Econometrica Supplementary Material

SUPPLEMENT TO "IDENTIFYING TECHNOLOGY SPILLOVERS AND PRODUCT MARKET RIVALRY": APPENDICES

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GIVEN THE LARGE NUMBER OF APPENDICES, there is an unusually heavy demand on mathematical notation in this paper. As a guide for the reader, we follow three rules. First, within the body of the text, we use a consistent set of symbols. Second, when the text draws from an appendix, we ensure that the notation is the same in that appendix and the text. Third, within each appendix, we use a consistent set of notation. However, some of the same symbols may be used in different appendices, subject to the constraint imposed by the second rule.

APPENDIX A: GENERALIZATIONS OF THE THEORETICAL MODEL

In this appendix, we describe three generalizations of the simple model presented in Section 2. First, we allow for a more general form of interaction between firms in technology and product market space (where there can be overlap), and also consider the N-firm case (rather than the three firm case). Second, we examine tournament models of R&D (rather than the non-tournament model in the baseline case). We show, with light modifications, that the essential insights of our simple model carry through to these more complex settings. Third, we allow the patenting decision to be an endogenous choice for the firm (rather than simply having patents as an empirical indicator of successfully produced knowledge from R&D). Although our main model predictions are robust, the extension to endogenous patenting implies that the partial derivative of patenting with respect to product market rivals' R&D (*SPILLSIC*) will be nonzero (it is zero in the basic model).

A.1. *General Form of Interactions in Technology and Product Market Space*

We begin with the general expression for flow profit

$$\pi^i = \pi^*(r_i, r_{-i}), \tag{A.1}$$

where r_{-i} is the vector of R&D for all firms other than i. In this formulation, the elements of r_{-i} captures both technology and product market spillover effects. To separate these components, we assume that (A.1) can be expressed as

$$\pi^i = \pi(r_i, r_{i\tau,} r_{im}), \tag{A.2}$$

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where

$$r_{i\tau} = \sum_{j \neq i} TECH_{ij} r_{ij}, \tag{A.3}$$

$$r_{im} = \sum_{j \neq i} SIC_{ij} r_{ij}, \tag{A.4}$$

and the partial derivatives are $\pi_1 > 0$, $\pi_2 \geq 0$, $\pi_3 \leq 0$, $\pi_{12} \gtrless 0$, $\pi_{13} \gtrless 0$, and $\pi_{23} \gtrless 0$. The technology spillover effect is $\pi_2 \geq 0$, and the business stealing effect is $\pi_3 \leq 0$. We do not constrain the effect of technology and product market spillovers on the marginal profitability of own R&D. Note that own R&D and product market spillovers are strategic substitutes if $\pi_{13} < 0$ and strategic complements if $\pi_{13} > 0$.

Equation (A.2) imposes constraints on (A.1) by partitioning the total effect of the R&D by each firm $j \neq i$ into technology spillovers $r_{i\tau}$ and product market rivalry spillovers r_{im}, and by assuming that the marginal contribution of firm j to each pool is proportional to its "distance" in technology and product market space, as summarized by $TECH_{ij}$ and SIC_{ij} (i.e., we assume that $\frac{\partial \pi^*}{\partial r_j}$ can be summarized in the form $\pi_2^i TECH_{ij} + \pi_3^i SIC_{ij}$ for each $j \neq i$).

Firm i chooses R&D to maximize net value

$$\max_{r_i} V_i = \pi(r_i, r_{i\tau}, r_{im}) - r_i.$$

Optimal R&D r_i^* satisfies the first-order condition

$$\pi_1(r_i^*, r_{i\tau}, r_{im}) - 1 = 0. \tag{A.5}$$

We want to study how (exogenous) variations in $r_{i\tau}$ and r_{im} affect optimal R&D. To do this, we choose an arbitrary subset of firms, S, and make compensating changes in their R&D such that either r_{im} or $r_{i\tau}$ is held constant. This allows us to isolate the impact of the spillover pool we are interested in. Consider a subset of firms denoted by $s \in S$, where $s \neq i$, and a set of changes in their R&D levels, $\{dr_s\}$, that satisfy the constraint $dr_{im} = \sum_{s \in S} SIC_{is}\, dr_s = 0$. These changes imply some change in the technology spillovers $dr_{i\tau} = \sum_{s \in S} TECH_{is}\, dr_s$, which, in general, will differ from zero (it can be either positive or negative depending on the $TECH$ and SIC weights). Now totally differentiate the first-order condition, allowing only r_s for $s \in S$ to change.[1] This gives

$$\pi_{11}\, dr_i + \pi_{12} \sum_{s \in S} TECH_{is}\, dr_s + \pi_{13} \sum_{s \in S} SIC_{is}\, dr_s = 0.$$

[1]We assume that the changes in R&D do not violate the restriction $r_s \geq 0$.

But the third summation is zero by construction ($dr_{im} = 0$), and the second summation is just $dr_{i\tau}$. So we get

$$\frac{\partial r_i^*}{\partial r_{i\tau}} = -\frac{\pi_{12}}{\pi_{11}}. \tag{A.6}$$

By similar derivation, we obtain

$$\frac{\partial r_i^*}{\partial r_{im}} = -\frac{\pi_{13}}{\pi_{11}}. \tag{A.7}$$

Equation (A.6) says that if we make compensating changes in the R&D such that the pool of product market spillovers is constant, the effect of the resulting change in technology spillovers has the same sign as π_{12}. This can be either positive or negative depending on how technology spillovers affect the marginal productivity of own R&D. Equation (A.7) says that if we make compensating changes in the R&D such that the pool of technology spillovers is constant, the effect of the resulting change in product market spillovers has the same sign as π_{13}—the sign depends on whether R&D by product market rivals is a strategic substitute or complement for the firm's own R&D.

Using the envelope theorem, the effects of $r_{i\tau}$ and r_{im} on the firm's market value are

$$\frac{\partial V_i^*}{\partial r_{i\tau}} = \pi_2 \geq 0,$$
$$\frac{\partial V_i^*}{\partial r_{im}} = \pi_3 \leq 0.$$

These equations say that an increase in technology spillovers raises the firm's market value, and an increase in product market rivals' R&D reduces it.

One remark is in order. There are multiple (infinite) different ways to change R&D in a subset of firms so as to ensure that the constraint $dr_{im} = 0$ is satisfied. Each of the combinations $\{dr_s\}$ that do this will imply a different value of $dr_{i\tau} = \sum_{s \in S} TECH_{is}\, dr_s$. Thus the discrete impact of such changes will depend on the precise combination of changes made, but the marginal impact of a change in $dr_{i\tau}$ does not depend on that choice.

A.2. *Tournament Model of R&D Competition With Technology Spillovers*

In this subsection, we analyze a stochastic patent race model with spillovers. We do not distinguish between competing firms in the technology and product markets because the distinction does not make sense in a simple patent race (where the winner alone gets profit). For generality, we assume that n firms compete for the patent.

Stage 2. Firm 0 has profit function $\pi(k_0, x_0, x_m)$. As before, we allow innovation output k_0 to have a direct effect on profits, as well as an indirect (strategic) effect working through x. In stage 1, n firms compete in a patent race (i.e., there are $n-1$ firms in the set m). If firm 0 wins the patent, $k_0 = 1$; otherwise, $k_0 = 0$. The best response function is given by $x_0^* = \arg\max \pi(x_0, x_m, k_m)$. Thus, second stage profit for firm 0, if it wins the patent race, is $\pi(x_0^*, x_m^*; k_0 = 1)$; otherwise, it is $\pi(x_0^*, x_m^*; k_0 = 0)$.

We can write the second stage Nash decision for firm 0 as $x_0^* = f(k_0, k_m)$ and the first stage profit as $\Pi(k_0, k_m) = \pi(k_0, x_0^*, x_m^*)$. If there is no strategic interaction in the product market, π^i does not vary with x_j, and thus x_i^* and Π^i do not depend directly on k_j. Recall that in the context of a patent race, however, only one firm gets the patent: if $k_j = 1$, then $k_i = 0$. Thus Π^i depends indirectly on k_j in this sense. The patent race corresponds to an (extreme) example where $\partial \Pi^i(k_i, k_j)/\partial k_j < 0$.

Stage 1. We consider a symmetric patent race between n firms with a fixed prize (patent value) $\Delta = \pi^0(f(1,0), f(0,1); k_0 = 1) - \pi^0(f(0,1), f(1,0); k_0 = 0)$. The expected value of firm 1 can be expressed as

$$V^0(r_0, r_m) = \frac{h(r_0, (n-1)r_m)\Delta - r_0}{h(r_0, (n-1)r_m) + (n-1)h(r_m, (n-1)r_m + r_0) + R},$$

where R is the interest rate, r_m is the R&D spending of each of firm 0's rivals, and $h(r_0, (n-1)r_m)$ is the probability that firm 0 gets the patent at each point of time given that it has not done so before (hazard rate). We assume that $h(r_0, (n-1)r_m)$ is increasing and concave in both arguments. It is rising in r_m because of spillovers. We also assume that $h\Delta - R \geq 0$ (expected benefits per period exceed the opportunity cost of funds).

The best response is $r_0^* = \arg\max V^0(r_0, r_m)$. Using the first-order condition for the maximization of $V^0(r_0, r_m)$, imposing symmetry and doing comparative statics, we obtain

$$\begin{aligned} \text{sign}\left(\frac{\partial r_0}{\partial r_m}\right) = \text{sign}\big\{&h_{12}\big(h\Delta(n-1) + r\Delta - R\big) + \big\{h_1(n-1)(h_1\Delta - 1)\big\} \\ &- \big\{h_{22}(n-1)(h\Delta - R)\big\} - h_2\big\{(n-1)h_2\Delta - 1\big\}\big\}, \end{aligned}$$

where subscripts on h denote partial (and cross) derivatives.

We assume $h_{12} \geq 0$ (spillovers do not reduce the marginal product of a firm's R&D) and $h_1\Delta - 1 \geq 0$ (expected net benefit of own R&D is nonnegative). These assumptions imply that the first three bracketed terms are positive. Thus, a sufficient condition for strategic complementarity in the R&D game ($\frac{\partial r_0}{\partial r_m} > 0$) is that $(n-1)h_2\Delta - 1 \leq 0$. This requires that spillovers not be "too large." If firm 0 increases R&D by one unit, this raises the probability that one of its rivals wins the patent race by $(n-1)h_2$. The condition says that the expected gain for its rivals must be less than the marginal R&D cost to firm 0.

