

# Randomize at your own Risk: on the Observability of Ambiguity Aversion\*

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## Abstract

Facing several decisions, people may consider each one in isolation or integrate them into a single optimization problem. Isolation and integration may yield different choices, for instance, if uncertainty is involved, and only one randomly selected decision is implemented. We investigate whether the random incentive system in experiments that measure ambiguity aversion provides a hedge against ambiguity, making ambiguity-averse subjects who integrate behave as if they were ambiguity neutral. Our results suggest that about half of the ambiguity averse subjects integrated their choices in the experiment into a single problem, whereas the other half isolated. Our design further enable us to disentangle properties of the integrating subjects' preferences over compound objects induced by the random incentive system and the choice problems in the experiment.

**JEL classifications:** C81, C91.

**Keywords:** hedging; random incentives; Ellsberg; ambiguity aversion; design of experiments; integration; isolation; narrow bracketing; narrow framing.

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# 1 Introduction

For most decision-makers, not all forms of uncertainty are equivalent. Following early contributions by [Knight \(1921\)](#) and [Keynes \(1921\)](#), [Ellsberg's \(1961\)](#) thought experiments demonstrated that many individuals do not reduce (subjective) uncertainty to (objective) risk, a behavior he termed *ambiguity aversion*, and is inconsistent with Subjective Expected Utility ([Savage, 1954](#)). However, [Raiffa \(1961\)](#) questioned the normative appeal of ambiguity-averse preferences, arguing that if choices can be made conditional on an objective randomization device, uncertainty will be reduced back to risk. How randomization interplays with ambiguity has been studied theoretically since then, with important recent contributions by [Saito \(2015\)](#) and [Ke and Zhang \(2020\)](#). The present paper investigates this interplay experimentally, while focusing on the challenge of observing ambiguity-averse preferences.

In most circumstances, identification of behavior requires the individual to make more than a single choice. Consider a situation in which the individual makes several risky decisions that will be simultaneously implemented. From a normative perspective, as only the final consequences matter, one should worry about the interactions between the choices. To do so, the individual can integrate all decisions into a single optimization problem. In practice, however, people tend to focus too much on each decision independently, exhibiting isolation or ‘narrow framing’ ([Tversky and Kahneman, 1981](#); [Kahneman and Lovallo, 1993](#); [Barberis et al., 2006](#); [Rabin and Weizsäcker, 2009](#)). Isolation is less cognitively demanding and gives the same solution as integration if the problem is linear (expected value maximization). However, nonlinearities in the objective function (e.g., risk aversion within expected utility) lead to different choices under integration and isolation. The analyst may not be able to interpret the observed choices without knowing whether the individual isolates or integrates.

A solution for the analyst in such cases, commonly used in Experimental Economics, is to let subjects make several decisions and to pay only one randomly selected decision – a mechanism known as the Random Incentive System (RIS). Under expected utility, both isolation and integration yield the same optimal strategy because probabilities (of the risky decisions and of the randomization device) are all dealt with linearly. However, once nonexpected utility is considered, isolating and integrating decisions may, again, yield different choices. In such cases, recovering preferences from observed choices becomes even more challenging and crucially depends on identifying assumptions.

To demonstrate the challenge, consider an analyst interested in eliciting an individual's ambiguity attitude toward the price of a specific stock in a remote country. The analyst observes choices in two choice problems. In the first, the individual chooses between a bet that pays \$10 if the stock price increases (ambiguous) and a bet that pays \$10 with a 50 percent chance (risky). The second choice problem is similar, except that now the ambiguous bet pays if the stock price decreases. To

determine which of the two chosen bets will be selected for payment, a fair coin is tossed. If the individual isolates their decisions, the analyst can identify their ambiguity attitude. If they choose twice the risky bet, their ambiguity aversion is revealed; if they choose twice the bet on the stock's price, their ambiguity seeking is revealed; and choosing one risky bet and one ambiguous bet implies that they believe one direction of change in the stock's price is more likely than the other. However, if the individual integrates their decisions, identifying ambiguity attitudes becomes more challenging. Even if the individual is ambiguity averse, they may (following [Raiffa](#)) reason that since only one of the choices is selected for payment by a fair coin, choosing to bet on the stock's price in both problems will guarantee them an exactly 50 percent chance of winning, irrespective of whether the stock's price increases or decreases. That is, the decision-maker can use the coin to completely hedge the ambiguity. Theoretically, this reasoning relies on the individual subjectively evaluating the bets such that the coin toss follows the realization of the stock's price fluctuation. In practice, the analyst neither knows whether the individual isolates or integrates, nor knows how the individual's perception of the order of uncertainty resolution affects their choices.

The interplay between risk and ambiguity poses a very practical challenge of the mere observation of ambiguity attitudes. Despite ample theoretical arguments suggesting that the usage of RIS makes it impossible to observe ambiguity aversion ([Bade 2015](#), first version 2011; [Kuzmics 2017](#), first version 2012; [Baillon et al. forthcoming](#), first version 2013; [Oechssler and Roomets, 2014](#); [Azrieli et al. 2018](#), first version 2014), most (but not all) of the experimental literature continues to use RIS to measure ambiguity attitudes, with varying degrees of success ([Trautmann and van de Kuilen, 2015](#)), suggesting that isolation may take place, at least in some cases. These issues highlight important tradeoffs involved in the design of experiments that measure behavioral preferences more broadly. In particular, could an analyst simultaneously identify different preference profiles? Do decision-makers isolate or integrate? Do decision-makers choose differently if randomization follows or precedes decisions and resolution of uncertainty?

The present paper provides the first direct evidence on the impact of RIS on the observability of ambiguity aversion. By doing so, it provides a test of whether people isolate their decisions when responding to various decision problems in experiments or behave as if they integrate them into a single decision. If they integrate, our experimental design allows us to further understand how people integrate risk with ambiguity, and especially whether the order matters, and if so, whether backward induction takes place.

We conducted a first experiment (the “main study”) with more than 400 subjects divided into five between-subject treatments. In all treatments, the subjects had to choose between known (risky) bets (on the color of a chip drawn from a bag containing one red and one blue chip) and unknown (ambiguous) bets (each chip could be red or blue, but the color composition of the chips in the bag was unknown).

The ambiguous bets yielded slightly higher payments than the risky bets, allowing us to identify strict ambiguity aversion. In the control, called *Single*, no RIS was employed. Subjects chose the color they wanted to bet on (blue or red) and then chose a bag (known or unknown). The proportion of subjects choosing the known bag in the Single control served as a baseline for strict ambiguity aversion. All other treatments described next employed RIS and were compared to this baseline, enabling us to gauge the impact of RIS on the observability of ambiguity aversion. In one treatment, called *Before*, subjects had to make choices for both colors (which bag to draw from if they bet on red, and if they bet on blue), and the choice they would be paid for was randomly determined (but was not disclosed to them) *before* their choices and before the ambiguity was resolved, i.e., before chips were drawn from the bags. Another treatment (*After*) reversed the order of the random incentives and the resolution of ambiguity, drawing chips from the bags before determining which choice would be paid. If people were to condition their evaluation of the bets on what is determined first and apply backward induction, satisfying [Azrieli et al.’s](#) (2018) Statewise Monotonicity condition, we would expect to observe more ambiguity aversion in the Before treatment than in the After treatment. Indeed, with backward induction, the ambiguity model is applied directly to the bets in the Before treatment, whereas it is applied to a hedge of bets in the After treatment. On the other hand, if they were indifferent between the timing of risk and ambiguity, satisfying Reversal of Order, the observed ambiguity aversion would not differ between Before and After. This way, these treatments enable us to focus on the subjects’ preferences over compound objects, specifically, how they integrate risk and ambiguity. Two additional treatments—Before-6 and After-6—included six choices to be able to identify various degrees of ambiguity aversion/seeking.

In the Single control, approximately 50% of the subjects exhibited strict ambiguity aversion. In all other treatments, the proportion of choices consistent with strict ambiguity aversion was between 25% and 29%. First, this result implies that experimental work that relies on RIS may have *underestimated* the prevalence of ambiguity aversion in the population. Second, our results show that whereas half of the ambiguity-averse subjects view each choice in isolation, the other half made choices as if they integrated all choices in the experiment into one meta-choice. Random incentives prevented the latter half (integrating) of the subjects from revealing their underlying ambiguity (averse) preferences. Specifically, the integrating subjects seem to satisfy Reversal of Order and not Statewise Monotonicity, which [Azrieli et al.](#) (2018) showed is equivalent to incentive compatibility of the RIS.

