

NON-CLAIRVOYANT DYNAMIC MECHANISM DESIGN

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We introduce a new family of dynamic mechanisms that restricts sellers from using future distributional knowledge. Since the allocation and pricing of each auction period do not depend on the type distributions of future periods, we call this family of dynamic mechanisms non-clairvoyant.

We develop a framework (bank account mechanisms) for characterizing, designing, and proving lower bounds for dynamic mechanisms (clairvoyant or non-clairvoyant). We use the same methods to compare the revenue extraction power of clairvoyant and non-clairvoyant dynamic mechanisms.

KEYWORDS: Dynamic mechanism design, revenue optimization, non-clairvoyance.

1. INTRODUCTION

DYNAMIC MECHANISM DESIGN OPTIMIZES A REPEATED AUCTION across its different time periods, rather than optimizing each period individually. Dynamic mechanisms yield more revenue and produce better allocation efficiency—so why are they not more widely adopted?

While dynamic mechanisms are powerful, three main considerations have limited their adoption in practice. First, dynamic mechanisms tend to be excessively detail dependent: They require the designer to have reliable forecasts of valuation distributions in all periods. Second, they often require all buyers to share their beliefs. Third, dynamic mechanisms are descriptively complex, with nonintuitive allocation and pricing rules. In this paper, we present a way to overcome these limitations and design more practical dynamic mechanisms.

To illustrate a concrete application for dynamic mechanisms, we consider the design of a sequential auction. Suppose that a firm wants to sell T products, one per period, over T time periods. Each product has public features, observable to the seller and all buyers, and a set of private features for each buyer, observable only to that buyer. Each

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buyer's valuation is a function of the publicly observable features and her set of privately observable features. For each period t , the publicly observable feature vector induces a common-knowledge distribution F_t over the buyer's valuations. In this model, there is no higher level prior from which the distributions are generated. The distributions F_1, \dots, F_T are adversarially chosen in the beginning and then gradually revealed to the seller.

Jackson and Sonnenschein (2007) and Manelli and Vincent (2007) observe that even when the valuation samples drawn from F_1, \dots, F_T are independent, the seller can optimize the auction by linking the decisions. For example, the seller can benefit by offering a buyer a higher price today in exchange for a discount tomorrow. In reality, many sellers implement this technique by offering discounts to buyers who have previously purchased their products. Papadimitriou, Pierrakos, Psomas, and Rubinstein (2016) quantified the revenue gap between the optimal Myerson auction, which sells a single item optimally in each time period, and an optimal auction that links the decisions across time periods. They showed that the latter auction could perform arbitrarily better than the former.

In this paper, we describe a concrete economic scenario and argue that the fully Bayesian uncertainty model of traditional dynamic mechanism design is insufficiently robust to address it in practice. We then describe a way to design more robust dynamic mechanisms by removing dependence on distributional beliefs about the future, using a mixed uncertainty model in which the seller has Bayesian uncertainty about the present and Knightian uncertainty about future periods.

Motivation and Economic Application. Consider a scenario in which advertisers (the buyers) use an advertising platform (the seller) to repeatedly purchase display advertisements (the products) on web pages such as news sites and blogs. For each page view, the advertising platform sends to a set of advertisers a real-time bid request containing the publicly observable features of that page view. The public features of the page view include the user's geolocation, device, and browser as well as the originating web page. Importantly, the public features also include a unique identifier (UserId) for the user who initiated the page view. The platform and the advertisers can independently verify the page-view data; hence, the public features are common knowledge among all agents.

Each advertiser also has a set of private features—derived from information they have collected previously—associated with the initiating UserId. An online merchant, for example, typically maintains a data base containing the UserIds of recent visitors to its own website. Upon receiving a bid request from the advertising platform, an advertiser can check whether the UserId in the bid request is one of its recent visitors and bids accordingly. This is called *remarketing*, and is one of the main sources of information asymmetry between the advertisers and the advertising platform.

Mixed Uncertainty Model. The scenario above introduces two sources of uncertainty into an auction: the public features (derived from the page view) and each advertiser's set of private features (derived from their own data bases).

A set of private features is collected by a single advertiser and includes demographic and behavioral information about the user who initiated the page view. This kind of information, once conditioned on the contextual information provided by the public features, is relatively stable. Therefore, it is possible to build good machine learning models that predict the distribution of bids as a function of contextual information—the public features, on which both parties can agree. Hence, a common Bayesian belief forms that is derived from the public features.

In contrast, an auction's public features depend on internet traffic patterns, which can change suddenly as a result of real-world events. For example, the exchange rate between

dollars and euros affects the magnitude of advertiser bids, an unexpected celebrity death generates spikes in search terms, or a major product launch by Apple generates a large number of queries for entirely new search terms. The unpredictable nature of internet traffic means that it is almost impossible to design a good Bayesian prior over public features. Hence, we can build more robust models of public features with Knightian uncertainty.

Non-Clairvoyance. If the seller cannot develop good forecasts, how can he verify that a mechanism is dynamic incentive compatible? To address this limitation, we introduce the concept of *non-clairvoyance*. Since the buyer's value distribution F_t only becomes available at time t when the publicly observable features of the product are revealed, a natural class of mechanisms is one in which the allocation and payment do not depend on distributional information about the future. We call such mechanisms *non-clairvoyant*. Conversely, we call mechanisms that do depend on knowledge of future distributions *clairvoyant*. Formally, a mechanism is non-clairvoyant if, for each period t , it maps distributions F_1, \dots, F_t and their resulting sampled types $\theta_1, \dots, \theta_t$ to an allocation and a set of payments.

How is dynamic incentive compatibility (DIC) defined in the non-clairvoyant sense? In traditional mechanism design, a mechanism is dynamic incentive compatible if the buyer is incentivized to truthfully report her current type *in expectation* over her types in future periods. A mechanism is non-clairvoyant dynamic incentive compatible if, for *any* period t and continuation future F_{t+1}, \dots, F_T , the buyer is incentivized to truthfully report her type in period t , assuming that she will report truthfully in future periods. This is a very strong notion, since we do not even require the agents and the designer to agree on the forecasts for future periods. However, the designer and the agents still agree on the distribution of the current period because they share the common Bayesian beliefs about the present. Recall from the example scenario above that all parties observe contextual information (the auction's public features) about the item being sold and use that information to form their beliefs about the present.

To understand the relative power of *non-clairvoyant* and *clairvoyant* mechanisms, we must consider two scenarios:

$$\text{scenario A : } F_1, \dots, F_t, F'_{t+1}, \dots, F'_T; \quad \text{scenario B : } F_1, \dots, F_t, F''_{t+1}, \dots, F''_T.$$

The designer of a non-clairvoyant mechanism must make the same allocations and payments for the first t periods in both scenarios, while the designer of a clairvoyant mechanism can tailor his allocation and payments to his knowledge of whether he is in scenario A or B.

Importance of ex post Individual Rationality. We depart from traditional dynamic mechanism design and impose ex post individual rationality (IR) instead of the more usual interim constraints. As a requirement for participation, the usual interim IR constraints would suffice. The stronger ex post version is rather a *self-imposed constraint* motivated by practical aspects of internet advertising. Most internet advertising contracts contain provisions that allow buyers to specify a hard limit on the maximum amount they will pay for any given item. This is crucial from a business perspective, as it allows advertisers to explicitly and reliably control their risk and thus lowers a barrier that might otherwise prevent new advertisers from joining the system.

In addition, ex post constraints are distribution independent and therefore more robust than interim constraints, which are taken in expectation over the type distributions.

Techniques. To design non-clairvoyant mechanisms and find the upper bound for the revenue of any clairvoyant mechanism, we use a framework that we call *bank account mechanisms*.

We show that for any clairvoyant mechanism that is dynamic incentive compatible and ex post individually rational, there is a bank account mechanism with the same properties that produces at least the same revenue (Lemma 3.3).

Bank account mechanisms have three important properties. First, bank account mechanisms are dynamic incentive compatible by design (Lemma 3.2). Second, the revenue produced by a bank account mechanism naturally decomposes into two parts: the intraperiod revenue, which can be bounded by the Myerson revenue for that period, and the interperiod revenue, which we call *bank account spend* (Lemma 4.1). Third, bank account mechanisms naturally lend themselves to the design of non-clairvoyant mechanisms. Formally, any non-clairvoyant dynamic mechanism can be written as a non-clairvoyant bank account mechanism (Lemma 5.2).

As a result, bank account mechanisms are a class of clairvoyant (or non-clairvoyant) dynamic mechanisms with simple structures. We can design bank account mechanisms to achieve the optimal revenue and welfare without loss of generality.

Main Results. Using bank account mechanisms, we characterize the optimal non-clairvoyant mechanism for selling one item per period, for two periods, to multiple buyers (Theorem 6.3). The mechanism is optimal in the sense that it guarantees $1/2$ of the optimal clairvoyant revenue—the best achievable ratio. We present an impossibility result (Theorem 5.1) showing that no non-clairvoyant mechanism can guarantee a better than $1/2$ fraction of the revenue of the optimal clairvoyant mechanism for all sequences of distributions.