Using the envelope theorem, we get $\frac{\partial V^0}{\partial r_m} < 0$. The intuition is that a rise in r_m increases the probability that firm m wins the patent. While it may also generate spillovers that raise the win probability for firm 0, we assume that the direct effect is larger than the spillover effect. For the same reason, $\frac{\partial V^0}{\partial k_m} = 0$. As in the non-tournament case, $\frac{\partial r_0}{\partial r_m} > 0$ and $\frac{\partial V^0}{\partial r_m} < 0$. The difference is that with a simple patent race, $\frac{\partial V^0}{\partial k_m}$ is zero rather than negative because firms only race for a single patent.[2]

A.3. *Endogenizing the Decision to Patent*

We generalize the basic non-tournament model to include an endogenous decision to patent. We study a two stage game. In stage 1, firms make *two* decisions: (1) the level of R&D spending, and (2) the "propensity to patent." The firm produces knowledge with its own R&D and the R&D by technology rivals. The firm also chooses the fraction of this knowledge that it protects by patenting. Let $\rho \in [0, 1]$ denote this patent propensity and let $\lambda \geq 1$ denote patent effectiveness, that is, the rents earned from a given innovation if it is patented relative to the rents if it is not patented. Thus $\lambda - 1$ represents the patent premium and θk is the rent associated with knowledge k, where $\theta = \rho\lambda + (1 - \rho)$. There is a fixed cost of patenting each unit of knowledge, c.

As in the basic model at stage 2, firms compete in some variable, x, conditional on their knowledge levels k. There are three firms, labeled 0, τ, and m. Firms 0 and τ interact only in technology space but not in the product market; firms 0 and m compete only in the product market.

Stage 2. Firm 0's profit function is $\pi(x_0, x_m, \theta_0 k_0)$. We assume that the function π is common to all firms. Innovation output k_0 may have a direct effect on profits, as well as an indirect (strategic) effect working through x.

The best response for firms 0 and m is given by $x_0^* = \arg\max \pi(x_0, x_m, \theta_0 k_0)$ and $x_m^* = \arg\max \pi(x_m, x_0, \theta_m k_m)$, respectively. Solving for second stage Nash decisions yields $x_0^* = f(\theta_0 k_0, \theta_m k_m)$ and $x_m^* = f(\theta_m k_m, \theta_0 k_0)$. First stage profit for firm 0 is $\Pi(\theta_0 k_0, \theta_m k_m) = \pi(\theta_0 k_0, x_0^*, x_m^*)$, and similarly for firm m. If there is no strategic interaction in the product market, $\pi(\theta_0 k_0, x_0^*, x_m^*)$ does not vary with x_m and thus Π^0 do not depend on $\theta_m k_m$. We assume that $\Pi(\theta_0 k_0, \theta_m k_m)$ is increasing in $\theta_0 k_0$, decreasing in $\theta_m k_m$, and concave.

Stage 1. Firm 0's knowledge production function remains as

$$k_0 = \phi(r_0, r_\tau), \tag{A.8}$$

[2]In this analysis, we have assumed that $k = 0$ initially, so, ex post, the winner has $k = 1$ and the losers $k = 0$. The same qualitative results hold if we allow for positive initial k.

where we assume that $\phi(\cdot)$ is non-decreasing and concave in both arguments and common to all firms. Firm 0 solves the following problem:

$$\max_{r_0,\rho_0} V^0 = \Pi\big(\theta_0\phi(r_0, r_\tau), \theta_m k_m\big) - r_0 - c\rho_0\phi(r_0, r_\tau). \tag{A.9}$$

The first-order conditions are

$$r_0 : \big(\Pi_1^0\theta_0 - c\rho_0\big)\phi_1^0 - 1 = 0, \tag{A.10}$$

$$\rho_0 : \Pi_1^0\phi^0(\lambda - 1) - c\phi^0 = 0, \tag{A.11}$$

where the subscripts denote partial derivatives and superscripts denote the firm. Comparative statics on equations (A.10) and (A.11) yield the following results for comparison with the baseline model[3]:

$$\frac{\partial r_0^*}{\partial r_\tau} = \frac{V_{\rho_0\rho_0}V_{r_0r_\tau} - V_{\rho_0r_0}V_{\rho_0\rho_\tau}}{-A} \gtrless 0, \tag{A.12}$$

where $V_{r_0r_\tau} \equiv \frac{\partial^2 V}{\partial r_0 r_\tau}$, etc., and $A = V_{r_0r_0}V_{\rho_0\rho_0} - V_{r_0\rho_0}^2 > 0$ by the second order conditions.

As in the basic model, the sign of $\frac{\partial r_0^*}{\partial r_\tau}$ depends on sign$\{\phi_{12}\}$ and the magnitude of Π_{11}. We also obtain:

$$\frac{\partial r_0^*}{\partial r_m} = \frac{V_{\rho_0\rho_0}V_{r_0\rho_m} - V_{\rho_0r_0}V_{\rho_0\rho_m}}{-A} \gtrless 0 \quad \text{depending on sign}\{\Pi_{12}\}, \tag{A.13}$$

$$\frac{\partial \rho_0^*}{\partial r_m} = \frac{V_{\rho_0\rho_0}V_{r_0r_m} - V_{\rho_0r_0}V_{\rho_0r_m}}{-A} \gtrless 0 \quad \text{depending on sign}\{\Pi_{12}\}. \tag{A.14}$$

In signing the above results, we use the fact that $V_{r_0r_0} < 0, V_{\rho_0\rho_0} < 0, V_{\rho_0r_0} > 0$ (provided Π_{11} is "sufficiently small") and the other cross partials which are: $V_{r_0r_\tau} = \frac{\phi_{12}}{\phi_1} + \theta_0^2\phi_1^0\phi_2^0\Pi_{11}$; $V_{r_0r_m} = \theta_0\theta_m\phi_1^0\phi_1^m\Pi_{12}$; $V_{r_0\rho_\tau} = 0$; $V_{r_0\rho_m} = (\lambda - 1) \times \theta_0 k_m\phi_1^0\Pi_{12}$; $V_{\rho_0r_\tau} = (\lambda - 1)\theta_0 k_0\phi_2^0\Pi_{11}$; $V_{\rho_0r_m} = (\lambda - 1)k_0\theta_m\phi_1^m\Pi_{12}$; $V_{\rho_0\rho_\tau} = 0$; and $V_{\rho_0\rho_m} = (\lambda - 1)^2 k_0 k_m\phi_2^0\Pi_{12}$.

The basic results of the simpler model go through. First, an increase in technology spillovers (r_τ) has an ambiguous sign on own R&D spending (equation (A.12)). Second, after some algebra, we can show that sign$\{\frac{\partial r_0^*}{\partial r_m}\}$ = sign$\{\Pi_{12}\}$ provided that Π_{11} is "sufficiently small." An increase in product market rivals' R&D raises own R&D if they are strategic complements (conversely for strategic substitutes) (equation (A.13)). Third, from the knowledge production function (A.8), it follows that technology spillovers raise firm 0's knowledge stock, $\frac{\partial k_0^*}{\partial r_\tau} \geq 0$, and product market rivals' R&D has no effect on it, $\frac{\partial k_0^*}{\partial r_m} = 0$. Finally,

[3]This is not a full list of the comparative statics results.

the impacts on the value of the firm follow immediately by applying the envelope theorem to the value equation (A.9): namely, $\frac{\partial V_0^*}{\partial r_\tau} \geq 0$ and $\frac{\partial V_0^*}{\partial r_m} \leq 0$.

The new result here is that an increase in the R&D by firm 0's product market rivals will affect the firm's propensity to patent, $\frac{\partial \rho_0^*}{\partial r_m}$ (equation (A.14)). After some algebra, we can show that sign $\frac{\partial \rho_0^*}{\partial r_m} = \text{sign}\, \Pi_{12}$, provided that Π_{11} is "sufficiently small." Thus, if there is strategic complementarity ($\Pi_{12} > 0$), an increase in product market rivals' R&D raises the firm's propensity to patent (the opposite holds for strategic substitution). The intuition is that, under strategic complementarity, when rivals increase R&D spending (thus their stock of knowledge), this increases the marginal profitability of firm 0's R&D and thus the profitability of patenting (given the fixed cost of doing so). Thus R&D by product market rivals raises both R&D spending and patent propensity of firm 0.[4]

APPENDIX B: DATA APPENDIX

B.1. *The Patents and Compustat Databases*

The NBER patents database provides detailed patenting and citation information for around 2,500 firms (as described in Hall, Jaffe, and Trajtenberg (2005) and Jaffe and Trajtenberg (2002)). We started by using the NBER's match of the Compustat accounting data to the USPTO data between 1970 and 1999,[5] and kept only patenting firms, leaving a sample size of 1,865. These firms were then matched into the Compustat Segment ("line of business") Data Set, keeping only the 795 firms with data on both sales by four digit industry and patents, although these need not be concurrent. For example, a firm that patented in 1985, 1988, and 1989, had Segment data from 1993 to 1997, and accounting data from 1980 to 1997, would be kept in our data set for the period 1985 to 1997. The Compustat Segment Database allocates firm sales into four digit industries each year using firm's descriptions of their sales by lines of business. See Villalonga (2004) for a more detailed description.

Finally, this data set was cleaned to remove accounting years with extremely large jumps in sales, employment, or capital, signaling merger and acquisition activity. When we removed a year, we treated the firm as a new entity and gave it a new identifier (and therefore a new fixed effect) even if the firm identifier (CUSIP reference) in Compustat remained the same. This is more general than including a full set of firm fixed effects, as we are allowing the fixed effect to change over time. We also removed firms with less than four consecutive years of data. This left a final sample of 715 firms to estimate the model on,

[4]Since product market rivals' R&D does not affect knowledge production by firm 0, this result for the propensity to patent also applies to the number of patents taken out by firm 0.

[5]We dropped pre-1970 data as being too outdated for our 1980s and 1990s accounts data.

with accounting data for at least some of the period 1980 to 2001 and patenting data for at least some of the period between 1970 and 1999. The panel is unbalanced, as we keep new entrants and exiters in the sample.

The main variables we use are as follows (Compustat mnemonics are in parentheses). The book value of capital is the net stock of property, plant, and equipment (*PPENT*) and employment is the number of employees (*EMP*). R&D (*XRD*) is used to create R&D capital stocks following inter alia Hall, Jaffe, and Trajtenberg (2005). This uses a perpetual inventory method with a depreciation rate (δ) of 15%. So the R&D stock, G, in year t is $G_t = R_t + (1-\delta)G_{t-1}$, where R is the R&D flow expenditure in year t and $\delta = 0.15$. For the first year we observe a firm, we assume it is in steady state, so $G_0 = R_0/(\delta + g)$ where $g =$ the steady stage growth rate of the R&D stock, G. We use sales as our output measure (*SALE*), but also compare this with value added specifications. Industry price deflators were taken from Bartelsman, Becker, and Gray (2000) until 1996 and then the BEA four digit NAICS Shipment Price Deflators thereafter. For Tobin's Q, firm value is the sum of the values of common stock, preferred stock, and total debt net of current assets (*MKVAF*, *PSTK*, *DT*, and *ACT*). The book value of capital includes net plant, property and equipment, inventories, investments in unconsolidated subsidiaries, and intangibles other than R&D (*PPENT*, *INVT*, *IVAEQ*, *IVAO*, and *INTAN*). Tobin's Q was winsorized by setting it to 0.1 for values below 0.1 and at 20 for values above 20 (see Lanjouw and Schankerman (2004)).