We conducted a follow-up study to further investigate insights from recent models in the literature. We varied when the unknown bag was constructed in the Single and the Before treatments. Following the arguments in [Saito](#) (2015) and [Ke and Zhang](#) (2020), randomization is rendered ineffective if the ambiguous bag is constructed after randomization takes place. In *Single-Bag Later* (*Single-BL*), the bag was prepared

after the subjects made their decision, removing the possibility for ambiguity-averse subjects to hedge by mentally randomizing during decision time (as proposed by Raiffa, 1961). We found no evidence that behavior in Single-BL differed from the behavior observed in the replication of the Single control in the follow-up study. Subjects did not seem to mentally randomize. We also varied when the bag was constructed in two variants of the Before treatment: after the random incentives, or after the random incentives and the decision. Following Saito (2015) and Ke and Zhang (2020), this should have eliminated the impact of the random incentives. It did not seem to be the case, however, as substantially fewer subjects exhibited ambiguity aversion in these two variants of the Before treatment than in Single and Single-BL.

Our results provide a valuable lesson on the challenge/impossibility of observing the complete spectrum of ambiguity preferences. One can design an experiment to observe ambiguity aversion (e.g., our baseline treatment with no random incentives). However, an experimenter who wishes to deliver richer and finer measurements of ambiguity attitudes runs the risk that some ambiguity-averse subjects will choose as if they integrate the decisions and hedge the ambiguity, making their choices indistinguishable from those of subjective expected utility decision-makers. Experimenters face a tradeoff between richer measurements that may distort the behavior of a substantial proportion of participants, and simple measurements that cannot reveal the full spectrum of behavior without imposing empirically problematic identifying restrictions.

## 2 Main Study

### 2.1 The tasks

All subjects faced two bags, K(nown) and U(nknown).<sup>1</sup> Each bag contained two poker chips that could be either red or blue. Bag K contained exactly one blue chip and one red chip, while the composition of bag U was unknown. One chip would be drawn from each bag. The two bags, with bag K (opened) and bag U (closed), were presented to all subjects individually.<sup>2</sup> After they finished reading the instructions, they received a choice sheet with two choice problems. In the Red choice problem, subjects had to choose between winning €10 if a *red* chip was drawn from bag K and winning €10.20 if a *red* chip was drawn from bag U. In the Blue choice problem, the choice was between winning €10 if a *blue* chip was drawn from bag K and winning €10.20 if a *blue* chip was drawn from bag U. Varying the payments slightly allows the subject to express strict preference, a technique introduced by Epstein and Halevy (2019).

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<sup>1</sup>In the experiment, they were referred to as bags A and B, respectively.

<sup>2</sup>Subjects were not told who had prepared bag U.

We ran a control (*Single*) and four treatments: *Before*, *After*, *Before-6* and *After-6*.

- *Single* control: Subjects first selected the choice problem – Red or Blue – that they wanted to determine their payment. Then, in the chosen problem, they made a choice between the two bets.
- *Before* and *After* treatments: Subjects made a decision in both choice problems displayed on the choice sheet (see Figure 2.1). The experimental tasks are identical in these two treatments, but they differ in the timing of incentivizing, as explained in subsection 2.2.
- *Before-6* and *After-6* treatments: Subjects made a decision in six choice problems displayed on the choice sheet (see Figure 2.2). The design in these treatments is closer to choice lists (also called multiple price lists): the three choice problems for betting on red (blue) can be considered three sub-choice problems in a choice list. Nevertheless, choice problems Red 1 and Blue 4 are the same as the problems in the *Before* and *After* treatments. Hence, comparing proportions of subjects choosing bag K in these two problems in the *Before-6* and *After-6* treatments with those in the other treatments informs us whether introducing more choice problems further affects the measurement of ambiguity aversion. The additional choice problems allowed for richer and more refined classification of ambiguity attitudes.

Figure 2.1: The two choice problems

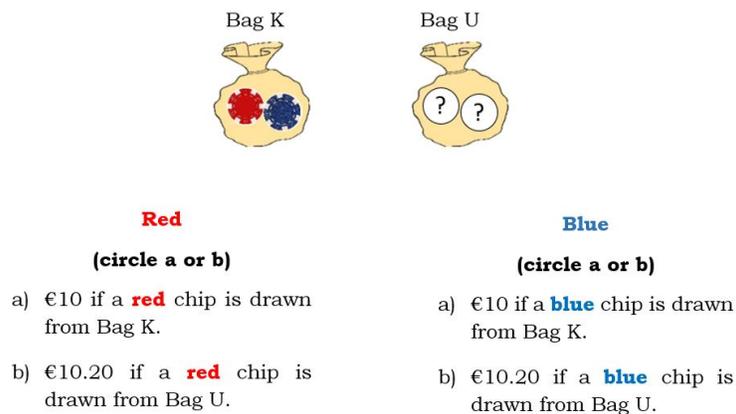


Figure 2.2: The choice problems in additional treatments



**Red 1 (circle a or b)**

- a) €10 if a **red** chip is drawn from Bag K.
- b) €10.20 if a **red** chip is drawn from Bag U.

**Red 2 (circle c or d)**

- c) €10 if a **red** chip is drawn from Bag K.
- d) €10 if a **red** chip is drawn from Bag U.

**Red 3 (circle e or f)**

- e) €10.20 if a **red** chip is drawn from Bag K.
- f) €10 if a **red** chip is drawn from Bag U.

**Blue 4 (circle a or b)**

- a) €10 if a **blue** chip is drawn from Bag K.
- b) €10.20 if a **blue** chip is drawn from Bag U.

**Blue 5 (circle c or d)**

- c) €10 if a **blue** chip is drawn from Bag K.
- d) €10 if a **blue** chip is drawn from Bag U.

**Blue 6 (circle e or f)**

- e) €10.20 if a **blue** chip is drawn from Bag K.
- f) €10 if a **blue** chip is drawn from Bag U.

## 2.2 Incentives and ordering

As one of the main treatment manipulations, we varied the order of randomization, (subjects') choice-making, and uncertainty resolution. To ensure fair and transparent implementation, we randomly selected one subject in each experimental session as the *implementer*, whose tasks differed across treatments.

- Single control: First, all subjects made their choices. Then the implementer resolved the uncertainty by drawing one chip from each bag and announcing the color of each chip. Subjects were paid according to the outcome of their chosen bet in their chosen choice problem. The timeline in this treatment is hence: Choices - Chip draw.
- Before and Before-6 treatment: Before the subjects received the choice sheet, the implementer implemented random incentives by rolling a 6-sided die. Three sides of the die were marked red and the others marked blue. The implementer threw the die for all subjects and put the corresponding choice problem into

sealed envelopes (choice problems Red or Blue in the Before treatment, and choice problems 1 to 6 in the Before-6 treatment). Each subject drew an envelope, knowing that it contained the choice problem that would determine their payment. Then, they received the choice sheet and made their choices in all choice problems. Once the experimenters collected the choice sheets from all subjects, the subjects were asked to open their envelopes to find out which choice problem would be played for real. Finally, the implementer resolved the uncertainty. Subjects were paid according to their choice in the choice problem that was included in their envelope. The timeline in this treatment is hence: Randomization - Choices - Chip draw. With this procedure, it was maximally salient that the random incentives determining which choice problem matters for payment preceded the resolution of uncertainty concerning the draws from the bags. This implementation followed our proposal in [Baillon et al. \(forthcoming\)](#), which was previously implemented in [Epstein and Halevy \(2019\)](#).

- After and After-6 treatment: Once all subjects were done with the experiment, the implementer drew a chip from each bag and announced the color of each chip. Afterward, the implementer rolled the die for each subject. Subjects were paid according to their choice in the choice problem selected by die roll. The timeline in this treatment is hence: Choices - Chip draw - Randomization. The instructions describing this process made it salient that the random incentives followed the resolution of uncertainty.

Experimental instructions for all treatments are included in [Appendix.C](#). In total, 27 experimental sessions were conducted. Seventeen experimental sessions were randomly assigned to Single, Before, and After. Ten additional sessions were conducted at different times and randomly assigned to Before-6 and After-6. The sessions lasted on average 25 minutes. To ensure that all sessions would take about the same time, we asked the subjects in the Single control to answer an additional questionnaire after they completed the choice sheet. It was specified that this questionnaire was unrelated to payment. The subjects of the Before and After treatments answered a shorter questionnaire. The Implementers (27 subjects) received a fixed payment of €10. All other subjects (84, 87, 87, 87, and 89 in the Single, Before, After, Before-6 and After-6 treatments, respectively) were guaranteed a show-up fee of €5 and a variable amount (€0, €10, or €10.20) depending on their choices.

## 2.3 Predictions

In this subsection, we provide intuitive explanations of what we can predict for each treatment. [Section 4](#) derives these predictions formally. Our analysis focuses on *strictly ambiguity-averse (SAA)* subjects (i.e., subjects who strictly prefer risk to ambiguity). We look at whether their choices are *revealing strict ambiguity aversion*

( $SAA^r$ , i.e., they are willing to give up a positive amount to remove ambiguity). An incentive compatible experiment should guarantee that  $SAA$  subjects are  $SAA^r$ . Preference conditions to define  $SAA$  and  $SAA^r$ , as well as all other properties used in this subsection are provided in Section 4.

Table 2.1 summarizes our predictions concerning  $SAA^r$  choices. That is, selecting Bag K: in their chosen choice problem in the Single control; in both choice problems in the After and Before treatments; and in Red 1 and Blue 4 of the After-6 and Before-6 treatments.

