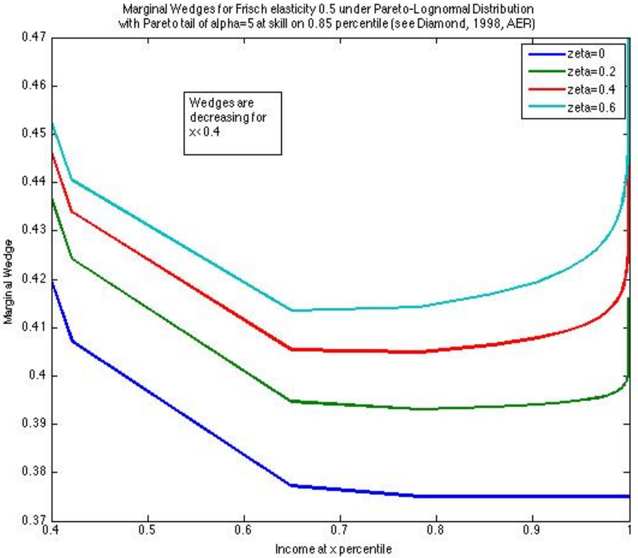
$$\boxed{\hat{W}_1(\theta_1) > \hat{W}_2(\theta) \text{ all } \theta}$$

with $\hat{W}_1(\theta_1)$ strictly increasing in θ_1 (progressivity)

Kapicka Restud 2013 (Pareto)



Diamond, 1998, AER (Pareto-Lognormal)



Intuition for progressivity and wedge dynamics

- **Positive effect of LBD on W_t**

  - reduction in y_t reduces future cost of incentives through

  (a) impulse responses ($I_{t \to s}$ increasing in y_t under Ben-Porath specification)

  (b) distribution of future types (handicaps increasing in future types)

  - importance of future cost of incentives declines with $t \longrightarrow W_t$ *decline* with t

- **Progressivity**

  - handicaps increasing in types $\longrightarrow$ reduction in cost of incentives most pronounced for higher types

Conclusions

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  useful tool for positive and normative analysis of environments in which

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  $\longrightarrow$ important qualitative and quantitative implications

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  - dynamic matching (Fershtman and Pavan, 2015)

- (Much) more remains to be done

  - partial commitment

  - interaction of time-varying information w. population dynamics (e.g., Garrett 2013)

  - endogenous disclosure (Calzolari and Pavan (2006a,b), Kamenica and Gentzkow, 2011,...)

  - empirics (Handel, Hendel, Whinston, 2014, Einav et al., 2015...)

Thank You!

Mechanisms and Principal's problem

- direct mechanism $\chi = \langle \mathbf{q}, \mathbf{p} \rangle$, with $q_t : \Theta^t \to \mathcal{Q}$ and $p_t : \Theta^t \to \mathbb{R}$
- principal designs χ to maximize

$$\mathbb{E}\left[\sum_t \delta^{t-1}(p_t - c(q_t))\right]$$

subject to

$$\mathbb{E}\left[\sum_t \delta^{t-1}(\theta_t q_t - p_t) \mid \theta_1\right] \geq 0 \quad \text{for all } \theta_1 \in \Theta \tag{IR-1}$$

$$\mathbb{E}\left[\sum_{s \geq t} \delta^{s-1}(\theta_s q_s - p_s) \mid \theta^t\right] \geq \mathbb{E}\left[\sum_{s \geq t} \delta^{s-1}(\theta_s q_s^\sigma - p_s^\sigma) \mid \theta^t\right] \tag{IC-t}$$

$$\text{for all } \sigma, \text{ all } \theta^t = (\theta_1, \ldots, \theta_t) \in \Theta^t$$

IC-IR-simple

ICFOC: Proof Sketch

- Agent's payoff in terms of state representation:

$$\mathbb{E}\left[\sum_t \delta^{t-1}(\theta_t q_t - p_t) \mid \theta_1\right] = \tilde{\mathbb{E}}\left[\sum_t \delta^{t-1}(\tilde{q}_t(\theta_1,\varepsilon^t)Z_t(\theta_1,\varepsilon^t) - \tilde{p}_t(\theta_1,\varepsilon^t)) \mid \theta_1\right]$$

- Thus,

$$V_1(\theta) = \max_{\hat{\theta}} U(\hat{\theta};\theta)$$

where

$$U(\hat{\theta};\theta) \equiv \tilde{\mathbb{E}}\left[\sum_t \delta^{t-1}\left(\tilde{q}_t(\hat{\theta},\varepsilon^t)Z_t(\theta_1,\varepsilon^t) - \tilde{p}_t(\hat{\theta},\varepsilon^t)\right) \mid \theta\right]$$