B.2. *Other Variables*

The construction of the spillover variables is described in detail in Section 3 of the main paper. About 80% of the variance of *SPILLTECH* and *SPILLSIC* is between firm and 20% is within firm. When we include fixed effects, we are, of course, relying on the within firm time series variation for identification. Industry sales were constructed from total sales of the Compustat database by four digit industry code and year, and merged to the firm level in our panel using each firm's distribution of sales across four digit industry codes.

B.3. *Using the Tax Treatment of R&D to Construct Instrumental Variables*

B.3.1. *Methodology*

To fix ideas, consider our basic model for firm productivity and abstract away from all other variables except own R&D and the technology spillover term. Similar issues arise for the other three equations, subject to additional complications noted below:

$$\ln Y_{it} = \beta_1 \ln\left(\sum_{j\neq i} TECH_{ij} R_{jt-1}\right) + u_{it}. \tag{B.1}$$

We are concerned that $E[u_{it} \ln(\sum_{j \neq i} TECH_{ij} R_{jt-1})] \neq 0$, so OLS is inconsistent, and consider instrumental variable techniques. Note that R&D spillovers are entered lagged at least one period and that fixed effects and other covariates are also included. Given these considerations, the existing literature has argued that the bias on a weakly exogenous variable is likely to be small.

We consider two candidate instrumental variables (z) based on R&D-specific supply side shocks: firm and state-wide R&D tax credits. The Hall–Jorgenson user cost of capital for firm i, ρ_{it}^{U}, is

$$\rho_{it}^{\mathrm{U}} = \frac{(1 - D_{it})}{(1 - \tau_{st})} \left[I_t + \delta - \frac{\Delta p_t}{p_{t-1}} \right], \tag{B.2}$$

where D_{it} is the discounted value of tax credits and depreciation allowances, τ_{st} (which is shorthand for $\tau_{s_i t}$) is the rate of corporation tax (which has a state as well as a federal component), I_t is the real interest rate, δ is the depreciation rate of R&D capital, and $\frac{\Delta p_t}{p_{t-1}}$ is the growth of the R&D asset price. Since $[I_t + \delta - \frac{\Delta p_t}{p_{t-1}}]$ does not vary between firms, we focus on the tax price component of the user cost, $\rho_{it}^{\mathrm{P}} = \frac{(1-D_{it})}{(1-\tau_{st})}$.

We decompose the variation of ρ_{it}^{P} into two broad channels: "firm level," ρ_{it}^{F}, based on firm-level interactions with the federal tax rules, and "state level," ρ_{it}^{S}. We use the state-by-year R&D tax-price data from Wilson (2009), who quantified the impact of state-level tax credits, depreciation allowances, and corporation taxes. The firms in our data benefit differentially from these state credits depending on in which state their R&D is located. Tax credits are for R&D performed within the state that can be offset against state-level corporation tax liabilities. State-level corporation tax liabilities are calculated on total firm profits allocated across states according to a weighted combination of the location of firm sales, employment, and property. Hence, any firm with an R&D lab within the state is likely to be both liable for state corporation tax (due to its employees and property in the state) and eligible for an offsetting R&D tax credit. Hence, inventor location appears to provide a good proxy for eligibility for state-level R&D tax credits.[6]

We estimate the spatial distribution of a firm's inventors from the USPTO patents file. The state component of the tax price is therefore

$$\rho_{it}^{\mathrm{S}} = \sum_{s} \theta_{ist} \rho_{st}^{\mathrm{S}}, \tag{B.3}$$

[6]State-level R&D tax credits can be generous, and vary differentially over states and time. For example, the five largest R&D-doing states had the following tax credit histories: California introduced an 8% credit in 1987, raised to 11%, 12%, and 15% in 1997, 1999, and 2000, respectively; Massachusetts, New Jersey, and Texas introduced 10%, 10%, and 4% rates in 1991, 1994, and 2000, respectively; while Michigan has never introduced an R&D tax credit.

where ρ^{S}_{st} is the state-level tax price and θ_{ist} is firm i's 10-year moving average share of inventors located in state s.

The second component of the tax price is based solely on federal rules (ρ^{F}_{it}) and is constructed following Hall (1992) and Bloom, Criscuolo, Hall, and Van Reenen (2008). The "Research and Experimentation" tax credit was first introduced in 1981 and has been in continuous operation and subject to many rule changes. It has a firm-specific component for several reasons. First, the amount of tax credit that can be claimed is based on the difference between actual R&D and a firm-specific "base." From 1981 to 1989, the base was the maximum of a rolling average of the previous three years' R&D. From 1990 onward (except 1995–1996, when the tax credit lapsed), the base was fixed to be the average of the firm's R&D to sales ratio between 1984 and 1988, multiplied by current sales (up to a maximum of 16%). Start-ups were treated differently, initially with a base of 3%, but modified each year. Second, if the credit exceeds the taxable profits of the firm, it cannot be fully claimed and must be carried forward. With discounting, this leads to a lower implicit value of the credit for tax exhausted firms. Third, these firm-specific components all interact with changes in the aggregate tax credit rate (25% in 1981, 20% in 1990, 0% in 1995, etc.), deduction rules, and corporate tax rate (which enters the denominator of equation (B.2)).

We implement the IV approach by first projecting the endogenous variable (R&D) on the instruments in the first stage. Table A.I shows the results from this estimation and demonstrates that the instruments have considerable power. Column (1) has the basic results, column (2) adds time dummies, and column (3) also adds firm dummies. Even in the most general regression of the final column of Table A.I that is used in the later stages, the F-statistics are over 16. From this, we calculate the value of R&D predicted by these tax

TABLE A.I

PREDICTING R&D USING FEDERAL AND STATE R&D TAX CREDITS

Dependent Variable:	(1) ln(R&D)	(2) ln(R&D)	(3) ln(R&D)
ln(State tax credit component of R&D user cost$_{i,t}$)	−3.828 (0.265)	−4.250 (0.204)	−0.398 (0.174)
ln(Federal tax credit component of R&D user cost$_{i,t}$)	−1.672 (0.231)	−0.387 (0.114)	−0.440 (0.085)
Firm fixed effects	No	Yes	Yes
Year dummies	No	No	Yes
Joint F-test of the tax credits	137.15	304.56	16.28
No. observations	14,971	14,971	14,971

Notes: Standard errors (in brackets) are robust to arbitrary heteroskedasticity and allow for first-order serial correlation using the Newey–West procedure.

credits, R_{it}^{TAX}, and then generate the stock of this using the standard perpetual inventory method, G_{it}^{TAX}. For each firm, we then weight up other firms' tax-credit predicted R&D stocks using *SIC* and *TECH* to create the distance weighted instruments *TECHTAX* and *SICTAX*. For example, $TECHTAX_{it} = \sum_{j\neq i} TECH_{ij} G_{jt}^{\text{TAX}}$. We then use $\ln(TECHTAX)$ and $\ln(SICTAX)$ as instrumental variables for $\ln(SPILLTECH)$ and $\ln(SPILLSIC)$ in the final stage regressions as presented in the last columns of Tables III, V, and VI. For the citation weighted patent regressions, we use a Negative Binomial equation, so to allow for endogeneity we take a control function approach (e.g., Aghion, Bloom, Blundell, Griffith, and Howitt (2005)). We estimate the first stages for $\ln(SPILLTECH)$ and $\ln(SPILLSIC)$ and then include a fifth-order series expansion of each of the residuals from these first stages in the final column of Table IV. We correct the standard errors using 1,000 bootstrap replications over firms.

We also considered an alternative approach of using *TECH* and *SIC* weighted ρ_{it}^{S} and ρ_{it}^{F} as instrumental variables. However, the unbalanced nature of the panel makes this very unattractive, as the value of the instruments changes as new firms exit and enter the sample. This generates a positive bias between R&D the user cost of R&D. For example, imagine that a firm j enters a market where firm i, an incumbent, already operates. Then, for some firm i for which $TECH_{ij} > 0$, there will be a rise in $SPILLTECH_{it}$ since there is now another firm doing R&D in its technology space. But its $TECH_{ij}$ weighted R&D user cost measure will also rise, since the values of ρ_{jt}^{S} and ρ_{jt}^{F} for firm j are zero pre-entry (since they are missing) but strictly positive post entry.

B.3.2. *Exogeneity of R&D Tax Credit Policy Changes*

A concern is that changes to the R&D tax credit may be endogenous. Could states respond to falls in R&D by increasing the tax credit rate, for example? One check was to conduct experiments lagging the tax credit instruments one and two periods, which led to qualitatively similar results. But we also investigated this issue further by reviewing the literature on U.S. state R&D and corporate tax rates. Three facts are clear from this literature:

(1) State-level tax credits have been gradually introduced across states over time. The first state R&D tax credit was introduced in Minnesota in 1982 following the introduction of the Federal Tax Credit in 1981, with 31 states having introduced credits by the end of 2005 (Wilson (2005)), and 38 states by 2010. The generosity of these credits has also been gradually trending up over time, rising from 6.25% in 1982 to 7.9% in 2005.

(2) The cross-sectional variation in credit rates is extremely large relative to the mean and growth rates of the average rate. The cross-sectional standard deviation of credit rates is more than twice their mean. For example, the credit rates range from 2.5% in South Carolina and Minnesota to 20% in Arizona and Hawaii. These rates also change frequently over time within states. For

example, California changed its rate five times between its introduction in 1987 and 2010.

(3) The level and timing of introduction of the credits—which provides the empirical identification in our estimation given our firm and time dummies—seems to be uncorrelated with any observables after controlling for state and year fixed effects. Papers that have tried to explain the evolution of state-level corporate tax credits have found that aggregate variables (such as the federal credit rate) have some explanatory power, but local economic or political variables do not seem important (e.g., Chirinko and Wilson (2008a, 2008b)). This partly reflects the long time delays in passing tax credits through state legislature, and also the fact that the costs of many of these tax credits are small so that their adoption tends not to be strongly driven by budgetary concerns.

To investigate this issue further, we regressed the rate of the state R&D tax credit on lagged state R&D expenditures, a full set of state and year dummies, and the lagged value of the state tax credit (to control for dynamics). Using a variety of specifications with and without these controls, we could not find any predictive power for lagged state-level R&D expenditure or GDP per capita (a crude proxy for productivity) on current R&D tax credits. That is, prior state-level R&D expenditure does not seem to predict current levels of state R&D tax credits.

In summary, while state-level R&D tax credits have been rising since the early 1980s, this has happened at differential rates and levels across states, with these state-by-year differences in generosity seemingly uncorrelated with lagged economic or political variables. This suggests that the current level of R&D tax credits provides pseudo-random variation to identify corporate R&D expenditure in a regression including firm and year fixed effects.

