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DECOMPOSING THE GROWTH OF TOP WEALTH SHARES

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What drives the dynamics of top wealth inequality? I answer this question through an accounting framework that decomposes the growth of the share of aggregate wealth owned by a top percentile into three terms: a *within* term, which is the average wealth growth of individuals initially in the top percentile relative to the economy; a *between* term, which accounts for individuals entering and exiting the top percentile due to changes in their relative rankings; and a *demography* term, which accounts for death and population growth. I obtain closed-form expressions for each term in a wide range of random-growth models in the continuous-time limit. Evidence from the *Forbes* 400 list suggests that the between term accounts for half of the recent rise in top wealth inequality.

KEYWORDS: Wealth inequality, Pareto distribution, accounting decomposition.

1. INTRODUCTION

The share of aggregate wealth owned by the wealthiest households in the U.S. has increased greatly in the past fourty years (Saez and Zucman, 2016, Smith et al., 2021). We know very little about what drives this phenomenon. One hypothesis is that wealthy households have grown faster than the rest of the economy (Piketty, 2014). However, this view neglects the fact that the composition of households at the top of the wealth distribution has dramatically changed over time: for instance, less than 10% of the households in the 1983 *Forbes* list of the richest households in the United States were still in the list thirty years later.

In this paper, I study theoretically and empirically the effect of these composition changes on the dynamics of top wealth shares. More precisely, I propose a novel accounting framework to decompose the growth of the average wealth owned by households in a top percentile. When individual wealth is normalized by the average wealth in the economy, this is equivalent to decomposing the growth of a top percentile wealth share.

The accounting framework breaks down the growth of the average wealth in a top percentile into three terms. The first term (*within*) is the average wealth growth of households that are initially in the top percentile. The second term (*between*) measures the contribution of households entering or exiting the top percentile due to higher-or-lower-than-average wealth growth. The third term (*demography*) measures the contribution of households entering or exiting the top percentile due to death or population growth. Intuitively, this accounting decomposition identifies three distinct drivers of a rise in the share of aggregate wealth owned by a top percentile: a high growth rate of existing fortunes relative to the economy (*within*), an inflow of new fortunes in the top percentiles (*between*), or a low rate of population renewal (*demography*).

Using continuous-time methods, I obtain simple analytical expressions for the within, between, and demography terms in a wide range of random-growth models. These analytical expressions shed light on the underlying forces driving the dynamics of top wealth shares.

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They can also help researchers to quantify the effect of composition changes in the absence of panel data. In particular, I show that, when wealth follows a diffusion process, the between term is given by $1/2(\zeta-1)\nu^2$, where ν denotes the idiosyncratic of wealth around the percentile threshold and where ζ denotes the local Pareto exponent of the wealth distribution. In the absence of inheritance, the demography term is given by $-(\delta+\eta)/\zeta$, where δ denotes the death rate and where η denotes the population growth rate.

I then turn to a number of empirical applications. I first use my accounting framework to decompose the growth of the share of wealth owned by households in the Forbes 400 list. This is a natural application for my framework because it allows me to track individual households over time. The annual growth rate of the Forbes 400 wealth share, which averages to 3.9% between 1983 and 2017, can be decomposed as follows: the average within term is 3.0%, the average between term is 2.5%, and the average demography term of -1.5%. Put differently, without the between term (which captures the contribution of the dispersion in wealth growth among wealthy households), the annual growth rate of the Forbes 400 wealth share would have been 1.4% instead of 3.9%.

I document substantial heterogeneity in the relative importance of each term over time. In particular, while the between term plays an outsized role at the beginning of the sample, its importance has gradually decayed over time, from 3.3% in the 1980s to 1.5% in the 2010s. I decompose this decline using my analytical expression for the between term in a diffusion model: most of the decline is driven by a thickening of the right tail of the wealth distribution (ζ decreases from 1.6 to 1.4), rather than a decrease in the dispersion of wealth growth (ν only decreases from 0.27 to 0.25). Put differently, the decline in the between term mostly reflects the fact that, as wealth inequality gradually increased, it became harder for new fortunes to displace existing ones.

I then use my analytical results to quantify the between term for the top 1%, 0.1%, and 0.01% of the wealth distribution over the 20th century, for which panel data are not available. I construct a yearly estimate of ζ, the local Pareto exponent, from cross-sectional data on the wealth distribution. I construct a yearly estimate of ν, the standard deviation of wealth growth, using the standard deviation of firm-level returns. I then combine these two estimates to obtain a yearly estimate of a "model-implied" between term, $1/2(\zeta-1)\nu^2$. I find that this between term matches the inverted U-shape of top wealth inequality over the 20th century. It first peaked during the Great Depression, remained low during World War II and the postwar economic boom, before peaking again during the technological revolutions of the 1980s and 1990s. Overall, this exercise suggests that low-frequency fluctuations in the between term are a central force behind the fluctuations in wealth inequality observed during the 20th century.

Finally, I discuss the implication of the accounting framework for wealth mobility. I focus on a simple notion of downward mobility: the average time a wealthy household remains in a top percentile. I prove that a rise in the average wealth growth of top households (*within*) decreases mobility, while a rise in the dispersion of their wealth growth (*between*) increases mobility. Together with my empirical findings, this suggests that the recent rise in top wealth inequality is associated with a permanent rise in downward wealth mobility, even as wealth inequality continues to increase.

