From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share*

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Abstract

In the US, large firms now account for a greater share of economic activity, new firms are being created at slower rates, and workers are receiving a smaller share of GDP. Changes in population growth provide a unified quantitative explanation. A decrease in population growth lowers firm entry rates, shifting the firm-age distribution towards older firms. Firm aging accounts for i) the concentration of employment in large firms, ii) and trends in average firm size and exit rates, key determinants of firm entry rates. Feedback effects from firm demographics generate two-thirds of the effect. Prior to the decrease, entry rates rose steadily reflecting the earlier baby boom. The glut of firms due to the baby boom lead to rich transitional dynamics within the feedback effects, accounting for more than half the total change. Baby boom induced changes in the firm-age distribution provide a driving force for the post-WWII rise and fall in the aggregate labor share. Ignoring changes in population growth attributes all the long run decline in entry rates to a decrease in firm exit rates, which in reality have been only one third as large.

J.E.L. Codes: J11, E13, E20, L16, L26
Keywords: Firm Dynamics, Demographics, Entrepreneurship, Concentration, Labor Share

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1 Introduction

Three long-term changes in the US economy have attracted a great deal of attention. First, economic activity is increasingly being concentrated in larger firms. Second, the rate of firm creation has steadily declined. Third, the share of GDP going to labor, once thought to be stable, has declined. What are the economic forces driving these long-term changes? This paper looks at the role of population growth and firm demographics.

Our analysis begins by highlighting the importance of changing firm demographics in driving these aggregate trends. The mass of US firms has been aging. Had it not been for firm aging, employment concentration would have, if anything, declined. Concentration has increased because an aging firm distribution shifts weights towards older firms, which have higher levels of concentration. We document that changing firm demographics can also account for two related changes: the increase in average firm size and the decline in the aggregate firm exit rate. The patterns exhibited by these variables are similar to concentration. In the absence of firm aging, the change in these variables, if anything, would have gone in the opposite direction.

Because firm exit rates by age have changed little, the probability that a new firm survives to a particular age has also changed little. Therefore, the aging of firms is a result of the dramatic fall in the rate at which new firms are created. This decline in the rate of firm entry can be analyzed through the lens of a simple accounting identity,\footnote{This identity comes from the definition of average firm size, $e \equiv N/M$, where $N$ is the number of workers and $M$ is the number of firms. It follows that the growth rate in the number of firms equals the growth rate in the number of workers minus the growth rate of average firm size, $\dot{M} = \dot{N} - \dot{e}$. The growth in the number of firms also depends on firm entry rate $\lambda$ and firm exit rate $\xi$, $\dot{M} = \lambda - \xi$. Combining these two equations leads to identity (1). We measure $\dot{N}$ using labor force growth. Other measures of $\dot{N}$ are discussed in Appendix B.}

$$\text{entry rate} = \text{labor force growth} - \text{growth in average size} + \text{exit rate}. \quad (1)$$

Standard models of firm dynamics have the property that for a constant growth rate of the labor force, the change in the other two components of this identity converge to zero. Therefore, changes in labor force growth are a natural candidate to explain changes in the firm entry rate. Holding the exit rate and average firm
size constant, can a change in labor force growth explain the observed drop in firm entry rates? No. Figure 1 shows US civilian labor force growth rates by decade. The rise and fall of labor force growth mostly reflects the earlier baby boom. Since the 1970s, labor force growth has declined by 2 percentage points (pp), which is only one-third of the 6pp decline in the entry rate. The remaining two-thirds is attributed to changes in the exit rate and changes in the growth rate of average firm size.

We show that changes in labor force growth feed back to changes in both the aggregate exit rate and average firm size. Consider an increase in labor force growth. The increase in labor supply must be met by a corresponding increase in labor demand. Incumbent firms are limited by scale, so they cannot absorb the entire increase in labor supply. The residual labor demand must therefore be absorbed by entry of new firms. The increase in firm entry shifts the firm-age distribution towards younger firms, which have higher exit rates and lower size.

To be consistent with the data, the changes in labor force growth should change aggregate variables while maintaining relative stability of exit rates by age. While this property holds along a steady state, it is not clear that it carries over to transitions. This distinction is of interest because the growth in average firm size in the data is non-zero, indicating that the US economy is going through
a transition. The theoretical challenge is to determine whether the transition path is consistent with these empirical patterns. We derive sufficient conditions for the existence of such a path in a general equilibrium framework that incorporates standard models of perfect competition and models of imperfect competition featuring both constant and variable markups.

The transitional dynamics of firm entry depend on the entire history of past entry. Firm entry fills the gap between labor supply and incumbent labor demand. Therefore, entry depends on total labor demand by incumbents in each age group, which in turn is determined by past entry, and age profiles of exit and average size. We characterize this history dependence with a dynamic entry equation, which relates current entry to the distributed lag of past entries. The dynamic nature of entry implies that changes in current entry affect future entry, through the firm-age distribution.

Can changes in labor force growth, combined with feedback from firm demographics, quantitatively generate the secular changes experienced by the US economy? What is the role of the feedback effect? What is the role of transitional dynamics, and therefore the importance of the baby boom? How do we expect entry rates and firm demographics to evolve from here on? We can answer these questions using the dynamic entry equation. To obtain labor demand by incumbents in each age group, we estimate a stochastic process for firm employment that is consistent with key facts about firm size, firm growth and exit. We then feed the labor force series into the dynamic entry equation and iterate forward. We make use of historical labor force data which goes back to 1940. Doing so allows us to obtain the 1978 age distribution that is consistent with the baby boom.

Going back to the entry rate equation above, changes in labor force growth have a direct effect on entry, but also an indirect effect as they can impact average firm size and exit rates. This occurs mainly through a change in the age distribution of firms, as older firms are larger and have lower exit rates. We find that, taking into account the changes in the age distribution of firms, the decline in labor force growth explains the majority of the observed decline in firm entry rates from 1978 to 2014.

The entry rate decline can be further decomposed into long-run and transitional effects. The long-run effect corresponds to the difference in entry rates across steady states with different population growth rates. It follows from iden-
tity (1) that, in addition to the direct effect of population growth, the long-run effect depends on how changes in population growth affect the exit rate. We find that the long-run elasticity of entry rates to population growth is in the order of 1.5 for the US economy. This implies that the 2pp drop in labor force growth generates roughly a 3pp drop in entry rates. The long-run effect corresponds to 45 percent of the 6pp decline in the entry rate.

The long-run effect implicitly assumes a history of constant labor force growth prior to its decrease, so the resulting firm-age distributions do not capture the glut of firms born due to the baby boom. The baby boom shows up in the transitional effects in two ways, captured by the exit rate and growth in average size terms in identity (1). The transitional firm-age distribution is younger than the long-run distribution in 1978 immediately after the rise in labor force growth, and older than the long-run distribution in 2014 once the glut of firms have aged. These changes together lead to a 1pp effect from exit along the transition. Additionally, entry rates are not only high but rising in 1978, fueled by the incorporation of baby-boomers into the labor force. As a consequence, average firm size is falling during that period, which feeds back to even higher entry rates. The opposite trend occurs in 2014 as the glut of the firms created during the entry boom are aging, implying positive average size growth. These transitional changes in average firm size contribute about 2pp to the change in entry rates, which together with the transitional effect on exit rates explain 55 percent of the total decline in entry rates.

We next turn to the labor share. A recent set of papers document two facts about labor shares: (i) firm-level labor shares are negatively related to firm size and (ii) almost all of the decline in the aggregate labor share is due to reallocation from high to low labor share units, rather than a decline within labor share units. A decline in labor force growth lowers the aggregate labor share in a way that is consistent with these facts. By shifting the the firm-age distribution towards older firms, declining labor force growth reallocates the share of value added towards larger firms which have lower labor shares. Furthermore, the time-series pattern of the aggregate labor share implied by labor force growth is consistent with the

\[ \text{Hartman-Glaser, Lustig and Xiaolan (2019)} \] documents this pattern by showing that the capital share has been increasing for the largest public firms in the US. \text{Autor, Dorn, Katz, Patterson and Van Reenen (2020)} document the same pattern using US Census Data. \text{Kehrig and Vincent (2021)} document the reallocation for manufacturing establishments.
Declining population growth also has implications for job reallocation. As noted by Decker, Haltiwanger, Jarmin and Miranda (2014), older firms create, destroy, and reallocate jobs at lower rates. Therefore, firm aging also acts as a force that contributes to declining aggregate job reallocation rates. We find that, for the 1978 to 2014 period, firm aging induced by population growth accounts for 48 percent of the observed decline in job creation, 38 percent of the decline in job destruction, and 32 percent of the decline in job reallocation.

Our paper finds that changes in the age distribution of firms resulting from labor force dynamics have played a major role in accounting for the observed trends in exit rates, average firm size and concentration. The literature has considered other alternative hypotheses to explain the observed trends, such as an increase in economies of scale, higher entry costs and lower rates of diffusion. We evaluate, in the context of our model, some of the implications of these alternative forces. Our findings suggest that while these alternative explanations can contribute to explain some of the documented facts, they are not consistent with others. As an example, an increase in entry cost contributes to explain a decline in entry rates but, contrary to our documented evidence, leads to an increase in average firm size and decline in exit rates, conditional on age. As a general implication, and going back to our accounting identity, any alternative explanation that abstracts from the role of changes in population growth must end up attributing all the long run decline in entry rates to a decrease in firm exit rates of the same magnitude, which in reality has only been one third as large.


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3 Reedy and Strom (2012) document a related long-term decline in job creation by new businesses.
across geographic areas and industries. Decker, Haltiwanger, Jarmin and Miranda (2014), Hathaway and Litan (2014b) and Pugsley and Şahin (2018) document the aging of the firm distribution and link it to declining firm entry.

A different strand of the literature has documented trends in the aggregate labor share and the rise in concentration. Karabarbounis and Neiman (2014) find that the decline in the labor share is primarily a within-industry rather than a cross-industry phenomenon. Grullon, Larkin and Michaely (2019) document increased concentration across most U.S. industries, whereas Barkai (2020) and Autor et al. (2020) both document a positive correlation between industry concentration and the decline in the labor share. Our paper shows that population growth and firm demographics play an important role in linking these empirical findings. We show that the changes observed in the age distribution of firms played a major role in explaining the rise in concentration. Given the widely documented negative relationship between firm size and labor share (Hartman-Glaser et al., 2019; Autor et al., 2020; Kehrig and Vincent, 2021), firm aging induced by population growth also lowers the aggregate labor share by reallocating the share of value added towards larger firms which have lower labor shares. A related observation that has gained considerable attention is that of the rise in markups (De Loecker, Eeckhout and Unger, 2020). Our framework is consistent with rising markups provided older firms have higher markups.

To the best of our knowledge, ours is the first paper that jointly explains the transitional dynamics of entrepreneurship, concentration, and the labor share. We are not the first paper to propose the decline in labor force growth as an explanation for the decline in firm entry rates. Karahan, Pugsley and Şahin (2018) provide empirical evidence of a causal relationship between changes in population growth and the entry rate across US states. The authors then explore the role

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4Our framework shows that it is possible to generate an increase in concentration without decreasing competition. Rossi-Hansberg, Sarte and Trachter (2020) also show that increasing concentration at the aggregate level need not be due to declining competition. They present evidence that the positive trend observed in national product-market concentration becomes a negative trend when focusing on measures of local concentration.

5Contemporaneous work by Peters and Walsh (2019) finds this to be the case in US data. They measure markups using the Longitudinal Business Database and derive steady state implications of declining population growth in a model of creative destruction.