In the Single control, we expect  $SAA$  subjects to reveal their ambiguity aversion.<sup>3</sup> For the other treatments,  $SAA$  and  $SAA^r$  may differ because the other treatments involve several choice problems and a randomization device over these choice problems. The question, then, is whether subjects will isolate each choice problem in an experiment or integrate all choice problems in an experiment as a single compound object. We refer to the former view as *isolation* and the latter as *integration*.<sup>4</sup>

The prediction of the isolation view is straightforward: all  $SAA$  subjects should also be  $SAA^r$  in all treatments. Under isolation, subjects treat each choice problem as if it were the only choice problem that determines their payoffs; therefore, their reported preferences would coincide with their preferences in each choice problem. There is strong evidence in the economic and financial literature for isolation, also known as *narrow framing* or *narrow bracketing* (Tversky and Kahneman, 1981; Kahneman and Lovallo, 1993; Barberis et al., 2006; Rabin and Weizsäcker, 2009). If subjects isolate choice problems, then all treatments are incentive compatible.

Predictions for the integration view are more nuanced and depend on both the incentive structure used by the experimenter and the subjects' preferences over compound objects. In this case, their choices will not always reveal their 'true' preferences over acts. Next, we develop our predictions for integrating subjects in each treatment. In the After treatment, consider the strategy of choosing Bag U in both choice problems. If a red chip is drawn from Bag U, then the chance of winning €10.20 is 50%, which is the probability that the Red choice problem is selected. If a blue chip is drawn, the chance of winning is also 50%, the probability that the Blue choice problem is selected. To sum up, regardless of which chip is drawn from U, the probability of winning €10.20 is 50%. By comparison, choosing Bag K twice induces a 50% chance of winning €10. Hence, choosing Bag U twice is not more ambiguous (in the integration view) than choosing Bag K twice, and it yields a higher payment (€10.20 instead of €10). It follows that choosing Bag K in both choice problems is dominated by at least one other strategy (choosing Bag U twice) under integration (see Section

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<sup>3</sup>We explore this further in the follow-up study. There, we consider the possibility that some subjects mentally randomized which color to choose, thus hedging the ambiguity.

<sup>4</sup>In the literature, "integration" has often been used to refer to integration of outcomes in experiments with the decision-maker's wealth. We use "integration" to refer to the integration of decision problems within the experiment.

4). Consequently, integration predicts that *SAA* subjects will not be *SAA<sup>r</sup>*. This point is very closely related to Raiffa’s (1961) argument and has recently been made by [Bade \(2015, first version 2011\)](#); [Kuzmics \(2017, first version 2012\)](#); [Baillon et al. \(forthcoming, first version 2013\)](#); [Azrieli et al. \(2018, first version 2014\)](#) and [Oechssler and Roomets \(2014\)](#).

In the Before treatment, we can differentiate two cases. First, if subjects are not sensitive to the order in which risk and uncertainty are resolved, i.e., if they satisfy *Reversal of Order*, then the Before treatment is equivalent to the After treatment and the same prediction applies. That is, integration combined with *Reversal of Order* also predicts that *SAA* subjects will not be *SAA<sup>r</sup>*. But subjects may satisfy another property, that [Azrieli et al. \(2018\)](#) called *Statewise Monotonicity*. This property requires that preferring some bets to alternative bets (e.g., preferring bet *a* to bet *b* and bet *c* to bet *d*) implies preferring a randomization over the former to a randomization over the latter (preferring a 50-50 chance of *a* or *c* to a 50-50 chance of *b* or *d*). In other words, they first evaluate the bets (with their ambiguity preferences) and then consider the randomization over the subjective values they assigned to the bets. Hence, *Statewise Monotonicity* directly implies incentive compatibility in the Before treatment ([Baillon et al., forthcoming](#); [Azrieli et al., 2018](#)) and under this property, *SAA* subjects should be *SAA<sup>r</sup>*. Note that *SAA* subjects cannot satisfy both *Reversal of Order* and *Statewise Monotonicity* because it would lead to contradictions (violations of transitivity or stochastic dominance).

Our theoretical arguments carry through to the After-6 and Before-6 treatments. These treatments are closer to choice lists, and we can study whether an increase in the number of tasks affects, for instance, subjects’ tendency to isolate or integrate choice problems. Analyzing the four additional choice problems included in these treatments also allows us to infer whether *SAA* subjects who integrate in the After treatment, choose an ambiguity neutral strategy (as predicted by [Bade \(2015\)](#) for instance) or if they even become ambiguity seeking. Experimental papers regularly find substantial ambiguity seeking. It is interesting to investigate whether it might be due to the incentive mechanism or if the mechanism only makes ambiguity-averse subjects appear neutral.<sup>5</sup>

In summary, the proportion of *SAA<sup>r</sup>* subjects in the *Single* control establishes a

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<sup>5</sup>Note that we concentrate on subjects whose choices revealed strict ambiguity aversion by choosing the known bag in the *Single* control. Choosing to bet on the unknown bag in the *Single* control is uninformative about the subject’s ambiguity attitude: it is consistent with ambiguity neutrality and believing that the probability of drawing a red chip equals the probability of drawing a blue chip; it could result from believing that the chip drawn is more likely to be of one color than another, while still being ambiguity neutral; it is consistent with being ambiguity averse but preferring to bet on one color than another in the unknown bag; it could result from being ambiguity averse but having an ambiguity premium lower than 20 cents; or it could result from being ambiguity seeking. In other words, while choosing K identifies strict ambiguity aversion, choosing U does not reveal the subject’s ambiguity attitude.

Table 2.1: Summary of Predictions for *SAA* subjects  
Treatments

	<i>Single</i>	<i>After &amp; After-6</i>	<i>Before &amp; Before-6</i>
Isolation	<i>SAA<sup>r</sup></i>	<i>SAA<sup>r</sup></i>	<i>SAA<sup>r</sup></i>
Integration + Statewise Monotonicity	<i>SAA<sup>r</sup></i>	not <i>SAA<sup>r</sup></i>	<i>SAA<sup>r</sup></i>
Integration + Reversal of Order	<i>SAA<sup>r</sup></i>	not <i>SAA<sup>r</sup></i>	not <i>SAA<sup>r</sup></i>

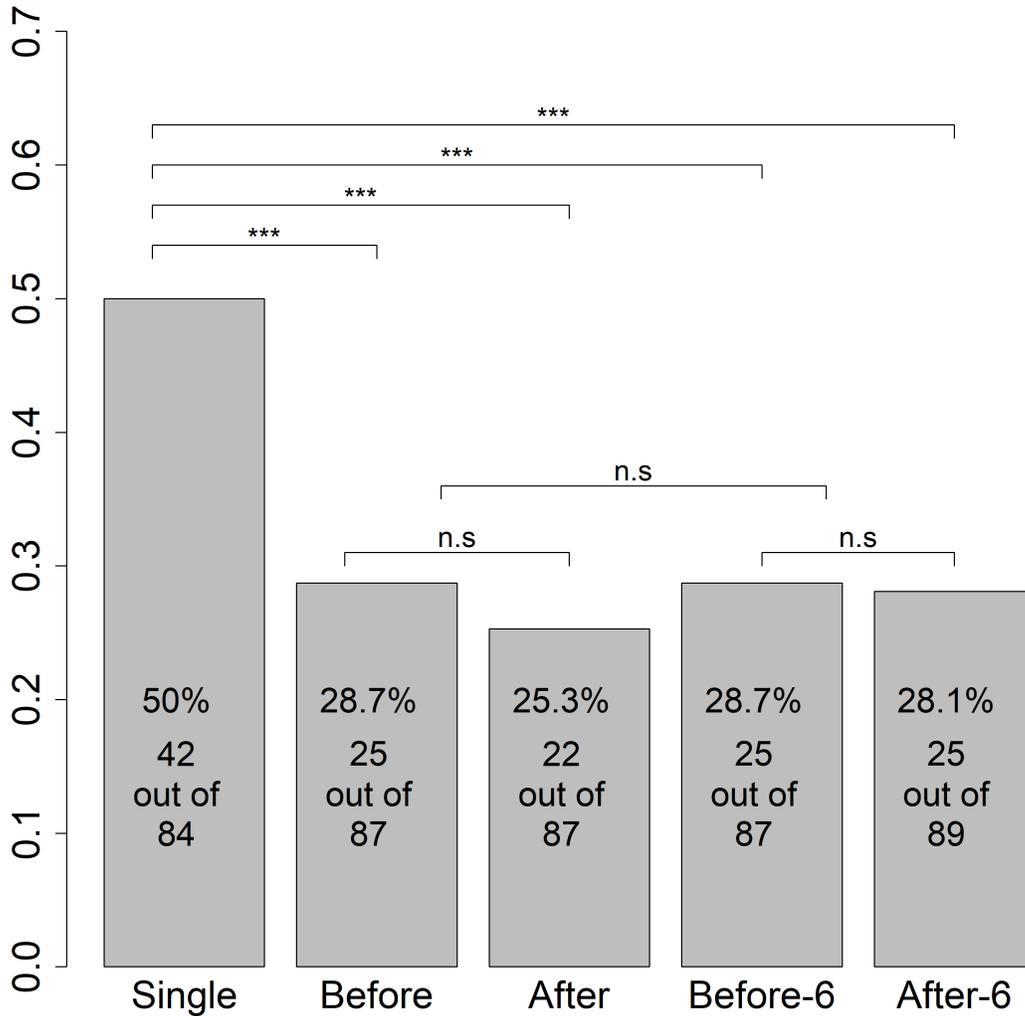
baseline for the rate of *SAA* subjects in the population. In the After treatment, the integration view predicts that the rate of *SAA<sup>r</sup>* subjects drops with respect to the Single control, whereas isolation predicts it does not change. In the Before treatment, both isolation and integration with Statewise Monotonicity predict the same rate of *SAA<sup>r</sup>* subjects as in the Single control, while integration with Reversal of Order (and in the absence of Statewise Monotonicity) predicts a drop of *SAA<sup>r</sup>* subjects. Overall, we can, therefore, expect the highest rate of *SAA<sup>r</sup>* subjects in the Single control, followed by the Before treatment (where integration need not alter choices), and the lowest rate in the After treatment. The After-6 and Before-6 treatments are expected to produce similar results as the After and Before treatments, respectively, unless the presence of additional choices affect subjects' propensity to isolate or integrate.