The result described above is a special case of a more general construction that holds for any number of periods (Theorem 6.1). We describe a non-clairvoyant mechanism for selling one item per period, for any number of periods, which we call the NONCLAIRVOYANTBALANCE mechanism. The NONCLAIRVOYANTBALANCE mechanism produces at least $1/5$ of the revenue achievable by the optimal clairvoyant mechanism. Since the optimal dynamic mechanism can produce arbitrarily many times more revenue than the optimal static mechanism, $1/5$ of the revenue of the optimal dynamic auction is often much more revenue than the optimal static auction can produce.

In each period, the NONCLAIRVOYANTBALANCE mechanism sells $1/5$ of the item using the Myerson auction for the distribution in that period and $2/5$ of the item in a plain second-price auction. We use a dynamic mechanism to sell the remaining $2/5$ of the item. For each agent, we compute a parameter b^i to represent her *bank balance* as a function of her previous reports and the previous distributions. Then we run a modification of the optimal money-burning auction of Hartline and Roughgarden (2008). The Myerson auction component captures the revenue that can be obtained within each individual period. A combination of the second-price and the money-burning components captures the gains from interperiod interactions.

Finally, in Theorem 5.4, we demonstrate how to overcome the impossibility result in Theorem 5.1 when the number of periods (T) is large, the distributions have a uniform upper bound, and their expectations are bounded away from zero. For that regime, we present an asymptotically optimal non-clairvoyant bank account mechanism.

Roadmap. We spend the first half of the paper discussing the single-agent environment, since the analysis is simpler and the notation lighter. In Section 2, we describe

our model and introduce the notion of a non-clairvoyant mechanism. In Section 3, we describe a framework (bank account mechanisms) for designing dynamic non-clairvoyant mechanisms. In Section 4, we instantiate this framework to obtain a non-clairvoyant three-approximation for a single agent. In Section 5, we show that there is no better than two-approximation without additional assumptions on the distributions. In the same section, we show that with a large number of periods and well behaved distributions, it is possible to obtain asymptotically optimal mechanisms. Finally, we extend our construction to multiple agents in Section 6.

2. REPEATED AUCTIONS MODEL

Notation. Given a vector (x_1, \dots, x_T) , we will use subscript x_t to denote a single element and superscript x^t to denote a prefix (x_1, \dots, x_t) .

Auction Setup. The standard dynamic mechanism design setting with a finite time horizon describes an economic setup in which a designer repeatedly selects an outcome over T periods based on the reports by strategic agents. For the sake of clarity, the first part of our paper focuses on the single-agent case and then extends it to the multiple-agent case in Section 6. In each period $t \in [T]$, the agent has type $\theta_t \in \Theta \subseteq \mathbb{R}_+$, which is drawn from a distribution F_t independently across time steps. Her valuation for outcome $x_t \in [0, 1]$ is given by $\theta_t \cdot x_t$.¹

Our assumption that the agent types are independent across time steps is inspired by our main application in internet advertising: Each time a page view arrives, the advertiser's value is a function of the publicly observable features from the page view (e.g., as geographic and demographic information) plus some private features observable to the advertiser (e.g., browser cookies).² The publicly observable features determine the distribution from which the agent's type is sampled, while the private features determine the realization of the type. Unless the advertiser is starting a new campaign, she already has an established notion of value for the combination of cookie and demographic information, so the allocation for one page view will not affect the value of others. We consider implementing dynamic mechanisms with a short span (say a few hours or a day) in which there is a large enough volume of queries that we can reap the benefit of dynamic queries, but the time span is short enough for the valuations to remain stable. Concerns about valuations that shift over time arise when we try to apply dynamic mechanisms over large time spans when the market is likely to move. This issue, however, lies beyond the scope of the current paper.

Continuing the description of the model, the following events occur in each period t :

- (i) The agent learns her type $\theta_t \sim F_t$.
- (ii) The agent reports type $\hat{\theta}_t$ to the designer.
- (iii) The designer implements an outcome $x_t \in [0, 1]$ and charges the agent p_t .
- (iv) The agent obtains utility $u_t = \theta_t \cdot x_t - p_t$.

¹Note that the methodology in this paper extends to a generic outcome space \mathcal{O} as long as the valuation function $v : \Theta \times \mathcal{O} \rightarrow \mathbb{R}$ has a convex structure: Given two outcomes $x_1, x_2 \in \mathcal{O}$ and a parameter $\lambda \in (0, 1)$, there is one outcome $x_\lambda \in \mathcal{O}$ such that for every type $\theta \in \Theta$, we have $v(\theta, x_\lambda) = \lambda \cdot v(\theta, x_1) + (1 - \lambda) \cdot v(\theta, x_2)$.

²Browser cookies are small pieces of data sent from websites and stored in the user's web browser so that when the user revisits the same website, the cookies can be used to identify the user's previous actions. Such data are encrypted and can only be read by the website that placed them. For example, if a user visits an online merchant, a cookie is placed on his browser. In an auction, only the advertiser corresponding to that merchant will be able to read that cookie, making it a private signal.

The final utility of the agent is additive and without discounting across all periods, that is, $\sum_{t=1}^T u_t$.

A mechanism can be described in terms of an outcome and a pricing function, which map the distributional knowledge of the designer $F^T = (F_1, F_2, \dots, F_T)$ and the history of reports $\hat{\theta}^t = (\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_t)$ to an outcome x_t and a payment p_t ,

- outcome, $x_t : \Theta^t \times (\Delta\Theta)^T \rightarrow [0, 1]$,
- payment, $p_t : \Theta^t \times (\Delta\Theta)^T \rightarrow \mathbb{R}$,

where Θ is the space of types for the agents and $\Delta\Theta$ is the set of distributions over Θ . We use a semicolon to separate the report and distribution parameters: $x_t(\hat{\theta}^t; F^T)$ and $p_t(\hat{\theta}^t; F^T)$. We omit the distributional parameters F^T when clear from the context and write the outcome and payments simply as $x_t(\hat{\theta}^t)$ and $p_t(\hat{\theta}^t)$.

We define the utility of the agent with type θ_t in step t given a history of reports $\hat{\theta}^t$ and the designer distribution knowledge F^T as

$$u_t(\theta_t; \hat{\theta}^t; F^T) = \theta_t \cdot x_t(\hat{\theta}^t; F^T) - p_t(\hat{\theta}^t; F^T).$$

Again, we omit F^T when clear from context.

Incentive Constraints. We adopt the traditional notion of incentive compatibility (IC) in dynamic settings, namely *dynamic incentive compatibility (DIC)*, where agents have incentives to report their types truthfully in each period. This can be defined easily by backward induction: In the last period, regardless of the history to date, it should be incentive compatible for an agent to report her true type. This corresponds to the usual notion of IC in (static) mechanism design:

$$\theta_T = \arg \max_{\hat{\theta}_T} u_T(\theta_T; \hat{\theta}^{T-1}, \hat{\theta}_T; F^T) \quad \forall \hat{\theta}^{T-1}, \theta_T \in \Theta^T.$$

In the penultimate period, it should be incentive compatible for the agent to report her true type given that she will report her true type in the following period:

$$\begin{aligned} \theta_{T-1} = \arg \max_{\hat{\theta}_{T-1}} [& u_{T-1}(\theta_{T-1}; \hat{\theta}^{T-2}, \hat{\theta}_{T-1}; F^T) \\ & + \mathbb{E}_{\theta_T \sim F_T} [u_T(\theta_T; \hat{\theta}^{T-2}, \hat{\theta}_{T-1}, \theta_T; F^T)]] \quad \forall \hat{\theta}^{T-2}, \theta_{T-1} \in \Theta^{T-1}. \end{aligned}$$

Proceeding by backward induction for all periods, we require

$$\theta_t = \arg \max_{\hat{\theta}_t} u_t(\theta_t; \hat{\theta}^{t-1}, \hat{\theta}_t; F^T) + U_t(\hat{\theta}^{t-1}, \hat{\theta}_t; F^T) \quad \forall t \in [T], \hat{\theta}^{t-1}, \theta_t \in \Theta^t, \quad (\text{DIC})$$

where the second term is the *continuation utility*, that is, the expected utility obtained from the subsequent periods of the mechanism:

$$U_t(\hat{\theta}^t; F^T) = \mathbb{E}_{\theta_{t+1}, \dots, \theta_T \sim F_{t+1}, \dots, F_T} \left[\sum_{\tau=t+1}^T u_\tau(\theta_\tau; \hat{\theta}^t, \theta_{t+1}, \dots, \theta_\tau; F^T) \right].$$

By the time consistency of optimal choices in multiperiod decision theory, the agent's expected overall utility $U_0 = \mathbb{E}[\sum_t u_t(\theta_t; \hat{\theta}^t; F^T)]$ under DIC mechanisms is maximized when the agent reports truthfully in each period.

Individual Rationality. We also enforce individual rationality (IR) constraints, which require that the total payments never exceed the total declared values. Inspired by our main motivation, we enforce those constraints *ex post*, that is, in every realization of the agent types. The stronger *ex post* version is a self-imposed constraint that allows advertisers to explicitly control their risk. We refer to this constraint as *ex post individual rationality*:

$$\sum_{t=1}^T u_t(\theta_t; \theta^t; F^T) \geq 0 \quad \forall \theta^T \in \Theta^T. \tag{EPIR}$$

Revenue Optimization. We focus on the problem of maximizing revenue subject to the (DIC) and (EPIR) constraints. Fixing a set of distributions F^T , we can define the revenue-optimal mechanism for those distributions as

$$\text{REV}^*(F^T) = \max \mathbb{E}_{\theta^T \sim F^T} \left[\sum_{t=1}^T p_t(\theta^t; F^T) \right] \quad \text{s.t. (DIC) and (EPIR)}. \tag{RMAX}$$

Static Mechanisms. Informally, a mechanism is static if the allocation and pricing functions x_t, p_t at time t depend only on the distributional knowledge F^T and the reported type θ_t in that period.