6Hathaway and Litan (2014c) document a correlation between declining firm entry rates and population growth across geographic regions. Other explanations for the decline in entrepreneurship include the decline in corporate taxes (Neira and Singhania, 2018), the decline in interest
of labor force growth as a driver for the fall in the entry rate in the steady state of a Hopenhayn (1992a)-style model. Our paper builds on this idea. We identify the key equations that underlie the process by which population growth affects the entry rate through the age structure of firms. We provide sufficient conditions under which these equations apply to a large class models, including models of perfect and imperfect competition, and also hold along transitions. Quantitatively, transitional dynamics played a major role in explaining the magnitude of the decrease in entry rates. \(^7\)

In addition, our parsimonious framework allows us to track the impact of the changes in labor force growth on the age distribution of firms, which, as we extensively document in the paper, played a critical role in explaining the fall in exit rates, rise in average firm size and concentration, and the fall in the aggregate labor share. Therefore, we provide a unified quantitative explanation for a set of trends that have been treated as separate in the literature. In particular, the trends in concentration and the aggregate labor share (also the related trend in markups) were treated as separate from business dynamism trends (entry, exit, average size and job reallocation). Our paper shows not only that these trends are related, but that the driving force of population growth ties them together in a way that is consistent with micro evidence. \(^8\)

The rest of the paper is organized as follows. Section 2 presents the data. Section 3 presents the theoretical framework. Section 4 presents the quantitative rates, (Liu, Mian and Sufi, forthcoming; Chatterjee and Eyigungor, forthcoming), and skill-biased technical change (Salgado, 2019; Jiang and Sohail, 2017). \(^7\)

The revision of both of our papers has proceeded in parallel. A more recent version of their paper, Karahan, Pugsley and Şahin (2021), includes some discussion of a transition. We provide a richer quantitative analysis of transitional dynamics, along with analytical expressions that allow us to (i) separate long-run effects from transitional effects, and (ii) decompose transitional entry rates into their various components. \(^8\)

Complementary explanations include an increase in the span of control (Aghion, Bergeaud, Boppart, Klenow and Li, 2019), and a decline in knowledge diffusion between frontier and laggard firms (Akçigit and Ates, 2021). Explanations specific to the labor share decline include a slowdown in productivity (Grossman, Helpman, Oberfield and Sampson, 2021), an increase in firm-level volatility (Hartman-Glaser, Lustig and Xiaolan, 2019), the treatment of intangible capital (Koh, Sántaëlálía-Llopis and Zheng, 2020), the decline in the relative price of capital (Karabarbounis and Neiman, 2014), capital accumulation (Piketty and Zucman, 2014), import competition and globalization (Elsby, Hobijn and Sahin, 2013), and corporate taxes (Kaymak and Schott, 2018). One related, but distinct, explanation is that of the aging of the workforce (Liang, Wang and Lazear, 2018; Kopecky, 2017; Engbom, 2017). More broadly, population aging has been linked to slower growth in advanced economies; see Cooley and Henriksen (2018).
findings. Section 5 discusses further implications of our framework, including the labor share, job reallocation, and alternative explanations. Section 6 concludes.

2 Motivating Facts

This section documents the importance of changes in firm demographics in explaining the trends in the US economy. The share of firms over 10 years of age has experienced a steady increase from around 32 percent in 1986 to 48 percent in 2014. We first discuss the importance of this channel in regard to aggregate trends and next extend the analysis to trends within sectors. Our main data source is the Business Dynamics Statistics (BDS) produced by the US Census Bureau. The BDS has near universal coverage of private sector firms with paid employees. The BDS data covers years 1977 to 2014. We follow the common practice of dropping the first year of BDS data due to suspected measurement error (e.g. Moscarini and Postel-Vinay, 2012). Thus our sample covers the 1978 to 2014 period.

![Figure 2](image)

**Figure 2**

Notes. Average firm size is number of workers per firm. Concentration is the share of employment in firms with 250+ employees.

The role of firm aging in aggregate trends. Figure 2 shows the time series evolution of the aggregate exit rate, average firm size and concentration in US data. The aggregate exit rate in the US has declined from about 9.5 percent to 7.5 percent. Average firm size has increased from about 20 employees to 24 employees.
Aggregate concentration, measured as the share of employment in firms with 250 or more employees, has increased from 51.6 percent to 57.3 percent.\textsuperscript{9} Table 1 reports the linear time trend for each aggregate variable. The aggregate trend is statistically significant for all three variables. The annual trends can be multiplied by 36 to calculate the implied change in each variable since 1978. In percentage terms, the exit rate has declined by 15 percent, average size has increased by 17 percent, and concentration has increased by 11 percent.

Table 1: Time trends in average firm size, exit rates and concentration.

<table>
<thead>
<tr>
<th></th>
<th>Aggregate</th>
<th>Controlling for Sector</th>
<th>Controlling for Age</th>
<th>Controlling for Age &amp; Sector</th>
<th>Controlling for Age, Sector &amp; Interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EXIT RATE:</strong></td>
<td>−0.043***</td>
<td>−0.036***</td>
<td>0.011</td>
<td>0.006*</td>
<td>0.006*</td>
</tr>
<tr>
<td>(0.008)</td>
<td>(0.004)</td>
<td>(0.007)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td><strong>AVERAGE FIRM SIZE:</strong></td>
<td>0.423</td>
<td>0.771</td>
<td>0.972</td>
<td>0.939</td>
<td>0.957</td>
</tr>
<tr>
<td>(0.0007)</td>
<td>(0.015)</td>
<td>(0.021)</td>
<td>(0.040)</td>
<td>(0.011)</td>
<td></td>
</tr>
<tr>
<td><strong>CONCENTRATION:</strong></td>
<td>0.159***</td>
<td>0.172***</td>
<td>−0.069***</td>
<td>−0.149***</td>
<td>−0.149***</td>
</tr>
<tr>
<td>(0.013)</td>
<td>(0.017)</td>
<td>(0.023)</td>
<td>(0.018)</td>
<td>(0.012)</td>
<td></td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.838</td>
<td>0.95</td>
<td>0.99</td>
<td>0.783</td>
<td>0.985</td>
</tr>
<tr>
<td>Observations</td>
<td>37</td>
<td>333</td>
<td>307</td>
<td>2430</td>
<td>2430</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.817</td>
<td>0.948</td>
<td>0.985</td>
<td>0.937</td>
<td>0.973</td>
</tr>
<tr>
<td>Observations</td>
<td>37</td>
<td>315</td>
<td>307</td>
<td>2193</td>
<td>2193</td>
</tr>
</tbody>
</table>

*** p < 0.01; ** p < 0.05; * p < 0.1

Notes. The table reports $\beta_1$ from the regression equation $y_t = \beta_0 + \beta_1 y e a r + X' \beta + e_t$, where the dependent variable is exit rate, average size or concentration. The unit of observation is a year-sector cell in specification (2), a year-age cell in specification (3), and a year-age-sector cell in specifications (4) and (5). Specification (2) is weighted by 1978 sectoral activity, which corresponds to firm weights for average size and exit rate regressions and to employment weights for the concentration regression. Specification (3) is weighted by 2014 age activity, which correspond to firm weights for all regressions. Specifications (4) and (5) are weighted by the product of sectoral and age activity weights. We use age weights from 2014 because the sample in that year contains data for all age groups. The age weights are also multiplied by the inverse of the number of observations in the age group to correct for the bias that results from some age groups having more observations than others.

The US economy has undergone massive reallocation of economic activity at the sectoral level, primarily reflecting the reallocation from manufacturing to service sectors due to structural change. It could be the case that changing sectoral composition might be responsible for the observed aggregate trends. The sec-

\textsuperscript{9} The increase in concentration is robust to the firm size cutoff. For size cutoffs of 5, 10, 20, 50, 100, 250, 500, 1000, 2500, 5000, and 10,000 employees, the share of employment increased by 1.6, 3.1, 4.3, 5.4, 6.0, 5.7, 5.1, 4.6, 3.9, 3.1, and 2.4 percentage points, respectively.
ond column of Table 1 shows that the aggregate trends are not being driven by changes in sectoral composition over the sample period. The inclusion of sector controls in fact deepens the puzzle: it increases the magnitude of the trend for average size and concentration.\textsuperscript{10} This result reflects the fact that the US service sectors are less concentrated and have smaller firms than the manufacturing sectors. A strong negative trend persists for the exit rate after the inclusion of sector controls.

The picture looks very different once we look at the data conditional on firm age. The third column of Table 1 shows that the inclusion of firm-age controls flips the sign of the trend for all three variables. Exit rate now has a slight positive trend, while average size and concentration have a negative trend. This finding is robust to the inclusion of further controls for sector (column 4) and interaction between age and sector (column 5).\textsuperscript{11}

Figure 3 shows data on exit rates, average size and concentration conditional on firm age. The data exhibit two key features. First, firm-age bins do not reflect

\textsuperscript{10}This is similar to the finding in Decker et al. (2014), who use detailed industry-level data and find that the decline in job reallocation would have been larger if industrial composition had remained unchanged.

\textsuperscript{11}The results for concentration are robust to size threshold. Table C-2 of Appendix C reports results for size thresholds of 500, 1000, 2500, 5000 and 10,000 employees.
the aggregate trend for all three variables. This is the reason why the trends in Table 1 flip sign with the inclusion of age controls. The negative trend conditional on age for average size and concentration is mostly coming from a trend amongst older age groups. Young age groups exhibit either a mild trend or no trend.\textsuperscript{12} The second feature is that there are clear age profiles for each variable. Firm exit rates decrease with age. Average size and concentration increase with age. Taken together with results in Table 1, these features of the data imply that a shift in the firm-age distribution towards older firms — firm aging — is a force that plays a major role in driving aggregate trends.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{firm_entry_rate.png}
\caption{Firm Entry Rate}
\label{fig:firm_entry_rate}
\end{figure}

\textit{Notes.} The entry rate from 1963 to 1977 is linearly interpolated.

Given that exit rates by firm age have changed little, firm aging must be the result of a decline in firm entry. Figure 4 shows that the firm entry rate in the US has declined since 1978. We extend the entry rate series further back in time. The entry rate from 1940 to 1962 comes from the now discontinued Survey of Current Business. The entry rate from 1963 to 1977 is linearly interpolated.\textsuperscript{13} After a transitory spike in entry rates during World War II, entry rates steadily increased up to the 1970s and then exhibited a steady decline.

\textbf{Firm aging within sectors.} While the aggregate trends are not necessarily reflected within every sector, this section documents that the force of firm aging

\textsuperscript{12}Table C-1 in the Appendix reports the linear time trend within each age bin.

\textsuperscript{13}The apparent increase in that period is consistent with the increase in the entry rate for establishments documented by Karahan, Pugsley and Şahin (2018).
plays a role in each of the seven major sectors reported in BDS data. Panel A of Table 2 reports that the aggregate patterns of declining entry and the subsequent increase in firm aging are present at the sectoral level. As in aggregate data, the entry rate in each sector has a statistically significant negative trend. The aggregate entry rate declined on average by 0.16pp per year, while the sectoral declines range from 0.11pp to 0.38pp. The share of firms over 10 years of age in each sector has a statistically significant positive trend. The share of firms over 10 years of age increased on average by 0.57pp per year in the aggregate, while the sectoral increases range from 0.41pp to 1.46pp.