B.4. *Sample Selection Issues*

To be in the final regression sample, a firm has to have at least one patent granted (since 1963) in order to construct our measure of technological closeness (which requires patent information), so this does screen out firms who never patent. However, we do not require that a firm has several lines of business, only that it has some data in the Compustat Segment File (CSF), which contains the breakdown of sales across four digit industry classes. Note that some firms operate solely in one class and are recorded as such (and we use their information). Nearly all firms in Compustat which have a patent are also in the CSF, so this is not a source of sample selection.

The main source of selection is that we use the Compustat database that covers only publicly listed firms. This is because capital investment, R&D, and other important variables are not required reporting items for privately listed firms in the US. R&D is heavily concentrated on listed firms, however, so our sample accounts for a large proportion of the entire R&D in the United States. For example, in 1995, the sum of R&D expenditures in our sample was $79bn, while the total industry R&D in the U.S. was estimated as $130bn by the NSF, so we cover about 62% of the total.

Of course, it would be ideal to know the exact R&D of every firm, but this is not necessary for implementation of our technique. As we have shown in Appendix A.1, our comparative static results hold for any triple of firms. All we need is the thought experiment of increasing *SPILLTECH* while holding *SPILLSIC* constant (and vice versa).

There could be biases if, for a given firm, we omit a relevant spillover. Certainly, the absence of other R&D performing firms will mean that we underestimate the R&D relevant spillovers for some firms. In general, it is unclear whether this would bias our estimates systematically in any one direction. If this generated classical measurement error, with random under-counting of spillovers, it would cause an attenuation bias toward zero. Thus there is the possibility that we underestimate the strength of both positive technology and negative product market spillovers. If this is so, our conclusion that both types of spillovers operate, one positive and the other negative, is strengthened.

Nonrandom exit relating to unobservables is very difficult to control for with existing techniques. Evidence from nonparametric control function techniques to control for selection in production functions suggests that the main form of bias comes from conditioning on a balanced panel (e.g., Olley and Pakes (1996)). Since our panel is unbalanced—we keep all entrants and exiters—this mitigates this problem.

B.5. *The Bureau Van Dijk (BVD) Database*

As noted in Section 6.3.2, the finance literature has debated the extent to which the breakdown of firm sales into four digit industries from the Compustat Segment Data Set is reliable. To address this concern, we used an alternative data source, the BVD (Bureau Van Dijk) database.

B.5.1. *Description*

The BVD data for the United States is obtained from Dun and Bradstreet (D&B), which collects the data to provide credit ratings and to sell as a marketing database. These credit ratings are used to open bank accounts, and are also required for corporate clients by most large companies (e.g., Wal-Mart and General Electric) and the Federal Government, so almost all multiperson establishments in the U.S. are in the D&B database. Since these data are commercially used and sold for various financial and marketing purposes, they are regularly quality checked by D&B. In Europe, the BVD data come from the National Registries of companies (such as Companies House in the U.K.), which have statutory requirements on reporting for all public and private firms. We used the primary and secondary four digit industry classes for every subsidiary within a Compustat firm that could be matched to BVD to calculate distribution of employment across four digit industries (essentially summing across all the global subsidiaries) as a proxy for sales by four digit industries.

The U.S. data report one primary four digit industry code and an ordered set of up to six secondary four digit industry codes. We allocated employment across sectors for an individual firm by assuming that 75% of activity was in the primary industry code, 75% of the remainder was in the main secondary code, 75% of this remainder was in the next secondary industry code, and so on, with the final secondary industry code containing 100% of the ultimate residual. In the European data, firms report one primary industry code and as many secondary industry codes as they wish (with some firms reporting over 30) but without any ordering. Employment was allocated assuming that 75% of employees were in the primary industry code and the remaining 25% were split equally among the secondary industry codes. Finally, employment was added across all industry codes in every enterprise in Europe and the U.S. owned by the ultimate Compustat parent to compute a four digit industry activity breakdown.

B.5.2. *Matching to Compustat*

We successfully matched three quarters of the Compustat firms in the original sample. The matched firms were larger and more R&D intensive than the non-matched firms. Consequently, these matched firms accounted for 84% of all employment and 95% of all R&D in the Compustat sample, so that, judged by R&D, the coverage of the BVD data of the Compustat sample was very good. The correlation between the Compustat Segment and BVD Data Set measures is reasonably high. The correlation between the sales share of firm i in industry k between the two data sets is 0.503. The correlation of $\ln(SPILLSIC)$ across the two measures is 0.592. The within-firm over-time variation of $\ln(SPILLSIC)$, which identifies our empirical results given that we control for fixed effects, reassuringly rises to 0.737. In terms of average levels, both measures are similar, with an average *SIC* of 0.0138 using the Compustat measure and 0.0132 using the BVD measure. The maximum number of four digit industries for one of our firms, General Electric, is 213.

As an example of the extent of similarity between the two measures, the Compustat and BVD *SIC* correlations for the four firms examined in the Case Study discussed in Appendix D below are presented in Table A.II. As can be seen, the two measures are similar; IBM and Apple (PC manufacturers) are highly correlated on both measures, and Motorola and Intel (semiconductor manufacturers) are also highly correlated. But the correlation across these two pairs is low. There are also some differences; for example, the BVD based measure of *SIC* finds that IBM is closer in sales space with Intel and Motorola ($SIC = 0.07$) than the Compustat-based measure ($SIC = 0.01$). This is because IBM uses many of its own semiconductor chips in its own products, so this is not included in the sales figures. The BVD based measure picks these up because IBM's three chip-making subsidiaries are tracked in BVD's ICARUS

TABLE A.II

AN EXAMPLE OF *SPILLTEC* AND *SPILLSIC* FOR FOUR MAJOR FIRMS

	Correlation	IBM	Apple	Motorola	Intel
IBM	*SIC* Compustat	1	0.65	0.01	0.01
	SIC BVD	1	0.55	0.02	0.07
	TECH	***1***	***0.64***	***0.46***	***0.76***
Apple	*SIC* Compustat		1	0.02	0.00
	SIC BVD		1	0.01	0.03
	TECH		***1***	***0.17***	***0.47***
Motorola	*SIC* Compustat			1	0.34
	SIC BVD			1	0.47
	TECH			***1***	***0.46***
Intel	*SIC* Compustat				1
	SIC BVD				1
	TECH				***1***

Notes: The cell entries are the values of $SIC_{ij} = (S_i S_j')/[(S_i S_i')^{1/2}(S_j S_j')^{1/2}]$ (in normal script) using the Compustat Line of Business sales breakdown ("SIC Compustat") and the Bureau Van Dijk database ("*SIC* BVD"), and $TECH_{ij} = (T_i T_j')/[(T_i T_i')^{1/2}(T_j T_j')^{1/2}]$ (in ***bold italics***) between these pairs of firms.

data even if their products are wholly used within IBM's vertically integrated chain.

B.5.3. *Coverage*

The industry coverage was broader in the BVD data than the Compustat Segment Data Set. The mean number of distinct four digit industry codes per firm was 13.8 in the BVD data (on average, there were 29.6 enterprises: 18.2 in Europe and 11.4 in the U.S.) compared to 4.6 in the Compustat Segment files. This confirms Villalonga's (2004) finding that the Compustat Segment Data Set underestimates the number of industries in which a firm operates.

APPENDIX C: ALTERNATIVE DISTANCE METRICS

C.1. *Robustness to Aggregation*

The standard Jaffe measure, $TECH^{\mathrm{J}}$, differs from the Jaffe covariance (and Exposure) measure, $TECH^{\mathrm{J\text{-}COV}}$, because it is an (uncentered) correlation, rather than covariance, between the vectors F_i and F_j: $TECH^{\mathrm{J}}_{ij} = \frac{F_i F_j'}{(F_i F_i')^{1/2}(F_j F_j')^{1/2}}$. We now show that this normalization has the advantage that it makes the index less sensitive to the aggregation of technology fields. We refer to this property as "robustness to aggregation."

To see this formally, consider the case where technology fields $Y-1$ and Y are aggregated, and define the $1\times(Y-1)$ vector $F_i^* = (F_{i1},\ldots,F_{i,Y-2},F_{iY-1} + F_{i,Y-2})$. The new $TECH^{\text{J-COV}}$ index can be expressed as

$$F_i^* F_j^{*\prime} = F_i F_j' + (F_{i,Y-1}F_{jY} + F_{j,Y-1}F_{iY}) \geq F_i F_j',$$

and strict inequality holds if each firm operates in at least one of the two aggregated fields, $Y-1$ and Y. This shows that $TECH^{\text{J-COV}}$ increases as a consequence of aggregation. This is an unattractive property. The standard $TECH^{\text{J}}$ measure mitigates this aggregation bias because it normalizes by the standard deviations of the vectors F_i^* and F_j^*, which also increase (since $F_i^* F_i^{*\prime} = F_i F_i' + 2F_{i,Y-1}F_{iY} \geq F_i F_i'$).

Define $\phi_{ij} = \frac{F_{i,Y-1}F_{jY}+F_{j,Y-1}F_{iY}}{F_i F_j'}$, $\phi_{ii} = \frac{2F_{i,Y-1}F_{iY}}{F_i F_i'}$, and $\phi_{jj} = \frac{2F_{j,Y-1}F_{jY}}{F_j F_j'}$. Letting asterisks denote the index based on the aggregated technology fields, it follows immediately that

$$TECH^{\text{J}^*} = \lambda TECH^{\text{J}},$$
$$TECH^{\text{J-COV}^*} = \theta TECH^{\text{J-COV}},$$

where $\lambda = \frac{1+\phi_{ij}}{\{(1+\phi_{ii})(1+\phi_{jj})\}^{1/2}} < \theta = 1 + \phi_{ij}$. That is, aggregation leads to a smaller percentage increase in $TECH^{\text{J}}$ than in the $TECH^{\text{J-COV}}$ index. This is the sense in which the Jaffe index is less sensitive to aggregation than the Jaffe covariance measure.[7]

Note that it is straightforward to generalize the results in this subsection to the case where more than two technology fields are aggregated, and to the case where several subsets of fields are aggregated. An example of the latter is moving from, say, four digit to three digit classification of fields.

C.2. *Mahalanobis*

To explain the calculation of the Mahalanobis normed measure we need to define some notation. First, we let $T = [T_1', T_2', \ldots, T_N']$ denote the $(426, N)$ matrix where each column contains a firm's patent shares in the 426 technological classes. Second, we define a normalized $(426, N)$ matrix $\tilde{T} = [T_1'/(T_1T_1')^{1/2}, T_2'/(T_2T_2')^{1/2}, \ldots, T_N'/(T_NT_N')^{1/2}]$, in which each column is simply normalized by the firm's patent share dot product. Third, we define the $(N\times N)$ matrix $TECH = \tilde{T}'\tilde{T}$. This matrix $TECH$ is just the standard Jaffe

[7]In general, we cannot know whether $\lambda \lesseqgtr 1$. This depends on the specific distributions of firm R&D across technology fields. It is possible that aggregation reduces the $TECH^{\text{J}}$ index. But we can say that, if aggregation increases that measure, the increase will be proportionately smaller than for the $TECH^{\text{J-COV}}$ index.