Under this definition, the revenue optimization problem restricted to static mechanisms becomes separable: The optimal solution consists of applying for each period t the optimal mechanism for that period, that is, the mechanism $x_t(\theta_t; F_T), p_t(\theta_t; F_t)$ that maximizes $\mathbb{E}_{\theta_t \sim F_t}[p_t(\theta_t, F_t)]$ subject to single-period IC and IR.

We define $\text{REV}^S(F^T)$ as the revenue of the optimal static mechanism. Since the static problem is more constrained, we clearly have $\text{REV}^*(F^T) \geq \text{REV}^S(F^T)$. Papadimitriou et al. (2016) show that the ratio $\text{REV}^*(F^T)/\text{REV}^S(F^T)$ can be arbitrarily large.

2.1. Non-Clairvoyant Mechanism Design

Informally, a mechanism is *non-clairvoyant* if it depends on distributional knowledge about the present and past, but not the future. In other words, the contract offered by the designer cannot condition the outcome of a certain period on the information about future periods.

Non-Clairvoyant Direct Mechanism. To make this notion precise, we define the notion of a non-clairvoyant direct mechanism that corresponds to a direct mechanism where the allocation and pricing function depends only on the types and type distributions up to the current period,

- outcome, $x_t : \Theta^t \times (\Delta\Theta)^t \rightarrow [0, 1]$,
- payment, $p_t : \Theta^t \times (\Delta\Theta)^t \rightarrow \mathbb{R}$.

Note that the only change with respect to the previous definition is the superscript of $\Delta\Theta$. This can alternatively be phrased as a progressive measurability restriction imposed on top of standard direct mechanisms.

Non-Clairvoyant DIC. In the non-clairvoyant setting, the designer is constrained not to condition on future type distributions. Agents, however, are allowed to choose strategies that depend on knowledge they have over the anticipated future type distributions.

In a non-clairvoyant dynamic incentive compatible mechanism, the agents should weakly prefer to report their true type θ_i given past reports $\hat{\theta}^{t-1}$, realized distributions F^t , and anticipated future distributions (F_{t+1}, \dots, F_T) . In other words, condition (DIC) should hold for any continuation future,

$$\begin{aligned} \theta_i &= \arg \max_{\hat{\theta}_i} u_i(\theta_i; \hat{\theta}^{t-1}, \hat{\theta}_i; F^t) + U_i(\hat{\theta}^{t-1}, \hat{\theta}_i; F^T) \\ &\forall F^T, t \in [T], \hat{\theta}^{t-1}, \theta_i \in \Theta^t, \end{aligned} \tag{NCDIC}$$

where the second term, U_i , is the *non-clairvoyant continuation utility*.³

Non-Clairvoyant Revenue Maximization. We defined $\text{REV}^*(F^T)$ as the optimal revenue of a dynamic auction for a sequence of distributions F^T without imposing the non-clairvoyant constraints. We call this quantity the *optimal clairvoyant revenue* for F^T .

The optimal non-clairvoyant revenue for a sequence F^T is not well defined because, due to the non-clairvoyance constraint, the incentive constraint is not separable across different distribution sequences. Instead, we will define a non-clairvoyant revenue approximation.

Given a certain non-clairvoyant DIC mechanism \mathcal{M} , we define its revenue on a sequence of distributions F^T in the natural way:

$$\text{REV}^{\mathcal{M}}(F^T) = \mathbb{E}_{\theta^T \sim F^T} \left[\sum_t P^{\mathcal{M}}(\theta^t; F^t) \right].$$

We say that the non-clairvoyant dynamic mechanism \mathcal{M} is an α -approximation to the clairvoyant benchmark if, for all sequences of distributions F^T ,

$$\text{REV}^{\mathcal{M}}(F^T) \geq \frac{1}{\alpha} \cdot \text{REV}^*(F^T).$$

The main question in this paper is whether we can design non-clairvoyant mechanisms that provide good approximations. The optimal static mechanism is non-clairvoyant, but the example on the first page of Papadimitriou et al. (2016) shows that it fails to guarantee any approximation α . Given that fact, it is not clear in principle if we can obtain $\alpha < \infty$ at all.

Finally, while there is no notion of the optimal non-clairvoyant mechanism for a sequence F^T , we can define optimality in a maximin sense. Given T and a family of distributions \mathcal{F} , we define the maximin optimal non-clairvoyant mechanism as (where $0/0 = 1$)

$$\sup_{\mathcal{M}} \inf_{F^T \in \mathcal{F}^T} \frac{\text{REV}^{\mathcal{M}}(F^T)}{\text{REV}^*(F^T)} \quad \text{s.t. (NCDIC), (EPIR), } \mathcal{M} \text{ non-clairvoyant.} \tag{MAXIMIN}$$

In Section 5, we derive the optimal maximin mechanism for two periods and an asymptotically optimal maximin mechanism for $T \rightarrow \infty$.

³Note that we slightly abused the notation of U_i here, because the non-clairvoyant continuation utility should be defined with the periodic utilities not depending on any future distributions.

3. BANK ACCOUNT MECHANISMS

In this section, we define a general family of auctions called *bank account mechanisms*. We choose this name since they are based on a thought experiment in which a buyer “deposits” part of her utility in an account as an investment, which will result in a more favorable auction in future periods. The idea of a bank account is only an abstract device used in the construction of the mechanism and not a real entity that buyers reason about. We initially present our definition in the standard clairvoyant setting, where there is a fixed sequence of distributions F^T , and the functions of the mechanism can depend on all these distributions. To avoid excess notation, we omit distribution dependence.

Our auction will have two salient features: (i) Each period depends on the previous periods *only* through a single scalar variable called the *balance*, and (ii) in this framework, the designer needs to specify single-period auctions that are single-period incentive compatible together with a valid balance update policy. That is, once a valid balance update policy is in place, all the designer needs to worry about are the single-period IC constraints.

We have a bank account mechanism B in terms of the following functions for each period.

- A static single-period mechanism $x_t^B(\theta_t, b)$, $p_t^B(\theta_t, b)$ parameterized by a balance $b \in \mathbb{R}_+$ that is (single-period) *incentive compatible* for each b , that is,

$$\begin{aligned} \theta_t \cdot x_t^B(\theta_t, b) - p_t^B(\theta_t, b) &\geq \theta_t \cdot x_t^B(\theta'_t, b) - p_t^B(\theta'_t, b) \\ \forall b \in \mathbb{R}_+, \theta_t, \theta'_t \in \Theta. \end{aligned} \tag{IC}$$

Note that we do not require the mechanism to be (single-period) individually rational. We also require the utility of the agent to be *balance independent in expectation*, that is,

$$\begin{aligned} \mathbb{E}_{\theta_t}[\theta_t \cdot x_t^B(\theta_t, b) - p_t^B(\theta_t, b)] &\text{ is a nonnegative constant} \\ &\text{not depending on } b. \end{aligned} \tag{BI}$$

- A balance update policy $b_t^B(\theta_t, b)$ that maps the previous balance and the report to the current balance, satisfying the *balance update* conditions

$$0 \leq b_t^B(\theta_t, b) \leq b + \theta_t \cdot x_t^B(\theta_t, b) - p_t^B(\theta_t, b) \quad \forall b \in \mathbb{R}_+, \theta_t, \in \Theta. \tag{BU}$$

Given the balance update functions, we can define $b_t : \Theta^t \rightarrow \mathbb{R}_+$ recursively as

$$b_0 = 0 \quad \text{and} \quad b_1(\theta_1) = b_1^B(\theta_1, 0) \quad \text{and} \quad b_t(\theta^t) = b_t^B(\theta_t, b_{t-1}^B(\theta^{t-1})),$$

which allows us to define a dynamic mechanism in the standard sense as

$$x_t(\theta^t) = x_t^B(\theta_t, b_{t-1}^B(\theta^{t-1})), \quad p_t(\theta^t) = p_t^B(\theta_t, b_{t-1}^B(\theta^{t-1})).$$

The following example illustrates how a bank account mechanism works.

EXAMPLE 3.1: Consider a setting with a single buyer, two periods, and one item being sold per period. The following outline describes a non-clairvoyant, incentive compatible bank account mechanism.

- Period 1: Elicit type $\hat{\theta}_1$ of the buyer, give the item for free, and update the balance as $b_1 = \hat{\theta}_1$.

- Period 2: Charge $s_2 = \min(\mathbb{E}_{\theta_2 \sim F_2}[\theta_2], b_1)$ in advance, and run a second-price auction with reserve r such that

$$\mathbb{E}_{\theta_2 \sim F_2}[\max(0, \theta_2 - r)] = s_2.$$

First, we note that the mechanism is non-clairvoyant since it uses no information about F_2 in the first period. To verify that it satisfies non-clairvoyant DIC, we note that for any anticipated type distribution F_2 , the agent weakly prefers to report her true type θ_1 in the first period. Since reporting truthfully is optimal for any distribution F_2 , reporting truthfully is also optimal without knowledge of F_2 .