## 2.4 Results

Figure 2.3 presents the proportion of *SAA<sup>r</sup>* subjects in each treatment. The *SAA<sup>r</sup>* proportion in both the Before (28.7%) and After (25.3%) treatments is significantly lower than in the Single control (50%, p-values < 0.01 in the proportion test). The *SAA<sup>r</sup>* proportion is slightly higher in the Before treatment than in the After treatment, but the difference was not significant (p-value = 0.73 in the proportion test). Our results suggest that about half of the *SAA* subjects are *SAA<sup>r</sup>* due to isolation. The proportion of *SAA* subjects who satisfy integration and Statewise Monotonicity is negligible in our sample. Our results suggest that the introduction of RIS reduces the reported ambiguity aversion by almost half.

At first glance, one may worry that our results are generated by random choice. If subjects randomly choose (with equal probability) between options in every choice problem, then we would mechanically have 50% of ambiguity-averse subjects in the Single control and 25% in the Before and After treatments. But random choice would also imply that all possible choice patterns in Figure 2.1 (that is, KK, KU, UK, and UU) are equally likely. As can be seen in Table 2.4, this is not the case. For each treatment, the  $\chi^2$  test rejects equal proportion of the four choice patterns (p-values < 0.01). For instance, only a few subjects chose U in the Red problem and K in the

Figure 2.3: Proportion of  $SAA^r$  in each treatment



Notes: Each bar represents the number (and proportion) of  $SAA^r$  participants in each treatment. Square brackets indicate proportion test results between corresponding treatments (or groups of treatments).

\*\*\* means significant at 1%, and n.s stands for non-significant.

Figure 2.4: Choice pattern in Before and After treatments

		Chosen bag in			
		Red and Blue problems			
		KK	KU	UK	UU
Treatments	After	22	24	6	35
	Before	25	16	9	37

Blue problem. As predicted, the most chosen pattern is UU, i.e., preferring Bag U in both choice problems.<sup>6</sup> One could wonder whether the UU subjects really hedged or whether they actually became ambiguity seeking. Treatments Before-6 and After-6 provide further evidence.

The  $SAA^r$  proportion is not different between Before-6 and After-6 (p-value = 1 in proportion test). Further, the  $SAA^r$  proportions in the 6-choice-problem treatments are not different than those in the 2-choice-problem treatments (p-value = 0.86 in the proportion test). Therefore, in our experimental setting, presenting more (than two) choice problems does not further decrease the rate of  $SAA^r$ . It did not seem to increase subjects' propensity to isolate or integrate.

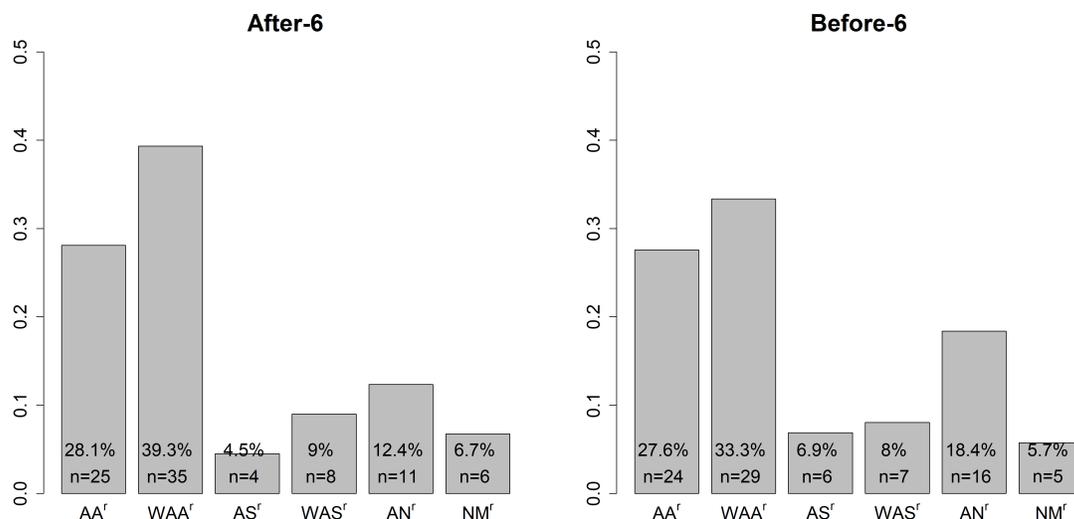
We can also use the additional choices to further classify subjects. For instance, choosing Bag U in Red 3 and in Blue 6 reveals ambiguity seeking. Choosing Bag U in Red 1 and Blue 4 but Bag K in Red 2 and Blue 5 (the latter two choices corresponding to the Ellsberg Paradox, as often implemented) reveals a weak dislike for Bag U or being pretty much indifferent. Such subjects can be called weakly ambiguity averse. In Appendix A, we describe how each choice pattern indicates a type of behavior: *ambiguity aversion* ( $AA^r$ ), *weak ambiguity aversion* ( $WAA^r$ ),<sup>7</sup> *ambiguity seeking* ( $AS^r$ ), *weakly ambiguity seeking* ( $WAS^r$ ), *ambiguity neutral* ( $AN^r$ ), and *non-monotonic* ( $NM^r$ ).

Figure 2.5 presents the ambiguity attitude categorization in the After-6 and Before-6 treatments. The distribution across categories does not differ between the

<sup>6</sup>Fifty-seven out of the 174 subjects (32.75%) in Before and After chose KU or UK. These choice patterns are consistent with various ambiguity attitudes (and with choosing U in the Single control, as noted in footnote 5), possibly suggesting that many subjects believed that one color was more likely to be drawn from the Unknown bag than another color. 70% of them chose KU, which is consistent with a belief that drawing a blue chip from the Unknown bag is more likely than drawing a red chip. Supportive evidence for asymmetric beliefs can also be found by examining the color choices in the Single control: among the 42 subjects who chose U, 32 (76%) had selected the blue choice problem and only 10 (24%) the Red choice problem. Oechssler et al. (2019) also reported a large proportion of participants exhibiting such asymmetric beliefs or color preferences. Alternatively, the position of the choices, which was not randomized, could have created this asymmetry.

<sup>7</sup>Note that  $WAA^r$  is also consistent with ambiguity neutrality. In the After-6 treatment,  $WAA^r$  FOSD  $AA^r$  for integrating subjects. If the subject satisfies Reversal of Order then the same holds for Before-6.

Figure 2.5: Ambiguity attitude categorization in After-6 and Before-6



two treatments (p-value = 0.85 in the Fisher’s exact test). Also, figure 2.5 clearly rejects random choice, knowing that random choice would result in 75% of  $NM^r$  (see Table A.1 in Appendix A). Finally, non- $SAA^r$  subjects were mostly not ambiguity seeking but rather weakly ambiguity averse or ambiguity neutral. This suggests that the RIS does not make people ambiguity seeking, but, as predicted, makes them display less ambiguity aversion.

### 3 Follow-up Study

#### 3.1 Goals and predictions

Thus far, we have investigated the interaction of the random incentive system with the timing of uncertainty, i.e., two stages, as in most traditional ambiguity models. Recently, Saito (2015) and Ke and Zhang (2020) (henceforth SKZ) highlighted the importance of a third stage: when Nature ‘chooses’ a *scenario*. In our setting, this stage corresponds to the timing when Bag U is constructed. SKZ differentiate this stage from the stage of uncertainty resolution, which in our setting corresponds to the time when the color of the chip drawn from Bag U is revealed. Ambiguity aversion in these models reflects the decision-maker’s expectation that Nature will select the least favorable scenario (in our setting, an unfavorable Bag U composition). Following this line of reasoning, the time in which Nature ‘chooses’ is crucial. Randomization may be an effective hedge against ambiguity if Nature has played first, but it provides no hedge if Nature plays after the randomization. This follows because Nature

could select the urn composition that is the least favorable given the randomization outcome.