Related Literature. This paper is motivated by a large empirical literature documenting the dynamics of top wealth shares in the U.S. (Kopczuk and Saez, 2004; Saez and Zucman, 2016; Kuhn et al., 2020). This literature tends to interpret the recent rise in top wealth shares as a rise in the wealth growth of top households relative to the economy (e.g., Piketty, 2014, Mian et al., 2020). In particular, Saez and Zucman (2016) define a "synthetic saving rate" based on the difference between the growth of the average wealth in top percentiles and the average return of top individuals. My paper clarifies that this synthetic saving rate is actually the sum of an "actual" household saving rate and two other terms due to composition effects: a between term (due to the dispersion of wealth growth) and a demography term (due to birth, death, and population growth).

Recent empirical papers on wealth inequality stress the importance of idiosyncratic shocks for wealth inequality. Benhabib et al. (2011) and Benhabib et al. (2015) examine the stationary wealth distribution in an economy with idiosyncratic returns. Fagereng et al. (2016) and Bach et al. (2017) emphasize the heterogeneity in asset returns among households in Norway and Sweden, respectively. Benhabib et al. (2019) examine the role of idiosyncratic volatility through the lens of a quantitative model.[1] Campbell et al. (2019) decompose the change in the variance of log wealth into a term due to differences in expected wealth growth and a term due to differences in unexpected wealth shocks. Relative to this literature, the contribution of this paper is to quantify the role of these idiosyncratic shocks for the growth of top wealth shares.

This work also adds to the theoretical literature that studies inequality through the lens of random-growth models (Wold and Whittle, 1957; Jones, 2015; Luttmer, 2012; Gabaix et al., 2016; Jones and Kim, 2018). A fundamental tool in this literature is the Kolmogorov forward equation, which expresses the dynamics of the wealth density in terms of the dynamics of individual wealth. My accounting framework corresponds to an "integrated" version of this equation, as it expresses the dynamics of top wealth shares in terms of the dynamics of individual wealth.[2]

Finally, my accounting framework is related to Baily et al. (1992), Foster et al. (2008), and Melitz and Polanec (2015), who decompose the change of firm productivity in the economy. My paper can be seen as an extension of these frameworks to decompose the change of an average quantity *for a particular subgroup of the population*, a top percentile. This requires accounting not only for the entry and exit of individuals in and out of the economy (*demography*) but also for the flow of existing individuals in and out of the top percentile (*between*).

Roadmap. The rest of the paper is organized as follows. Section 2 presents the accounting framework. Section 3 characterizes its continuous-time limit in a wide range of random-growth models. In Section 4, I apply this framework to decompose the growth of the Forbes 400 wealth share, and in Section 5, I examine the role of the between term for the top 1%, 0.1%, and 0.01% in the U.S. over the 20th century. Section 6 discusses the implications of my findings for the relationship between inequality and mobility, and Section 7 concludes.

2. ACCOUNTING FRAMEWORK

This section presents an accounting framework to decompose the growth of the average wealth in a top percentile. When wealth is *normalized* by the average wealth in the economy, this can be interpreted as a decomposition for the top percentile wealth *share*. To develop intuition, Section 2.1 first covers the case without demographic changes (i.e., without death or birth), while Section 2.2 covers the more general case with demographic changes.

2.1. *Case without Demographic Changes*

For the sake of intuition, I first present the accounting framework without demographic changes. Let $t \in \{0, 1\}$ denote time and let Ω denote the set of individuals in the economy,

[1] Existing theories to explain the concentrated portfolios of the rich include moral hazard, expertise, taste, or asymmetric information (see, e.g., Di Tella, 2016; Haddad et al., 2014; Eisfeldt et al., 2017; Roussanov, 2010).

[2] A related paper is Steinbrecher and Shaw (2008), who characterizes the dynamics of quantiles in continuous-time.

which is fixed over time (for now). Consider a given top percentile $p \in (0,1]$ of the wealth distribution (e.g. the top $p = 1\%$) and let $\mathcal{P}_t \subset \Omega$ denote the subset of individuals who are in the top percentile at time t.

Denote w_{it} the wealth of individual i at time t. As individuals experience heterogeneous wealth growth between $t = 0$ and $t = 1$, the set of individuals in the top percentile typically changes over time; that is, $\mathcal{P}_0 \neq \mathcal{P}_1$. More precisely, denoting $\mathcal{O}$ the set of individuals who get out of the top percentile during the time period and $\mathcal{I}$ the set of individuals who get in, we can write $\mathcal{P}_1 = (\mathcal{P}_0 \setminus \mathcal{O}) \cup \mathcal{I}$.[3]