Table 2: Entry and Firm Aging by Sector

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</thead>
<tbody>
<tr>
<td>Entry Rate</td>
<td>−0.155</td>
<td>−0.378</td>
<td>−0.110</td>
<td>−0.200</td>
<td>−0.139</td>
<td>−0.134</td>
<td>−0.157</td>
<td>−0.170</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.030)</td>
<td>(0.018)</td>
<td>(0.008)</td>
<td>(0.015)</td>
<td>(0.013)</td>
<td>(0.017)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Share of 11+ Firms</td>
<td>0.571</td>
<td>1.463</td>
<td>0.377</td>
<td>0.919</td>
<td>0.414</td>
<td>0.473</td>
<td>0.579</td>
<td>0.807</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.096)</td>
<td>(0.044)</td>
<td>(0.018)</td>
<td>(0.022)</td>
<td>(0.035)</td>
<td>(0.033)</td>
<td>(0.014)</td>
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</tr>
</thead>
<tbody>
<tr>
<td>Exit Rate</td>
<td>−18%</td>
<td>−36%</td>
<td>−17%</td>
<td>−16%</td>
<td>−7%</td>
<td>−20%</td>
<td>−9%</td>
<td>−21%</td>
</tr>
<tr>
<td>Avg. Firm Size</td>
<td>43%</td>
<td>79%</td>
<td>36%</td>
<td>18%</td>
<td>100%</td>
<td>68%</td>
<td>38%</td>
<td>56%</td>
</tr>
<tr>
<td>Concentration</td>
<td>16%</td>
<td>29%</td>
<td>9%</td>
<td>14%</td>
<td>56%</td>
<td>46%</td>
<td>25%</td>
<td>31%</td>
</tr>
</tbody>
</table>

Notes. All estimates in Panel A are statistically significant with $p < 0.01$. The estimates for the entry rate regressions indicate annual trends for years 1978-2014. The share of 11+ firms refers to firms over 10 years of age. The regressions with firms aged 11+ indicate the annual trend for years 1987-2014. Panel B shows the percentage change in the corresponding variable, relative to its 1978 level, if aging is the only force.

Panel B of Table 2 summarizes how aging at the sector level affects exit rates, average size and concentration. We measure the contribution of aging as the percentage change in a variable, relative to 1978, if aging was the only force. It is calculated as

$$\text{Change if Aging Only} = \frac{(\text{Uncond. Trend} - \text{Trend Cond. on Age}) \times 36}{\text{Level}_{1978}}. \quad (2)$$

14 For brevity, we do not report the Agriculture and Mining sectors. These sectors each represent less than 1% of the aggregate in terms of firms and employment. The aggregate pattern of the decline in the entry rate and increase in aging also holds in both these sectors.
This statistic indicates the total change in a variable net of the changes within age groups. As an example, consider the contribution of aging to the change in average firm size in the aggregate economy. As reported in Table 1 earlier, the unconditional trend in average size is positive (column 1), while the trend conditional on age is negative (column 3). In the absence of firm aging, average size in the economy would have declined. However, because average size actually increased, firm aging more than undoes the decline within age bins. As a result, the Change if Aging Only statistic, 43 percent, is greater than the total change in average size in the data, 17 percent.

The aggregate patterns of changes in exit rate, average size and concentration driven by firm aging can be seen in all the major sectors. While there is heterogeneity in magnitudes across sectors, firm aging is a force that is consistently present and quantitatively important at the sectoral level. If aging was the only force the exit rate in the aggregate would have declined by 18 percent, while the sectoral declines range from 7 percent to 36 percent. Aging alone would have generated an increase in average size of 43 percent and in concentration of 16 percent in the aggregate. The corresponding sectoral increases range from 18 percent to 100 percent for average size, and 9 percent to 56 percent for concentration.

Having established that firm aging has been a major force in recent decades, we next present a framework that helps organize and interpret the empirical evidence.

3 Theoretical Framework

This section presents a tractable theoretical framework that isolates the effects of population growth on firm aging. The framework captures in a transparent way the feedback effects of population growth and permits comparisons of the effects along steady states and transitions. We first provide the basic structure that is common to a class of models, and then give some specific examples. For clarity of exposition, we keep the basic structure as simple as possible. The class of models presented here share the property that equilibrium allocations, when conditioned on firm age, are invariant to nonstationary changes in population.

There is a fixed endowment of a labor $N_t$, which is inelastically supplied and also the numeraire. Firms are confronted with an aggregate state $z$ (e.g. price
index) and an idiosyncratic state $s$. The aggregate state $z$ is determined as part of the equilibrium. The idiosyncratic state $s$ follows a Markov process with conditional distribution $F(s_{t+1}|s_t)$, which we assume is continuous and nondecreasing. Let $\pi(s, z)$ denote the profits of a firm with idiosyncratic state $s$ when the aggregate state is $z$, with $n(s, z)$ denoting employment. We assume both functions are increasing in $s$ and $z$. Firms have a common discount factor $\beta$. Note that, for fixed $z$, the Markov process for a firm’s state $s_{it}$ and the function $n(s, z)$ determine the evolution of firm employment. In turn, this Markov process and the profit function $\pi(s, z)$ play a key role in determining the properties of firm survival, as explained below.\(^{15}\)

The technology for entry of a new firm is as follows. Upon paying a cost of entry of $c_e$ units of labor, entrants draw their initial productivity from a fixed distribution $G$. The productivity draws are independent across entrants and time. This assumption implies that potential entrants are ex-ante identical and get differentiated ex-post as a result of their initial draws and the stochastic evolution. While admittedly extreme, it captures in a stylized way the large amount of uncertainty faced by potential entrants. As discussed below, this assumption implies a perfectly elastic supply of potential entrants and will play a key role in our analysis. Later on, in Section 5, we consider the potential role of ex-ante heterogeneity.

3.1 Examples

Our formulation is general and can encompass models of perfect and imperfect competition. We provide three examples below. For each example, we map the aggregate state $z$, profit function $\pi(s, z)$, and employment function $n(s, z)$ to model fundamentals.

**Perfect competition.** Firms produce a homogeneous good with labor input $n$ and production technology $q(s, n)$, where $s$ can be interpreted as a productivity

\(^{15}\)It is worth noting that the basic framework can easily be extended in several ways. We can include (i) multiple factors of production, with the assumption of some common aggregator, (ii) R&D can be introduced assuming the Markov process for the firm’s state is affected by the resources (e.g. labor) employed in R&D, (iii) the stochastic process for $s_{it}$ need not be Markov, and thus firm-age effects or learning as in Jovanovic (1982) can be easily introduced. In particular, in the quantitative model used in Section 4, the state is Markov but only conditional on a firm’s permanent type drawn at birth.
shock. Assume $q$ is increasing in $s$, supermodular and strictly concave. In addition there is a fixed cost of production $c_f$ which captures labor overhead. The model is the standard entry and exit model considered in the literature based on Hopenhayn (1992a). Let $z$ be the price of the output good in units of labor, the numeraire. Profits are given by

$$\pi(s, z) = \max_n zq(s, n) - n - c_f$$

and employment $n(s, z)$ is the unique maximizer. Given the above assumptions it follows immediately that both $\pi(s, z)$ and $n(s, z)$ are increasing in $s$ and $z$.

**Monopolistic competition with constant markups.** Each firm $i$ produces a differentiated good with a linear production function, $q(i) = s(i)n(i)$, a fixed cost $c_f$ in units of labor, and faces a CES demand, as in Melitz (2003). In particular, the representative consumer has preferences over intermediate goods given by the aggregator

$$U = \left( \int c(i)^\eta \, di \right)^{1/\eta}$$

where $0 < \eta < 1$, and $c(i)$ denotes per capita consumption of firm $i$'s output. First order conditions for the choice of $c(i)$ are given by

$$U^{1-\eta}c(i)^{\eta-1} = \theta p(i),$$

where $\theta$ is the multiplier of the budget constraint of the consumer and $p(i)$ is the price of good $i$. Revenues per consumer are given by

$$p(i)c(i) = U^{1-\eta}\theta^{-1}c(i)^\eta$$

$$= U^{1-\eta}\theta^{-1}(s(i)n(i)/N)^\eta.$$

$^{16}$An alternative equivalent formulation is that $U$ represents a final good produced by perfectly competitive firms with the production function given by the aggregator above. In that case, $\theta^{-1}$ is the price of the final good.
Given a population of consumers \( N \), and letting \( z \equiv NU\theta^{-\frac{1}{1-\eta}} \), total firm revenues are \( z^{1-\eta} (sn)^\eta \). Profits are given by

\[
\pi(s, z) = \max_n z^{1-\eta} (sn)^\eta - n - c_f
\]

and employment \( n(s, z) \) is the unique maximizer. It follows immediately that both \( \pi(s, z) \) and \( n(s, z) \) are increasing in \( s \) and \( z \).

**Monopolistic competition with variable markups.** There is a continuum of firms each producing a different variety of a final good with quality \( s(i) \). Preferences of the representative consumer are given by the CES aggregator:

\[
U = \left( \int s(i) c(i)^\eta di \right)^{\frac{1}{\eta}}
\]

where \( 0 < \eta < 1 \) parameterizes the elasticity of substitution between varieties and \( c(i) \) denotes consumption per capita of good \( i \). Firms produce with linear technology \( q(i) = n(i) \). Suppose the marginal cost of producing an existing product by a firm of quality \( s \) is one. An outside imitator can make this product at marginal cost \( 1 < p(s) < 1/\eta \), as in the literature based on Grossman and Helpman (1991). All costs are expressed in units of labor. As a consequence of limit pricing by potential imitators, the price of a good of quality \( s \) will thus be equal to \( p(s) \). If \( p(s) \) is an increasing function, markups will increase in \( s \). Order the preferences of the representative consumer by product quality and let \( \psi(s) \) denote the measure of firm qualities. We have

\[
U = \left( \int sc(s)^\eta d\psi(s) \right)^{\frac{1}{\eta}}.
\]

This gives individual demand functions of the form

\[
c(s) = U\theta^{-\frac{1}{1-\eta}} (s/p(s))^{\frac{1}{1-\eta}},
\]

where \( \theta \) is the multiplier of the budget constraint of the consumer.
Profits of a firm of quality $s$ are

$$\pi(s, z) = z \left(\frac{s}{p(s)}\right)^{\frac{1}{1-\eta}} \left(p(s) - 1\right) - c_f,$$

where $z = NU \theta^{-\frac{1}{1-\eta}}$ and $c_f$ is overhead labor. Likewise, total output and employment $n(s, z)$ of firm of quality $s$ equals $Nc(s)$, so

$$n(s, z) = z \left(\frac{s}{p(s)}\right)^{\frac{1}{1-\eta}}.$$

Assuming $s/p(s)$ is increasing in $s$, it follows immediately that both $\pi(s, z)$ and $n(s, z)$ are increasing in $s$ and $z$. Note also that with these assumptions, higher quality firms are larger, i.e. employ more workers, and have higher markups.

### 3.2 Equilibrium

Given a deterministic path for the aggregate state $z_t = \{z_\tau\}_{\tau \geq t}$, the present discounted value of a firm is given by the Bellman equation:

$$v(s, z_t) = \max \left\{ 0, \pi(s, z_t) + \beta EV(s', z_{t+1} | s) \right\}.$$

The value of exit is normalized to zero, while the right-hand side under the maximization is the continuation value for the firm. It is easy to show that, when nonzero, the continuation value is increasing in $s$ and $z_t$. Let

$$s_t^* = \inf \{ s | \pi(s, z_t) + \beta EV(s', z_{t+1} | s) > 0 \}. \quad (3)$$

A firm is shut down iff $s \leq s_t^*$.\(^{17}\)

Prior to entry, the expected value of an entrant net of the entry cost is

$$v^e(z_t) = \int v(s, z_t) G(s) - c_e. \quad (4)$$

Let $\psi_t$ denote the measure of firms operating at time $t$, where for a set $A$ of idiosyncratic states, $\psi_t(A)$ measures the magnitude of firms that have $s_{it} \in A$.

\(^{17}\)Under some regularity conditions (see Hopenhayn, 1992b) it can be guaranteed that such a unique finite threshold $s_t^*$ exists. These conditions amount to profits being negative for sufficiently low quality $s$, and that shocks are persistent.
Given an initial measure $\psi_0$, the exit thresholds $s^*_t$ together with mass of entrants $m_t$ uniquely determine the sequence of measures $\{\psi_t\}$ recursively as follows. Define

$$
\psi_{t+1} (A) = m_{t+1} \left( \int_{s \in A, s \geq s^*_{t+1}} dG (s) \right) + \int \int_{s \in A, s \geq s^*_{t+1}} dF (s | x) \, d\psi_t (x). \quad (5)
$$

The first term on the right-hand side corresponds to entrants, excluding those that exit immediately, while the second term corresponds to incumbents after the realization of new productivities, excluding those that exit.