In the main study, Bag U was always constructed first, before subjects even entered the lab. This is common practice in experimental economics, as it avoids any form of suspicion toward the experimenter. Ambiguity aversion may arise from pessimism about a malevolent Nature, but it should not be confounded by the intervention of the experimenter. Following SKZ’s line of reasoning, participants could hedge against the ambiguity by mentally randomizing (randomly pick a color, as proposed by Raiffa, 1961) in the Single control and/or by using the RIS in the Before and After treatments. In this follow-up study, we further investigate the impact of the time in which Bag U was constructed. Note that these arguments assume that the participants do not satisfy reversal of orders, being sensitive to the timing of the three stages.

First, we replicated the Single control with one change: subjects were asked to seal their choice in an envelope as soon as they made it, so that no one could know what they have chosen. It was opened by the experimenter at the conclusion of the experiment to determine the subject’s payment. This change was not expected to have any impact on the Single control but ensured comparability with the treatments described next. The timeline of the Single control was thus: Bag construction - Choice - Chip draw.

We introduced a new treatment, called *Single-Bag-Later (Single-BL)*, with the following timeline: Choice - Bag construction - Chip draw. For this treatment, subjects sealing choices in an envelope avoided suspicion concerns. It prevented the experimenters from knowing how to decrease the subjects’ payoff when preparing Bag U. (Nature could still ‘know’ how to pick an unfavorable scenario). Comparing Single with Single-BL informed us if the subjects mentally randomized. If some did in Single, and therefore did not exhibit ambiguity aversion, we would expect more ambiguity aversion in Single-BL where mentally randomizing is futile. Recall that this line of reasoning assumes no reversal of orders.

We also introduced two new versions of the Before treatment.

- *Before-Bag (Before-B)*, with the timeline: Randomization - Bag construction - Choice - Chip draw;
- *Before-Bag-Later (Before-BL)*, with the timeline: Randomization - Choice - Bag construction - Chip draw.

The timelines of these two treatments differed from the timeline of Before in the main study by moving the construction of Bag U to after the randomization.<sup>8</sup> The randomization in these two treatments offered no hedge against ambiguity according to the

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<sup>8</sup>In the follow-up study, we did not roll a die for the RIS as we did in the main study, but asked participants to pick one of eight envelopes, four containing to the Blue choice problem and the other four containing the Red choice problem. See Appendix C for the other deviations from the main study due to health-safety measures.

Table 3.1: Summary of Predictions for SAA subjects

	Treatments			
	<i>Single</i>	<i>Single-BL</i>	<i>Before-B</i>	<i>Before-BL</i>
isolation	$SAA^r$	$SAA^r$	$SAA^r$	$SAA^r$
SKZ + mental randomization	not $SAA^r$	$SAA^r$	not $SAA^r$	$SAA^r$
SKZ + no mental randomization	$SAA^r$	$SAA^r$	$SAA^r$	$SAA^r$
Reversal of Order + mental randomization	not $SAA^r$	not $SAA^r$	not $SAA^r$	not $SAA^r$
Reversal of Order + no mental randomization	$SAA^r$	$SAA^r$	not $SAA^r$	not $SAA^r$

SKZ interpretation, which therefore predicted Before-B and Before-BL to have the same proportion of  $SAA^r$  as Single and Single-BL, respectively (mental randomization could still play a role). By contrast, Reversal of Orders would predict lower rates of  $SAA^r$  in Before-B and Before-BL. For completeness, recall that isolation predicts no difference between the four treatments.<sup>9</sup>

## 3.2 Procedure and incentives

The sessions for study 2 were run between July 2020 and October 2020, when the Dutch government relaxed the measures taken against COVID-19. We modified the experimental procedure in the additional treatments to effectively follow the hygiene protocol.

When entering the lab, all subjects had to sanitize their hands. The subjects saw the two bags. They were told that bag K (bag A in the experiment) already contained one red chip and one blue chip, whereas bag U (bag B in the experiment) was empty. The experimenter announced when he/she would put two chips into it, but the subjects did not know the color composition.

- *Single* control: the experimenter filled Bag U with two chips after all subjects entered the lab but before they made their choices.
- *Single-BL* control: the experimenter filled Bag U with two chips after all subjects made their choices.

In these two treatments, the subjects received a choice envelope together with the choice sheet. The experimenter asked each participant to put their choice sheet into

<sup>9</sup>In table 3.1, isolation provides the same prediction as SKZ without mental randomization in the follow-up study. Nevertheless, when comparing Single and Before in the main study, they give different predictions.  $SAA^r$  subjects in Before are compatible with isolation but not with SKZ.

the choice envelope once they made their choice so that no one else could know what they had chosen. At the end of all sessions, one subject was invited to draw a chip from each bag (without looking). The choice envelopes were opened at the payment desk by the experimenter and the subjects could not revise their choices after learning about the color draws. The subjects were paid the outcome of their chosen bet in their chosen choice problem.

In the following two treatments, each subject picked one sealed *ticket envelope* upon entering the lab (after hand sanitation). They read in the instructions that the envelope contained the choice problem that would determine their payment and were told not to open the *ticket envelope* until instructed.

- *Before-B* treatment: The experimenter filled Bag U with two chips after all subjects entered the lab but before they made their choices or opened the ticket envelope. The experimenter first checked that the choice envelope was properly sealed and then asked the subject to open the ticket envelope.
- *Before-BL* treatment: The subjects first sealed their choice sheets in the choice envelope and opened the ticket envelope. Then, the experimenter filled Bag U with two chips.

Similarly, the subjects were told to seal their choice sheet in the choice envelope, which was later opened by the experimenter at the payment desk. One subject was invited to draw a chip from each bag (without looking). Subjects were paid the outcome of their chosen bet in the choice problem that was in their ticket envelopes. Experimental instructions for all treatments are included in Appendix C.

In total, 215 subjects participated in the follow-up study in 42 small experimental sessions with no more than 9 subjects per session to ensure sufficient social distancing. Based on a power calculation using the effect size in the main experiment, our initial plan was to collect 80 observations per treatment. However, due to the second wave of COVID-19 in October 2020 and the more stringent social distancing measures in the Netherlands, we had to stop the data collection prematurely.

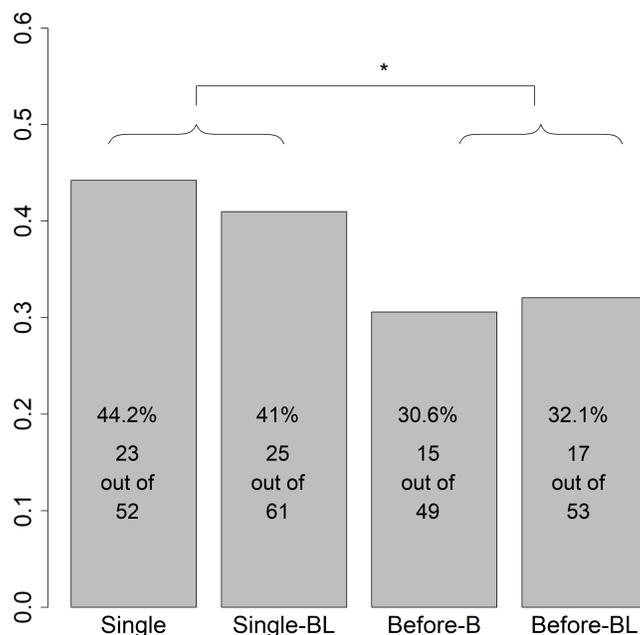
Subjects were randomly assigned to the four treatments. The sessions lasted on average 20 minutes. Subjects were guaranteed a show-up fee of €5 and a variable payment (€0, €10, or €10.20) depending on their choices and the outcome of the bets.

### 3.3 Results

Figure 3.1 presents the proportion of  $SAA^r$  subjects in the follow-up experiment. Compared with the main study, we observe slightly less ambiguity aversion in the Single control. We conjecture that the uncertainty involved in participating in an in-person experiment during a pandemic led to a small underrepresentation of ambiguity-averse subjects in the follow-up study.

Although none of the pairwise comparisons was significant in the proportion tests, the proportion of  $SAA^r$  subjects in Before-B and Before-BL (combined) is 11% lower than in the Single and Single-BL treatments combined (p-value = 0.093 in the proportion test). In other words, roughly 25% (11% decrease divided by 42.5% ambiguity-averse subjects in Single) of the ambiguity-averse participants seemed to satisfy reversal of order. The others are compatible with isolation or with the SKZ interpretation, since in both the Before treatments, the bag was filled *after* the randomization.<sup>10</sup> Further analysis of the Bayesian posteriors is contained in Appendix B.