This equality implies that the growth of the total wealth in the top percentile can be decomposed into two terms:

$$\frac{\sum_{i\in\mathcal{P}_1} w_{i1} - \sum_{i\in\mathcal{P}_0} w_{i0}}{\sum_{i\in\mathcal{P}_0} w_{i0}} = \underbrace{\frac{\sum_{i\in\mathcal{P}_0} w_{i1} - \sum_{i\in\mathcal{P}_0} w_{i0}}{\sum_{i\in\mathcal{P}_0} w_{i0}}}_{\text{Within}} + \underbrace{\frac{\sum_{i\in\mathcal{I}} w_{i1} - \sum_{i\in\mathcal{O}} w_{i1}}{\sum_{i\in\mathcal{P}_0} w_{i0}}}_{\text{Between}}. \tag{1}$$

The first term (*within*) holds constant the composition of individuals in the top percentile: it is the wealth growth of individuals who are initially in the top percentile—whether or not they remain in the top percentile by the end of the period. The second term (*between*) accounts for composition changes in the top percentile: it depends on the difference between the wealth of individuals getting into the top percentile and the wealth of individuals getting out of it, relative to the initial wealth of individuals in the top.

By the very definition of a top percentile, the between term is always greater than (or equal to) zero. This can be seen as a selection bias: only individuals with a high enough wealth growth remain in the top percentile over time. As a result, the growth of wealth in the top percentile is systematically higher than the wealth growth of individuals initially in the top percentile.

2.2. *Case with Demographic Changes*

For the sake of simplicity, the preceding analysis considered the case of a fixed population. In reality, the population of individuals in the economy changes over time. This generates additional composition changes in the top percentile. I now extend my accounting framework to account for these demographic changes.

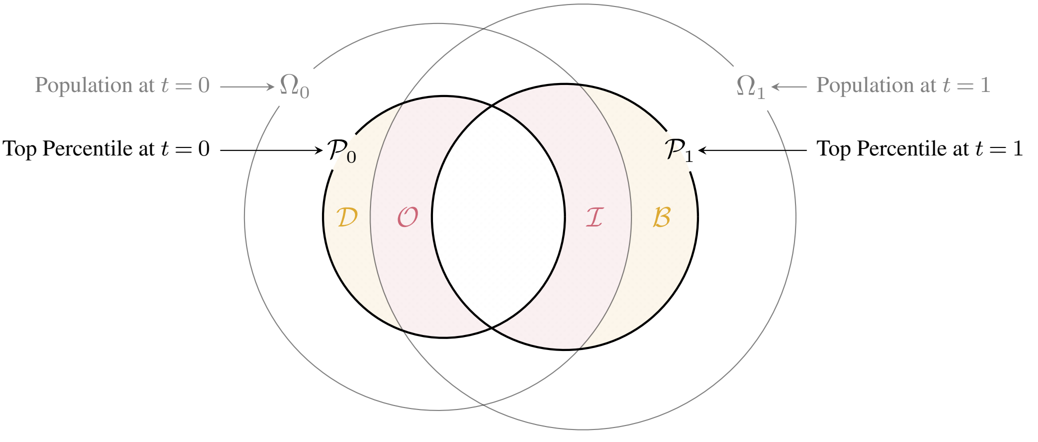
Formally, I now allow the set of individuals in the economy, Ω_t, to change over time; that is, $\Omega_0 \neq \Omega_1$. As a result, the composition of individuals in the top percentile can now change for two distinct reasons: either because of the heterogeneity in wealth growth among existing individuals (*between*) or because individuals enter and exit the economy over time (*demography*).

More precisely, the set of individuals who enter the top percentile can now be decomposed into two groups: $\mathcal{I}$, the set of individuals who were already in the economy at time $t = 0$ (i.e., who get into the top because of a higher-than-average wealth growth), and $\mathcal{B}$, the set of individuals who were not in the economy at time $t = 0$ (i.e., who are "born" into the top percentile).

Similarly, the set of individuals who exit the top percentile can now be decomposed into two groups: $\mathcal{O}$, the set of individuals who remain in the economy at $t = 1$ (i.e., who get out of the

[3]Here, and elsewhere, "$\mathcal{G}_1 \setminus \mathcal{G}_2$" denote the set of individuals in the set $\mathcal{G}_1$ who are not in the set $\mathcal{G}_2$.

FIGURE 1.—Venn Diagram of Composition Changes in a Top Percentile



[a]The figure represents Ω_t, the set of all individuals in the economy, and $\mathcal{P}_t \subset \Omega_t$, the subset of individuals in the top percentile, at time $t \in \{0, 1\}$. The intersections of these four sets define the four subsets used in the accounting decomposition: $\mathcal{I} = (\mathcal{P}_1 \setminus \mathcal{P}_0) \cap \Omega_0$, the set of individuals who get in the top percentile from another part of the distribution; $\mathcal{B} = \mathcal{P}_1 \setminus \Omega_0$, the set of individuals who are born into the top percentile; $\mathcal{O} = (\mathcal{P}_0 \setminus \mathcal{P}_1) \cap \Omega_1$, the set of individuals who get out of the top percentile but remain in the economy; and $\mathcal{D} = \mathcal{P}_0 \setminus \Omega_1$, the set of individuals initially in the top percentile who exit the economy.

top because of a lower-than-average wealth growth), and $\mathcal{D}$, the set of individuals who exit the economy at $t = 1$ (i.e., who "die").[4]

Figure 1 plots a Venn diagram to illustrate the relationship between these four sets. Note that these four sets fully summarize the composition changes in the top percentile between $t = 0$ and $t = 1$; that is, $\mathcal{P}_1 = (\mathcal{P}_0 \setminus (\mathcal{O} \cup \mathcal{D})) \cup \mathcal{I} \cup \mathcal{B}$.