Let $M_t \equiv \int d\psi_t (s)$ denote the total mass of firms. The resource constraint requires that

$$
\int n (s, z_t) \, d\psi_t (s) + \int c_f \, d\psi_t (s) + m_t c_e = N_t. \quad (6)
$$

The first term is productive labor demand, the second is overhead labor and the third is labor utilized for creation of entrants. The right-hand side represents total labor, which is inelastically supplied.

An equilibrium, given a sequence $\{N_t\}$ and an initial measure $\psi_0$, consists of shutdown thresholds $\{s^*_t\}$, mass of entrants $\{m_t\}$, measures of firms $\{\psi_t\}$ and aggregate states $z_t = \{z_t\}$ such that:

(i) Entry: No rents for entrants, $\nu^e (z_t) \leq 0$ and $\nu^e (z_t) \, m_t = 0$;

(ii) Exit: Shutdown thresholds are given by equation (3);

(iii) Law of motion: The sequence $\psi_t$ is generated recursively by equation (5), given the initial measure $\psi_0$;

(iv) Resource constraint (6) holds.

We focus on equilibria with strictly positive entry, which is the relevant case in reality. When labor $N_t$ grows at a constant rate, there is a unique level of the aggregate state for all periods, denoted $z^*$, that satisfies the zero net value condition for entrants. The unique allocation that supports this value $z^*$ as an equilibrium can be constructed along the lines of the stationary equilibrium in Hopenhayn (1992a).\footnote{The extension of this argument to the monopolistic competition case with constant and variable markups is in Appendix D.}
when labor is growing at non-constant rates, as would be the case along a transi-
tion path.\textsuperscript{19} The intuition behind the equilibrium construction is as follows. Given
the value $z^*$, there is a perfectly elastic supply of potential entrants. Adjustments
along the extensive margin of entry guarantee that the market clearing condition
(6) holds in every period, provided that the implied number of entrants is never
negative.\textsuperscript{20} The next section provides sufficient conditions for strictly positive en-
try in every period that are needed to develop the existence argument. Readers
who want to skip the details can jump to Section 3.2.2, which discuss the key
equilibrium properties used in our quantitative analysis.

### 3.2.1 Existence of Equilibrium

For existence, we need to show that the four equilibrium conditions hold in every
period. Let $z^*$ be such that the entry condition holds every period, $v^e(z^*) = 0.$
Let $s^*$ be the corresponding shutdown threshold, so that the exit condition holds
every period. Given $\psi_0$, construct the sequence $\psi_t$ recursively such that the law of
motion holds.

It remains to verify that the resource constraint holds. We begin by construct-
ing survival probabilities by age. Let $S_a$ denote the probability that an entrant
survives at least $a$ periods, i.e. that the state $s_{i\tau} \geq s^*$ for ages $\tau$ from 0 to $a$. Let
$\bar{\psi}_a$ denote the cross-sectional probability distribution of productivities for firms in
the cohort of age $a$. These can be obtained recursively as follows:

1. Let $S_0 = (1 - G(s^*))$. Let $\bar{\psi}_0(ds) = G(ds)/S_0$ denote the distribution of
entrant productivity draws conditional on $s \geq s^*$.

\textsuperscript{19}The equilibrium features interest rates that are invariant to changes in population growth. This
can be rationalized by assuming that all agents are risk neutral, or alternatively by considering a
small open economy. Our analysis should then be considered an approximation to equilibrium
behavior in a model with variable interest rates. In our numerical calculations, we verify that the
fluctuations in the implied path for aggregate consumption in such an economy are small, and
under standard levels of curvature imply small changes in interest rates. If there is an aggregate
trend in productivity growth, the aggregate state would grow at a rate proportional to productivity
trend growth.

\textsuperscript{20}The key assumption in the following construct is a perfectly elastic supply of new firms at the
cost of entry $c_e$. This follows from the assumption that all firms draw from the same distribution
$G(s)$ regardless of the number of entrants, so that there is no ex-ante heterogeneity. As usual
in models where there is a linear margin for adjustment, it is changes along this margin that
guarantee market clearing while keeping constant other margins. An analogue is the familiar case
of a perfectly competitive industry with perfectly elastic supply at minimum average cost, where
all changes in demand would be met by changes in the number of entrants at this constant price.
2. Let \( S_a = S_{a-1} \int P(s_a \geq s^*|s_{a-1}) d\tilde{\psi}_{a-1}(s_{a-1}) \), where the term under the integral is the probability that a firm in cohort \( a-1 \) is not shut down in the next period, and let

\[
\tilde{\psi}_a (ds) = \frac{\int P(ds_a|s_{a-1}) d\tilde{\psi}_{a-1}(s_{a-1})}{S_a/S_{a-1}}.
\]

Next, let \( \tilde{e}_a \equiv \int (n(s,z^*) + cf) d\tilde{\psi}_a \) denote the average firm size of the age \( a \) cohort. Let \( E_{tA} \) denote total employment by that cohort at time \( t \). In addition to average firm size \( \tilde{e}_a \), total employment \( E_{tA} \) depends on the original mass of entrants in that cohort and their survival rate,

\[
E_{tA} = m_{t-A} S_a \tilde{e}_a.
\]

Total employment by incumbents (i.e. excluding new entrants) at time \( t \) is the sum of employment by cohorts with age greater than one, \( E^I_t = \sum_{i}^{t} E_{tA} \). On adding \( E^I_t \) and total employment by entrants \( m_t (S_0 \tilde{e}_0 + c_e) \), we recover the resource constraint

\[
N_t = m_t (S_0 \tilde{e}_0 + c_e) + E^I_t.
\]

Given that the aggregate state equals \( z^* \) in every period, \( S_a \) and \( \tilde{e}_a \) are known at time \( t \). Because \( m_{t-A} \) and therefore \( E^I_t \), are also known at time \( t \), the only unknown in the above equation is \( m_t \). It follows that equation (7) implicitly determines \( m_t \) such that the resource constraint holds. If \( m_t \) is strictly positive, all equilibrium conditions hold and the existence argument is complete. This occurs provided that \( E^I_t < N_t \) in every period \( t \). The following proposition provides sufficient conditions for strictly positive entry.

**Proposition 1** (Sufficient Conditions for Strictly Positive Entry). A unique equilibrium with strictly positive entry exists if \( N_t \) is a nondecreasing sequence and \( S_a \tilde{e}_a \) is non-increasing.

The intuition is as follows. Because \( N_t \) is a nondecreasing sequence, a sufficient condition for \( E^I_{t+1} < N_{t+1} \), which guarantees strictly positive entry in period \( t+1 \),
is that \( E_{t+1}^I < N_t \). Note that

\[
N_t = m_t S_0 \tilde{e}_0 + m_{t-1} S_1 \tilde{e}_1 + \ldots + m_0 S_t \tilde{e}_t + m_t c_e
\]

\[
E_{t+1}^I = m_t S_1 \tilde{e}_1 + m_{t-1} S_2 \tilde{e}_2 + \ldots + m_0 S_{t+1} \tilde{e}_{t+1}.
\]

Therefore \( E_{t+1}^I \) is the inner product of the same vector of the mass of entrants as \( N_t \), with a forward shift in the corresponding terms \( S_a \tilde{e}_a \) and without the entry cost term \( m_t c_e \). A sufficient condition for \( N_t - E_{t+1}^I > 0 \) every period is that \( S_a \tilde{e}_a \) decreases with \( a \). For a given cohort, this condition is equivalent to saying that the total employment of the cohort is decreasing in age. In the data, survival rates are decreasing in \( a \) but average size of a cohort, when properly calibrated, is increasing. Therefore the sufficient condition holds when shutdown rates are sufficiently high to offset the growth in average size.\(^{21}\) While the conditions in Proposition 1 are sufficient for existence of equilibrium, they are not necessary. If these conditions are violated, it simply needs to be verified that entry is positive every period.

### 3.2.2 Properties of the Equilibrium

Now that we have established the existence of the equilibrium, we discuss its properties.

**Corollary 1** (Invariance of Age Profiles). Exit rates by age, average firm size by age, and size distributions by age are invariant to changes in population growth.

This corollary follows because firm exit decisions and optimal scale of operation do not change as population grows at non-constant rates. By equation (7), the invariance of age profiles implies that the mass of entrants \( m_t \) is linear in \( N_t \).

**Corollary 2** (Dynamic Entry Equation). The mass of entrants in equilibrium is given by

\[
m_t = \frac{N_t - \sum_{a=1}^{t} m_{t-a} S_a \tilde{e}_a}{S_0 \tilde{e}_0 + c_e}.
\]

\(^{21}\)In the model, this property is easy to verify given the stochastic process for the idiosyncratic shocks \( s_t \) and the shutdown threshold \( s^* \). Models that assume permanent productivity shocks and exogenous exit trivially satisfy this condition. The same holds true for the models where productivity shocks are redrawn with some probability from the same distribution as entrants, e.g. Mortensen and Pissarides (1994).
In equilibrium, changes in \(N_t\) are accommodated along the extensive margin by changes in entry. The dynamic entry equation also shows that entry is history dependent: current entry \(m_t\) depends on the distributed lags of past entry \(m_{t-a}\). History dependence implies that a one-time shock to firm entry today will have persistent effects on future entry through the mass of incumbents \(m_{t-a}\) in future periods.

### 3.3 Rates of Entry and Exit

In this section we examine the link between the age distribution of firms and aggregate rates of entry and exit. We then explore how changes in population growth affect aggregate variables via the age distribution along steady states and transitional paths.

The mass of aggregate exit at time \(t\), denoted \(X_t\), is the sum of exit masses of different age cohorts. Exit of firms of age \(a\) equals the difference in survival rates \(S_{a-1} - S_a\) multiplied by the size of the cohort at entry, \(m_{t-a}\). We follow here the convention that the age at which a firm is shut down corresponds to the age at which the firm last drew its idiosyncratic state. The model allows for entrants to exit immediately without producing. The mass of immediate exits \(m_t(1 - S_0)\) are excluded from aggregate exit. It follows that the mass of aggregate exit is given by

\[
X_t = \sum_{a=1}^{t} m_{t-a} (S_{a-1} - S_a).
\]

The number of firms at \(t-1\) is given by

\[
M_{t-1} = \sum_{a=1}^{t} m_{t-a} S_{a-1}.
\]

Let \(\tilde{\xi}_a \equiv (S_{a-1} - S_a) / S_{a-1}\) denote the probability of exit at age \(a\). The aggregate exit rate \(\xi_t \equiv X_t / M_{t-1}\) can be expressed as the weighted average of exit rates of different cohorts,

\[
\xi_t = \sum_{a=1}^{t} \left( \frac{m_{t-a} S_{a-1}}{\sum_{a=1}^{t} m_{t-a} S_{a-1}} \right) \tilde{\xi}_a,
\]

where the term in brackets is the share of firms of age \(a-1\) in the total mass of
firms at time $t-1$. Exit rates by age do not change with population growth by the Invariance of Age Profiles Corollary. Therefore, the aggregate exit rate is only a function of the age distribution of firms, which in turn is determined by past entry. This formula highlights the role of firm demographics in determining the aggregate exit rate. Because the exit rates are different across firm ages, a change in the age distribution of firms affects the aggregate exit rate. The exception, of course, is when exit rates are the same for all cohorts. In that case, the aggregate exit rate is independent of the age distribution.