This phenomenon is a general one: In any non-clairvoyant DIC mechanism, agents weakly prefer to report truthfully even if they have \mathcal{F} -Knightian uncertainty about the future distributions, that is, the buyer knows the distribution F_t from which her type at time t is drawn, but for future types at time $t' > t$, the buyer only knows that $F_{t'} \in \mathcal{F}$ for a certain feasible set of distributions \mathcal{F} .

In what follows, we will abuse notation by dropping the superscript B and refer to $x_t(\theta')$ and $x_t(\theta_t, b_{t-1})$ interchangeably. Our first theorem is that any bank account mechanism satisfies (DIC) and (EPIR). In fact, it also satisfies slightly stronger versions of those properties, which we discuss in Appendix A.1 of the Supplemental Material (Mirrokni, Paes Leme, Tang, and Zuo (2020)).

LEMMA 3.2: *Any bank account mechanism satisfying (IC), (BI), and (BU) is dynamic incentive compatible (DIC) and ex post individually rational (EPIR).*

A formal proof of Lemma 3.2 is given in Appendix A.2 of the Supplemental Material. Here we highlight the intuition for why this mechanism is incentive compatible. We note that when deciding on a strategy in each period, the agent needs to worry about two things: (i) the utility she obtains in this period, which corresponds to $\theta_t \cdot x_t - p_t$, and (ii) how her strategy in this period will affect the subsequent periods.

The condition (IC) guarantees that reporting her true type maximizes the agent's utility in the current period. The only reason that the agent might consider deviating is to improve her utility in future periods.

The only way that the agent can affect a future period is through the balance. The key idea behind bank account mechanisms is that the (BI) condition makes the agent indifferent (in expectation) to what the balance b_t will be in future periods. Those two facts together guarantee that the mechanism is (DIC).

Finally, (EPIR) follows from summing the condition (BU) over all periods.

The reason that we focus on bank account mechanisms and why they are useful both in designing optimal dynamic mechanisms and proving lower bounds is that any dynamic incentive compatible and ex post individually rational mechanism can be converted into a bank account mechanism without loss of revenue or welfare. Therefore, in designing or characterizing the revenue-optimal mechanism, it is enough to focus on the subclass of bank account mechanisms. Formally, we have the following lemma.

LEMMA 3.3: *Given any dynamic mechanism $(x_t, p_t)_t$ satisfying (DIC) and (EPIR), there exists a bank account mechanism with at least the same revenue and at least the same welfare.*

The proof has three main components: (i) We start by transforming a generic (DIC) and (EPIR) mechanism into a mechanism that is still (DIC) and in which the agent has zero

utility in all but the last period, where her utility is nonnegative. We call this a *payment frontloading* mechanism. (ii) The next step is a symmetrization lemma that transforms the mechanism such that if two histories result in the same expected total utility in subsequent periods, their allocation and payments are also the same. (iii) The final step shows an isomorphism between payment-frontloading symmetric mechanisms and bank account mechanisms. In all those transformations, the revenue and welfare of the mechanism are guaranteed to never decrease.

4. A NON-CLAIRVOYANT THREE-APPROXIMATION

We describe the central mechanism used in the paper for a single-buyer case and any number of periods. The mechanism is a non-clairvoyant three-approximation to the revenue of the optimal clairvoyant mechanism. In Section 5, we adapt this mechanism to obtain the maximin optimal mechanism for two periods as well as an asymptotically optimal mechanism for $T \rightarrow \infty$ periods with well behaved distributions. Below in Section 6, we generalize this result to the case of multiple buyers. Our main result is a non-clairvoyant mechanism that is a three-approximation to the revenue of the optimal clairvoyant mechanism.

NONCLAIRVOYANTBALANCE Mechanism. The mechanism is a combination of three bank account mechanisms. In each period t , we have a uniform combination of the following three mechanisms.

- (i) *Give for free.* Allocate the item regardless of agent type and charge her nothing. Increase the balance by her value:

$$x_t^F = 1, \quad p_t^F = 0, \quad b_t^F = b_{t-1} + \theta_t.$$

- (ii) *Posted price.* Define a target expected utility to be $s_t = \min(\mathbb{E}_{\theta_t \sim F_t}[\theta_t], 3b_{t-1})$. Charge the agent this amount in advance independent of her report and deduct this amount from the balance. Then choose a price r_t such that the expected utility of the agent under r_t is s_t , that is, $\mathbb{E}_{\theta_t \sim F_t}[(\theta_t - r_t)^+] = s_t$. Since $s_t \leq \mathbb{E}[\theta_t]$, the price r_t must be nonnegative. Run a posted-price auction with this price:

$$x_t^P = \mathbf{1}\{\theta_t \geq r_t\}, \quad p_t^P = s_t + r_t \cdot \mathbf{1}\{\theta_t \geq r_t\}, \quad b_t^P = b_{t-1} - s_t.$$

- (iii) *Myerson’s auction.* Find the posted price r_t^* that maximizes the revenue that can be obtained from this period, that is, $r_t^* = \arg \max_r r \cdot \Pr[\theta_t \geq r]$ and post price r_t^* :

$$x_t^M = \mathbf{1}\{\theta_t \geq r_t^*\}, \quad p_t^M = r_t^* \cdot \mathbf{1}\{\theta_t \geq r_t^*\}, \quad b_t^M = b_{t-1}.$$

We describe the mechanism in each period as a uniform combination of those three,

$$x_t = \frac{1}{3}[x_t^F + x_t^P + x_t^M] \quad p_t = \frac{1}{3}[p_t^F + p_t^P + p_t^M] \quad b_t = \frac{1}{3}[b_t^F + b_t^P + b_t^M],$$

where the functions above are functions of θ_t , b_{t-1} , and F_t , that is, $x_t(\theta_t, b_{t-1}; F_t)$, $p_t(\theta_t, b_{t-1}; F_t)$, and $b_t(\theta_t, b_{t-1}; F_t)$.

It is straightforward that this mechanism is non-clairvoyant, since the allocation and payment rule in period t depends only on θ_t , F_t , and the balance b_{t-1} carried from the previous periods, which is itself a function of θ^{t-1} , F^{t-1} . The mechanism is also ex post

individually rational and dynamic incentive compatible, since it is a bank account mechanism.

Next, we show that this mechanism is a three-approximation. It will be useful to define the notion of the *spend* and use it to show a revenue decomposition lemma. We define the spend as

$$s_t(b_{t-1}) = \max\left(0, -\min_{\theta_t} \theta_t \cdot x_t(\theta_t, b_{t-1}) - p_t(\theta_t, b_{t-1})\right). \tag{Spend}$$

LEMMA 4.1—Revenue Upper Bound: *The revenue of any bank account mechanism with a spend function s_t can be bounded by $\mathbb{E}[\sum_t s_t(\theta^t)]$ plus the revenue of the optimal static mechanism.*

PROOF: Letting $p'_t = p_t - s_t$, we have $\text{REV} \leq \mathbb{E}[\sum_t s_t(\theta^t)] + \mathbb{E}[\sum_t p'_t(\theta^t)]$. Since s_t is constant with respect to θ^t , (x_t, p'_t) is a single-period incentive compatible mechanism. Moreover, by the definition of s_t , we have $\theta_t \cdot x_t - p'_t = \theta_t \cdot x_t - p_t + s_t \geq 0$, so (x_t, p'_t) is single-period individually rational. Therefore, its revenue $\mathbb{E}_{\theta_t \sim F_t}[p'_t(\theta_t, b_{t-1})]$ can be bounded by the revenue of the optimal single-period static mechanism for that distribution. Q.E.D.

THEOREM 4.2: *The revenue of the NONCLAIRVOYANTBALANCE mechanism is at least 1/3 of the revenue of the optimal dynamic mechanism.*

PROOF: The revenue of the NONCLAIRVOYANTBALANCE mechanism is

$$\text{REV} = \mathbb{E}\left[\sum_t \frac{1}{3} [p_t^F(\theta^t) + p_t^P(\theta^t) + p_t^M(\theta^t)]\right].$$

Clearly, $\mathbb{E}[\sum_t p_t^M(\theta^t)]$ is the revenue of the optimal static mechanism, in this case the Myerson auction. Thus, by Lemma 4.1, all we need to prove is that $\mathbb{E}[\sum_t p_t^P(\theta^t) + p_t^F(\theta^t)]$ is greater than the sum of the spends of any optimal bank account mechanism. We will show the stronger statement that for any realization of types θ^T , we have

$$\sum_t p_t^P(\theta^t) + p_t^F(\theta^t) \geq \sum_t s_t(b_{t-1}(\theta^{t-1})). \tag{*}$$

Since the realization of the random variables is fixed, let us abbreviate the balance, payment, and spend in the generic bank account mechanism by b_t , p_t , and s_t . If u_t is the utility of the buyer in period t , define $u'_t = u_t + s_t$. By equations (BU), (Spend), and (BI), we know that $0 \leq u'_t \leq \theta_t$, $s_t \leq b_{t-1}$, and $s_t \leq \lambda_t = \mathbb{E}_{\hat{\theta}_t \sim F_t}[u'_t(\theta^{t-1}, \hat{\theta}_t)]$ so

$$b_t \leq b_{t-1} + u'_t - s_t, u'_t \leq \theta_t, s_t \leq \min(\lambda_t, b_{t-1}). \tag{BAL}$$

The way to select u'_t and s_t to optimize $\sum_t s_t$ subject to (BAL) is to use the greedy algorithm that always makes u'_t as large as possible, that is, $u'_t = \theta_t$, and always spend as much as possible, that is, $s_t = \min(\lambda_t, b_{t-1})$. It should be clear from the principle of local optimality that it is never useful to delay spending the outstanding balance. Finally, note that the NONCLAIRVOYANTBALANCE mechanism exactly implements the optimal greedy policy scaled by a factor of 1/3: The give for free mechanism adds $\frac{1}{3}\theta_t$ to the balance, and the posted-price mechanism consumes $\min(b_{t-1}, \frac{1}{3}\lambda_t)$, proving (*). These two facts together prove the theorem. Q.E.D.