I introduce a few more notations before presenting the full accounting decomposition. For any set $\mathcal{G}$, denote $n_{\mathcal{G}}$ the number of individuals in $\mathcal{G}$ relative to the number of individuals in $\mathcal{P}_1$, and denote $w_{\mathcal{G},t}$ the average wealth of individuals in $\mathcal{G}$ at time t. Finally, denote q_1 the wealth of the last person in the top percentile at time $t = 1$ (i.e., the top $p-$quantile). The following proposition generalizes the accounting decomposition (1) in the presence of demographic changes.

PROPOSITION 1: *The growth of the average wealth in a top percentile can be decomposed into three terms:*

$$\frac{w_{\mathcal{P}_1,1} - w_{\mathcal{P}_0,0}}{w_{\mathcal{P}_0,0}} = \underbrace{\frac{w_{\mathcal{P}_0 \setminus \mathcal{D},1} - w_{\mathcal{P}_0 \setminus \mathcal{D},0}}{w_{\mathcal{P}_0 \setminus \mathcal{D},0}}}_{\textit{Within}} + \underbrace{\underbrace{n_{\mathcal{I}} \frac{w_{\mathcal{I},1} - q_1}{w_{\mathcal{P}_0,0}}}_{\textit{Inflow}} + \underbrace{n_{\mathcal{O}} \frac{q_1 - w_{\mathcal{O},1}}{w_{\mathcal{P}_0,0}}}_{\textit{Outflow}}}_{\textit{Between}}$$

[4]I use the terms birth and death liberally: they simply refer to composition changes in the overall population observed by the econometrician. In particular, in the case in which top shares are constructed from a rotating panel of individuals, birth and death would refer to individuals entering and exiting the panel over time.

$$+\underbrace{\underbrace{n_{\mathcal{B}}\frac{w_{\mathcal{B},1}-q_1}{w_{\mathcal{P}_0,0}}}_{\textit{Birth}}\underbrace{-n_{\mathcal{D}}\frac{\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}}w_{\mathcal{D},0}-q_1}{w_{\mathcal{P}_0,0}}}_{\textit{Death}}\underbrace{-(1-n_{\mathcal{P}_0})\frac{\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}}w_{\mathcal{P}_0,0}-q_1}{w_{\mathcal{P}_0,0}}}_{\textit{Population Growth}}}_{\textit{Demography}}. \quad (2)$$

The first term (*within*) corresponds to the growth of the average wealth of individuals initially in the top percentile. Note that, because some individuals in the top percentile die from one period to the next, the within term now only accounts for the wealth growth of individuals who remain in the economy (i.e., in $\mathcal{P}_0 \setminus \mathcal{D}$).

The second term (*between*) measures the contribution of composition changes in the top percentile due to individuals present in the economy at $t = 0$ and $t = 1$. It is the sum of an inflow and an outflow term. To understand the expression for each term, consider an individual below the percentile threshold who gets into the top percentile after a higher-than-average wealth growth. Because the total number of individuals in the top percentile is constant, they displace the last individual in the top percentile, with wealth q_1. Overall, this *inflow* increases the average wealth in the top percentile by the fraction of individuals getting into the top percentile, $n_{\mathcal{I}}$, times the difference between the average wealth of entrants and the wealth at the percentile threshold, $w_{\mathcal{I},1} - q_1$.

Symmetrically, consider an individual above the percentile threshold who drops out of the top percentile after a lower-than-average wealth growth. Because the total number of individuals in the top percentile is constant, they are replaced by the individual just below the percentile threshold, with wealth q_1. Overall, this *outflow* increases the average wealth in the top percentile, relative to change in the average wealth of top households initially in the top, by the fraction of individuals dropping out of the top percentile, $n_{\mathcal{O}}$, times the difference between wealth at the threshold and the average wealth of dropouts, $q_1 - w_{\mathcal{O},1}$.

Note that, in the presence of demographic changes, the fraction of existing individuals getting in the top percentile (i.e., in $\mathcal{I}$) is typically different from the fraction of individuals getting out of it (i.e., in $\mathcal{O}$). This makes it is essential to compare the wealth of each individual getting in or out of the top percentile to q_1, the wealth at the percentile threshold.

The third term (*demography*) measures the contribution of composition changes in the top percentile due to individuals entering or exiting the economy between $t = 0$ and $t = 1$. It is the sum of three terms that, respectively, account for birth, death, and population growth. The first term (*birth*) is positive; it accounts for the contribution of individuals who are born into the top percentile. Each birth into the top percentile increases wealth in the top percentile by the difference between the wealth of the newborn and the wealth at the percentile threshold. The second term (*death*) is negative; it accounts for the contribution of individuals in the top percentile who die during the time period. Each death decreases wealth in the top percentile by the difference between the (time adjusted) wealth of the deceased individual and the wealth at the percentile threshold. Finally, the third term (*population growth*) is negative; it accounts for the contribution of population growth: since a top percentile contains a constant fraction of the population, an increase in population size increases the number of individuals in the top percentile. This decreases the average wealth in the top percentile by the difference between the (time adjusted) average wealth in the top percentile and the wealth at the percentile threshold.

