SUPPLEMENT TO "ESTIMATING THE EFFECTS OF A TIME-LIMITED EARNINGS SUBSIDY FOR WELFARE-LEAVERS"

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We present a simple dynamic model for welfare and work participation among long term welfare recipients in the presence and absence of the Self Sufficiency Project (SSP) earnings supplement. We compare the time profiles of the optimal reservation wage in the presence and absence of the subsidy.

KEYWORDS: Job search, optimal reservation wage.

THEORETICAL APPENDIX: A SIMPLE MODEL OF WORK AND WELFARE PARTICIPATION

A. *Model in the Absence of SSP*

WE CONSIDER A DISCRETE TIME SEARCH MODEL with time measured in months. Individuals are risk neutral and discount the future at the monthly interest rate r. Net income if on welfare is b. Net income if working at wage w is $w - c$. Each month, an individual receives a single job offer with probability λ, drawn from a distribution with density $f(w)$ and cumulative density $F(w)$, with $\ell \leq w \leq m$. The job destruction rate is δ. Optimal behavior is characterized by a value function $U(w)$, representing the value of holding a job that pays w, and by a value V^0, representing unemployment. To derive $U(w)$, note that for an individual who is currently holding a job with wage w, expected value next month is

$$\lambda(1 - F(w))\big\{(1 - \delta)E[U(\omega)|\omega > w] + \delta V^0\big\} \\ + \big(1 - \lambda(1 - F(w))\big)\{(1 - \delta)U(w) + \delta V^0\}.$$

The first term in this expression represents the outcome if an offer is obtained (which occurs with probability λ) and it pays more than the current wage (which occurs with probability $1 - F(w)$). In this case, with probability $1 - \delta$ the job survives to the end of the month and with probability δ it ends right away. The second term represents the outcome if no acceptable offer is obtained, in which case with probability $1 - \delta$ the existing job survives and with probability δ it ends. With some rearrangement, this expression becomes

$$\delta V^0 + (1 - \delta)U(w) + \lambda(1 - \delta)\int_w^m (U(\omega) - U(w))f(\omega)\,d\omega.$$

Thus,

$$U(w)=\frac{w-c}{1+r}+\frac{1}{1+r}\Big\{\delta V^0+(1-\delta)U(w)$$
$$+\lambda(1-\delta)\int_w^m (U(\omega)-U(w))f(\omega)\,d\omega\Big\}$$

or

$$\text{(A1)}\qquad U(w)=\frac{w-c}{r+\delta}+\frac{\delta}{r+\delta}V^0+\lambda\frac{1-\delta}{r+\delta}\int_w^m (U(\omega)-U(w))f(\omega)\,d\omega.$$

To derive the value of unemployment, note that if an individual is currently unemployed and will accept a job paying at least R, then (using the same arguments as above) expected value next month is

$$\lambda(1-F(R))\big\{(1-\delta)E[U(\omega)|\omega>R]+\delta V^0\big\}$$
$$+\big(1-\lambda(1-F(R))V^0\big).$$

This can be rewritten as

$$V^0+\lambda(1-\delta)\int_R^m (U(\omega)-V^0)f(\omega)\,d\omega.$$

Thus,

$$V^0=\frac{b}{1+r}+\frac{1}{1+r}\Big\{V^0+\lambda(1-\delta)\int_R^m (U(\omega)-V^0)f(\omega)\,d\omega\Big\}$$

or

$$\text{(A2)}\qquad V^0=\frac{b}{r}+\lambda\frac{1-\delta}{r}\int_R^m \{U(\omega)-V^0\}f(\omega)\,d\omega.$$

Standard arguments imply that the optimal reservation wage R has the property that $U(R)=V^0$. With this substitution, a comparison of (A1) and (A2) reveals that $R=b+c$.

