Robust Bayesian inference for set-identified models

Raffaella Giacomini and Toru Kitagawa

"Robust Bayesian inference for set-identified models" by Raffaella Giacomini and Toru Kitagawa developed a set of new tools for robust Bayesian inference in set-identified parametric models and demonstrated their value in the analysis of Structural Vector Autoregressions (SVARs).

The paper considers models with data-revisable priors for the reduced-form parameters and set-valued mappings defining the set of structural-parameters values corresponding to each reduced-form parameter value through the specified parametric model. As the latter implies conditional independence between the structural parameters and the data, given the reduced-form parameter, prior distributions of the structural parameters are updated by the data only through the update on the prior distribution of the reduced-form parameters. Thus, the priors for the set-identified structural parameters are decomposed into a data-revisable prior for the reduced-form parameters and a data-unrevisable conditional prior for the structural parameters, given the reduced-form parameters. The key idea is to allow for ambiguous beliefs (multiple priors) for the unrevisable component of the prior, thereby constructing a robust class of priors and a corresponding class of well-defined posteriors from which to conduct finite-sample formal Bayesian inference with large-sample frequentist validity. The priors of the structural parameters conditional on the reduced-form parameters are allowed to be arbitrary. They are only required to assign probability one to the set of structural parameter values corresponding to the value of the reduced-form parameter they are conditioned on.

The paper presents a series of results providing different ways to summarize the set of posterior distributions. It provides diagnostic tools for testing plausibility of identifying restrictions and also diagnostic tools for testing informativeness of identifying restrictions and of priors. One of the results specifies the posterior lower and upper probabilities for a parameter of interest being in a set in terms of the posterior of the reduced-form parameter. Thus, despite multiplicity of the prior distributions, the posterior lower and upper probabilities can be simulated using random draws from the posterior of reduced-form parameters. Another result shows that, under some conditions, the set of posterior means of a parameter of interest is the Aumann expectation of the convex hull of the identified set. Using the theory of random sets, this problem reduces to approximating the support function corresponding to the convex hull of the identified set and again, the set of posterior means of parameter of interest can be simulated using random draws from the posterior of reduced-form parameters. Furthermore, the paper developed a counterpart of the highest posterior density region in the context of robust Bayesian inference, robust credible region, that covers the parameter of interest with a given percentage, using the posterior lower probability. Next, the paper examined the large sample frequentist properties of its target quantities, showing that when the identified set is convex, the set of posterior means is a consistent estimator of the true identified set and the robust credible region has the correct frequentist asymptotic coverage for the true identified set. If the identified set is not convex, the method provides posterior inference about its convex hull.

The paper applied the general framework to develop robust Bayesian inference in the context of impulse-response analysis in SVARs with under-identifying zero restrictions and/or sign restrictions and obtained the set of posterior means as well as the smallest robust credible region with a given credibility for the pointwise impulse response function. The results show the usefulness of the new tools for quantifying the sensitivity of non-robust Bayesian inference to the choice of an unrevisable prior. Overall, the paper provided a new and inspiring framework of analysis for set identified models together with relevant theory and an application that convincingly illustrates the potential of the approach.