Now consider entry rates. We define $m_t S_0$ as the mass of entrants.\footnote{If we had assumed that all entrants must produce for at least one period, then $S_0 = 1$ and $m_t$ would be measured entry.} Let $e_t \equiv N_t / M_t$ denote average employment. The rate of growth in the number of firms is

$$\frac{M_t}{M_{t-1}} = \frac{N_t}{N_{t-1}} \frac{e_{t-1}}{e_t}. \quad (10)$$

The mass of firms $M_t$ can be decomposed into the mass of surviving incumbents plus the mass of entrants,

$$M_t = (1 - \xi_t) M_{t-1} + m_t S_0.$$

Solving for $M_t$ in (10) and substituting in the above equation gives the following expression for the entry rate, $\lambda_t \equiv m_t S_0 / M_{t-1}$,

$$\lambda_t = \left( \frac{N_t}{N_{t-1}} \frac{e_{t-1}}{e_t} - 1 \right) + \xi_t. \quad (11)$$

This expression is the discrete-time version of identity (1). It says that the entry rate is increasing in population growth, decreasing in average employment growth, and increasing in the exit rate. This expression also shows how a change in the entry rate feeds back into itself. Consider an increase in labor force growth $N_t / N_{t-1}$. This leads to an initial increase in entry which shifts the age distribution towards younger firms. The aggregate exit rate $\xi_t$ is a weighted average of cohort exit rates $\tilde{\xi}_a$ and age weights. Because exit rates are decreasing in age, a larger share of young firms leads to a higher aggregate exit rate, and consequently with higher entry. In addition, changes in average employment further impact the entry rate. The initial rise in entry rates increases the share of younger firms which
tend to be smaller. This lowers average employment and, from equation (11), fur-
ther increases the rate of entry. Thus, entry rates increase by more than the initial
increase in the rate of population growth.

**Long-Run vs. Adjustment Path.** Suppose we are in a steady state in with pop-
ulation growing at a constant rate $g$. Because average employment $e_t$ is constant
along the steady state, the cohort entry weights $m_{t-a}$ decay as a function of age
at the rate $1 + g$, so $m_{t-a} = (1 + g)^{-a} m_t$. The aggregate exit rate along the steady
state, denoted $\xi^{SS}$, follows from (9),

$$\xi^{SS} = \sum_{a=1}^{\infty} \left( \frac{(1 + g)^{-a} S_{a-1}}{\sum_{a=1}^{\infty} (1 + g)^{-a} S_{a-1}} \right) \tilde{\xi}_a,$$

which is independent of $t$. This equation shows that the long run exit rate $\xi^{SS}$ is
intimately related to rate of population growth $g$. The aggregate exit rate is an
average of the exit rates of different cohorts $\tilde{\xi}_a$, weighted by the corresponding
share of surviving firms in each cohort. It is easy to see that, for higher values
of $g$, the weights decrease faster with age $a$. Empirically, younger firms exit at
higher rates. Therefore higher population growth, which shifts weight towards
younger firms, is associated with higher long-run aggregate exit rates.

**Proposition 2** (Long-run Multiplier Effect). Assume exit rates by age $\tilde{\xi}_a$ are decreasing
in cohort age $a$. Then the long-run aggregate exit rate $\xi^{SS}$ is increasing in population
growth $g$.

Because average employment $e_t$ is constant in a steady state, the entry rate (11)
in the steady state, denoted $\lambda^{SS}$, is also independent of $t$.\footnote{The same holds in a model where productivity shocks are fully persistent or randomly re-
drawn from the same distribution as the one faced by entrants (as in Mortensen and Pissarides,
1994), average firm size is constant so the above formula applies. In particular this means that the
rate of entry is independent of history and only depends on current population growth. If exit
rates are not age dependent, the same will also be true for exit.} We have

$$\lambda^{SS} = g + \xi^{SS}.$$  \hspace{1cm} (13)

The entry rate equals the sum of the population growth rate and the exit rate. The
intuition is simple. Entrants must replace the exiting firms. In addition, because
average employment is constant, the total mass of firms needs to grow at the
rate of population growth, \( g \), to clear the labor market. Therefore, there must be enough entry to also create this extra employment. When exit rates are decreasing in age, as occurs in practice, a change \( \Delta g \) in the rate of population growth leads to a change in the long-run aggregate exit rate \( \Delta \xi^{SS} \) of the same sign, as implied by Proposition 2. Therefore, the effect of population growth on the long-run rate of entry is amplified, \( \Delta \lambda^{SS} > \Delta g \). We refer to this as the long-run multiplier effect. In our quantitative analysis we find that it is in the order of 1.5 for the US economy.

The effect of transitional dynamics can be illustrated by considering the impact of a period of rising labor force growth, as exhibited by the US economy after the baby boom. Compared to the steady state with constant population growth, rising population growth over time shifts the firm-age distribution toward younger cohorts. This results in aggregate exit rates greater than, and average size lower than, the steady state. Even in the hypothetical case that the rate of population growth is constant at its peak after the initial rise in the 1970s, the US economy should have seen a decrease in entry rates, as the aggregate exit rate and average firm size converge to their corresponding long-run values. Section 4.1.1 explores the role of the baby boom by decomposing changes in US entry rates into long-run vs. transitional effects.

4 Quantitative Analysis

The theory section described a class of models under which firm demographics variables by age are invariant to changes in the growth rate of the labor force. This class of models share a common difference equation for entry that can be used to answer a number of quantitative questions. Can changes in labor force growth, combined with feedback from firm demographics, quantitatively generate the secular changes experienced by the US economy? What is the role of the feedback effect? What is the role of transitional dynamics, and therefore the importance of the baby boom?

The dynamic entry equation (8) determines the evolution of the firm-age distribution given an exogenous labor force series, the cost of entry, an initial age distribution, and an age profile for exit rate and average size. In principle, one could iterate the dynamic entry equation forward without further need of a model. However, while there is reliable data on labor force growth, there is limited data
on the initial age distribution and firm demographics variables. Specifically, not all firms born before 1978 have an age assigned to them in BDS data. This implies that (i) the 1978 firm-age distribution is unknown, and that (ii) average size by age $\bar{e}_a$ and survival probabilities by age $S_a$ for firms born before 1978 are also unknown.

We use a structural model to fill the gaps in the data. In particular, the model generates a stochastic process for employment. The employment process consists of the distribution of entrant employment, the evolution of employment over time and an exit rule. Therefore, the employment process implies values for the age profile of exit and average size. In order to obtain the 1978 firm-age distribution, we make use of historical labor force data which goes back to 1940. We assume the US economy was in a steady state in 1940, implying that the 1940 firm-age distribution corresponds to the stationary distribution of the employment process. We then feed data on labor force growth and iterate the dynamic entry equation forward to obtain the firm-age distribution in 1978. Doing so allows us to obtain the 1978 age distribution that is consistent with historical labor force growth, and in particular, the baby boom.\(^{24}\)

The parameterization we present is consistent with any economy that generates the same employment process and falls within the theoretical framework laid out in Section 3. We present the parameters for two such economies, the perfect competition economy and the monopolistic competition with constant markup economy.\(^{25}\) These two economies are isomorphic with respect to the implied firm dynamics, once the parameters are appropriately reinterpreted.

The model period is set to one year. The time discount factor $\beta$ is set to 0.96. The steady-state labor force growth rate $g$ prior to 1940 is set to a standard value of one percent. The aggregate state $z^*$ and the idiosyncratic state $s$ enter the profit function multiplicatively, which leads to an identification problem. We normalize the aggregate state $z^*$ to unity to get around this identification problem, as in Hopenhayn and Rogerson (1993). The curvature of the revenue function is parameterized by $\alpha$, which is set to the standard value of 0.64. In the perfect com-

\(^{24}\)An alternative approach to specifying a structural model that implies a stochastic process for employment is to use information on the evolution of the left censored group of firms in conjunction with labor force growth to recover the 1978 age distribution along with average size and exit rates by age. That approach yields similar results.

\(^{25}\)Appendix E discusses how to map this exercise to the variable markup economy.
petition economy, the parameter $\alpha$ represents the degree of decreasing returns to scale in the production function of a firm, with $q(n,s)$ equal to $sn^\alpha$, which can be interpreted as the managerial span of control. In the economy featuring monopolistic competition with constant markup, $\alpha$ maps to the elasticity parameter $\eta$. The value of $\eta$ equal to 0.64 implies that the elasticity of substitution, $1/(1-\eta)$, is close to its standard value of 3.

The idiosyncratic state $s$ determines firm size in the model. In addition to the differences in the idiosyncratic state, we introduce permanent differences across firms by allowing firms in the economy to be of two types, high and low.\(^{26}\) High-type firms have (i) a higher long-run mean of their Markov processes and (ii) a higher labor overhead. The high-type firms roughly correspond to “superstar” firms. In the estimated model, these firms account for about 5 percent of firms, 30 percent of employment, and grow to be 7 times larger than low-type firms on average.

The Markov process followed by the idiosyncratic state $s$ is AR(1),

$$\log(s_{t+1}) = (1-\rho)\mu_i + \rho \log(s_t) + \varepsilon_{t+1}; \quad \varepsilon_{t+1} \sim N(0,\sigma^2_\varepsilon), \quad (14)$$

with $\mu_i$ as the long-run mean that varies with type $i \in \{\ell, h\}$, $\rho$ as the persistence, and $\sigma^2_\varepsilon$ as the variance of shocks. Firms draw their type after paying the entry cost $c_e$. Once drawn, the type is fixed throughout the lifetime of the firm. The probability of drawing the high type is parameterized by $\omega_h$. In addition to drawing their type, entrants draw their idiosyncratic state from a lognormal distribution $G$ with mean $\mu_G$ and variance $\sigma^2_G$. The state $s$ then evolves according to the corresponding AR(1) process. The evolution of employment at the firm level depends on the idiosyncratic state $s$, labor overhead of each type, $c_{f\ell}$ and $c_{fh}$, and an endogenous exit threshold for each type, $s^*_\ell$ and $s^*_h$.

There are ten additional parameters that need to be determined: $c_e, \mu_G, \sigma_G$.

\(^{26}\)As pointed out by Luttmer (2011), permanent differences can help generate the frequency with which firms achieve a large size at a relatively young age in the data. These differences across firms allow the model to be consistent with the observation in Haltiwanger, Jarmin and Miranda (2013) that firm characteristics such as age have explanatory power even after controlling for firm size. Permanent differences also make the model consistent with the observation that not all firms of the same size react equally to changes in the economic environment (Holmes and Stevens, 2014; Sterk \& et al., 2021), a feature of the model that will play a role in Section 5 when we discuss alternative explanations.
\( \rho, \sigma \epsilon, \mu \ell, \mu h, c f \ell, c f h \) and \( \omega h \). We jointly estimate these parameters using a Generalized Method of Moments estimator. The estimator minimizes the distance between moments from the beginning of the BDS sample and their model counterparts. In particular, we target the 1978 to 1983 time-series average of the firm entry rate, entrant size, concentration of entrants, 5-year firm growth and exit rates, firm share of sizes 1 to 4, and employment share of firms with 250+, 1000+ and 10,000+ employees.