5. MAXIMIN OPTIMAL MECHANISMS

Here we derive the maximin optimal mechanism for two periods. We start by showing that there is an inherent gap between clairvoyant and non-clairvoyant mechanisms. Formally, we show that no non-clairvoyant mechanism can provide a better than two-approximation to the clairvoyant benchmark. Then we present a two-period mechanism that achieves this approximation. Later, we also present an asymptotically optimal non-clairvoyant bank account mechanism for $T \rightarrow \infty$ when the distributions are well behaved.

Our lower bound is based on the following idea. Consider a pair of distributions F_1, F_2 and two possible situations: (i) only one item with distribution F_1 and (ii) an item with distribution F_1 followed by another item of distribution F_2 . The non-clairvoyant mechanism must allocate the same way in both cases. If the non-clairvoyant mechanism receives a second item, it can allocate and charge a payment for it; however, if not, its revenue will be that obtained from the first item.

Recall that given a sequence of distributions F^T , we denote by $\text{REV}^*(F^T)$ the revenue of the optimal clairvoyant mechanism. Given a non-clairvoyant mechanism M defined by $x_t(\theta^t; F^t)$ and $p_t(\theta^t; F^t)$, we define its revenue on a sequence of distributions F^T by $\text{REV}^M(F^T)$. Given the definitions, we prove the following lower bound.

THEOREM 5.1—Lower Bound: *For every $\delta > 0$, there are distributions F_1, F_2 such that for every non-clairvoyant mechanism M ,*

$$\text{either } \text{REV}^M(F_1) \leq \frac{1+\delta}{2} \text{REV}^*(F_1) \quad \text{or} \quad \text{REV}^M(F_1, F_2) \leq \frac{1+\delta}{2} \text{REV}^*(F_1, F_2).$$

In particular, if a non-clairvoyant mechanism is an α -approximation to the clairvoyant benchmark, then $\alpha \geq 2$.

The central ingredient in the proof (given in Appendix C of the Supplemental Material) is a characterization of non-clairvoyant mechanisms as bank account mechanisms. We define a non-clairvoyant bank account mechanism as a bank account mechanism with the restriction that the allocation and payment function at time t must depend only on the balance b_t , the reported type θ_t , and the sequence of distributions F^t until the current period. In other words, it is simply a bank account mechanism that is not allowed to depend on distributional knowledge about the future.

Our main characterization is that any non-clairvoyant mechanism can be written as a non-clairvoyant bank account mechanism with the same revenue:

LEMMA 5.2: *Given any non-clairvoyant dynamic mechanism satisfying (nCDIC) and (EPIR), there exists a non-clairvoyant bank account mechanism with the same revenue.*

The characterization in Lemma 5.2 is a non-clairvoyant analogue of Lemma 3.3. Although their proofs share some similarities, there are new challenges to overcome due to the restrictions imposed by non-clairvoyance: Notably, the proof of Lemma 3.3 starts by changing the original mechanism to an equivalent payment-frontloading mechanism. This clearly violates non-clairvoyance; thus, any non-clairvoyant reduction must avoid this step. Additionally, in the proof of Lemma 3.3, we symmetrize the mechanism around the concept of partially realized utility, which is not well defined for non-clairvoyant mechanisms. To overcome these problems, we will use two ideas. The first is a strong property implied by non-clairvoyance, which is the fact that the continuation utility must be constant in the reported type (Lemma C.1). The second idea is to symmetrize the mechanism

by resampling types of previous periods conditioned on a certain event, which in a way resembles the Myersonian ironing procedure.

Maximin Optimal Mechanism. We now present a two-period non-clairvoyant bank account mechanism that achieves the optimal approximation for two periods. The mechanism uses the same components as the NONCLAIRVOYANTBALANCE mechanism in Section 4, but with different probabilities. In the first period, we run a uniform combination of the *give for free* and *Myerson* mechanisms,

$$x_1 = \frac{1}{2}[x_1^F + x_1^M], \quad p_1 = \frac{1}{2}[p_1^F + p_1^M], \quad b_1 = \frac{1}{2}[b_1^F + b_1^M],$$

using the notation defined in Section 4. For the second period, we use a uniform combination of *Myerson* and the *posted-price* auction, with the difference that s_t in the posted-price mechanism is now defined as $s_2 = \min(2b_1, \mathbb{E}_{\theta_2 \sim F_2}[\theta_2])$. We have

$$x_2 = \frac{1}{2}[x_2^P + x_2^M], \quad p_2 = \frac{1}{2}[p_2^P + p_2^M], \quad b_2 = \frac{1}{2}[b_2^P + b_2^M].$$

THEOREM 5.3: *The mechanism above is a non-clairvoyant two-approximation to the two-period clairvoyant benchmark. Hence, it is an optimal solution to the problem (MAXIMIN) for $T = 2$ periods.*

PROOF: By Theorem 5.1, the solution of the problem (MAXIMIN) is at most $1/2$, so to show that the mechanism defined above is optimal, we need to argue that it is a two-approximation. This follows from using the same revenue decomposition used in the proof of Theorem 4.2. For two periods, it is easy to explicitly write the revenue upper bound of any (clairvoyant) mechanism in Lemma 4.1: It is at most the static revenue (i.e., $\mathbb{E}[p_1^M(\theta_1)] + \mathbb{E}[p_1^M(\theta_2)]$) plus $\mathbb{E}_{F_1}[\min(\theta_1, \mathbb{E}_{F_2}[\theta_2])]$, which corresponds to the maximum of $\mathbb{E}[s_1 + s_2]$ subject to the constraint (BAL) in the proof of Theorem 4.2. The non-clairvoyant mechanism described obtains exactly half of that revenue, where the Myerson component obtains half of the optimal static revenue and the *posted-price* mechanism in the second period obtains at least $\frac{1}{2}\mathbb{E}_{F_1}[\min(\theta_1, \mathbb{E}_{F_2}[\theta_2])]$. *Q.E.D.*

In Theorem 6.3, we show that it is also possible to obtain a two-period maximin optimal mechanism for any number of buyers.

Asymptotically Optimal Mechanism. The lower bound in Theorem 5.1 holds when the number of periods is small or when there are many periods, but the distributions change dramatically over time. In contrast, we show that it is possible to design non-clairvoyant mechanisms that are asymptotically optimal as $T \rightarrow \infty$ as long as the distributions are reasonably well behaved. We will focus on the class of bounded distributions with means bounded away from zero. For constants $0 < \epsilon < \bar{v}$, consider

$$\mathcal{F}_{\epsilon, \bar{v}} := \{F \in \Delta(\mathbb{R}_+) \text{ s.t. } \mathbb{E}_{\theta \sim F}[\theta] \geq \epsilon \text{ and } \Pr_{\theta \sim F}[\theta > \bar{v}] = 0\}.$$

The asymptotically optimal mechanism will be a combination of the *give for free* mechanism defined in Section 4 and a throttled allocation based on the balance, which we call the *spend mechanism*, defined as

$$x_t^S = \min\left\{1, \frac{b_{t-1}}{(1 - q_t)\mu_t}\right\}, \quad p_t^S = x_t^S \cdot \mu_t, \quad b_t^S = b_{t-1} + x_t^S \cdot \theta_t - p_t^S,$$

where $\mu_t = \mathbb{E}_{\theta_t \sim F_t}[\theta_t]$ and $q_t = \sqrt{t \ln t} - \sqrt{(t-1) \ln(t-1)}$. Given the two components, we define the ASYMPOPTIMAL mechanism as a combination of *give for free* and the *spend mechanism* with proportions q_t and $1 - q_t$, respectively:

$$x_t = q_t \cdot x_t^F + (1 - q_t) \cdot x_t^S, \quad p_t = q_t \cdot p_t^F + (1 - q_t) \cdot p_t^S, \quad b_t = q_t \cdot b_t^F + (1 - q_t) \cdot b_t^S.$$

From the structure, it is simple to see that ASYMPOPTIMAL is a non-clairvoyant bank account mechanism and hence satisfies (NCDIC) and (EPIR) by default. The next theorem shows that it obtains asymptotically optimal revenue.

THEOREM 5.4: *For any positive numbers $\epsilon < \bar{v}$, there is a constant $C_{\epsilon, \bar{v}}$ depending only on those parameters such that the revenue of the ASYMPOPTIMAL mechanism is at least*

$$\text{REV} \geq \sum_{t=1}^T \mu_t - C_{\epsilon, \bar{v}} \cdot \sqrt{T} \ln T \tag{5.1}$$

whenever $F_t \in \mathcal{F}_{\epsilon, \bar{v}}$ for all t . In particular, the optimal solution to (MAXIMIN) tends to 1 as $T \rightarrow \infty$ when the distributions are restricted to $\mathcal{F}_{\epsilon, \bar{v}}$.