Figure 3.1: Proportion of  $SAA^r$  in the follow-up study



Notes: Each bar represents the number (and proportion) of  $SAA^r$  participants in each treatment. Square brackets indicate proportion test results between corresponding treatments (or groups of treatments).

\* means significant at 10%.

## 4 Theoretical Analysis

In Sections 2.3 and 3.1, we provided intuitive explanations for our predictions. In this section we elaborate and present formal definitions and theoretical analysis. The first

<sup>10</sup>If we expect approximately 55% of ambiguity-averse participants to isolate (based on the main study, comparing Before to Single), then about 15 to 25% of the ambiguity-averse participants seem to follow SKZ's interpretation.

subsections derive predictions for the main study and subsection 4.5 for the follow-up study.

Consider a nondegenerate set  $\mathcal{M}$  of monetary amounts. Let  $S = \{RR, RB, BR, BB\}$  be the state space, where the first letter of each state indicates the color of the chip drawn from bag K ( $R$  for red,  $B$  for blue) and the second letter the color of the chip drawn from bag U. Denote by  $\hat{x} = [x_1, x_2, x_3, x_4]$  the *act* assigning monetary amounts  $x_1$  to  $RR$ ,  $x_2$  to  $RB$ ,  $x_3$  to  $BR$ , and  $x_4$  to  $BB$ . Let us denote  $\tilde{x} = (x_1, \dots, x_n)$  the *lottery* that yields  $x_1, \dots, x_n \in \mathcal{M}$  with equal chance. Lotteries are used to model the RIS, i.e., that each of the  $n$  choices of the experiment will be paid with equal chance.

We will further consider two types of *compound objects*. The first type is  $[\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4]$ , which assigns lotteries to events (often called an Anscombe-Aumann act). The second type is  $(\hat{x}_1, \dots, \hat{x}_n)$ , which yields  $n$  possible acts with equal chance, with  $n$  finite. The set of all possible acts is denoted  $\mathcal{A}$ , that of lotteries  $\mathcal{L}$  and that of compound objects  $\mathcal{C}$ . With a slight abuse of notation, degenerate compound objects will be referred to by the corresponding act or lottery.

**Definition 1.** An act  $[x_1, x_2, x_3, x_4]$  is *ambiguous* if  $x_1 \neq x_2$  or  $x_3 \neq x_4$ ; otherwise, it is *unambiguous*.

Unambiguous acts are constant in the color of the chip drawn from bag U.

## 4.1 Choices and types of preferences

Let  $\succsim$  be the subject's *weak preferences* defined over  $\mathcal{A}$ , with  $\sim$  and  $\succ$  being the symmetric and asymmetric parts, as usual. These are the preferences that the experimenter would like to learn about. We assume  $[x, x, y, y] \sim [y, y, x, x]$  for all  $x, y$ , which means that the subject has no intrinsic preferences for betting on red or blue in bag K, where both colors have an objective 50% chance of being drawn. In other words, the subject exhibits no color preference. Let  $\succsim^r$  be the reported (revealed) preferences over acts in the experiment.

**Definition 2.** A treatment is *incentive compatible* if  $[x_1, x_2, x_3, x_4] \succsim [y_1, y_2, y_3, y_4]$  implies **not**  $[y_1, y_2, y_3, y_4] \succ^r [x_1, x_2, x_3, x_4]$ .

Incentive compatibility means that subjects do not report preferences in the experiment that contradict their true preferences. It is not an equivalence because not all preferences ( $\succsim$ ) are elicited in the experiment. Indeed, it is the experimenters' discretion how much of  $\succsim$  they want to measure. One of the lessons of the current investigation (which we shall return to in the conclusion) is that there is a tradeoff between incentive compatibility and the domain on which  $\succsim^r$  is revealed.

If subjects view each choice problem in an experiment in *isolation*, then  $\succsim^r = \succsim$  in each choice of the experiment. Hence, if subjects satisfy isolation, all treatments

are incentive compatible. This result is almost tautological but is necessary to understand the results of our experiments. By contrast, in the *integration* view, revealed preferences  $\succsim^r$  concern compound objects  $\mathcal{C}$ . We therefore need to define what such preferences could be. Let  $\succsim^*$  be the subject's preferences over compound objects. We follow Azrieli et al. (2018) in defining admissible extensions of  $\succsim$ .

**Definition 3.**  $\succsim^*$  is an *admissible extension* of  $\succsim$  over  $\mathcal{C}$  if  $\hat{x}_1 \succ \hat{x}_2 \Leftrightarrow \hat{x}_1 \succ^* \hat{x}_2$  for all acts in  $\mathcal{A}$ .

In the remainder of this section, we assume that  $\succsim^*$  is an admissible extension of  $\succsim$ . Whether an experiment is incentive compatible will then depend on the properties of the incentive scheme and properties of  $\succsim^*$ . We assume  $\succsim^*$  to be transitive and complete. For consistency,  $[x, x, y, y] \sim^* (x, y)$  and all lotteries that are objectively equivalent lead to indifference.<sup>11</sup> Preferences  $\succsim^*$  are also assumed to satisfy *first-order stochastic dominance* (FOSD), i.e.,  $[\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4] \succ^* [\tilde{y}_1, \tilde{y}_2, \tilde{y}_3, \tilde{y}_4]$  whenever  $\tilde{x}_i$  strictly first-order stochastically dominates  $\tilde{y}_i$  for all  $i$ . We consider two properties of  $\succsim^*$ .

**Definition 4.**  $\succsim^*$  satisfies *Statewise Monotonicity* if  $(\hat{x}_1, \dots, \hat{x}_n) \succ^* (\hat{y}_1, \dots, \hat{y}_n)$  whenever  $\hat{x}_i \succ^* \hat{y}_i$  for all  $i$ .

**Definition 5.**  $\succsim^*$  satisfies *Reversal of Order* if  $[(x_{11}, \dots, x_{1n}), \dots, (x_{41}, \dots, x_{4n})] \sim^* ([x_{11}, \dots, x_{41}], \dots, [x_{1n}, \dots, x_{4n}])$ .

The Statewise Monotonicity property was proposed by Azrieli et al. (2018). It implies that a compound object is preferred when its second stage acts are all *subjectively* better. Unlike the commonly assumed FOSD property, which says that an act yielding *objectively* better outcomes is always preferred, Statewise Monotonicity implicitly implies that the decision-maker first considers what is subjectively better in the second stage and therefore separates the evaluation of the two stages in a compound object. In this sense, Statewise Monotonicity is close to *compound independence* proposed by Segal (1990) (who studies compound objective lotteries), or recursivity in more general settings.<sup>12</sup>

Next, we analyze the incentive compatibility of our treatments under different properties of  $\succsim^*$ . We are interested in strict ambiguity aversion, which is characterized by preferring bag K for both colors, even at the cost of a small payment. Note that

<sup>11</sup>For instance,  $(x, y) \sim^* (y, x) \sim^* (y, x, x, y)$ .

<sup>12</sup>Note that we define here Statewise Monotonicity for compound objects of the type  $(\hat{x}_1, \dots, \hat{x}_n)$  but it could also be defined for Anscombe-Aumann acts. The difference between Statewise Monotonicity and FOSD for Anscombe-Aumann acts is the same as the difference between Machina and Schmeidler's (1995) Axiom 4 (Substitution) and Axiom 5 (FOSD of Preferences). The former axiom states that the decision-maker always prefers replacing one lottery of an act by a preferred lottery, while the latter requires that the decision-maker always prefers replacing one lottery of an act by an objectively-better lottery.

this pattern of choice is sufficient for the identification of ambiguity aversion but is not necessary. It is possible that the premium some ambiguity-averse subjects are willing to pay is smaller than .20, so they choose bag U. Other subjects may believe that one of the two colors is much more likely than the other (but still be uncertain about the exact likelihood). In what follows, we focus on strict ambiguity aversion. As pointed out by [Epstein and Halevy \(2019\)](#), in experiments, we can at best observe weak preferences. So if subjects prefer a bet on a red chip from the known bag with a prize of 10 to a bet on red chip from the unknown bag, with a prize of 10.20, and assuming they prefer more money to less and have transitive preference, we can conclude that they strictly prefer a bet on the known bag when both prizes are equal.

**Definition 6.** Preferences  $[10, 10, 0, 0] \succsim [10, 0, 10, 0]$  and  $[0, 0, 10, 10] \succsim [0, 10, 0, 10]$ , with at least one strict, exhibit *strict ambiguity aversion (SAA)*.

In the analysis below, we will focus on *SAA* subjects. Let  $SAA^r$  refer to subjects who are “reporting  $[10, 10, 0, 0] \succsim^r [10.20, 0, 10.20, 0]$  and  $[0, 0, 10, 10] \succsim^r [0, 10.20, 0, 10.20]$ .”