While the between term is always greater than (or equal to) zero, the demography term tends to be lower (or equal to) zero.[5] Note that, in the absence of demographic changes, the demography term is zero, and therefore the accounting decomposition reverts to Equation (1).

[5] See Section 3 for a more formal statement.

Average Wealth versus Top Wealth Share. The accounting framework presented in Proposition 1 decomposes the growth of the average wealth in a top percentile. When individual wealth is *normalized* by the average wealth in the economy, this is equivalent to decomposing the growth of the top percentile wealth *share* (this comes from the fact that the growth of the top wealth share is equal to the growth of the average normalized wealth in the top percentile).[6] In this case, the within term should be interpreted as the wealth growth of households initially in the top percentile relative to the growth of the average wealth in the economy.

Arithmetic versus Logarithmic Growth. The accounting framework presented in Proposition 1 decomposes the *arithmetic* growth of the average wealth in a top percentile; that is, $(w_{\mathcal{P}_1,1} - w_{\mathcal{P}_0,0})/w_{\mathcal{P}_0,0}$. In Appendix B, I extend this accounting framework to decompose its *logarithmic* growth; that is, $\log(w_{\mathcal{P}_1,1}/w_{\mathcal{P}_0,0})$. This makes it easier to aggregate the result of the decomposition over multiple time periods. Still, the two decompositions give very similar results over short horizons (e.g., a year). In fact, in a wide range of random growth models, these decompositions converge to the same limit as the time horizon tends to zero. The next section characterizes this limit analytically.

3. ANALYTICAL RESULTS

This accounting framework can be seen as an "integrated" version of the Kolmogorov forward equation. I now use this connection to characterize the limit of the accounting framework as the time horizon tends to zero. I obtain closed-form expressions for the within, between, and demography terms in a wide range of random growth models. These expressions shed light on the mechanical forces driving the dynamics of top wealth shares. They can also be useful to quantify the between and demography terms in the absence of panel data.

The exposition follows the previous section: Section 3.1 starts with a baseline model without demographic changes (i.e., fixed population) to focus on the between term, while Section 3.2 considers a more general model with demographic changes (i.e., death and population growth) to focus on the demography term.

3.1. *Case without Demographic Changes*

3.1.1. *Diffusion Model*

Wealth Dynamics. Consider an economy populated by a continuum of agents with mass one that is fixed over time. Time $t \in [0, \infty)$ is continuous. I assume that the law of motion of the wealth of individual i at time t, w_{it}, follows a Markov diffusion process:

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_t(w_{it})\,\mathrm{d}t + \nu_t(w_{it})\,\mathrm{d}B_{it}, \tag{3}$$

where B_{it} is a standard idiosyncratic Brownian motion for individual i. The geometric drift $\mu_t(\cdot)$ and geometric volatility $\nu_t(\cdot)$ of wealth are allowed to depend on the wealth level.

The following set of regularity conditions ensures that the wealth density at any time $t \geq 0$, denoted by g_t, is smooth, positive on $\mathbb{R}_+$, and with finite mean: (i) the initial wealth density, $g_0 : \mathbb{R}_+ \to \mathbb{R}_+$, has finite mean; (ii) $\mu_t(\exp(.))$ and $\nu_t(\exp(.))$ possess bounded continuous

[6] Formally, denoting S_t the share of aggregate wealth owned by a top percentile p at time t, we have $S_t = p \cdot w_{\mathcal{P}_t,t}/w_{\Omega_t,t}$. This implies that the growth of S_t is equal to the growth of the normalized wealth in the top percentile, $w_{\mathcal{P}_t,t}/w_{\Omega_t,t}$.

derivatives of all orders; and (iii) idiosyncratic volatility never vanishes; that is, there exists $\epsilon > 0$ such that $\nu_t(.) \geq \epsilon$.[7]

Top Percentile. For a given top percentile $p \in (0,1)$ of the wealth distribution, denote q_t the wealth of the last person in the top percentile (i.e., the top p quantile) and $w_{\mathcal{P}t}$ the average wealth in the top percentile p. The following proposition characterizes the dynamics of the average wealth in the top percentile p in terms of the dynamics of individual wealth.

PROPOSITION 2: *Assuming that wealth follows the diffusion process* (3)*, the average wealth in the top percentile, $w_{\mathcal{P}t}$, follows the law of motion:*

$$\frac{\mathrm{d}w_{\mathcal{P}t}}{w_{\mathcal{P}t}} = \underbrace{\mathrm{E}^{g_t(\cdot)\cdot}\left[\mu_t(w)|w \geq q_t\right]\mathrm{d}t}_{\textit{Within}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{pw_{\mathcal{P}t}}\nu_t(q_t)^2\,\mathrm{d}t}_{\textit{Between}}, \tag{4}$$

where $\mathrm{E}^{g_t(\cdot)\cdot}$ denotes the wealth-weighted expectation with respect to the wealth density g_t.[8]

This proposition characterizes the asymptotic limit of the accounting framework (2) as the time period tends to zero. In particular, the proposition shows that the between term is a "first order" term; that is, it does not become negligible over short time horizons. The *within* term is the wealth-weighted average geometric drift in the top percentile group. The *between* term depends on the variance of wealth growth at the percentile threshold, $\nu_t(q_t)^2$, but also on the shape of the wealth distribution, $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$.