(1986) uncentered correlation measure between firms i and j, in which each element is the measure $TECH_{ij}$, exactly as defined in (3.6) above. Fourth, we define a $(N, 426)$ matrix $\tilde{X} = [T'_{(:,1)}/(T_{(:,1)}T'_{(:,1)})^{1/2}, \ldots, T'_{(:,426)}/(T_{(:,426)}T'_{(:,426)})^{1/2}]$ where $T_{(:,i)}$ is $(1, N)$ and is the ith row of T. This matrix $\tilde{X}$ is similar to $\tilde{T}$, except it is the normalized patent class shares across firms rather than firm shares across patent classes. Finally, we can define the $(426, 426)$ matrix $\Omega = \tilde{X}'\tilde{X}$ in which each element is the standard Jaffe (1986) uncentered correlation measure between patent classes (rather than between firms). So, for example, if patent classes i and j coincide frequently within the same firm, then Ω_{ij} will be close to 1 (with $\Omega_{ii} = 1$), while if they never coincide within the same firm Ω_{ij} will be 0.

The Mahalanobis normed technology closeness measure is defined as $TECH^M = \tilde{T}'\Omega\tilde{T}$. This measure weights the overlap in patent shares between firms by how close their different patents shares are to each other. The same patent class in different firms is given a weight of 1, and different patent classes in different firms are given a weight between 0 and 1, depending on how frequently they overlap within firms across the whole sample. Note that if $\Omega = I$, then $TECH^M = TECH$. Thus, if no patent class overlaps with any other patent class within the same firm, then the standard Jaffe (1986) measure is identical to the Mahalanobis norm measure. On the other hand, if some patent classes tend to overlap frequently within firms—suggesting they have some kind of technological spillover—then the overlap between firms sharing these patent classes will be higher.

APPENDIX D: CASE STUDIES OF PARTICULAR FIRMS

There are numerous case studies in the business literature of how firms can be differently placed in technology space and product market space. Consider, first, firms that are close in technology but sometimes far from each other in product market space (the bottom right hand quadrant of Figure 1). Table A.II shows IBM, Apple, Motorola, and Intel: four highly innovative firms in our sample. We show results for *SPILLSIC* measured both by the Compustat Segment Database and by the BVD Database. These firms are close to each other in technology space as revealed by their patenting. IBM, for example, has a *TECH* correlation of 0.76 with Intel, 0.64 with Apple, and 0.46 with Motorola (the overall average *TECH* correlation in the whole sample is 0.038—see Table IX). The technologies that IBM uses for computer hardware are closely related to those used by all these other companies. If we examine *SIC*, the product market closeness variable, however, there are major differences. IBM and Apple are product market rivals with a *SIC* of 0.65 (the overall average *SIC* correlation in the whole sample is 0.015—see Table IX). They both produce PC desktops and are competing head to head. Both have presences in other product markets, of course (in particular, IBM's consultancy arm is a major segment of its business), so the product market correlation is not perfect. By contrast, IBM (and Apple) have a very low *SIC* correlation with Intel and

Motorola (0.01) because the latter firms mainly produce semiconductor chips, not computer hardware. IBM produces relatively few semiconductor chips, so is not strongly competing with Intel and Motorola for customers. The *SIC* correlation between Intel and Motorola is, as expected, rather high (0.34) because they are both competitors in supplying chips. The picture is very similar when we look at the measures of *SIC* based on BVD instead of Compustat, although there are some small differences. For example, IBM appears closer to Intel (BVD $SIC = 0.07$) because IBM produces semiconductor chips for in-house use. This is largely missed in the Compustat Segment data, but will be picked up by the BVD data (through IBM's chip-making affiliates).

At the other end of the diagonal (top left hand corner of Figure 1), there are many firms that are in the same product market, but use quite different technologies. One example from our data set is Gillette and Valance Technologies, which compete in batteries, giving them a product market closeness measure of 0.33. Gillette owned Duracell but did no R&D in this area (its R&D was focused mainly on personal care products such as the Mach 3 razor and Braun electronic products). Valence Technologies used a new phosphate technology that radically improved the performance of standard lithium ion battery technologies. As a consequence, the two companies have little overlap in technology space ($TECH = 0.01$).

A third example is the high end of the market for hard disks, which are sold to computer manufacturers. Most firms based their technology on magnetic technologies, such as the market leader, Segway. Other firms (such as Phillips) offered hard disks based on newer, holographic technology. These firms draw their technologies from very different areas, yet competed in the same product market. R&D done by Phillips is likely to pose a competitive threat to Segway, but it is unlikely to generate useful knowledge spillovers for Segway.

APPENDIX E: ENDOGENOUS CHOICE OF TECHNOLOGY CLASSES

E.1. *The Basic Approach*

One way to provide a microfoundation for a distance metric for technological closeness is to draw explicitly on the economic geography literature. Simple economic geography models can show how firms may optimally choose the geographic location of their plants to benefit from potential spillovers and natural advantages leading to agglomeration and co-agglomeration patterns. We draw heavily on this work (in particular, Ellison and Glaeser (1997), Ellison, Glaeser, and Kerr (2007, 2010)) to consider a more microfounded model of our empirical measures of $TECH_{ij}$. However, rather than choosing which geographical classes to locate in, we will consider a firm's choice of which *technological* areas to locate in and consider co-location patterns in this dimension.

E.2. *Agglomeration and Co-Agglomeration Measures*

Consider extending the model in Section 2 to allow for a period 0 where each firm i chooses to direct its R&D across particular technological classes,

$\tau = 1, \ldots, \Upsilon$. A firm can choose to invest in one or more technological classes by establishing some R&D labs (denoted lab l) in these different classes. Assume that there is a fixed number of R&D employees in each lab, e_l. When choosing a technological profile, a firm will consider a number of factors, including the underlying technological opportunities in the class (common to all firms[8]), R&D lab l's ability in a particular field, as well as the other labs who have already located in this area as there are potentially spillovers.

We will model this explicitly below in a firm location model, but we first define some key indexes. A raw technological concentration measure for firm i is

$$G_i = \sum_{\tau=1}^{\Upsilon} (T_{i\tau} - x_\tau)^2,$$

where $T_{i\tau}$ is the proportion of firm i's R&D employment (or equivalently, the proportion of R&D labs) in technology area τ, and x_τ is the share of total R&D employment in the economy in technology area τ. Ellison and Glaeser (1997) suggested an agglomeration measure of the form

$$\gamma_i \equiv \frac{G_i \Big/ \left(1 - \sum_\tau x_\tau{}^2\right) - H_i}{1 - H_i}, \tag{E.1}$$

where H_i is a "Herfindahl Index" reflecting how concentrated are a firm's R&D labs.[9]

Ellison and Glaeser (1997) also suggested a co-agglomeration measure. In our context, consider a group of I firms, and let w_i be firm i's share of the group's total R&D employment. Let $T_1, \ldots, T_\Upsilon$ be the share of R&D employment in the group of I firms in each technology area. G is the raw geographic concentration for the I firm group ($G = \sum_\tau (T_\tau - x_\tau)^2$) and H is the Herfindahl Index of the I-firm group ($H = \sum_i w_i^2 H_i$). The co-agglomeration measure is

$$\gamma^C \equiv \frac{G \Big/ \left(1 - \sum_\tau x_\tau{}^2\right) - H - \sum_{i=1}^{I} \gamma_i w_i^2 (1 - H_i)}{1 - \sum_{i=1}^{I} w_i^2}. \tag{E.2}$$

[8]Thus the number of potential inventions is higher in some areas (like bio-pharmaceuticals) than others (like cement).

[9]$H_i = \sum_\tau (1/e_{l\tau})^2$, where $e_{i\tau}$ is the employment in lab l in technology class τ for firm i. Since we have assumed that all labs are equally sized, $H_i = \sum_\tau (1/L_i)^2$, where L_i is the number of R&D labs owned by firm i. Obviously, if a firm has only one R&D lab, it will have a high degree of measured agglomeration, as it can only locate in one class, but this is a rather artificial type of specialization and the presence of H_i in equation (E.1) corrects for this.

The particular form of this is motivated by relating the index to an explicit location choice model.

PROPOSITION E.1: *In an I-firm probabilistic location choice model, suppose that the indicator variables $\{u_{l\tau}\}$ for whether the lth lab locates in technological area τ satisfy $E(u_{l\tau}) = x_\tau$ and*

$$\mathrm{Corr}(u_{l\tau}, u_{l'\tau}) = \begin{cases} \gamma_i & \text{if labs } l \text{ and } l' \text{ both belong to firm } i, \\ \gamma_0 & \text{if labs } l \text{ and } l' \text{ belong to different firms;} \end{cases}$$

then $E(\gamma^{C}) = \gamma_0$.

The value of Proposition E.1 is that, under the given assumptions, the co-agglomeration index in equation (E.2), based on empirically observable measures, recovers an estimate of the unobserved "deep parameter," γ_0, which is relevant to the spillover effect.

E.3. *A Model of the Choice of Technological Class*

To simplify notation, we focus on the case of two firms, so $I = 2$. In this case, the co-agglomeration measure γ^{C} will be[10]

$$\gamma^{C} \equiv \frac{\sum_{\tau}(T_{1\tau} - x_\tau)(T_{2\tau} - x_\tau)}{1 - \sum_{\tau} x_\tau^2}. \tag{E.3}$$

Labs are indexed by $l \in L_1 \cup L_2$, with L_1 being the labs in firm 1 and L_2 the labs in firm 2. Firms choose where to locate labs between Y technology classes. Spillovers imply that lab l's profits are a function of the other labs' location decisions. If there is a potentially positive spillover between labs l and l', they will tend to be located in the same technology class.

We follow Ellison and Glaeser (1997), and Ellison, Glaeser, and Kerr (2007) in considering "all or nothing" spillovers, as this reduces substantially the problem of multiple equilibria in the lab location game. In particular, define a partition ω of $L_1 \cup L_2$ to be a correspondence $\omega : L_1 \cup L_2 \rightrightarrows L_1 \cup L_2$ such that $l \in \omega(l)$ for all l and $l' \in \omega(l) \Longrightarrow \omega(l) = \omega(l')$. Suppose lab location decisions are the outcome of a game in which the labs choose technology classes in some exogenously specified order and lab l's profits from locating in technology area τ are given by

$$\ln(\pi_{l\tau}) = \ln(x_\tau) + \sum_{l' \in \omega(l)} I(l_{l'} \neq \tau)(-\infty) + \varepsilon_{l\tau}. \tag{E.4}$$

[10]Note that this will be specific to any given pair of firms (i.e., γ^{C}_{12}), but we drop the subscripts for simplicity.