- For fixed $\hat{\theta}$,

$$\frac{d}{d\theta}U(\hat{\theta};\theta) = \tilde{\mathbb{E}}\left[\sum_t \delta^{t-1}\tilde{q}_t(\hat{\theta},\varepsilon^t)I_t \mid \theta\right]$$

- Envelope theorem then gives result
- Corollary: q pins down V_1 up to constant even if ε publicly observable $\Rightarrow$ Eso-Szentes' irrelevance result ▸ ICFOC

Integral Monotonicity: Proof sketch

- Fix t and θ^{t-1}.
- Let $U(\hat{\theta};\theta)$ = continuation utility of period-t type θ from one-stage deviation to $\hat{\theta}$.
- Markov and full support → IC equivalent to

$$V(\theta) \equiv U(\theta;\theta) = \max_{\hat{\theta}} U(\hat{\theta};\theta) \quad \text{all } \theta \in \Theta.$$

- Equivalently,

$$\hat{\theta} \in \arg\max_{\theta} \left\{ U(\hat{\theta};\theta) - V(\theta) \right\} \quad \text{for all } \hat{\theta} \in \Theta.$$

- ICFOC implies that, for $\hat{\theta}$ fixed, $g(\theta) = U(\hat{\theta},\theta) - V(\theta)$ is Lipschitz with $g'(\theta) = U_2(\hat{\theta},\theta) - V'(\theta) = U_2(\hat{\theta},\theta) - U_2(\theta,\theta)$ a.e., so

$$g(\hat{\theta}) - g(\theta) = \int_{\theta}^{\hat{\theta}} [U_2(\hat{\theta},x) - U_2(x,x)]dx,$$

- Because $U_2(\hat{\theta},x) = D_t((\theta^{t-1},x);\hat{\theta})$, $\hat{\theta}$ maximizes $g(\theta)$ iff (Int-M).

Int-M

Existence

- Let $g(\mathbf{q}) = \mathbb{E}\left[\sum_t \delta^{t-1}\left(q_t \cdot \left(\theta_t - \frac{1-F_1(\theta_1)}{f_1(\theta_1)} I_t\right) - c(q_t)\right)\right]$ and consider

$$\sup_{\mathbf{q}\in L_2} g(\mathbf{q}) \quad \text{s.t. (Int-M)}$$

  where $L_2 = L_2(\mathbb{R}^T)$ is space of square integrable processes with discounted measure, $\mathbf{q} \in L_2$ iff $\|\mathbf{q}\| = \mathbb{E}\left[\sum_t \delta^{t-1} q_t^2\right] < \infty$.

- Assume $c(q) \geq q^2$ for $|q| > \bar{q}$, for some $\bar{q}$
- Then $g(\mathbf{q}) \to -\infty$ as $\|\mathbf{q}\| \to \infty$.
- Moreover, g is concave and Gateux differentiable, and feasible set is closed, convex, and nonempty since defined by bounded linear operators.
- So supremum is achieved, because in a Hilbert space, every concave Gateux-differentiable functional that is "minus infinite at infinity" achieves its maximum on a closed convex set.
- robust

Handicap Dynamics – Proof sketch

- Recall that $\mathbb{E}[I_t \mid \theta_1] = \frac{d}{d\theta_1}\mathbb{E}[\theta_t \mid \theta_1]$.
- Thus,

$$\mathbb{E}\left[\frac{1-F_1(\theta_1)}{f_1(\theta_1)}I_t\right] = \mathbb{E}\left[\frac{1-F_1(\theta_1)}{f_1(\theta_1)}\mathbb{E}[I_t \mid \theta_1]\right] = \int_{\underline{\theta}}^{\bar{\theta}}(1-F_1(\theta_1))\mathbb{E}[I_t \mid \theta_1]d\theta_1$$

$$= (1-F_1(\theta_1))\mathbb{E}[\theta_t \mid \theta_1]|_{\theta_1=\underline{\theta}}^{\theta_1=\bar{\theta}} + \int_{\underline{\theta}}^{\bar{\theta}} f_1(\theta_1)\mathbb{E}[\theta_t \mid \theta_1]d\theta_1$$

$$= \mathbb{E}[\theta_t] - \mathbb{E}[\theta_t \mid \underline{\theta}] \to 0$$

  by ergodicity.
- If F monotone (FOSD),

$$\mathbb{E}[\theta_t] - \mathbb{E}[\theta_t \mid \underline{\theta}] \geq 0$$

- If, in addition, $F_1 = \pi$, then

$$\mathbb{E}\left[\frac{1-F_1(\theta_1)}{f_1(\theta_1)}I_t\right] - \mathbb{E}\left[\frac{1-F_1(\theta_1)}{f_1(\theta_1)}I_s\right] = \mathbb{E}[\theta_s \mid \underline{\theta}] - \mathbb{E}[\theta_t \mid \underline{\theta}] \leq 0$$

  for $t > s$.

handicap-dynamics