Heuristic Derivation. I now present a heuristic derivation of Proposition 2 to provide more intuition on these analytical expressions for the within and between terms (see Appendix A for a formal proof which relies on the Kolmogorov forward equation).

Let us consider the economy during t and $t+\Delta t$, where Δt is a short period of time. At the first order in Δt, the diffusion process for wealth can be approximated by the following binomial process:

$$w_{it+\Delta t} = \begin{cases} (1+\mu_t(w_{it})\Delta t + \nu_t(w_{it})\sqrt{\Delta t})w_{it} & \text{with probability half,} \\ (1+\mu_t(w_{it})\Delta t - \nu_t(w_{it})\sqrt{\Delta t})w_{it} & \text{otherwise.} \end{cases} \tag{5}$$

Applying the accounting framework (2) on this binomial process between t and $t+\Delta t$ gives that the growth of the average wealth in the top percentile can be decomposed into a within and a between term.

The within term is defined in (2) as the wealth-weighted average wealth growth of individuals initially in the top percentile. This gives $\mathrm{E}^{g_t(\cdot)\cdot}\left[\mu_t(w)|w \geq q_t\right]\Delta t$.

The between term is defined in (2) as the sum of an inflow and an outflow term. The inflow term is measured as the wealth of individuals who enter the top percentile after receiving a positive wealth shock minus the wealth of the percentile threshold, relative to the total wealth in the top percentile. This gives[9]

$$\text{Inflow} = \frac{1}{pw_{\mathcal{P}t}}\int_{\frac{q_t}{1+\nu_t(q_t)\sqrt{\Delta t}}}^{q_t}\left(\left(1+\nu_t(q_t)\sqrt{\Delta t}\right)w - q_t\right)\frac{1}{2}g_t(w)\,\mathrm{d}w + o(\Delta t)$$

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[7] See the proof of Proposition 2 in Appendix A.

[8] That is, $\mathrm{E}^{g_t(\cdot)\cdot}\left[\mu_t(w)|w \geq q_t\right] = \left(\int_{q_t}^{\infty}\mu_t(w)wg_t(w)\,\mathrm{d}w\right) / \left(\int_{q_t}^{\infty}wg_t(w)\,\mathrm{d}w\right)$.

[9] Here, and elsewhere, "$o(\Delta t)$" denotes an expression that converges faster than Δt; that is, $\lim_{\Delta t \to 0}|o(\Delta t)|/\Delta t = 0$.

$$= \left(\frac{1}{2} \frac{g_t(q_t) q_t}{p} \nu_t(q_t) \sqrt{\Delta t} \right) \cdot \left(\frac{1}{2} \frac{q_t}{w_{\mathcal{P}t}} \nu_t(q_t) \sqrt{\Delta t} \right) + o(\Delta t),$$

where the second line approximates the integral using the midpoint rule. As in the accounting framework (2), the inflow term can be written as the product of two terms: the first term corresponds to the relative mass of individuals who enter the top percentile between t and $t + \Delta t$ while the second term corresponds to the average effect of an entrant on the growth of the average wealth in the top percentile.[10]

Symmetrically, the outflow term is measured as the difference between the wealth at the percentile threshold and the wealth of individuals who exit the top percentile after receiving a negative wealth shock. Applied to the binomial process (5), this definition gives

$$\begin{aligned} \text{Outflow} &= \frac{1}{p w_{\mathcal{P}t}} \int_{q_t}^{\frac{q_t}{1-\nu_t(q_t)\sqrt{\Delta t}}} \left(q_t - \left(1 - \nu_t(q_t)\sqrt{\Delta t}\right) w \right) \frac{1}{2} g_t(w) \, \mathrm{d}w + o(\Delta t) \\ &= \left(\frac{1}{2} \frac{g_t(q_t) q_t}{p} \nu_t(q_t) \sqrt{\Delta t} \right) \cdot \left(\frac{1}{2} \frac{q_t}{w_{\mathcal{P}t}} \nu_t(q_t) \sqrt{\Delta t} \right) + o(\Delta t). \end{aligned}$$

As in the accounting framework (2), the outflow term can be written as a product of two terms: the first term corresponds to the fraction of individuals who exit the top percentile between t and $t + \Delta t$ while the second term corresponds to the average effect of an exit on the growth of the average wealth in the top percentile.[11]

Note that the inflow term is equal to the outflow term at the first order in Δt. Combining these two expressions gives that the between term is equal to $1/2 \cdot g_t(q_t) q_t^2 / (p w_{\mathcal{P}t}) \cdot \nu_t(q_t)^2 \Delta t$ at the first order in Δt, which concludes the proof.