B. *Model with SSP*

In the presence of SSP there are three value functions: $V_i(t)$, the value of welfare participation if not yet SSP-eligible, t months after assignment; $U_e(w,d)$, the value of a job paying a wage w if SSP-eligible with d months of elapsed eligibility; and $V_e(d)$, the value of not working if SSP-eligible with

d months of elapsed eligibility. From revealed preference arguments we have the inequalities

$$V_i(t) \geq V_i(t+1) \geq V^0, \quad \text{with } V_i(13) = V^0,$$
$$U_e(w,d) \geq U_e(w,d+1) \geq U(w), \quad \text{with } U_e(w,37) = U(w),$$
$$V_e(d) \geq V_e(d+1) \geq V_0, \quad \text{with } V_e(36) = V^0.$$

Following the same line of argument as used in the derivation of $U(w)$, the value for a job paying wage w after d months of elapsed eligibility is

$$\text{(A3)} \qquad U_e(w,d) = \frac{w - c + s(w)}{1+r} + \frac{1-\delta}{1+r}\max\{U_e(w,d+1), V_e(d+1)\} + \frac{\delta}{1+r}V_e(d+1) + \lambda\frac{1-\delta}{1+r}\int_w^m \{U_e(\omega,d+1) - U_e(w,d+1)\}f(\omega)\,d\omega,$$

where $s(w)$ is the SSP subsidy for a worker with wage w, and allowance has been made for the fact that in a nonstationary environment, an individual may take a job in month d that she will quit in month $d+1$. Similarly, the value of nonemployment while still SSP-eligible is

$$\text{(A4)} \qquad V_e(d) = \frac{b}{1+r} + \frac{1}{1+r}V_e(d+1) + \lambda\frac{1-\delta}{1+r}\int_{R_e(d)}^m \{U_e(\omega,d+1) - V_e(d+1)\}f(\omega)\,d\omega,$$

where $R_e(d)$ is the reservation wage for an SSP-eligible person with d months of elapsed eligibility, and the value of nonemployment for those who are not yet eligible for SSP is

$$\text{(A5)} \qquad V_i(t) = \frac{b}{1+r} + \frac{1}{1+r}V_i(t+1) + \lambda\frac{1-\delta}{1+r}\int_{R_i(t)}^m \{U_e(\omega,1) - V_i(t+1)\}f(\omega)\,d\omega,$$

where $R_i(t)$ is the reservation wage in month t for people who are offered SSP but are not yet eligible.

The optimal reservation wage for an SSP-eligible person satisfies the equality $U_e(R_e(d),d) = V_e(d)$. Substituting this into (A3) and (A4) and working backward from month $d = 36$ (i.e., the last month of SSP eligibility), it is readily shown that $R_e(d) = R_e$, where $R_e + s(R_e) = b + c$.

ADDITIONAL FIGURES

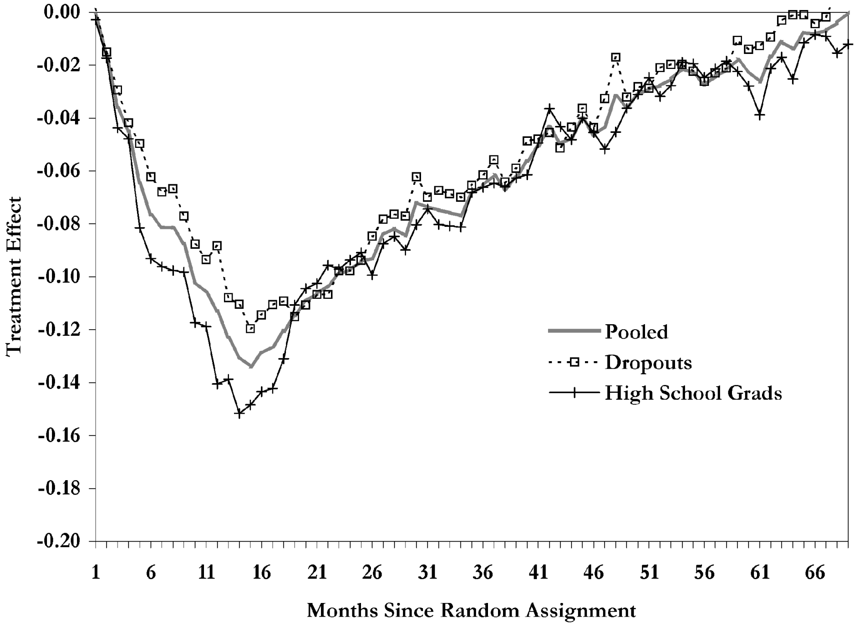

FIGURE A.—Treatment effects on welfare participation by education.