## 4.2 Single control

In the Single control, by selecting one color and then one bag for that color, subjects can effectively choose among four acts:  $[10, 10, 0, 0]$ ,  $[10.20, 0, 10.20, 0]$ ,  $[0, 0, 10, 10]$ , and  $[0, 10.20, 0, 10.20]$ . There are no compound objects. Hence, the isolation and the integration views coincide. A subject who reports  $[10, 10, 0, 0] \succsim^r [10.20, 0, 10.20, 0]$  or  $[0, 0, 10, 10] \succsim^r [0, 10.20, 0, 10.20]$  (selecting one of the two colors and choosing bag K) is therefore *SAA*. As discussed above, the proportion of bag K choices in the Single control provides a lower bound and a benchmark for the rate of *SAA* subjects in the population.

## 4.3 After treatment

In the After treatment, combinations of subjects’ choices in the two choice problems generate four compound objects. With the RIS lottery following the resolution of uncertainty, each compound object is an Anscombe-Aumann act.

- Always choosing bag K ( $[10, 10, 0, 0] \succsim^r [10.20, 0, 10.20, 0]$  and  $[0, 0, 10, 10] \succsim^r [0, 10.20, 0, 10.20]$ ) reveals a preference for the following compound object  $[(10, 0), (10, 0), (0, 10), (0, 10)]$ , because, irrespective of the color drawn from bag K, there is a 50% chance that the RIS selects it as the winning color. Since the order of outcomes is immaterial in 50-50 lotteries, the compound object can also be written as  $[(10, 0), (10, 0), (10, 0), (10, 0)]$ .

- Always choosing bag U ( $[10.20, 0, 10.20, 0] \succsim^r [10, 10, 0, 0]$  and  $[0, 10.20, 0, 10.20] \succsim^r [0, 0, 10, 10]$ ) reveals a preference for  $[(10.20, 0), (10.20, 0), (0, 10.20), (0, 10.20)]$ . The reasoning is the same as in the first case: once the color of the chip drawn from bag U is determined, with a 50% chance this color will match the winning color determined by the RIS. But outcomes are higher, and this choice pattern yields  $[(10.20, 0), (10.20, 0), (0, 10.20), (0, 10.20)]$ .
- Choosing bag K for red and bag U for blue reveals a preference for  $[(10, 0), (10, 10.20), (0, 0), (0, 10.20)]$ .
- Choosing bag U for red and bag K for blue reveals a preference for  $[(0, 10.20), (0, 0), (10, 10.20), (10, 0)]$ .

By FOSD,  $[(10.20, 0), (10.20, 0), (0, 10.20), (0, 10.20)] \succ^* [(10, 0), (10, 0), (0, 10), (0, 10)]$ . Hence, subjects who employ an integration view and satisfy FOSD would never exhibit  $[10, 10, 0, 0] \succsim^r [10.20, 0, 10.20, 0]$  and  $[0, 0, 10, 10] \succsim^r [0, 10.20, 0, 10.20]$ , even if they satisfy SAA. In other words, the incentive scheme in the After treatment is not incentive compatible for subjects with these preferences.

**Proposition 7.** *RIS with the lottery stage after the resolution of uncertainty is not incentive compatible if subjects integrate the choice problems in the experiment and satisfy FOSD.*

Although it is theoretically possible for subjects to integrate while violating FOSD, it is rather difficult to envision such a case in practice: for subjects who are able to perceive the whole experiment as a compound object, it is hard to imagine that they would fail to choose the dominant option. The next proposition provides predictions of subjects' preferences on their reported preferences in the After treatment.

**Proposition 8.** *In the After treatment: (i) SAA subjects who satisfy isolation are  $SAA^r$ ; (ii) SAA subjects who satisfy integration are not  $SAA^r$ .*

#### 4.4 Before treatment

In the Before treatment, the RIS lottery stage precedes the resolution of uncertainty. The integration view gives four possible compound objects that can be simply described as  $(\hat{x}_1, \hat{x}_2)$  for some  $\hat{x}_1$  and  $\hat{x}_2$ . For instance, the combination of choosing bag K in both choice problems ( $[10, 10, 0, 0] \succsim^r [10.20, 0, 10.20, 0]$  and  $[0, 0, 10, 10] \succsim^r [0, 10.20, 0, 10.20]$ ) gives  $([10, 10, 0, 0], [0, 0, 10, 10])$ , whereas the combination of always choosing bag U gives  $([10.20, 0, 10.20, 0], [0, 10.20, 0, 10.20])$ . For subjects who employ the integration view while satisfying Statewise Monotonicity,

they will first evaluate the second stage acts according to their true preferences  $\succsim$  over acts, and Statewise Monotonicity ensures that their reported preferences  $\succsim^r$  reveal their true preferences. For instance, *SAA* subjects who satisfy Statewise Monotonicity will prefer (in the sense of  $\succ^*$ )  $([10, 10, 0, 0], [0, 0, 10, 10])$  to all other possible compound objects. Hence, the incentive scheme in the Before treatment is incentive compatible for subjects who satisfy Statewise Monotonicity under the integration view.

**Proposition 9.** *RIS with the lottery stage before the resolution of uncertainty is incentive compatible if subjects satisfy isolation or integration with Statewise Monotonicity.*

Interestingly, if subjects satisfy Reversal of Order, they exhibit  $[(10, 0), (10, 0), (0, 10), (0, 10)] \sim^* ([10, 10, 0, 0], [0, 0, 10, 10])$  and  $[(10.20, 0), (10.20, 0), (0, 10.20), (0, 10.20)] \sim^* ([10.20, 0, 10.20, 0], [0, 10.20, 0, 10.20])$ . Consider an integrating *SAA* subject who satisfies Statewise Monotonicity and Reversal of Order:  $([10, 10, 0, 0], [0, 0, 10, 10]) \succsim^* ([10.20, 0, 10.20, 0], [0, 10.20, 0, 10.20])$  together with Reversal of Order imply  $[(10, 0), (10, 0), (0, 10), (0, 10)] \succsim^* [(10.20, 0), (10.20, 0), (0, 10.20), (0, 10.20)]$ , hence a violation of FOSD. Similarly, an integrating *SAA* subject who satisfies FOSD and Reversal of Order (Statewise Monotonicity) must violate Statewise Monotonicity (Reversal of Order). This leads to the following proposition:

**Proposition 10.** *For integrating SAA subjects: FOSD, Statewise Monotonicity and Reversal of Order cannot hold simultaneously.*

As mentioned before, we assume that all integrating subjects satisfy FOSD, therefore integrating *SAA* subjects violates either Statewise Monotonicity or Reversal of Order. If they satisfy Reversal of Order, then they treat the Before treatment similarly to the After treatment and cannot be *SAA*<sup>r</sup>, as seen in Proposition 6.

**Proposition 11.** *In the Before treatment: (i) SAA subjects who satisfy isolation are SAA<sup>r</sup>; (ii) SAA subjects who satisfy integration with Statewise Monotonicity are SAA<sup>r</sup>; (iii) SAA subjects who satisfy integration with Reversal of Order are not SAA<sup>r</sup>.*

## 4.5 Scenario selection

In the main study and the previous subsections, we considered only two stages (subjective uncertainty and RIS), which allowed us to investigate the interaction between various properties of subjects' preferences and the incentive mechanism employed by the experimenter. As mentioned above, [Saito \(2015\)](#) and [Ke and Zhang \(2020\)](#) also included a stage of scenario selection (when "Nature plays"). A subject who is ambiguity averse and their preferences are represented by *Maxmin expected utility* (MEU)

(Gilboa and Schmeidler, 1989) can be thought of as having a set of priors and evaluating each act by the lowest expected utility attained on this set. In other words, ambiguity aversion is modeled by Nature picking the least advantageous prior. In *Double maxmin expected utility (DMEU)* (Ke and Zhang, 2020), Nature can ‘play’ twice. First, it chooses a set of priors, called a *scenario*, from a collection. The subject makes a decision and then Nature chooses a prior from the scenario. Randomization at the decision time can render Nature’s first move ineffective but not the second. We follow the notation in Ke and Zhang (2020) and derive predictions for a subject represented by the DMEU for both studies.

Let us consider a compound object  $(\hat{x}_1, \dots, \hat{x}_n)$ . DMEU assigns it the value

$$\min_{M \in \mathcal{M}} \sum_{i=1}^n \frac{1}{n} \min_{\mu \in M} \sum_{s \in S} \mu(s) u(x_{is})$$

where  $\mathcal{M}$  is the *scenario set* and  $x_{is}$  is the monetary amount that act  $\hat{x}_i$  assigns to state  $s$ . Each scenario is a set of priors  $M$ . Let  $\mu_t$  refer to the probability measure over  $S$  induced by having  $t$  blue chips in Bag U. If Bag U is prepared first, then a plausible scenario set is  $\mathcal{M}_{ex} = \{\{\mu_0\}, \{\mu_1\}, \{\mu_2\}\}$ . The subject has in mind 3 scenarios, each representing a singleton prior over  $S$ . Ke and Zhang (2020) call this case *ex-ante MEU*. If Bag U is prepared after randomization, then the plausible scenario set becomes the singleton  $\mathcal{M}_{post} = \{\{\mu_0, \mu_1, \mu_2\}\}$ . It corresponds to *ex-post MEU*.<sup>13</sup>

The number of blue chips in Bag U has no impact on the DMEU evaluation of  $([10, 10, 0, 0], [0, 0, 10, 10])$ , which is  $0.5u(10)$  for both ex-ante and ex-post MEU. However, with the scenario sets specified above, ex-ante MEU assigns  $0.5u(10.20)$  to  $([10.20, 0, 10.20, 0], [0, 10.20, 0, 10.20])$ , whereas ex-post MEU assigns it 0. The DMEU model predicts ambiguity aversion only if Nature is perceived to play after any randomization. Subjects may have different scenario sets than assumed in this example, but the reasoning remains the same. In ex-ante MEU, the scenario set is made of singletons, and randomization is effective against ambiguity. By contrast, in ex-post MEU, randomization is ineffective in hedging against ambiguity, and ambiguity aversion may be displayed.