Application. I now discuss two ways in which Proposition 2 can be useful for applied researchers. First, for researchers with access to panel data (i.e., who can directly measure the between term in the data), the proposition suggests a way to further decompose the between term into a term due to the dispersion of wealth shocks, $\nu_t(q_t)^2$, and a term due to the shape of the wealth distribution, $g_t(q_t) q_t^2 / (p w_{\mathcal{P}t})$. Along these lines, I later use this decomposition to shed light on the decline of the between term observed for the Forbes 400 wealth share in Section 4 as well as on the difference between the between term observed in the U.S. and in China in Appendix E.

Second, for researchers without access to panel data (i.e., who cannot measure the between term in the data), Proposition 2 provides a method to construct a "model–implied" between term. Indeed, the shape of the wealth distribution, $g_t(q_t) q_t^2 / (p w_{\mathcal{P}t})$, can be estimated using cross-sectional data while the level of wealth volatility, $\nu_t(q_t)$, can often be proxied using external sources. I use this method to construct a "model–implied" between term for top wealth percentiles over the 20th century in Section 5.

Stationary Case. One of the most pervasive regularities in economics is that many distributions, including the wealth distribution, have a Pareto tail; that is, their densities satisfy

$$g(w) \sim C w^{-\zeta - 1} \text{ as } w \to \infty, \tag{6}$$

where $C > 0$ is a constant and $\zeta > 0$ is called the Pareto exponent.[12]

[10] These two terms correspond to $n_{\mathcal{I}}$ and $(w_{\mathcal{I},1} - q_1)/w_{\mathcal{P}_0,0}$, respectively.
[11] These two terms correspond to $n_{\mathcal{O}}$ and $(q_1 - w_{\mathcal{O},1})/w_{\mathcal{P}_0,0}$, respectively.
[12] Here, and elsewhere, "$f(x) \sim g(x)$" for two functions f and g means $\lim_{x \to \infty} f(x)/g(x) = 1$.

Under the following set of assumptions, the process (3) converges to a stationary distribution with a Pareto tail: (i) the drift and volatility of wealth are constant over time (i.e., $\mu_t(w) = \mu(w)$ and $\nu_t(w) = \nu(w)$); (ii) they have finite limits as wealth tends to infinity (i.e., $\mu(w) \to \mu < 0$ and $\nu(w) \to \nu$ as $w \to \infty$); and (iii) the wealth drift is large enough at wealth tends to zero (i.e., $\mu(w) - \nu(w)^2/2 \geq K|\ln w|^\beta$ near 0, for some $K > 0$ and $\beta > 1$). Moreover, in this case, the Pareto exponent of the wealth distribution ζ is higher than one (i.e., the distribution has a finite mean).[13]

In this case, the expression for the between term takes a particularly simple form: taking the limit $p \to 0$ in Proposition 2 gives

$$0 = \underbrace{\mu \,\mathrm{d}t}_{\text{Within}} + \underbrace{\frac{\zeta - 1}{2}\nu^2 \,\mathrm{d}t}_{\text{Between}}. \tag{7}$$

This expression says that the between term is asymptotically constant in the right tail and that it only depends on the shape of the wealth distribution through its Pareto exponent ζ. This comes from the fact that, for a distribution with a Pareto tail, the ratio between the mass of individuals around a percentile threshold and the mass of individuals in the top percentile, $g(q)q/p$, converges to ζ as $p \to 0$, while the ratio between wealth at a top percentile threshold and the average wealth above the threshold, $q/w_{\mathcal{P}t}$, converges to $1 - 1/\zeta$.[14] As a result, the product between the two terms, $g(q)q^2/p$, converges to $\zeta - 1$.

Moreover, this expression says that the between term *increases* with the Pareto exponent ζ (i.e., it decreases with the level of wealth inequality). This happens for two reasons. First, when inequality is high, the wealth distribution is more spread out around the percentile threshold, which decreases the fraction of individuals who enter or exit the top percentile per unit of time. Second, and more importantly, when inequality is high, wealth at the percentile threshold is small relative to the average wealth in the top percentile, which decreases the contribution of each entry or exit to the top percentile.[15] In the limit where the Pareto exponent ζ becomes close to 1 (Zipf's law), wealth at the percentile threshold becomes infinitesimally small compared to the average wealth in the top percentile, and therefore the between term is close to zero.

Additionally, note that Equation (7) can also be used to pin down the Pareto exponent ζ of the stationary distribution. The corresponding expression, $\zeta = 1 - 2\mu/\nu^2$, is well known in the inequality literature.[16] The key insight is that this expression can be interpreted as a balance equation for the share of wealth owned by a top percentile: the steady state Pareto exponent ζ must be such that the (negative) within term μ is exactly compensated by the (positive) between term $1/2(\zeta - 1)\nu^2$.

Finally, this expression is useful to obtain a back-of-the-envelope approximation for the between term. In the U.S., the Pareto exponent of the wealth distribution is approximately $\zeta = 1.5$.[17] As a result, for an idiosyncratic volatility of wealth growth $\nu = 0.15$, we can expect the between term to be around 0.5% per year. Doubling the idiosyncratic volatility to $\nu = 0.30$

[13]The existence of the stationary wealth distribution follows from Karlin and Taylor (1981). The fact that it has a Pareto tail follows from Gabaix et al. (2016). We could replace assumption (iii) by a reflecting boundary at some wealth level.

[14]Here, and elsewhere, I remove the t subscript to refer to quantities associated with the stationary wealth distribution.