We highlight that while the bound in Theorem 5.4 depends on the parameters ϵ and \bar{v} , the mechanism itself does not require any knowledge of those parameters. The key observation in the proof is that the stochastic process defined by the balance is a submartingale with bounded differences,

$$\mathbb{E}[b_t] = \mathbb{E}[b_{t-1}] + q_t \cdot \mu_t, \quad |b_t - b_{t-1}| \leq \bar{v}.$$

Therefore, the Azuma–Hoeffding inequality will guarantee that for sufficiently large t , the balance will be above $(1 - q_t)\mu_t$, and the spend mechanism will be able to allocate with probability 1, extracting the full surplus.

PROOF OF THEOREM 5.4: We start by observing that the balance is a submartingale with bounded differences since

$$b_t - b_{t-1} = q_t \theta_t + (1 - q_t) \cdot x_t^S \cdot (\theta_t - \mu_t).$$

As the second term has zero expectation, we have that $\mathbb{E}[b_t | b_{t-1}] = b_{t-1} + q_t \mu_t$. For the difference bound, observe that $\forall \theta_t \in [0, \bar{v}]$,

$$-(1 - q_t)\mu_t \leq b_t - b_{t-1} \leq \bar{v} - (1 - q_t)\mu_t.$$

Therefore, the random variable $\tilde{b}_t = b_t - \sum_{s=1}^t q_s \mu_s$ is a martingale with $|\tilde{b}_t - \tilde{b}_{t-1}| < \bar{v}$. Applying Azuma–Hoeffding inequality (Azuma (1967)) for any $y > 0$,

$$\Pr \left[b_t < \sum_{s=1}^t q_s \mu_s - y \right] = \Pr[\tilde{b}_t < -y] \leq \exp\left(-\frac{y^2}{2t\bar{v}^2}\right). \tag{5.2}$$

Next, we use this concentration result to argue that the spend mechanism is throttled with vanishing probability. The mechanism is only throttled when $b_{t-1} < (1 - q_t)\mu_t \leq \bar{v}$. For $t > \exp(4\bar{v}^2/\epsilon^2)$, the expected balance is much larger than \bar{v} :

$$\mathbb{E}[b_t] = \sum_{s=1}^t q_s \mu_s \geq \epsilon \sum_{s=1}^t q_s = \epsilon \sqrt{t \ln t} \geq 2\bar{v} \sqrt{t \ln t} \geq \bar{v} \sqrt{t \ln t} + \bar{v}. \tag{5.3}$$

The concentration inequality in (5.2) can be used to guarantee that the balance will be above \bar{v} with very high probability and the allocation of the spend mechanism will not be throttled:

$$\begin{aligned} \Pr[x_t^S < 1] &= \Pr[b_{t-1} < (1 - q_t)\mu_t] = \Pr\left[\tilde{b}_{t-1} < (1 - q_t)\mu_t - \sum_{s=1}^{t-1} q_s\mu_s\right] \\ &\leq \Pr\left[\tilde{b}_{t-1} < -\left(\sum_{s=1}^t q_s\mu_s - \bar{v}\right)\right] \stackrel{(5.3)}{\leq} \Pr[\tilde{b}_{t-1} < -\bar{v}\sqrt{t \ln t}] \\ &\stackrel{(5.2)}{\leq} \exp(-\ln t/2) = 1/\sqrt{t}. \end{aligned}$$

The expected revenue of the mechanism given history (F_1, \dots, F_T) can be bounded by

$$\text{REV} = \sum_{t=1}^T (1 - q_t)\mathbb{E}[x_t^S \cdot \mu_t] \geq \sum_{t=1}^T \mu_t - \sum_{t=1}^T q_t\mu_t - \bar{v} \sum_{t=1}^T \Pr[x_t^S < 1].$$

The first term $\sum_{t=1}^T \mu_t$ corresponds to the highest achievable welfare, and it is a clear upper bound on the performance of any mechanism. We now bound the remaining terms. For the second term, we have

$$\sum_{t=1}^T q_t\mu_t \leq \bar{v} \sum_{t=2}^T q_t = \bar{v}\sqrt{T} \ln T.$$

For the third term, we only have a meaningful bound of $\Pr[x_t^S < 1]$ when $t > t_0 := \exp(4\bar{v}^2/\epsilon^2)$. For the earlier periods, we use the trivial bound of $\Pr[x_t^S < 1] \leq 1$, obtaining

$$\sum_{t=1}^T \Pr[x_t^S < 1] \leq t_0 + \sum_{t=t_0+1}^T 1/\sqrt{t} \leq 2\sqrt{T} + \exp(4\bar{v}^2/\epsilon^2).$$

The last three inequalities together conclude the proof of the bound in equation (5.1). To see that the mechanism is asymptotically optimal, note that the optimal revenue is at most $\sum_{t=1}^T \mu_t$ and that $\sum_{t=1}^T \mu_t \geq \epsilon T$. Therefore,

$$\begin{aligned} \frac{\text{REV}}{\text{REV}^*} &\geq \frac{\text{REV}}{\sum_{t=1}^T \mu_t} \geq 1 - \frac{\bar{v}\sqrt{T} \ln T + 2\bar{v}\sqrt{T} + \bar{v} \exp(4\bar{v}^2/\epsilon^2)}{\sum_{t=1}^T \mu_t} \\ &\geq 1 - \frac{\bar{v}\sqrt{T} \ln T + 2\bar{v}\sqrt{T} + \bar{v} \exp(4\bar{v}^2/\epsilon^2)}{\epsilon T} \geq 1 - C_{\epsilon, \bar{v}} \frac{\ln T}{\sqrt{T}}. \end{aligned} \quad \text{Q.E.D.}$$

6. MULTIPLE BUYERS

In this section, we extend our results to multiple-buyer cases. Our decision to focus on a single buyer was driven by the desire to keep the notation as simple as possible and focus on the complications introduced by non-clairvoyance. Once the single-buyer case is understood, most of the results presented thus far extend to the multiple-buyer setting. Our

characterization results (Lemma 3.3 and Lemma 5.2) extend with essentially no change in the proofs. The lower bound also naturally extends. The only major difference is in the extension of the NONCLAIRVOYANTBALANCE mechanism. We need to keep a balance for every buyer, so the state will be a vector. As a consequence, we must reason about utility trade-offs not only across time periods, but also across buyers. In the single-buyer case, we solved this problem by decreasing the posted price of the buyer based on her bank balance in a greedy manner. Here, instead, we will need to be more careful and decide which auction to use based on the result of an optimization program. This program will resemble what is often called the optimal money-burning auction (Hartline and Roughgarden (2008)).

A formal definition of the mechanism design problem for multiple buyers is given in Appendix D.1 of the Supplemental Material. It is the natural extension of the single-buyer model with the incentive notion, which we call dynamic Bayesian IC,

$$\theta_t^i = \arg \max_{\hat{\theta}_t^i} \mathbb{E}_{\theta_{-t}^i} [u_t^i(\theta_t^i; \hat{\theta}^{t-1}, (\theta_{-t}^i, \hat{\theta}_t^i); F^T) + U_t^i(\hat{\theta}^{t-1}, (\theta_{-t}^i, \hat{\theta}_t^i); F^T)] \quad (\text{DBIC})$$

$$\forall i \in [n], t \in [T], \hat{\theta}^{t-1}, \theta_t^i \in \Theta^t,$$

where $U_t^i(\hat{\theta}^t; F^T)$ is the expected total utility of a buyer from periods $t + 1$ to T if her history of reports up to period t is $\hat{\theta}^t$ and all the buyers report truthfully from period $t + 1$ onward.

The notion of the bank account mechanism can also be naturally extended to multiple buyers. The balance is now an n -dimensional variable $b \in \mathbb{R}_+^n$, and the mechanism in each period is a static incentive compatible mechanism parameterized by b satisfying the multiple-buyer version of conditions (BI) and (BU). We refer to Appendix D.2 of the Supplemental Material for the details.

6.1. A Non-Clairvoyant Five-Approximation for Multiple Buyers

We then extend the NONCLAIRVOYANTBALANCE mechanism defined in Section 4 to multiple-buyer cases.

We start by observing that Lemma 4.1 still holds in the multiple-buyer case. The revenue of any bank account mechanism can be bounded by the revenue of the optimal static mechanism plus the sum of spends $\mathbb{E}[\sum_t \sum_i s_i^j(\theta^t)]$ (see Appendix D.2 of the Supplemental Material). A natural strategy given this lemma is to combine the optimal static mechanism (in this case, the Myerson auction) with the mechanism that tries to spend as much as possible from the bank accounts. To this end, we replace the *give for free* mechanism by a *second-price auction*, and we replace the *posted-price* by the *money-burning* mechanism (Hartline and Roughgarden (2008)).

