Comment on Andrews (1991) “Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimation”*

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This comment includes a solution to a problem in Section 8 in Andrews (1991) and points out a method to generalize the mean-squared error (MSE) bounds appearing in Andrews (1988) and Andrews (1991).

Section 8 of Andrews (1991) extends results in previous sections of the paper for fourth-order stationary processes to nonstationary processes using the results in the working paper version of the paper, Andrews (1988). Assumption A* in Section 8 is not sufficient for the stated results because it fails to include a condition in (2.11) of Andrews (1988). Specifically, as in Andrews (1988, p. 12), let P_1 be a distribution such that $\{V_t\}$ is a mean zero, second-order stationary sequence under P_1 with autocovariance function $\{\Gamma_1 (j) : j = 0, \pm 1, \ldots\}$ and $\{V_t\}$ has spectral density function $f_1 (\lambda) = (2\pi)^{-1} \sum_{j=-\infty}^{\infty} \Gamma_1 (j) e^{-ij\lambda}$. Assumption A* in Andrews (1991) should be augmented by Assumption A**, which requires $-\Gamma_1 (j) \leq \mathbb{E}(V_t V_{t+j}') \leq \Gamma_1 (j)$ for all $t \geq 1$, $j \geq -t+1$, as in (2.11) of Andrews (1988). The term $b'fb$ that appears in the second summand of the bound on the MSE in the equation on p. 840 of Andrews (1991) should be $b'f_1b$ with $f_1 = f_1 (0)$ as in Lemma 1(b) and Theorem 1(b) of Andrews (1988) ($b'f_1b$ corresponds to f_{1b} in Andrews (1988) where $f_{1b} = f_{1b}(0)$ and $f_{vb}(\lambda) = (2\pi)^{-1} \sum_{j=-\infty}^{\infty} b'\Gamma_v (j) be^{-ij\lambda}$ with $v = 1$). (Note that Assumption A** is not needed for the extension of Proposition 1(b) of Andrews (1991) to nonstationary time series.)

The optimality results based on a minimax MSE criterion for kernels and bandwidths sequences under nonstationarity established by Andrews (1988) and the MSE upper bound established by Andrews (1991) in Section 8 can be generalized as follows. Andrews expressed the bounds

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in terms of the distributions of two different second-order stationary processes. The bounds apply to a class of nonstationary processes. The two distributions provide upper and lower bounds, respectively, to the autocovariances of the nonstationary processes in the class. This class can be enlarged if the two distributions are taken to be those of some nonstationary processes that satisfy certain restrictions [e.g., piecewise local stationarity as in Casini (2021)], thereby allowing for more variability of $\mathbb{E}(V_t V'_{t-j})$ and serial dependence of $\{V_t\}$. The resulting minimax MSE bounds can also provide information on how nonstationarity influences the estimation bias. See Casini (2021) and Casini and Perron (2021) for formal details.

References

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