### Table 3

<table>
<thead>
<tr>
<th>Assigned</th>
<th>Value</th>
<th>Definition</th>
<th>Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.96</td>
<td>Discount factor</td>
<td>Standard</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.64</td>
<td>Curvature of revenue function</td>
<td>Standard</td>
</tr>
<tr>
<td>( g )</td>
<td>0.01</td>
<td>Labor force growth rate (SS)</td>
<td>Standard</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jointly Estimated</th>
<th>Value</th>
<th>SE</th>
<th>Definition</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_e )</td>
<td>0.013</td>
<td>(0.008)</td>
<td>Entry cost</td>
<td>13.14%</td>
<td>13.20%</td>
</tr>
<tr>
<td>( \mu_G )</td>
<td>-4.344</td>
<td>(0.723)</td>
<td>Mean of entrant dist.</td>
<td>5.96</td>
<td>5.55</td>
</tr>
<tr>
<td>( \sigma_G )</td>
<td>1.331</td>
<td>(0.462)</td>
<td>Std. dev. of entrant dist.</td>
<td>4.89%</td>
<td>5.19%</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.984</td>
<td>(0.001)</td>
<td>Persistence of AR(1)</td>
<td>73.86%</td>
<td>76.69%</td>
</tr>
<tr>
<td>( \sigma_\epsilon )</td>
<td>0.246</td>
<td>(0.041)</td>
<td>Std. dev. of AR(1) shocks</td>
<td>5-year exit rate</td>
<td>55.01%</td>
</tr>
<tr>
<td>( \mu_l )</td>
<td>-2.431</td>
<td>(0.849)</td>
<td>Low-type AR(1) long-run mean</td>
<td>Emp. share 10,000+</td>
<td>26%</td>
</tr>
<tr>
<td>( \mu_h )</td>
<td>-1.436</td>
<td>(1.756)</td>
<td>High-type AR(1) long-run mean</td>
<td>Firm share size 1 to 4</td>
<td>55.98</td>
</tr>
<tr>
<td>( c_f \ell )</td>
<td>2.299</td>
<td>(0.871)</td>
<td>Labor overhead low type</td>
<td>Emp. share 1000+</td>
<td>41.64%</td>
</tr>
<tr>
<td>( c_f h )</td>
<td>24.308</td>
<td>(5.860)</td>
<td>Labor overhead high type</td>
<td>Agg. concentration</td>
<td>52.35%</td>
</tr>
<tr>
<td>( \omega h )</td>
<td>0.359</td>
<td>(0.179)</td>
<td>Share of high-type entrants</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

**Notes.** All data and model moments are time averages from 1978 to 1983. The mapping to the monopolistic competition economy with constant markups is as follows. The parameter \( \alpha \) maps to the elasticity parameter \( \eta \), and the AR(1) processes describe the evolution of \( \eta \log(s_t) \).

When choosing the set of targeted moments, we tried to strike a balance between firm-weighted moments, which our theory shows are economically important, and employment-weighted moments, which might be more accurately measured in BDS data. The cost of entry primarily targets the entry rate. From the dynamic entry equation, matching the average entrant size is necessary to match the entry rate, so we target this moment. Average entrant size is determined primarily by \( \mu_G \), the mean of the entrant productivity distribution \( G \). The dispersion \( \sigma_G \) determines the thickness of the right tail of \( G \), and therefore tar-
gets the concentration of entrants. The dispersion of the productivity process \( \sigma_\varepsilon \) affects the weight on productivity gridpoints at which firms exit, so it primarily targets the 5-year exit rate. The persistence parameter \( \rho \) mainly determines the rate of convergence of the AR(1) processes to their respective long-run means, so we use it to target the 5-year growth rate of firms. High-type firms are larger than low-type firms. They matter most in terms of their employment shares, so we set their long-run mean \( \mu_{h} \), their labor overhead \( c_{fh} \) and their share at birth \( \omega_{h} \) to target three right-tail moments of the employment-weighted size distribution: the employment share of firm size 250+, 1000+, and 10,000+. Finally, the labor overhead of low-type firms is most important in determining the share of firms in the smallest size bin, the 1 to 4 employees category.

Table 3 reports the estimated parameters and standard errors, along with the fit to the targets. Figure 5 shows the match of the firm size distribution in the model and the data. The inclusion of firm types allows the model to better match the tails of the firm size distribution.

![Figure 5: 1978 Firm Size Distribution](image)

Table 4 reports firm demographics variables both in the model and the data. The age profiles of exit, size, and concentration in the model follow an expected pattern given that entrants start small and converge to a higher long-run mean: the probability of exit is decreasing in age, average firm size is increasing in age, and concentration increases with age. As mentioned earlier, all of these patterns are present in the data.\(^{27}\)

\(^{27}\)While the model generates the general patterns of exit rate, average firm size, and concentra-
Table 4: Age Profiles

<table>
<thead>
<tr>
<th>Age</th>
<th>Exit rate Data(%)</th>
<th>Exit rate Model(%)</th>
<th>Average firm size Data</th>
<th>Average firm size Model</th>
<th>Concentration Data(%)</th>
<th>Concentration Model(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>–</td>
<td>–</td>
<td>6.05</td>
<td>5.55</td>
<td>5.90</td>
<td>5.19</td>
</tr>
<tr>
<td>1</td>
<td>21.85</td>
<td>21.51</td>
<td>7.73</td>
<td>6.29</td>
<td>12.29</td>
<td>7.13</td>
</tr>
<tr>
<td>2</td>
<td>15.86</td>
<td>15.76</td>
<td>8.46</td>
<td>7.05</td>
<td>13.29</td>
<td>9.18</td>
</tr>
<tr>
<td>3</td>
<td>13.43</td>
<td>13.06</td>
<td>9.14</td>
<td>7.89</td>
<td>14.83</td>
<td>11.44</td>
</tr>
<tr>
<td>4</td>
<td>11.68</td>
<td>11.42</td>
<td>9.77</td>
<td>8.80</td>
<td>16.45</td>
<td>13.93</td>
</tr>
<tr>
<td>5</td>
<td>10.48</td>
<td>10.30</td>
<td>10.36</td>
<td>9.80</td>
<td>17.84</td>
<td>16.66</td>
</tr>
<tr>
<td>6-10</td>
<td>8.32</td>
<td>8.51</td>
<td>11.98</td>
<td>13.49</td>
<td>23.00</td>
<td>25.96</td>
</tr>
<tr>
<td>11-15</td>
<td>6.40</td>
<td>6.85</td>
<td>15.08</td>
<td>21.52</td>
<td>31.85</td>
<td>41.34</td>
</tr>
<tr>
<td>16-20</td>
<td>5.56</td>
<td>6.04</td>
<td>18.81</td>
<td>31.87</td>
<td>40.68</td>
<td>53.90</td>
</tr>
<tr>
<td>21-25</td>
<td>4.99</td>
<td>5.55</td>
<td>24.03</td>
<td>43.82</td>
<td>50.47</td>
<td>62.92</td>
</tr>
<tr>
<td>Above 25</td>
<td>4.29</td>
<td>4.93</td>
<td>81.59</td>
<td>78.51</td>
<td>78.91</td>
<td>76.23</td>
</tr>
</tbody>
</table>

Notes. Concentration is the share of employment in firms with 250+ employees within the age category divided by total employment in the age category. Data moments are the average across all years in the sample. Model moments are the time average for the same years as the data moments.

4.1 Firm Entry

Figure 6 presents the findings for the entry rate. We highlight three distinct non-targeted episodes that the model matches well. The first episode is related to World War II. The years around the war exhibited large fluctuations in the entry rate. The labor force growth series also exhibits large fluctuations around the same time, corresponding to large numbers of civilians leaving the labor force to join the war effort and then returning after the war. Through the lens of our model, these large labor force growth fluctuations translate into similarly large fluctuations in the entry rate. Second, the model generates the apparent increase in the entry rate before 1978. This increase is driven by the steady increase in...
labor force growth during the same time period, largely reflecting the baby boom generations entering the labor force.\textsuperscript{29}

![Graph showing labor force growth and entry rates over time.](image)

**Figure 6**

*Notes.* The entry rate from 1963 to 1977 is linearly interpolated.


Finally, the feedback effects from firm demographics amplify the approximately 2pp decline in the labor force growth between 1978 and 2014 into the approximately 6pp decline in the entry rate. As discussed earlier, in the model this amplification arises through an aging of the firm distribution: the share of firms over 10 years of age increases by 13pp. Given the age distribution is history dependent, the glut of firms born due to the baby boom will impact firm entry in the decades that follow. The next section discusses the importance of the baby boom for the post-1978 period.

\textsuperscript{29} Appendix B shows that the bulk of changes in labor force growth are accounted for changes in birth rates sixteen years prior, and can therefore be attributed to the baby boom. The contribution of changes in labor force participation to labor force growth is small despite two notable trends in participation rates throughout this period: the increase in female labor force participation since the 1950s, and the post-2000 decline in labor force participation of the young. The reason these trends do not show up more strongly in our decomposition is that, for each, there was a concurrent countervailing force. The effect of the increase in female labor force participation was dampened by a decline in male labor force participation. The effect of the decline in labor force participation of 16-24 year olds was dampened by an increase in the labor force participation of those aged 55 and above.
4.1.1 The Role of the Baby Boom

The first column of Table 5 begins by breaking down the post-1978 changes in the entry rate into the direct effect of labor force growth and the feedback effects of firm demographics using identity (1). The entry rate in the model declines by 6.08pp between 1978 and 2014. Of this decline, 30 percent (1.88pp) is accounted for by the decline in labor force growth. The 30 percent decline would occur even in the absence of firm demographics i.e. without differences in average size and exit rates by firm age. The remaining 70 percent is due to the feedback effect of firm demographics. An aging firm distribution pushes the aggregate exit rate down by 1.81pp and increases the growth rate of average firm size by 2.11pp. The direct effect of labor force growth and the feedback effect together add up to 5.80pp. The remaining 0.28pp is due to a residual arising from changes in labor allocated to the creation of entrants.30

Table 5: Decomposition: Steady State vs. Transitional Effects

<table>
<thead>
<tr>
<th></th>
<th>Total Change</th>
<th>Long-Run Effect</th>
<th>1978 Transition Effect</th>
<th>2014 Transition Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ LF Growth</td>
<td>−1.88</td>
<td>−1.88</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+ Δ Exit Rate</td>
<td>−1.81</td>
<td>−0.88</td>
<td>−0.30</td>
<td>−0.63</td>
</tr>
<tr>
<td>− Δ AFS Growth</td>
<td>2.11</td>
<td>0</td>
<td>1.16</td>
<td>0.95</td>
</tr>
<tr>
<td>+ Δ Residual</td>
<td>−0.28</td>
<td>0</td>
<td>−0.02</td>
<td>−0.26</td>
</tr>
<tr>
<td>Δ Entry Rate</td>
<td>−6.08</td>
<td>−2.76</td>
<td>−1.47</td>
<td>−1.84</td>
</tr>
</tbody>
</table>

Notes. All values are in percentage points. The total decline in the entry rate (6.08pp) is the sum of the long-run effect (2.76pp), the 1978 transition effect (1.47pp), and the 2014 transition effect (1.84pp). With entry costs denominated in units of labor, identity (1) holds approximately, \( \lambda \approx \hat{N} + \xi - \dot{c} \). The residual corresponds to changes in the growth rate of labor allocated towards the creation of entrants.

In order to discuss the role of the glut of firms born due to the baby boom, we further decompose the change in each variable into long-run and transitional effects. The long-run effect is the change that can be attributed to differences across

30When labor is needed for creation of entrants, average firm size in the model is not equal to population divided by the number of firms. In that case identity (1) holds exactly only in a steady state, when the mass of entrants grows at the same rate as population. Along a transition, the accounting identity has an additional term that captures the fact that the growth rate of the mass of entrants, and therefore labor allocated to creations of entrants, is different from the population growth rate.
two steady states: a 2014 steady state, which effectively assumes that labor force in the pre-2014 years grew at a constant rate equal to its 2014 value, and a 1978 steady state, which assumes that labor force in the pre-1978 years grew at a constant rate equal to its 1978 value.\textsuperscript{31} The long-run effect lowers entry by 2.76pp, which is 45 percent of the total decline. This reflects the direct effect of a decline in labor force growth, which accounts for 1.88pp, and the long-run multiplier effect of labor force growth on exit rates of 0.88pp presented in Proposition 2.