The first term on the right hand side of equation (E.4), x_τ, is the "fecundity" of the technology area, τ. The middle term reflects spillovers: a spillover exists between labs l and l' if $l' = \omega(l)$ and that, when spillovers exist, they are sufficiently strong to outweigh all other factors in a firm's decision about whether to locate a lab in this technology class. The third term is $\varepsilon_{l\tau}$, a Weibull distributed random shock that is independent across labs and locations.

Under these conditions it is possible to prove the following.

PROPOSITION E.2: *Consider the model of technological area choice described above.*

(a) *The Perfect Bayesian Equilibrium (PBE) is unique. In equilibrium, each lab l chooses technology class τ that maximizes $\ln(x_\tau) + \varepsilon_{l\tau}$ if no lab with $l' \in \omega(l)$ has previously chosen a technology class, and chooses the same technology class of previously located labs if some such labs have chosen this location.*

(b) *If $0 \le \gamma_0^{\mathrm{S}} \le \gamma_1^{\mathrm{S}}, \gamma_2^{\mathrm{S}}$ or $0 \le \gamma_0^{\mathrm{S}} \le \gamma_1^{\mathrm{S}}\gamma_2^{\mathrm{S}}$ and $0 \le \gamma_0 \le \min(1/L_1, 1/L_2)$, then there exist distributions over the set of possible partitions for which:*

$$\mathrm{Prob}\big(l' \in \omega(l)\big) = \begin{cases} \gamma_i^{\mathrm{S}} & \text{if labs } l \text{ and } l' \text{ both belong to firm } i, \\ \gamma_0^{\mathrm{S}} & \text{if labs } l \text{ and } l' \text{ belong to different firms.} \end{cases}$$

(c) *If the distribution satisfies the condition in part (b), then, in any PBE of the model, the agglomeration and co-agglomeration indexes satisfy $E(\gamma^{\mathrm{C}}) = \gamma_0^{\mathrm{S}}$ and $E(\gamma_i) = \gamma_i^{\mathrm{S}}$.*

For the proof, see Ellison, Glaeser, and Kerr (2007, Appendix A).

The proposition shows the conditions under which calculation of the co-agglomeration index γ^{C} is an unbiased estimate of the deep parameter γ_0^{S}. Thus, it gives some theoretical foundation of a distance metric that we use in our empirical work. Proposition E.2 shows that the framework developed has a degree of robustness to equilibrium selection.

In our context, equation (E.3) becomes

$$\gamma_{ij}^{\mathrm{C}} \equiv \frac{\displaystyle\sum_\tau (T_{i\tau} - x_\tau)(T_{j\tau} - x_\tau)}{1 - \displaystyle\sum_\tau x_\tau^2},$$

which is the alternative distance metric used in Section 6.1.

E.4. *Extensions*

The model is obviously specialized, but can be extended in various dimensions. First, the model can be extended to allow for other reasons for co-agglomeration patterns, such as "natural advantage." It is difficult to separately identify these from spillovers and, in general, the indexes will capture elements of both. In the context of our paper, we seek to separate spillovers from common clusters of technological opportunity by explicitly examining how shocks to a firm's R&D differentially affect other firms who are "close neighbors" (as indicated by γ^{C}) relative to those who are more distant. If there are genuine spillovers, the close neighbors will be more affected (e.g., in the productivity) than those who are distant. This would not be the case if natural advantage (clusters of common technological opportunity) were fixed. If these changed over time, then the identification problem would reappear. This is why we use tax-policy changes to R&D as instrumental variables, as we argue that these are orthogonal to such common technology shocks.

A second limitation is that the framework does not allow for product market rivalry. Section 2 shows how this can be allowed for in later stages of the game. It is harder to consider a framework for product choice. The Ellison–Glaeser framework is not well adapted for this, as firms will suffer a negative spillover and will want to locate away from where firms currently are, in general, rather than be close as in equation (E.4).

Finally, note that equation (E.4) is quite restrictive. Not only do the errors take an independent parametric form; we assume all classes are neither complements nor substitutes, so each lab can be seen as making a profit maximizing decision independent of the identity of the firm which owns the lab. We show how this can be relaxed in our Mahalanobis measure, which allows differential degrees of closeness between technology classes. If these were literally geographic distances, we could use the actual distance in travel times or miles, as in Duranton and Overman (2005).

APPENDIX F: ECONOMETRIC RESULTS FOR THREE HIGH-TECH INDUSTRIES

We used both the cross-firm and cross-industry variation (over time) to identify our two spillover effects. An interesting extension of the methodology outlined here is to examine particular industries in much greater detail. This is difficult to do, given the size of our data set. Nevertheless, it would be worrying if the basic theory was contradicted in the high-tech sectors, as this would suggest that our results might be due to biases induced by pooling across heterogeneous sectors. To investigate this, we examine in more detail the three most R&D intensive sectors where we have a sufficient number of firms to estimate our key equations: computer hardware, pharmaceuticals, and telecommunications equipment. *Computer hardware* covers *SIC* 3570 to 3577 (Computer and Office Equipment (3570), Electronic Computers (3571), Computer Storage Devices (3572), Computer Terminals (3575), Computer Communications Equipment (3576), and Computer Peripheral Equipment Not Elsewhere Classified (3577)). *Pharmaceuticals* includes Pharmaceutical Preparations (2834) and In Vitro and In Vivo Diagnostic Substances (2835). *Telecommunications*

TABLE A.III

ALTERNATIVE CONSTRUCTION OF SPILLOVER VARIABLES

Dependent Variable:	(1) Tobin's Q	(2) Cite-Weighted Patents	(3) Real Sales	(4) R&D/Sales
A. Baseline (Summarized From Tables III–VI)				
$\ln(SPILLTECH)_{t-1}$	0.381	0.468	0.191	0.100
	(0.113)	(0.080)	(0.046)	(0.076)
$\ln(SPILLSIC)_{t-1}$	−0.083	0.056	−0.005	0.083
	(0.032)	(0.037)	(0.011)	(0.034)
Observations	9,944	9,023	9,935	8,579
B. Constructing *SPILLSIC* Based on BVD Industries Instead of Compustat				
$\ln(SPILLTECH)_{t-1}$	0.313	0.482	0.100	0.056
	(0.108)	(0.093)	(0.052)	(0.078)
$\ln(SPILLSIC)_{t-1}$	−0.063	0.057	0.000	0.142
	(0.034)	(0.029)	(0.014)	(0.034)
Observations	7,269	6,696	7,364	6,445
C. Alternative Based on *SPILLTECH* (See Thompson and Fox-Kean (2005))				
$\ln(SPILLTECH)_{t-1}$	0.105	0.434	0.059	0.023
	(0.062)	(0.054)	(0.025)	(0.029)
$\ln(SPILLSIC)_{t-1}$	−0.063	0.028	0.002	0.021
	(0.033)	(0.039)	(0.013)	(0.019)
Observations	9,848	8,932	9,913	8,386
D. Using Firm Pairs With ($SIC < 0.1$ and/or $TECH < 0.1$)				
$\ln(SPILLTECH^{*})_{t-1}$	0.135	0.416	0.108	0.044
	(0.109)	(0.070)	(0.044)	(0.073)
$\ln(SPILLSIC^{*})_{t-1}$	−0.060	0.054	0.004	0.093
	(0.032)	(0.036)	(0.012)	(0.033)
Observations	9,944	9,023	9,935	8,579

Notes: Value equation in column (1) corresponds to Table III, column (2); the patents equation in column (2) corresponds to Table IV, column (2); the productivity equation in column (4) corresponds to Table V, column (2), and the R&D equation in column (3) corresponds to Table VI, column (2). Panel A summarizes results in Tables III–VI. Panel B uses the alternative method of constructing *SPILLSIC* based on BVD data (see Appendix B.5). Panel C uses a more disaggregated version of technology classes, $SPILLTECH^{\mathrm{TFK}}$, as suggested by Thompson and Fox-Kean (2005)). In Panel D, *TECH* and *SIC* are replaced with the value 0 for any pair of firms in which both *TECH* and *SIC* are above 0.1. Otherwise all specifications are the same as in Panel A.

Equipment covers Telephone and Telegraph Apparatus (3661), Radio and TV Broadcasting and Communications Equipment (3663), and Communications Equipment Not Elsewhere Classified (3669).

Table A.IV summarizes the results from these experiments. The results for computer hardware (Panel A) are qualitatively similar to the pooled results. Despite being estimated on a much smaller sample, *SPILLTECH* has a positive and significant association with market value and *SPILLSIC* a negative and significant association. There is also evidence of technology spillovers in the

TABLE A.IV

ECONOMETRIC RESULTS FOR SPECIFIC HIGH-TECH INDUSTRIES

Dependent Variable:	(1) Tobin's Q	(2) Cite-Weighted Patents	(3) Real Sales	(4) R&D/Sales
A. Computer Hardware				
$\ln(SPILLTECH)_{t-1}$	1.884	0.588	0.398	−0.462
	(0.623)	(0.300)	(0.221)	(0.220)
$\ln(SPILLSIC)_{t-1}$	−0.471	0.055	−0.000	0.317
	(0.157)	(0.813)	(0.111)	(0.107)
Observations	358	277	343	395
B. Pharmaceuticals				
$\ln(SPILLTECH)_{t-1}$	2.126	1.833	0.981	−0.733
	(0.735)	(0.861)	(0.273)	(0.448)
$\ln(SPILLSIC)_{t-1}$	−1.615	−0.050	−0.669	1.266
	(0.649)	(0.312)	(0.329)	(0.567)
Observations	334	265	313	381
C. Telecommunications Equipment				
$\ln(SPILLTECH)_{t-1}$	1.509	1.401	0.789	0.721
	(0.840)	(0.666)	(0.292)	(0.327)
$\ln(SPILLSIC)_{t-1}$	−0.125	0.016	0.095	−0.006
	(0.456)	(0.378)	(0.169)	(0.128)
Observations	405	353	390	450

Notes: Each panel (A, B, C) contains the results from estimating the model on the specified separate industries. Each column corresponds to a separate equation for the industries specified. The regression specification is the most general one used in the pooled regressions. Tobin's Q (column (1)) corresponds to the specification in column (2) of Table III; Cite-weighted patents (column (2)) correspond to column (2) of Table IV; real sales in column (3) corresponds to column (2) of Table V; R&D/Sales (column (4)) corresponds to column (2) of Table VI.

production function and the patenting equation. *SPILLSIC* is positive in the R&D equation, indicating strategic complementarity, and is not significant in patents or productivity regressions, as our model predicts.