We then define the multiple-buyer version of the NONCLAIRVOYANTBALANCE mechanism. As before, we will define three mechanisms that are parameterized by the balance b_t together with a balance update policy. As done in Section 4, we will count the spend as part of the payment:

- (i) *Second-price auction*. Allocate the item to the buyer with the highest type (breaking ties arbitrarily). Increase the balance of the top bidder by her utility. In other words, if we order the buyers such that $\theta_t^1 \geq \theta_t^2 \geq \dots \geq \theta_t^n$, then

$$\begin{aligned} x_t^{S,1} &= 1, & x_t^{S,j} &= 0; & p_t^{S,1} &= \theta_t^2, & p_t^{S,j} &= 0; \\ b_t^{S,1} &= b_{t-1}^1 + \theta_t^1 - \theta_t^2, & b_t^{S,j} &= b_{t-1}^j \end{aligned}$$

for $j \geq 2$. The mechanism guarantees the largest possible increase in bank balance.

- (ii) *Money-burning auction.* Given the bank account states b_{t-1} , we compute the single-period mechanism that maximizes the sum of the expected utilities of the buyers subject to each buyer i having utility of at most $\frac{5}{2}b_{t-1}^i$. That is, we want to compute the allocation and payment rule x_t^B, \tilde{p}_t^B satisfying Bayesian IC and IR and maximizing

$$\max \sum_i \mathbb{E}_{\theta_t} [\tilde{u}_t^{B,i}(\theta_t)] \quad \text{s.t.} \quad \mathbb{E}[\tilde{u}_t^{B,i}] \leq \frac{5}{2}b_{t-1}^i \quad \forall i \in [n], \text{ BIC, and IR.}$$

Money-burning mechanisms have this name since they correspond to the welfare maximization problem when the revenue obtained is burned. [Hartline and Roughgarden \(2008\)](#) provided a comprehensive study of such mechanisms and showed that they can be written as a virtual value maximization for a different notion of virtual values. In fact, we can deduce from their result that the solution to the problem above corresponds to the auction where we transform the values to the space of virtual values for utilities and run a (scaled) second-price auction in that space. Thus, in that sense, it is not very different from Myerson’s auction other than the fact that the notion of virtual values is nonstandard. Given such a solution, we define the money-burning mechanism using the allocation and payment obtained from the program, that is, $\tilde{x}_t^{B,i}, \tilde{p}_t^{B,i}$. Then its allocation, payment, and balance are defined as

$$x_t^{B,i} = \tilde{x}_t^{B,i}, \quad p_t^{B,i} = \tilde{p}_t^{B,i} + \mathbb{E}[\tilde{u}_t^{B,i}], \quad b_t^{B,i} = b_{t-1}^i - \mathbb{E}[\tilde{u}_t^{B,i}].$$

- (iii) *Myerson’s auction.* Run the static optimal auction given by x^M and p^M . Bank accounts are unchanged, that is, $b_t^{M,i} = b_{t-1}^i$.

The non-clairvoyant balance mechanism is the mechanism defined by

$$\begin{aligned} x_t^i &= \frac{1}{5}x_t^{M,i} + \frac{2}{5}x_t^{S,i} + \frac{2}{5}x_t^{B,i}, & p_t^i &= \frac{1}{5}p_t^{M,i} + \frac{2}{5}p_t^{S,i} + \frac{2}{5}p_t^{B,i}, \\ b_t^i &= \frac{1}{5}b_t^{M,i} + \frac{2}{5}b_t^{S,i} + \frac{2}{5}b_t^{B,i}. \end{aligned}$$

In Appendix E of the Supplemental Material, we argue that each component of the NONCLAIRVOYANTBALANCE mechanism can be implemented as a virtual value maximizer. Next, we provide an approximation guarantee with respect to the clairvoyant benchmark.

THEOREM 6.1: *The multiple-buyer version of the NONCLAIRVOYANTBALANCE mechanism is a non-clairvoyant five-approximation to the clairvoyant benchmark.*

Stronger Incentive Guarantees. While our mechanism produces at least 1/5 of the revenue of any dynamic Bayesian incentive compatible (DBIC) mechanism, it actually satisfies a stronger notion of IC: It is optimal for each agent to report her true type even if she knows the types of other agents in the period when she is reporting. This corresponds to

the notion of strong dynamic Bayesian IC:

$$\theta_t^i = \arg \max_{\hat{\theta}_t^i} u_t^i(\theta_t^i; \hat{\theta}^{t-1}, (\theta_t^{-i}, \hat{\theta}_t^i); F^T) + U_t^i(\hat{\theta}^{t-1}, (\theta_t^{-i}, \hat{\theta}_t^i); F^T) \tag{SDBIC}$$

$$\forall i \in [n], t \in [T], \hat{\theta}^{t-1}, \theta_t \in \Theta^t.$$

LEMMA 6.2: *The NONCLAIRVOYANTBALANCE mechanism satisfies (SDBIC).*

The proof is straightforward, but we include it in Appendix D.4 of the Supplemental Material for completeness.

6.2. Maximin Optimal Multiple-Buyer Mechanism for Two Periods

Finally, we adapt the two-period mechanism in Section 5 to multiple-buyer cases and demonstrate that it is still a two-approximation and is therefore maximin optimal. The mechanism has the same three components we used for the multiple-buyer version of NONCLAIRVOYANTBALANCE, except that the money-burning auction differs slightly in the coefficients (2 instead of 5/2) in the spend constraints:

$$\max \sum_i \mathbb{E}_{\theta_t} [\tilde{u}_t^B(\theta_t)] \quad \text{s.t.} \quad \mathbb{E}[\tilde{u}_t^B] \leq 2b_{t-1}^i \quad \forall i \in [n], \text{ IC and IR.}$$

Then the two-period version of the NONCLAIRVOYANTBALANCE mechanism is defined as

$$x_1^i = \frac{1}{2}[x_1^{M,i} + x_1^{S,i}], \quad p_1^i = \frac{1}{2}[p_1^{M,i} + p_1^{S,i}], \quad b_1^i = \frac{1}{2}[b_1^{M,i} + b_1^{S,i}],$$

$$x_2^i = \frac{1}{2}[x_2^{M,i} + x_2^{B,i}], \quad p_2^i = \frac{1}{2}[p_2^{M,i} + p_2^{B,i}], \quad b_2^i = \frac{1}{2}[b_2^{M,i} + b_2^{B,i}].$$

THEOREM 6.3: *The two-period version of the NONCLAIRVOYANTBALANCE mechanism is a non-clairvoyant two-approximation to the two-period clairvoyant benchmark. Hence, it is an optimal solution to problem (MAXIMIN) for T = 2 periods.*

A proof is included in Appendix D.4 of the Supplemental Material.

7. DISCUSSION

An important modeling assumption made in this paper is that the private information of the buyer is drawn independently in each period such that what the auctioneer learns about the buyer’s valuation in past periods becomes irrelevant given the distribution F_t . In particular, the model does not allow for persistent values. An important avenue of investigation is to define the notion of *non-clairvoyance* in a setting with correlations across time and determine whether it is still possible to approximate the optimal clairvoyant mechanism in such cases.

Another open problem is to design a single mechanism that is simultaneously a constant approximation to the optimal for a small number of periods and asymptotically optimal as $T \rightarrow \infty$. This paper shows that it is possible to obtain each property separately by using a different mechanism.

8. RELATED WORK

Dynamic Mechanism Design. The literature on dynamic mechanism design is too extensive to survey here: We refer to the survey by Bergemann and Said (2011) and another recent survey by Bergemann and Välimäki (2019) for comprehensive treatments of the subject. Here, we discuss a few representative papers in the literature.

For efficiency (social-welfare) maximization, Bergemann and Välimäki (2010) propose the dynamic pivot mechanism, which is a natural generalization of the Vickrey–Clarke–Groves (VCG) mechanism to a dynamic environment where agents receive private information over time, and Athey and Segal (2013) propose the team mechanism to achieve budget-balanced outcomes (see also Bergemann and Välimäki (2003, 2006), Cavallo, Parkes, and Singh (2006, 2009), Cavallo (2008)).

For revenue maximization, a line of research was initiated by Baron and Besanko (1984) and Courty and Li (2000) that studies the setting where the private information of agents varies over time. The latter show an optimal dynamic contract that “screens” the agents twice in a setting where agents initially have private information about the future distribution of their values (see also Boleslavsky and Said (2013), Akan, Ata, and Dana (2015) for “screening” in dynamic mechanism design).

Esó and Szentes (2007) study a closely related two-period model, where the agents only have a rough estimation of their private values of the item in the first round, and the seller can release additional signals to affect their values before selling the item in the second round. In a particular setting, they propose a handicap auction that shares some similar ideas with our bank account mechanism in each period: In a handicap auction, the agents buy their premiums from a menu offered by the seller in the first round based on their rough estimation of private values and then compete with each other under unequal conditions (premiums) in the second round after receiving additional signals from the seller. It is similar to our bank account mechanism in the sense that in both settings, the agents first buy some advantages/discounts for the next round via either premium costs (in handicap auctions) or spends (in bank account auctions) based on rough estimations of their values (prior distributions of each period in our case), and then compete under different levels of advantage after observing their realized values.

Pavan, Segal, and Toikka (2009, 2010, 2014) generalize the idea of Myerson (1981) to a multiperiod setting with dynamic private information and characterize IC in terms of necessary conditions and some sufficient conditions. Kakade, Lobel, and Nazerzadeh (2013) propose the virtual-pivot mechanism by combining ideas of “virtual values” for static optimal mechanism design (Myerson (1981)) and “dynamic pivot mechanisms” for dynamic efficient mechanism design (Bergemann and Välimäki (2010)). In particular, they show that the virtual-pivot mechanisms are optimal in certain dynamic environments that are “separable,” satisfy periodic *ex post* IC and IR, and have simple structure in multi-armed bandit settings (see also Battaglini (2005), Deb (2008) for settings with private values evolving through Markovian processes). Devanur, Peres, and Sivan (2019) and Chawla, Devanur, Karlin, and Sivan (2016) study the repeated selling of fresh copies of an item to a single buyer who has either fixed private value (Devanur, Peres, and Sivan (2019)) or evolving values (Chawla et al. (2016)) of the copies.