## 5 Related Experimental Literature

Experiments with RIS differ in two ways from single-task experiments: (i) a random device creates compound objects and (ii) more choices are present (e.g., on the answer sheet). In this paper, we mainly considered the impact of (i). For risky decisions, Cox et al. (2014) and Cox et al. (2015) found that (some) subjects do not isolate, and Freeman et al. (2019) document that when one of the alternatives involves certainty,

<sup>13</sup>Saito (2015) proposes a model that is a linear combination of ex-ante and ex-post MEU.

choice lists (with RIS) may underestimate risk aversion. Some studies controlled for (ii) by displaying all choice problems even in single-task treatments; see, for instance, [Starmer and Sugden \(1991\)](#) and [Cubitt et al. \(1998\)](#), who found (some) support for isolation (but [Freeman and Mayraz \(2019\)](#) show it fails in choice list when one alternative is certain). In our experiment, (ii) cannot explain the results because the answer sheet of the Single control also displayed both Red and Blue choice problems (since the subjects had to select which choice problem would determine their payment).

Our results provide the first direct evidence that RIS affects the measurement of ambiguity aversion. The finding could be expected from the theoretical literature but less so from the experimental literature. The experimental literature to date ([Dominiak and Schnedler, 2011](#); [Oechssler et al., 2019](#)) has relied on direct measurement of preference for randomization and its association to ambiguity attitude. That is, subjects were offered an objective tool (e.g., a coin toss) with which they can hedge the ambiguity, and the experimenter observed if they chose to employ it or how much they valued it. Both studies used a within-subject design, but employ different incentive schemes: [Dominiak and Schnedler \(2011\)](#) pay all decisions with trivial stakes (€0.10) – creating perfect insurance for subjects who integrate decisions, while [Oechssler et al. \(2019\)](#) pay one decision randomly – which may interact (as they acknowledge) with their basic research question and confound any result.<sup>14</sup> Both studies reported no relation between ambiguity aversion and preference for randomization. [Dominiak and Schnedler \(2011\)](#) found that only 6 out of 35 ambiguity-averse subjects were also randomization loving. [Oechssler et al. \(2019\)](#) asked subjects to make two choices that together could potentially hedge ambiguity but could not reveal strict preference for randomization. Approximately 50% of subjects made choices to hedge, a proportion that was independent of their ambiguity attitude. Crucially, since [Oechssler et al. \(2019\)](#) could not identify strict preference for randomization, this proportion is indistinguishable from indifference and choosing randomly. Our between-subject design, though considerably more expensive, bypasses these severe incentive problems. By focusing on a single measure – strict ambiguity aversion, we can investigate how it is affected by the potential for hedging. Our main study provides strong evidence that ambiguity-averse subjects realized the hedging opportunity, by comparing Before and After to Single.

[Oechssler et al. \(2019\)](#) also varied the order of the objective hedge and the determination of the ambiguous state of the world. They found no difference between these treatments. In their “alternative specification”, the authors also included a treatment in which, as in our Before treatment, the objective hedge preceded the decision. Again, they found no difference. Consistent with [Oechssler et al. \(2019\)](#), our main study supports reversal of order and finds no difference between Before and Af-

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<sup>14</sup>Their second experiment (called “alternative specification”) did not have additional tasks so did not require the usage of RIS, but did not provide a benchmark for ambiguity aversion, making it impossible to identify non-hedging ambiguity-averse subjects from ambiguity neutral ones.

ter. Moreover, our follow-up investigation further contributes to the field by varying when the bags were constructed, an aspect that was not considered previously. We find that the answer is more nuanced, and some subjects are sensitive to this aspect, as proposed by [Saito \(2015\)](#) and [Ke and Zhang \(2020\)](#).

## 6 Discussion

Our experiments test the impact of random incentives on observability of ambiguity aversion. Further, our design allows us to draw conclusions about strictly ambiguity-averse subjects, using the predictions displayed in [Table 2.1](#). From the Single control, we expect that about 50% of the subjects are  $SAA$ . Observing about 25% of  $SAA^r$  in the After and After-6 treatments in the main study suggests that half of the  $SAA$  subjects isolate, while the other half choose as if they integrate their decisions. Furthermore, the absence of difference between the After and Before treatments is compatible with Reversal of Order rather than Statewise Monotonicity.

The fact that half of the subjects integrate, or at least are influenced by the presence of random incentives, is worrisome, as ambiguity experiments may underestimate the prevalence of ambiguity aversion. It can be argued that our design made the possibility of integration especially salient and that subjects of more complex experiments are less likely to integrate their choices. Adding a few more choices (in the After-6 and Before-6 treatments) did not seem to influence the rate of integrating subjects, but even these treatments, with more choices, remained relatively simple relative to some experiments from the literature. Assessing the impact of a more complex design is tricky if one wants to keep the same benchmark as we had (one single choice). More complex experiments may also introduce other confounds, such as order effects and fatigue.

However, in the main study, we could not exclude the possibility that some subjects mentally randomized, and therefore our measure of  $SAA^r$  in the Single control was a lower bound for the level of strict ambiguity aversion in the population. The effects of the Before and After treatments are, therefore, also lower bounds of the effects of using RIS. In the follow-up study, however, we did not find that constructing the bag later increased the frequency of  $SAA^r$ ; hence, we found no experimental evidence that subjects mentally randomized in the SKZ framework. It can be that our manipulation was somewhat artificial. For many sources of ambiguity in real life, the timing of Nature’s play is well-defined and cannot be manipulated. Coupling this observation with the fact that the effect of RIS was smaller in the follow-up study (we conjecture it was mainly due to sample selection during the pandemic but other sources are possible too), we believe that the SKZ interpretation when the source of ambiguity, and especially its timing, naturally arises and deserves further investigation.

Our aim was to test the theoretical claims that random incentives can provide a hedge against ambiguity. We used rather extreme conditions to give these predictions a chance and to identify how large the problem of using random incentives in ambiguity experiments might be. When implementing the various treatments, we made the order between the resolution of uncertainty and of random incentives as salient as possible. We also displayed all choices on a single page, making hedging possibilities more salient. We tested whether the Before treatment and/or increasing the number of choices could reduce the prevalence of hedging. We leave for further research whether other manipulations (for instance, not displaying all choices on the same page, using filler tasks) could reduce hedging when random incentives are used.

We were genuinely surprised by the absence of a difference between the After and Before treatments. In a previously-circulating theoretical paper ([Baillon et al., forthcoming](#)), we conjectured that more subjects would exhibit ambiguity aversion in the Before treatment, especially if the difference between the two treatments was very salient. In other words, we expected (integrating) subjects to exhibit State-wise Monotonicity rather than Reversal of Order. Our instructions were written to enhance the perception of the order between risk and ambiguity. We also followed recommendations of [Johnson et al. \(2021\)](#) to use envelopes in the Before and Before-6 treatments. Making the prior selection of one-choice problem tangible (as the choice problem is already in the envelope) can help subjects condition on choice problems, i.e., help them isolate or exhibit Statewise Monotonicity. Our implementation of the Before treatments did not seem to have any effect on this.

The predictions of both studies were derived from models assuming deterministic, well-defined preferences toward choice objects. The interpretation of our results in terms of hedging relies on similar assumptions. We cannot reject that subjects were not actually hedging but, for instance, were using simple heuristics.

To conclude, one should recall that the main motivation to employ RIS in ambiguity experiments is to avoid relying on symmetric beliefs, i.e., not assuming that subjects consider both colors equally likely. Alternatively, letting subjects choose a color to bet on allows experimenters to unequivocally conclude that a subject preferring the unknown bag is ambiguity averse. However, experimenters must be willing to assume belief symmetry if they want to infer ambiguity seeking from the choice data. A general lesson from our empirical investigation is that if experimenters want to observe attitudes toward ambiguity that are more general than ambiguity aversion, they must tradeoff identification assumptions that have only partial empirical support: either assuming that subjects isolate or that they hold symmetric beliefs. In this respect, there are behavioral constraints to observability, much like has been acknowledged long ago in other experimental sciences, such as Physics and Psychology.

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