The pattern in pharmaceuticals is very similar, with the parameters being consistent with the predicted signs from the theory and statistically significant. Technology spillovers are also found in the production function and the patents equation, and there is also evidence of strategic complementarity, as indicated by the large coefficient on *SPILLSIC* in the R&D equation. We find a much larger, negative coefficient on *SPILLSIC* in the market value equation than in the pooled results, indicating substantial business stealing effects in this sector.

The results are slightly different in the telecommunications equipment industry. We also observe significant technology spillover effects in the market value equation and citation-weighted patents equations, but the coefficient on *SPILLTECH* is insignificant (although positive) in the productivity equation.

There is no evidence of significant business stealing or strategic complementarity of R&D in this sector, however.

Like the pooled sample, these findings on technological spillovers and business stealing are robust to treating R&D as endogenous. For example, in the IV estimation, the coefficients (*standard error*) on *SPILLTECH* and *SPILLSIC* in the market value equation for computer hardware are 2.314 (*0.668*) and −0.512 (*0.243*), respectively.[11]

Overall, the qualitative results from these high-tech sectors indicate that our main results are broadly present in those R&D intensive industries where we would expect our theory to have most bite. Technology spillovers are found in all three sectors, with larger coefficients than in the pooled results, as we would expect.[12] However, there is also substantial heterogeneity across the sectors. First, the size of the technology spillover and product market rivalry effects vary. Second, we find statistically significant product market rivalry effects of R&D on market value in two of the three industries studied. Finally, there is evidence of strategic complementarity in R&D for computers and drugs, but not for telecommunications.

APPENDIX G: COMPUTING PRIVATE AND SOCIAL RETURNS TO R&D

G.1. *Roadmap*

In this appendix, we show how to compute the private and social returns to R&D in the analytical framework developed in this paper. Section G.2 provides some basic notation, following the presentation in the empirical section of the paper, and derives some "reduced forms" after substituting out all the interactions operating through the spillover terms. The main results are in Section G.3, which calculates the general form of the marginal social and private returns to R&D to an arbitrary firm.

We define the marginal social return (MSR) to R&D for firm i as the increase in aggregate output generated by a unit increase in firm i's R&D stock (taking into account the induced changes in R&D by other firms). The marginal private return (MPR) is defined as the increase in firm i's output generated by a marginal increase in its R&D stock. In the general case, the rates of return for individual firms depend on the details of their linkages to other

[11]These same coefficients (*standard errors*) on *SPILLTEC* and *SPILLSIC* in the market value equation for pharmaceuticals and telecommunications equipment are 3.139 (*1.456*) and −1.317 (*1.427*), and 2.500 (*0.696*) and −0.113 (*0.540*), respectively.

[12]We also examined industry heterogeneity in terms of technology levels, defined as the average R&D/Sales ratio in the four digit industry. We interacted this with *SPILLTECH* and *SPILLSIC* in each of the Tobin's Q, patents, productivity, and R&D equations. The coefficients on spillovers tended to be larger in absolute magnitude, but only one of these eight interactions was significant at the 5% level (*SPILLTECH* in the productivity equation, which had a coefficient (*standard error*) of 1.035 (*0.497*)). This is mild evidence for the greater importance of spillovers in high-tech industries as in Table A.IV

firms in both the technology and product market spaces. For the computations presented in the text, we use the general formulas developed here, but we also show below that the key intuition can be understood by examining the special case where all firms are symmetric and there is no "amplification" effect (due to the presence of product market spillovers in the R&D equation). In this case, the wedge between the social and private returns can be either positive or negative, as it depends upon the importance of technology spillovers in the production function (φ_2) relative to product market rivalry effects in the market value equation (γ_3). Social returns will be larger when φ_2 is larger, and private returns will be larger as (the absolute value of) γ_3 rises. Both private and social returns increase in the effect of R&D on output (φ_1).

G.2. *Basic Equations*

The empirical specification of the model consists of four equations: R&D, Tobin's Q, productivity, and patents. For purposes of evaluating rates of return to R&D, we do not need the patent equation because there is no feedback from patents to these other endogenous variables in our model. Thus, for this analysis, we use only the R&D, market value, and productivity equations.

We examine the long-run effects in the model, setting $R_{it} = R_i$ and $Y_{it} = Y_i$ for all t, and $G_j = \frac{R_j}{\delta}$, where R is the flow of R&D expenditures, Y is output, G is the R&D stock, and δ is the depreciation rate used to construct G. The model can be written as

$$\ln R_i = \alpha_2 \ln \sum_{j\neq i} TECH_{ij} R_j + \alpha_3 \ln \sum_{j\neq i} SIC_{ij} R_j + \alpha_4 X_{1i} + \ln Y_i, \tag{G.1}$$

$$\ln(V/A)_i = \gamma_1 \ln(R/A)_i + \gamma_2 \ln \sum_{j\neq i} TECH_{ij} R_j + \gamma_3 \ln \sum_{j\neq i} SIC_{ij} R_j + \gamma_4 X_{2i}, \tag{G.2}$$

$$\ln Y_i = \varphi_1 \ln R_i + \varphi_2 \ln \sum_{j\neq i} TECH_{ij} R_j + \varphi_3 \ln \sum_{j\neq i} SIC_{ij} R_j + \varphi_4 X_{3i}, \tag{G.3}$$

where V/A is Tobin's Q, X_1, X_2, and X_3 are vectors of control variables (for ease of exposition, we treat them as scalars), and the depreciation rate δ gets absorbed by the constant terms in each of the equations (which we suppress for brevity). We then solve out the cross equation links with Y_i by substituting equation (G.3) into equation (G.1). This yields a new equation for R&D:

$$\ln R_i = \alpha_2' \ln \sum_{j\neq i} TECH_{ij} R_j + \alpha_3' \ln \sum_{j\neq i} SIC_{ij} R_j + \alpha_4' X_{1i}, \tag{G.4}$$

where $\alpha_1' = \frac{\alpha_1+\varphi_1}{(1-\varphi_1)}$, $\alpha_2' = \frac{\alpha_2+\varphi_2}{(1-\varphi_1)}$, $\alpha_3' = \frac{\alpha_3+\varphi_3}{(1-\varphi_1)}$, and $\alpha_4' = \frac{\alpha_4+\varphi_4}{(1-\varphi_1)}$. The model we use for the calculations in this appendix is given by equations (G.4), (G.2), and (G.3).

We take a first-order expansion of $\ln[\sum_{j\neq i} TECH_{ij}R_j]$ and $\ln[\sum_{j\neq i} SIC_{ij}R_j]$, approximating them in terms of $\ln R$ around some point, say, $\ln R^0$. Take first $f^i = \ln[\sum_{j\neq i} TECH_{ij}R_j] = \ln[\sum_{j\neq i} TECH_{ij}\exp(\ln R_j)]$. Approximating this nonlinear function of $\ln R$,

$$
\begin{aligned}
f^i &\simeq \left\{\ln \sum_{j\neq i} TECH_{ij}R_j^0 - \sum_{j\neq i}\left(\frac{TECH_{ij}R_j^0}{\sum_{j\neq i} TECH_{ij}R_j^0}\right)\ln R_j^0\right\} \\
&\quad + \sum_{j\neq i}\left(\frac{TECH_{ij}R_j^0}{\sum_{k\neq i} TECH_{ik}R_k^0}\right)\ln R_j \\
&\equiv a_i + \sum_{j\neq i} b_{ij}\ln R_j,
\end{aligned}
$$

where a_i reflects the terms in large curly brackets and b_{ij} captures the terms in parentheses in the last terms.

Now consider the term $g^i = \ln[\sum_{j\neq i} SIC_{ij}R_j]$. By similar steps,

$$
\begin{aligned}
g^i &\simeq \left\{\ln \sum_{j\neq i} SIC_{ij}R_j^0 - \sum_{j\neq i}\left[\frac{SIC_{ij}R_j^0}{\sum_{j\neq i} SIC_{ij}R_j^0}\right]\ln R_j^0\right\} \\
&\quad + \sum_{j\neq i}\left(\frac{SIC_{ij}R_j^0}{\sum_{k\neq i} SIC_{ik}R_k^0}\right)\ln R_j \\
&\equiv c_i + \sum_{j\neq i} d_{ij}\ln R_j.
\end{aligned}
$$

Using these approximations, we can write the R&D equation (G.4) as

$$
\ln R_i = \lambda_i + \sum_{j\neq i} \theta_{ij}\ln R_j + \alpha_4' X_{1i},
$$

where $\lambda_i = \alpha_2' a_i + \alpha_3' c_i$ and $\theta_{ij} = \alpha_2' b_{ij} + \alpha_3' d_{ij}$. Let $\lambda, \ln R$, and X be $N \times 1$ vectors, and define the $N \times N$ matrix

$$
H = \begin{bmatrix} 0 & \theta_{ij} \\ \theta_{ij} & 0 \end{bmatrix}.
$$

Then the R&D equation in matrix form is

$$\ln R = \Omega^{-1}\lambda + \alpha_4' \Omega^{-1} X_1, \tag{G.5}$$

where $\Omega = I - H$.

By a similar derivation, we can write the production function as

$$\ln Y_i = \psi_i + \varphi_1 \ln R_i + \sum_{j \neq i} \delta_{ij} \ln R_j + \varphi_4' X_{3i},$$

where $\psi_i = \varphi_2 a_i + \varphi_3 c_i$ and $\delta_{ij} = \varphi_2 b_{ij} + \varphi_3 d_{ij}$. Let ψ be an $N \times 1$ vector and define the $N \times N$ matrix

$$M = \begin{bmatrix} \varphi_1 & \delta_{ij} \\ \delta_{ij} & \varphi_1 \end{bmatrix}.$$

Then the production function in matrix form is

$$\ln Y = \psi + M \ln R + \varphi_4' X_3. \tag{G.6}$$

Finally, the market value equation can be expressed as

$$\ln(V/A)_i = \phi_i - \gamma_1 \ln A_i + \gamma_1 \ln R_i + \sum_{j \neq i} \omega_{ij} \ln R_j + \gamma_4' X_{2i},$$

where $\phi_i = \gamma_2 a_i + \gamma_3 c_i$ and $\omega_{ij} = \gamma_2 b_{ij} + \gamma_3 d_{ij}$. Letting ϕ be an $N \times 1$ vector and defining the $N \times N$ matrix

$$\Gamma = \begin{bmatrix} \gamma_1 & \omega_{ij} \\ \omega_{ij} & \gamma_1 \end{bmatrix},$$

the value equation in matrix form is

$$\ln(V/A) = \phi - \gamma_1 \ln A + \Gamma \ln R + \gamma_4 X_2. \tag{G.7}$$

The model is summarized by equations (G.5), (G.6), and (G.7).