Because the steady states assume a history of constant labor force growth, the resulting firm-age distributions do not capture the effect of the baby boom. The effect of the glut of firms due to the baby boom is instead captured by the transitional dynamics, which account for 55 percent of the total decline in the entry rate. The effect of transitional dynamics can be further broken down into differences between the 1978 steady state and the transition path in 1978, which we refer to as the 1978 transition effect, and the difference between the 2014 steady state and transition path in 2014, which we refer to as the 2014 transition effect. These effects are reported in the last two columns of Table 5.

The 1978 transition effect accounts for a 1.47pp drop in the entry rate. This occurs for two main reasons. First, the exit rate in the 1978 transition is 0.30pp higher than the exit rate in the 1978 steady state. This is because the 1978 firm age distribution is younger in the transition, as shown in Figure 7a, reflecting the pre-1978 rise in firm entry rates. Second, the growth in average firm size in the 1978 steady state is zero, while average size in the 1978 transition exhibits a growth rate of negative 1.16pp. This negative growth rate is due to the fact that the firm distribution is still getting younger in 1978, mostly reflecting the pre-1978 increase in labor force growth. The age distribution starts getting older only after after a few years of declining labor force growth. The resulting time series of average firm size is U-shaped, with the minimum point reached in 1980.

The 2014 transition effect accounts for the remaining 1.84pp drop in the entry rate. This can again be traced to the baby boom and firm aging. First, the glut of firms born due to the baby boom nearly four decades earlier have not yet died off. As shown in Figure 7b, these surviving firms imply that the firm-age distribution in the 2014 transition is older than the 2014 steady state. Consequently, the exit rate in the 2014 transition is 0.63pp lower than the exit rate in the 2014 steady

\footnote{The long-run effect corresponds to the effect in Karahan, Pugsley and Şahin (2018).}
state. Second, unlike the 2014 steady state, the firm distribution continues to age in the 2014 transition. As a result, average firm size continues to grow, which lowers the entry rate by an additional 0.95pp.32

Given the central role of age distributions in transition versus steady state comparisons, it is useful to contrast the corresponding age distributions to those in the data. While the empirical 1978 age distribution is unknown, we can compare statistics in 2014. Figure 7b shows the 2014 age distributions in the data, the transition and the steady state. The 2014 transitional distribution better approximates the data. This observation, taken together with the strength of the transitional effects, indicates that the glut of baby boom firms is an important feature of the data.

4.2 Exit Rate, Average Firm Size, and Concentration

The evolution of the firm-age distribution along the transitional path, combined with the age profiles of exit, size and concentration presented earlier, generates an aggregate time-series for each these variables. Figure 8 presents the time-series for each of these variables in the model and in the data. The data series are presented in two ways: the actual series and the changes in the variable that would occur if aging was the only force. As discussed earlier in equation (2), the ‘aging only’

32 Appendix F explore the implications for future entry rates by feeding labor force projections to the year 2060 from the BLS and iterating the model forward. Despite a projected decline in labor force growth, future entry rates are projected to bounce back by 1pp.
The impact of the aging force are well-captured by the model. Aging in the data, by itself, leads to an 18 percent decline in exit, a 43 percent increase in average firm size, and a 16 percent increase in concentration. These changes are close to the model-generated 16 percent decline in exit, 41 percent increase in average firm size, and 15 percent increase in concentration. The actual changes in the data are smaller than the changes due to aging only, indicating that within-age group effects also play a role in generating the aggregate time series, particularly for average firm size and concentration. Section 5 discusses what additional forces can help generate these changes within age groups.

5 Discussion

Labor share. In recent work, Hartman-Glaser et al. (2019), Autor et al. (2020) and Kehrig and Vincent (2021) find that almost all of the recent decline of the US aggregate labor share is due to reallocation of value-added from high to low labor share units, rather than a decline in labor share within units. These studies further document a negative relationship between firm size and labor share. It follows

\[ \text{If the Change if Aging Only statistic is calculated using trends with sector controls (columns 2 and 4 in Table 1), the numbers for the aggregate are similar. The aggregate exit rate declines by 14 percent, average size increases by 50 percent, and concentration increases by 23 percent.} \]
that the decline in the aggregate labor share is primarily due to reallocation of economic activity towards larger firms, i.e. increasing concentration. Firm aging induced by the slowdown in population growth provides a driving force that is consistent with these empirical findings. It leads to a reallocation of market shares towards older firms, which are larger and have lower labor shares.\footnote{This mechanism is consistent with labor share dynamics in the data: Kehrig and Vincent (2021) document that reallocation occurs towards units that lower their labor share, as opposed to those that have a low level of the labor share.}

Before we can illustrate the implications of firm aging as a driving force, we need to generate a negative relationship between firm size and labor share. We do so using overhead labor, as in Atkeson and Kehoe (2005).\footnote{The negative relationship between firm size and labor share can be generated in various ways without affecting the results. For example, the negative relationship could arise because larger firms choose technologies that are less labor intensive (Guimaraes and Gil, 2019), have higher markups (Autor et al., 2020), or have higher intellectual property products capital (Koh et al., 2020).} Total employment at the firm level includes both overhead and productive labor. With fixed overhead labor, labor shares decline with firm size.

Figure 9 plots the cumulative change in the aggregate labor share implied by firm aging. That figure also presents two measure of the corporate labor share, one from Karabarbounis and Neiman (2014) and another from Koh, Santaeulàlia-Llopis and Zheng (2020).
Firm aging generates the hump-shaped pattern seen in the Koh et al. (2020) series. From 1940 to 1980, the aggregate labor share increases with the entry rate because entrants are small in size, and therefore have higher labor shares. From 1980 onwards, as firms age and grow in size the share of firms with low labor shares increases, leading to a decline in the aggregate labor share, as in the Karabarbounis and Neiman (2014) series.

**Job creation, destruction and reallocation.** In addition to firm entry rates, the US has also experienced a decline in the job creation, destruction and reallocation rates. These rates exhibit age effects: older firms create, destroy and reallocate jobs at lower rates. Therefore, firm aging induced by labor force growth qualitatively generates a decline in aggregate job creation and destruction rates. To explore the quantitative role of firm aging, we take average job creation and destruction rates by age from BDS data and use the evolution of the firm-age distribution from the structural model to calculate aggregate job creation, destruction and reallocation rates.

Figure 10 shows the resulting time series, along with the data. A statistic that summarizes the role of firm aging is the ratio of the slope of the trendline from 1978 to 2014 in the composition time series to the slope of the trendline in the actual time series for the same years. By this measure, aging explains 48 percent of the decline in the job creation rate, 38 percent of the decline in the job destruction rate, and 32 percent of the decline in the job reallocation rate.

These numbers are larger than those found by previous empirical studies, such as Decker, Haltiwanger, Jarmin and Miranda (2014). The main reason for

36The Koh et al. (2020) measure is different because it accounts for changes in the way the Bureau of Economic Analysis treats intellectual property products. Prior to 1999, intellectual property was treated as a business or consumption expenditure. However, over time the BEA has started treating intellectual property as capital, affecting the measurement of the labor share.

37The job creation rate is the ratio of jobs created, either by entrants or continuers, to total jobs in a period. The job destruction rate is the ratio of jobs destroyed, either by exiting firms or by continuers, to total jobs. The job reallocation rate is equal to creation rate + destruction rate − abs(creation rate − destruction rate).

38Values by age group are the average from the first year the age group is observed to 2006. Value of age groups with multiple ages were assigned to the intermediate age (e.g. the mean of the 6 to 10 age group was assigned to age 8). The average of the Above 25 age group was assigned to all ages 31 and older.

39The numbers are larger if the trendlines stop in 2006. In this case, aging explains 66 percent of the decline in the job creation rate, 55 percent of the decline in the job destruction rate, and 57 percent of the decline in the job reallocation rate.
this difference is that the structural model allows us to unpack the left censored group in the data, so we can perform an analysis with a finer age distribution and over a longer period of time.

![Graphs showing job reallocation, creation, and destruction rates from 1980 to 2020.](image)

**Figure 10**

**High tech sectors.** Decker, Haltiwanger, Jarmin and Miranda (2020) document that business dynamism in the high tech sectors, inclusive of information and communication technology, behaved differently than the rest of the US economy. The rate of firm entry, exit and job reallocation in these sectors peaked about two decades later, in the late 1990s and early 2000s. Even though the high tech sectors only represent about 4 percent of firms and about 6 percent of employment (Goldschlag and Miranda, 2020), they played an important role in the US productivity surge and the subsequent slowdown (Byrne, Fernald and Reinsdorf, 2016). In our framework, average annual productivity growth inherits the hump shape of labor force growth and declines concurrently with firm entry rates. Therefore, while the model does generate a slowdown in productivity, it does not capture the outsized role of the high tech sectors in aggregate productivity dynamics. However, because the high tech sectors represent a small share of firms and aggregate employment, these sectors have little effect on patterns of aggregate business dynamism along the dimensions we study: firm entry, exit, average size and concentration.

**Ex-ante heterogeneity of firms and free entry.** Our analysis relies on the assumption of a perfectly elastic supply of firms. This leads to the very tractable
and transparent difference equation for entry rates (8). This section briefly explores the possible implications of departures from this assumption.

There are basically two forms of ex-ante heterogeneity to be considered: differences in the cost of entry or in the distribution of opportunities faced by entrepreneurs. In other words, better entrepreneurs can either be more efficient in the startup stage or more successful thereafter. Either of these two assumptions introduces curvature in the extensive margin of entry, dampening the fall in entry rates and giving a role to the intensive margin in the adjustment process. As a result, when labor force growth falls, the marginal entrepreneur (or idea) is positively selected, leading to a decrease in the equilibrium aggregate price $z$. In turn, this leads to a decrease in the average size and survival of incumbent firms. This force could help give a role to population growth in explaining the fall in average size observed for older firms in the data.\(^{40}\)

To further examine the potential role of ex-ante heterogeneity at the extensive margin, we take a closer look at the evolution of entry rates. As a working hypothesis, we assume that the quality of the marginal entrant is a function of the ratio of new firms to total employment. This is consistent with the existence of an entrepreneurial talent pool or arrival of new ideas that scale with the labor force.\(^{41}\)

Figure 11 presents the evolution of the ratio of new firms to total employment in the data, as well as the one implied by our model. The first thing to note is the huge range of variation between the starting and end points of our series, data and model, where the ratio of new firms to employment is reduced by more than one half. A large degree of curvature in the extensive margin within this range would have implied fairly dramatic changes at the intensive margin, which are not seen in our age conditional moments, at least for exit and concentration. The second thing to note, is that the data series and the one implied by the model, which by

\(^{40}\)In the data, there are no significant observed changes in average size or survival for younger firms. This could be explained if the source of heterogeneity were ex-ante differences in the distribution of productivity draws. Since younger firms are those that entered in periods of lower entry rates, this positive selection could have countered the overall drop in profitability.

\(^{41}\)It is illustrative to consider the extreme case where the share of new startups, e.g. ideas, are a constant fraction $\gamma$ of labor force $N(t)$. Letting $M(t)$ denote the number of firms and $m(t) = \gamma N(t)$ new entrants, it follows immediately that the rate of entry $m(t)/M(t)$ is proportional to average firm size $(1 - \gamma) N(t) / M(t)$. So a fall in entry rates would imply a fall in average firm size.
construction assumes no ex-ante heterogeneity, are quite close. Combining these two observations, suggests that our assumption of no ex-ante heterogeneity does not seem to be a bad approximation, at least within the ranges of entry observed.

![Figure 11: Ratio of New Firms to Employment](image)

**Alternative explanations.** Several alternative explanations have been given in the literature to explain the observed rise in concentration and decrease in entry rates. These can be roughly grouped into three categories: 1) Some form of non-neutral technological change that allows a set of firms, usually the most productive, to reach a larger scale. This contributes directly to the rise of concentration. 2) An increase in the cost of entry that leads to the reduction of entry rates. As a result, incumbents must grow to absorb the residual labor force, so average firm size increases. 3) A decrease in the rate of diffusion. This increases incumbency advantage, making it harder for new firms to grow, thus decreasing the payoffs to entry.\(^\text{42}\) All these alternative explanations assume either labor force is fixed or growing at a constant rate. We discuss below that, by ignoring the effect of labor force growth, alternative theories must overstate the role of firm exit in generating the observed fall in entry rates.