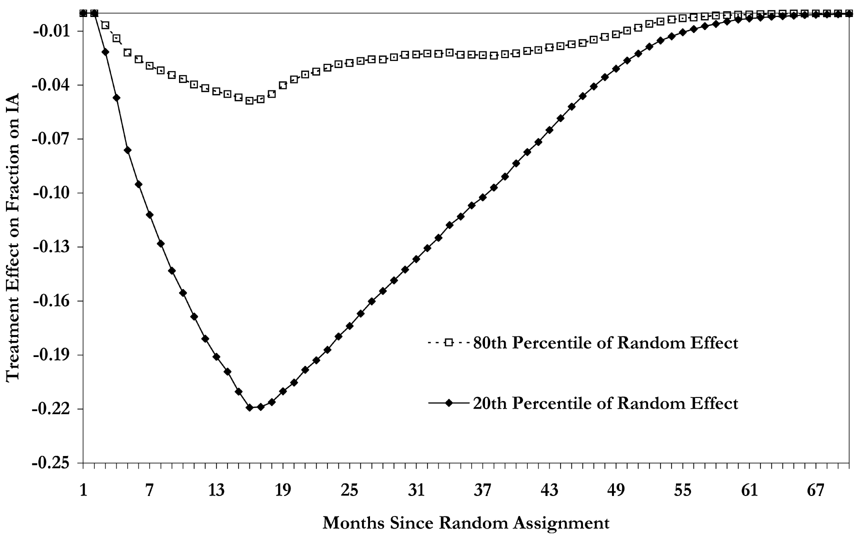

FIGURE B.—Simulated treatment effects at 20th and 80th percentiles of random effect.

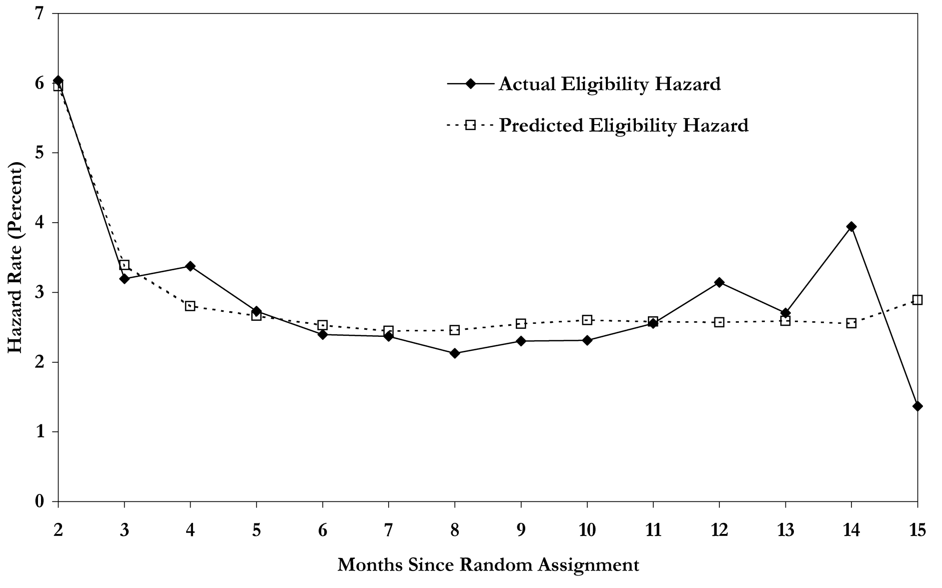

FIGURE C.—Actual and predicted hazard into eligibility.