One major difference between our setting and that with dynamic private information we just discussed above is that we have no initial private types for the agents, and the private types/values are independent of previous outcomes. Instead, we are able to guarantee *ex post* IR for a very general setting in our case, while weaker notions of IR (i.e., interim IR or individually rational in expectation) are adopted in most of the previous studies (except

for [Kakade, Lobel, and Nazerzadeh \(2013\)](#), which guarantees ex post IR for environments satisfying a separability condition).

There are more works primarily focused on the setting with dynamic populations and fixed information ([Parkes and Singh \(2003\)](#), [Gallien \(2006\)](#), [Board \(2008\)](#), [Pai and Vohra \(2008\)](#), [Said \(2008\)](#), [Gershkov and Moldovanu \(2009\)](#), [Board and Skrzypacz \(2010\)](#), [Gershkov and Moldovanu \(2010\)](#), [Said \(2012\)](#)). In particular, the notion of non-clairvoyance we introduced is similar in spirit to the online mechanism design setting studied by [Parkes and Singh \(2003\)](#) (for welfare maximization) and [Pai and Vohra \(2008\)](#) (for revenue maximization) in the sense that the designer has restricted information about dynamic arrival/departure (for online mechanisms) or dynamic prior distributions (for non-clairvoyant mechanisms) in future periods. In contrast to the settings with dynamic populations discussed above, however, our setting emphasizes the dynamic arrivals of perishable goods (e.g., ad impressions), while it is still general enough to capture the dynamic attendance of agents by setting periodic prior distributions to be $\Pr[v = 0] = 1$ when they are absent from the auction except that the agents have unlimited demands. [Hock \(2003\)](#) studies the revenue-maximization problem for selling homogeneous items to unit-demand buyers when the demand curve is unknown. In particular, he considers an approach of sequentially selling the items and setting the optimal price for the current buyer based on the demand curve estimated from bids of previous buyers, which is also related to our notion of non-clairvoyance.

Our work is closer to the line of inquiry initiated by [Papadimitriou et al. \(2016\)](#), who seek to design revenue-optimal auctions in the setting where items are sequentially sold to the same set of buyers over time. They first show that the problem of designing the optimal deterministic auction is NP-hard even for one agent and two periods, but they provide a polynomial time algorithm for the optimal randomized auction via a linear programming formulation for a constant number of buyers and correlated valuations. The formulation is exponential in the number of buyers and the support of the distribution of agent type profiles over time. If agents have independent types over periods, this also causes their formulation to become exponential in the number of periods. This problem is addressed by [Ashlagi, Daskalakis, and Haghpanah \(2016\)](#), who replace the linear programming formulation by a dynamic program and obtain a $(1 + \epsilon)$ -approximation that is polynomial in the number of periods for a single buyer with independent valuations. For multiple buyers, they provide a mathematical characterization but not an algorithm to solve it. Simultaneously and independently, we also provide a $(1 + \epsilon)$ -approximation for agents with independent valuations using dynamic programming in our unpublished manuscript ([Mirrokni, Paes Leme, Tang, and Zuo \(2016\)](#)).

Another closely related stream of literature is on the design of the dynamic mechanism in a time-discounted model where valuations of the buyers are drawn from an identical distribution in each step. This line was initiated by [Biais, Mariotti, Plantin, and Rochet \(2007\)](#) and [Krishna, Lopomo, and Taylor \(2013\)](#). [Belloni, Chen, and Sun \(2015\)](#) provide a characterization of the optimal mechanism by extending Myerson's ironing technique to dynamic settings. [Balseiro, Mirrokni, and Paes Leme \(2018\)](#) study the effect of imposing stronger constraints on the utilities of buyers and design closed-form mechanisms that approach the optimal in the limit. This line of literature is not comparable with our work: Their settings are independent and identically distributed (i.i.d.) over time (while we only assume independence), focus on a single buyer, and are based on a fixed-point formulation that is only possible in time-discounted models. While their model is more restricted, they are able to provide stronger guarantees and closed-form mechanisms.

Dynamic Mechanism Design Frameworks. One major contribution of our paper is the bank account framework, which provides a general framework to design (traditional or non-clairvoyant) dynamic mechanisms. In particular, incorporating this framework with ex post IR is technically challenging. Another major step in the development of the bank account framework is to show that all non-clairvoyant mechanisms can be cast in it. There have been other very interesting and useful frameworks, the oldest of which seems to be the *promised utility* framework of Thomas and Worrall (1990) (see Belloni, Chen, and Sun (2016) or Balseiro, Mirrokni, and Paes Leme (2018) for recent applications). In a more recent work, Ashlagi, Daskalakis, and Haghpahan (2016) design a framework based on revenue-utility trade-off functions. The results in both Thomas and Worrall (1990) and Ashlagi, Daskalakis, and Haghpahan (2016) accommodate ex post IR and are universal in the sense that the optimal mechanism is always contained in their class.

The main difference between bank accounts and promised utilities or revenue-utility trade-offs is that while the latter two are forward-looking (i.e., they define an optimal form for one period, given the optimal solution for the next), the bank account framework is backward-looking. It defines an allocation and pricing rule based on the past and not the future. To the best of our knowledge, this is the only framework capable of accommodating non-clairvoyance.

Online Supply and Scheduling. The term *clairvoyant* is borrowed from the scheduling literature, where it is typically used to refer to an algorithm that can “see the future” in the sense that it can know, for example, the total execution time of jobs not yet completed. It is also often used to describe an adversary that can predict all the algorithm actions, present and future. The concept of non-clairvoyance is typically used to refer to an algorithm that can perform a certain task well, regardless of having all the information.

In that sense, one can see our paper as an *online algorithm approach* to dynamic mechanism design. The study of incentives in problems where items arrive over time in an online manner was initiated by Babaioff, Blumrosen, and Roth (2010), who design auctions (and prove lower bounds) for problems where incentives are required to be maintained and we are required to allocate goods without information about what the total supply is. Goel, Mirrokni, and Paes Leme (2015) extend this to budgeted settings. The online supply problem is also studied from the perspective of revenue in both the Bayesian and prior-free settings by Mahdian and Saberi (2006) and Devanur and Hartline (2009). In this line of work, however, agents reveal their types at the beginning of the period, and the challenge is to guarantee a monotone allocation. Since types are only reported once, incentive constraints do not need to be enforced dynamically.

Robustness and Detail Independence. Non-clairvoyance can be seen as a form of robustification of dynamic mechanisms. By requiring the mechanism not to use any distributional information from future periods, we obtain mechanisms that are much less detail dependent, in the spirit of Wilson’s doctrine (Wilson (1987)). In this sense, we share the philosophy of Bergemann and Morris (2012) in their theory of robust dynamic mechanism design, which seeks to design mechanisms that work irrespective of beliefs that agents might have. While we make the mechanisms free of beliefs about the future, we still assume beliefs about the present (i.e., the seller in period t has forecast F_t for demand during that period). In that sense, we are more in line with Yogi Berra, who says, “It’s tough to make predictions, especially about the future.”

Simpler Mechanisms Without Backward Induction. The constraints imposed by non-clairvoyance naturally produce simpler mechanisms. To illustrate the simplicity of the NONCLAIRVOYANTBALANCE mechanism, it is useful to compare it with previous approaches to designing dynamic mechanisms. All previous approaches require some form of expensive preprocessing step. In Papadimitriou et al. (2016), the allocation and pricing are determined by the solutions of a large linear program that has one variable for each sequence of reports. If the distributions are independent, this requires a number of variables that are exponential both in the number of buyers and the number of periods. Another approach is to replace the linear program by a dynamic program that is solved via backward induction. This is the approach taken by Ashlagi, Daskalakis, and Haghpahan (2016) and by Mirrokni et al. (2016). The mechanism extracts a $(1 - \epsilon)$ fraction of the optimal revenue, but it is no longer exponential in the number of periods. In both cases, it is only analyzed for a single buyer. The mathematical characterization of the optimal mechanism for multiple buyers is also presented in Ashlagi, Daskalakis, and Haghpahan (2016), but it is not made algorithmic beyond a single buyer. Ashlagi, Daskalakis, and Haghpahan (2016) also propose a second mechanism that extracts at least $1/2$ of the optimal revenue but requires solving a simpler dynamic program and produces a simpler allocation rule; however, it still requires backward induction and only applies to one buyer.

Non-clairvoyance clearly prevents the designer from using any form of backward induction, since at period t , we do not know the distributions in future periods. In fact, we do not even know how many more items we will have to allocate. The NONCLAIRVOYANTBALANCE mechanism requires no backward induction: In each period t , it uses the distributions of the buyers in that period to construct an optimal auction (which is based on virtual values, following the Myersonian approach), a second-price auction, and a money-burning auction (which also admits a virtual value description).

In summary, we obtain an auction that requires no preprocessing and no backward induction. Moreover, each of its components is a virtual value maximizer.

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