\(^{42}\)Some examples of papers in the respective categories include 1) Aghion, Bergeaud, Boppart, Klenow and Li (2019), Hsieh and Rossi-Hansberg (2019), De Ridder (2020); 2) Gutiérrez, Jones and Philippon (forthcoming), De Loecker, Eeckhout and Mongey (2021); and 3) Luttmer (2010), Luttmer (2012), Akcigit and Ates (2021).
To put these theories in perspective it is useful to go back to our identity on entry rates:

$$\text{entry rate} = \text{labor force growth} - \text{growth in average size} + \text{exit rate}.$$  

In a long run steady state the growth in average size is zero so, holding fixed labor force growth, longstanding decreases in the entry rate must be associated with equally sized decreases in the long run exit rate. By equation (12), the same change in the aggregate exit rate can be obtained either by a change in labor force growth or by a proportional change in survival rates. In the absence of changes in labor force growth, the aggregate exit rate declines only due to an increase in survival rates. No matter what is the primitive source generating this increase in survival rates, it must be of the same order of magnitude as the fall in labor force growth needed to induce the same steady state decrease in the entry rate. So in order to generate the same long run impact as a 2 percent decline in labor force growth, to a first approximation, any alternative explanation must induce an average 2 percent decline in exit rates, conditional on age. The feedback effect from the change in the age distribution of firms results in an additional decrease of 1 percent in aggregate exit rates, as explained earlier, resulting in a total decline in exit rates of about 3 percent. In the data the total decline in exit rate is in the order of 1.5 percent, half of that amount. This is almost entirely due to the change in the age distribution, so once controlling for age, there is almost no decline.

To facilitate comparisons with alternative theories, we consider their implications within the context of our model. In each exercise we consider steady state comparisons and ignore transition paths. This is partly the assumption in many of the alternative models and also simplifies considerably the calculations. We take as benchmark the changes in steady state values in our model resulting from the 2 percent decrease in population growth rate observed in the data. These are reported in the third row of Table 6. For reference, the first two rows give the total changes from 1978 to 2014 observed in the data and our results for the benchmark model, when including transitional dynamics.

*Increase in returns to scale for the high type.*— This exercise is meant to capture

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43Needless to say, this is a rough approximation as many of these models have quite different structure.
Table 6: Alternative Explanations

<table>
<thead>
<tr>
<th></th>
<th>Unconditional</th>
<th>Conditional on age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Δ Entry rate</td>
<td>Δ Exit rate</td>
</tr>
<tr>
<td>Data (1978-2014)</td>
<td>−5.6</td>
<td>−1.6</td>
</tr>
<tr>
<td>Our model with transition</td>
<td>−6.1</td>
<td>−1.8</td>
</tr>
<tr>
<td>CHANGE RELATIVE TO 1978 SS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Our model no transition</td>
<td>−2.8</td>
<td>−0.9</td>
</tr>
<tr>
<td>Increase in returns to scale</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Higher entry cost</td>
<td>−2.7</td>
<td>−2.7</td>
</tr>
<tr>
<td>Decrease in mean reversion</td>
<td>−2.8</td>
<td>−2.8</td>
</tr>
</tbody>
</table>

Notes. All values are in percentage points. The highlighted cells indicate targeted moments. The values for Data (1978-2014) are calculated by multiplying the corresponding annual time trends by 36, the number of years in the sample.

a key element in the first group of theories, which is the increase in returns to scale for a selected set of firms. We restrict the increase in returns to scale to those firms in our model that have the potential to become superstar firms, which are the ones in the high-type group. Starting from our initial steady state, we set the increase in returns to scale so that concentration increases by 8.3pp in the new steady state. This is approximately the increase in concentration that we find in our own model, abstracting from the adjustment path. Table 6 reports our results for different key aggregates. The gray cell is the targeted increase in concentration. There is an increase in entry and exit rates, contrary to what we see in the data. This has an intuitive explanation, as the ability of more productive firms to expand more has a competitive adverse effect on those that do not enjoy the increased benefits of scaling and this increases their exit rates. Average firm size increases by more than the benchmark. This increase is explained by a 10.2pp increase in average size conditional on age, while in the benchmark it is zero and slightly negative in the data.

The results observed for this exercise are the net effect of the impact on different groups of firms. Those that benefit from the increase in returns to scale, have higher survival rates and grow. At the same time, the increased competitive pressure from these firms, leads to higher exit rates and downsizing of those firms that do not enjoy the benefits of larger scale, as occurs for those in the low-type group in our exercise. The positively affected group expanded

44The effect on the average size for this group is ambiguous, as firm’s downsize conditional on productivity, but there is better selection resulting from higher exit rates. The net effect in our experiment was just slightly positive.
considerably, leading to the high increase in average size conditional on age observed in Table 6. The exit rate conditional on age increases, reflecting the higher exit rates of the group of firms that do not enjoy the benefits of larger scale.

Increase in entry cost.— Starting at the original steady state, we increase the cost of entry so that in the new steady state the rate of entry decreases by a similar magnitude as in the benchmark. As anticipated, exit rates fall by the same magnitude, exceeding the values observed in the data. In addition we find increases of average firm size and concentration that are roughly three times as large as those in the data. While part of this increase is explained by the aging of firms, there is also a significant increase in average firm size and concentration conditional on age, which is of the opposite sign as the one found in the data. This is explained by the general equilibrium effects of a rise in entry costs that imply an increase in the size and profitability of incumbent firms.

Decrease in the rate of diffusion.— This exercise is meant to capture, in a reduced form way, the impact of a reduction in knowledge diffusion. There has been a growing literature on knowledge diffusion in macro models; see e.g. Lucas and Moll (2014) and Stokey (2020). In most models, knowledge diffusion contributes to mean reversion. In our model, the average gap in size between any two firms is reduced at a rate $(1 - \rho)$, which is a measure of mean reversion. We decrease the degree of mean reversion to match the fall in entry rates as in our benchmark model. Results are reported in Table 6. Exit rates decrease by the same amount as the drop in entry rates, which, again, exceeds the fall in the data. We find an increase in average firm size, close to the one observed in the data and a very large increase in concentration, substantially larger than the observed one. In spite of the increase in average size, there is a large drop when conditioned on age, which exceeds substantially the one observed in the data.45

In models of creative destruction, as in Aghion and Howitt (1992) and Klette and Kortum (2004), there is another force. When knowledge diffuses from one firm to the other, the former loses market share while the latter grows. The frequency of arrival of this event can be taken as a measure of the degree of knowledge diffusion. As this frequency decreases, so does the variance of innovations.

45Firms start small, so a decrease in mean reversion implies a slower convergence to the long run mean, explaining the decrease in average firm size. In the limit, as firms productivity converges to the same long run mean, average size actually increases because of general equilibrium effects ($z$ is higher in the new steady state).
in firm size. In turn, this leads to a decrease in concentration, not an increase. Regarding exit rates (and thus entry rate) there appears to be an ambiguous effect. On the one hand, a decrease in the variance lowers the option value for staying in the market, which raises the exit rate. On the other hand, a lower variance decreases the likelihood of entering the exit zone for surviving firms. In a series of simulations, with decreases in the standard deviation of the innovation ranging from 5 to 50 percent we found almost no effect on entry rates and a very strong reduction in concentration.\(^{46}\)

While each of these alternative forces cannot explain the above empirical regularities by themselves, it is possible that they can complement the role of population growth. (In particular, they might help explain the fall in average firm size and concentration, conditional on age.) As an example, a lower (not higher!) cost of entry, could mitigate the fall in entry rates and rise of average size that we find, and contribute to a fall in average size conditional on age. A decrease in mean reversion, as seen above, can help mitigate the rise in average firm size and contribute to a drop in age-conditional average size. A rise in returns to scale, as modelled here, lead to counterfactual increases in average firm size and concentration, both conditional on age and to a mild increase in entry and exit rates.

It is worth noting that these explanations might play out differently in models of creative destruction. Those models feature another channel: a fall in entry rates reduces the threat faced by incumbent firms. This effect in turn can contribute to a fall in exit rates. The effects on average firm size are likely to be ambiguous.\(^{47}\)

\(^{46}\)Take a simplified Klette and Kortum model where innovations for incumbents arrive at an exogenous rate \(\lambda\) and products are taken over at the rate \(\delta = \lambda + e\), where \(e\) is the rate of entry. The value of a firm with \(n\) products is \(vn\), where \(v = \pi / (r + \delta - \lambda) = \pi / (r + e)\). The equilibrium rate of entry \(e\) is the one that equates the value \(v\) to the cost of entry. The implied process for the innovations in firm size follows a Skellman process (difference between two Poisson distributions) with mean \(\lambda - \delta = -e\) and variance \(\lambda + \delta = 2\lambda + e\), both scaled by the current number of products of the firm \(n\). A decrease in diffusion can be interpreted in this model as a fall in \(\lambda\). Note that it has no effect on the entry rate, while decreasing concentration as a result of a fall in the variance of innovations.

\(^{47}\)While the number of products per firm will increase, there is likely to be fall in profitability and market size per product. In De Ridder (2020), where the driving force is an increase in productivity for a subset of firms, the former effect dominates and average firm size conditional on age increases.
Policy implications. As pointed out in the theory section, our analysis is consistent with various kinds of models of perfect and imperfect competition. Without further details, it is not possible to discuss policy recommendations. In the case of perfect competition, standard welfare theorems apply so the equilibrium is Pareto Optimal. In contrast, the rise in markups in our third example could have negative welfare implications as the share (not size) of high markup firms increases.\textsuperscript{48} Similar considerations can be made about the fall in entry. While the reduction in the number of firms could have resulted in a fall in measured productivity, as pointed out in the paper, the policy implications again are not obvious. In our competitive benchmark, this fall in entry is an optimal response to the decrease in population growth and policies aimed at mitigating this effect would have distorted extensive vs. intensive margins in the allocation of resources. In contrast, in a model where entrants have positive external effects (e.g. Luttmer, 2007) such kind of policies could be justified. More generally, if firms are considered knowledge, such policies might have a role in models of endogenous technical change based on Romer (1990).\textsuperscript{49}

6 Final Remarks

Recent decades have witnessed a decline in firm entry and exit rates, and an increase in employment concentration and average firm size. In contrast, none of these trends appear within firm-age bins. Therefore, the bulk of the aggregate change is explained by the aging of firms as a result of the decline in firm entry. We find that changes in population growth are a big driving force for changes in entry rates. While the direct effect of population demographics on the creation of new firms accounts for one-third of the total effect, the feedback from firm demographics accounts for the remaining two-thirds. The glut of firms born due to the baby boom generate rich transitional dynamics within these feedback effects, accounting for half of the total decline. The glut of firms impact outcomes far into

\textsuperscript{48}Using a model with variable markups, Edmond, Midrigan and Xu (2018) find sizable costs of aggregate markups. However, as in Baqae and Farhi (2019), they find that the observed rise in aggregate markups could have resulted in lower welfare costs as a result of the decrease in markup dispersion.

\textsuperscript{49}Note that our model can be extended by allowing firm productivity to be uniformly impacted by the total number of firms, without changing the equilibrium allocations.
the future. These firms are still around and continue to play a role in determining firm entry and related variables.

Several authors have emphasized the role of other potential factors, such as changes in economies of scale or the incentives for innovation. While other forces are likely to have contributed to explaining the observed trends in the evolution of aggregate productivity and firm dynamics, our analysis underscores that the effect of changes in population growth are too big a source of variation to omit.
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