G.3. *Deriving the Private and Social Return to R&D*

G.3.1. *General Case*

To derive the private and social rates of return to R&D, we consider the effect of a one percent increase in the stock of R&D by firm i. Since, in steady state, the stock is proportional to the flow of R&D ($G = \frac{R}{\delta}$), we can capture this effect by setting $d \ln R_i = \alpha_4' \, dX_{1i} = 1$ and zero for $j \neq i$.[13] Using the R&D

[13] We scale by 100 here—one percent is taken as 1. In the final calculations, the change in R&D stock is divided by 100.

equation (G.5), the absolute changes in R&D levels, after amplification, are given by the $N \times 1$ vector $dR = B_R \Omega^{-1} z^*$, where z^* is an $N \times 1$ vector with 1 in the ith position and zeroes elsewhere, and B_R is an $N \times N$ matrix with R_i in the ith diagonal position and zeroes elsewhere. From the production function (G.6), this induces changes in productivity (output, given the levels of labor and capital) which are given by $dY = B_Y M \Omega^{-1} z^*$, where B_Y denotes an $N \times N$ matrix with Y_j in the jth diagonal position ($j = 1, \dots, N$) and zeroes elsewhere.

This computation of the output effects is correct for the steady state analysis. Recall that we define the marginal social return (MSR) to R&D for firm i as the increase in aggregate output associated with a unit increase in firm i's R&D stock (not a unit increase in its R&D flow), taking into account the induced changes in R&D by other firms. Therefore, we need to divide the aggregate output gain by the increase in the stock of R&D for firm i and any other firms whose R&D is induced by the change, which is given by $dG'z = \frac{1}{\delta} dR'z$, where z is an $N \times 1$ vector of ones. Thus we can write the MSR as

$$\text{(G.8)} \qquad \text{MSR}_i = \frac{dY'z}{dG'z}.$$

Note that the MSR is a scalar.

The marginal private return (MPR) is defined as the increase in firm i's output generated by a unit increase in its R&D stock (any induced R&D by other firms is not relevant to this computation). The MPR consists of two parts. The first is the increase in firm i's output, given its levels of labor and capital. This increase is given by $z^{*\prime} dY$, where z^* is an $N \times 1$ vector with 1 in the ith position and zeroes elsewhere. In addition, the firm enjoys output gains through the business stealing effect. This will be reflected in an increase in the level of labor and capital used by the firm (holding the level of *productivity constant*). Thus we cannot compute business stealing gains directly from the effect of R&D in the production function.

To compute these gains, we exploit the impact of business stealing in the market value equation. To isolate the impact of business stealing (*SPILLSIC*) on market value, we hold the productivity level constant by "turning off" the effect of own R&D ($\gamma_1 = 0$) and *SPILLTECH* ($\gamma_2 = 0$). Define the $N \times N$ matrix

$$\Gamma^* = \begin{bmatrix} 0 & \omega^*_{ij} \\ \omega^*_{ij} & 0 \end{bmatrix},$$

where $\omega^*_{ij} = \gamma_3 d_{ij} \le 0$ ($j \ne i$). From (G.7), the induced percentage change in market value is

$$d \ln V^* = \Gamma^* \, d \ln G = \Gamma^* \Omega^{-1} z^*.$$

The change in market value associated with the business stealing effect, $d \ln V^*$, can be decomposed into two parts—a change in the level of output and shifts in the price-cost margin of the firm. To compute the private return to R&D in terms of output gains, we need to separate the estimated value effect of R&D into these output and price effects. We assume that a fraction σ of the overall change in market value is due to changes in output (the case $\sigma = 1$ corresponds to the case where the price-cost margin is constant—in particular, not affected by *SPILLSIC*). We discuss later how we choose the empirical value of σ for the computations. Using this value, we can write the absolute output changes associated with business stealing as $dY^* = \sigma B_Y \Gamma^* \Omega^{-1} z^*$.[14]

There is a change in output due to business stealing for each firm. The change for firm j is distributed to (or from) all other firms, in general, and we need to describe what that depends on. Consistent with the original formulation of *SPILLSIC*, we assume that the fraction of the overall loss by firm j which goes to firm i, which we call s_{ji}, depends on the closeness of the two firms, SIC_{ji}, and on how much firm i changes its R&D, which is what induces the redistribution, dR_i. Following our earlier derivation of the linear approximation to the system, we use

$$s_{ji} = \frac{SIC_{ji}\, dR_i}{\sum_{k\neq j} SIC_{jk}\, dR_k}.$$

As required, these weights add up to 1 over all recipient firms.

Let z^{**} denote an $N \times 1$ vector with $+1$ in the ith position and $-s_{ji}$ in the $j \neq i$ positions. Then we can write the total change in firm i's output as $dY'z^* + dY^{*\prime}z^{**}$. The first component is the direct gain in output by firm i, and the second component is the redistribution of output from other firms to firm i. The marginal private return to R&D is the total output gain by firm i divided by the increase in the R&D stock by firm i:

$$\text{MPR}_i = \frac{dY'z^* + dY^{*\prime}z^{**}}{dG'z^*}. \tag{G.9}$$

A comparison of the expressions for MSR and MPR, in equations (G.8) and (G.9), shows that we cannot say which is larger a priori. The MSR and MPR differ in three respects: (1) the MSR is larger because it includes productivity (output) gains from firms other than i due to technology spillovers in the numerator, (2) the MSR is smaller than the MPR because it also counts the full R&D costs of other firms (if there is amplification) in the denominator, and (3) the MSR is also smaller because the MPR counts output gains for the firm through the business stealing effects, while the social return excludes them.

[14]Note that if there is no amplification effect in R&D ($\Omega = I$), then all firms lose output to firm i. But when there is amplification, this need not be true, and, in fact, even firm i can end up losing output to other firms whose R&D was increased by amplification. It all depends on the pattern of amplification and firms' positions in product space (i.e., on Ω and Γ^*).

G.3.2. *Special Case: No R&D Amplification*

Consider the case where there is no R&D amplification effect ($\Omega = I$) and no *SPILLSIC* effect on output ($\varphi_3 = 0$). In this case, the earlier formula for dY reduces to

$$dY = \begin{pmatrix} \varphi_1 Y_1 & \delta_{12} Y_1 & \delta_{1N} Y_1 \\ \delta_{21} Y_2 & \varphi_1 Y_2 & \delta_{12} Y_1 \\ \delta_{N1} Y_N & \delta_{N2} Y_N & \varphi_1 Y_N \end{pmatrix} z^* = \begin{pmatrix} \delta_{1i} Y_1 \\ \delta_{2i} Y_2 \\ \varphi_1 Y_i \\ \delta_{Ni} Y_N \end{pmatrix},$$

where again z^* is an $N \times 1$ vector with 1 in the ith position and zeroes elsewhere. It follows that $dY'z = \varphi_1 Y_i + \sum_{j \neq i} \delta_{ji} Y_j$, so the marginal social return for firm i can be expressed as

$$\text{MSR}_i = \varphi_1 \frac{Y_i}{G_i} + \varphi_2 \frac{\sum_{j \neq i} b_{ji} Y_j}{G_i}.$$

The MSR depends on the coefficients of own R&D and technology spillovers in the production function, and the technology spillover linkages across firms. The term $\varphi_2 \frac{\sum_{j \neq i} b_{ji} Y_j}{G_i}$ captures the marginal impact of an increase in firm i's R&D stock on all other firms' output levels, which are mediated by the technology linkages between firm i and other firms.

In the fully symmetric case where all firms are identical both in size *and* in technology spillover linkages ($Y_i = Y_j$ and $b_{ji} = b$ for all i, j), this expression simplifies even further to

$$\text{MSR}_i = \frac{Y_i}{G_i} (\varphi_1 + \varphi_2). \tag{G.10}$$

We turn next to the marginal private return. Using the expression above for dY, we get $dY'z^* = \varphi_1 Y_i$. The second term involves dY^*, which is

$$dY^* = \sigma \begin{pmatrix} Y_1 & 0 & 0 \\ 0 & Y_2 & 0 \\ 0 & 0 & Y_N \end{pmatrix} \begin{pmatrix} 0 & \omega^*_{12} & \omega^*_{1N} \\ \omega^*_{21} & 0 & \omega^*_{2N} \\ \omega^*_{N1} & \omega^*_{N2} & 0 \end{pmatrix} z^* = \sigma \begin{pmatrix} \omega^*_{1i} Y_1 \\ \omega^*_{2i} Y_2 \\ \omega^*_{Ni} Y_N \end{pmatrix}.$$

Recalling that z^{**} denotes an $N \times 1$ vector with $+1$ in the ith position and $-s_{ji}$ in the $j \neq i$ positions, we get $dY^{*\prime} z^{**} = -\sigma \sum_{j \neq i} s_{ji} \omega^*_{ji} Y_j$. Combining these results and recalling that $\omega^*_{ji} = \gamma_3 d_{ji}$, the marginal private return for firm i can be written as

$$\text{MPR}_i = \varphi_1 \frac{Y_i}{G_i} - \sigma \gamma_3 \sum_{j \neq i} s_{ji} d_{ji} \frac{Y_j}{G_i}.$$

The MPR depends on the coefficient of own R&D in the production function and the coefficient of business stealing in the value equation, plus the product market linkages (these are embedded in both the s_{ji} and d_{ji} coefficients). In the fully symmetric case where all firms are identical in size *and* product market linkages, this simplifies to

$$\text{MPR}_i = \frac{Y_i}{G_i}(\varphi_1 - \sigma\gamma_3). \tag{G.11}$$

In this fully symmetric case, the ratio between the marginal social and private returns is

$$\frac{\text{MSR}}{\text{MPR}} = \frac{\varphi_1 + \varphi_2}{\varphi_1 - \sigma\gamma_3}. \tag{G.12}$$

The social return is larger than the private return if the coefficient of technology spillovers in the production function is larger than the coefficient of business stealing in the value equation in absolute value, adjusted by σ (i.e., $\varphi_2 > |\sigma\gamma_3|$). In the general case, however, the relative returns also depend on the position of the firm in both the technology and product market spaces.

As we pointed out earlier, to compute the private return to R&D in terms of output gains, we need to separate the estimated value effect of R&D into the output and price effects. The empirical computations of the private returns to R&D are done using the value $\sigma = \frac{1}{2}$. That is, we assume that half of the percentage change in the market value of a firm is due to changes in output and half to changes in its price-cost margin. This assumption can be microfounded. In particular, we analyzed an N-firm Cournot model with asymmetric costs—where firm i has unit cost c and all other firms have unit cost c' (no cost ranking is assumed). We can show that a marginal increase in R&D by firm i reduces the profit of all other firms, and that *at most half* of this reduction is due to changes in the output levels of those firms. This implies $\sigma \leq \frac{1}{2}$. The actual breakdown into changes in output and price-cost margins depends on the number of firms and the elasticity of demand. Using the assumption $\sigma = \frac{1}{2}$ is conservative in the sense that it provides an upper bound to the MPR, and thus a lower bound to the gap between MSR and MPR when that gap is positive (as we find empirically). Further details are available on http://cep.lse.ac.uk/textonly/_new/research/productivity/BSV_sigma_1March.pdf.

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