[15]As discussed in the heuristic proof of Proposition 2, these two terms correspond to $(g(q)q/p)\,\nu\sqrt{\Delta t} \to \zeta\nu\sqrt{\Delta t}$ and $1/2(q/w_{\mathcal{P}})\nu\sqrt{\Delta t} \to 1/2(1 - 1/\zeta)\nu\sqrt{\Delta t}$, which both increase in ζ.

[16]See, for instance, Gabaix (2009).

[17]See, for instance, Klass et al. (2006).

would quadruple the between term to 2% per year. As shown below, this is indeed close to the between term measured for the Forbes 400 wealth share.

3.1.2. *Extensions*

For the sake of simplicity, the preceding analysis focused on the case in which wealth follows a diffusion process. I now discuss the between term in three extensions of the baseline model, which are fully discussed in Appendix C.

Type Heterogeneity. The preceding analysis assumed that the drift and volatility of wealth only depended on the wealth level. In reality, wealth dynamics also depend on other characteristics, such as age or preferences.

To examine the effect of this type of heterogeneity, I consider an economy composed of several groups of individuals, where each groups differ in their drifts and idiosyncratic volatilities. In this economy, the within and between terms remain essentially the same: as in the baseline model, they depend, respectively, on the wealth-weighted average drift of individuals in the top percentile and on the average idiosyncratic variance of individuals at the percentile threshold. In particular, heterogeneous drifts only have second-order effects on the between term.

Aggregate Shocks. The preceding analysis assumed that individuals were only exposed to idiosyncratic shocks. In reality, some shocks tend to be correlated across individuals.

To examine the effect of correlated shocks, I consider an economy in which different types of individuals have heterogeneous exposures to an aggregate Brownian Motion. I find that, in this economy, the within term is stochastic: its exposure to the aggregate Brownian Motion is given by the wealth-weighted average exposure of individuals in the top percentile. The between term, however, is still deterministic; it depends on the idiosyncratic variance of individuals at the percentile threshold as well as on the cross-sectional variance of their exposures to the aggregate shock.

Jumps. The preceding analysis assumed that individual wealth followed a diffusion process (e.g., log normal innovations). This implied that during a short period of time, only individuals close to the percentile threshold could enter or exit the top percentile. In reality, the rapid rise of a few individuals at the top of the distribution suggests that jumps may play an important role in composition changes in the top percentile.

To understand how jumps (e.g., non-normal innovations) affect the formula for the between term, I consider an economy in which individual wealth follows a jump-diffusion process. In this case, the between term does not only depend on variance of (log) wealth growth and the wealth density at the percentile threshold: it also depends on all of the higher-order cumulants of wealth growth as well as on the higher-order derivatives of the wealth density around the percentile threshold.

When cumulants do not depend on the wealth level and when the stationary wealth distribution has a Pareto tail, the expression for the between term takes a particularly simple form as

$p \to 0$:[18]

$$\text{Between} = \frac{\zeta - 1}{2}\text{sd}^2\,\mathrm{d}t + \frac{\zeta^2 - 1}{3!}\cdot\text{skewness}\cdot\text{sd}^3\,\mathrm{d}t + \frac{\zeta^3 - 1}{4!}\cdot\text{excess kurtosis}\cdot\text{sd}^4\,\mathrm{d}t + \text{higher-order terms}\ldots \tag{8}$$

where $\zeta > 1$ denotes the Pareto exponent of the wealth distribution. While the first term is the same as the one obtained in the diffusion (log-normal) model, the other terms in the sum reflect the contribution of higher-order cumulants of log wealth growth, such as its skewness and excess kurtosis. In particular, positive skewness and positive excess kurtosis increase the between term relative to the diffusion case.

Note that the relative importance of higher-order cumulants increases with ζ; that is, it decreases with the level of wealth inequality. Intuitively, the higher the level of wealth inequality, the more spread out the wealth distribution and therefore the less likely it is to enter the top percentile far away from the percentile threshold. To take a quantitative example, going from a distribution with $\zeta = 2.5$ (the approximate Pareto exponent for the distribution of labor income) to $\zeta = 1.5$ (the approximate Pareto exponent for the distribution of wealth), the term due to the variance of wealth shocks is divided by 3, the term due to skewness is divided by 4, and the term due to excess kurtosis is divided by 6. I will return to this observation when examining the effect of skewness and kurtosis for the growth of the Forbes 400 wealth share in Section 4.

3.2. *Case with Demographic Changes*

I now extend the analytical framework discussed above to account for demographic changes. This allows me to obtain an analytical characterization for the demography term.

More precisely, I now assume that individuals die with a hazard rate $\delta_t > 0$, which is independent on wealth, and that population grows at rate $\eta_t > 0$. Both rates are continuous with respect to time. Moreover, I assume individuals are born with wealth density g_{Bt}, which is continuous with respect to time and such that the average wealth of newborn agents is finite; that is, $\int_{\mathbb{R}_+} w g_{Bt}(w)\,\mathrm{d}w < \infty$.

The following proposition characterizes the dynamics of the average wealth in the top percentile, $w_{\mathcal{P}t}$, in terms of the dynamics of individual wealth.

PROPOSITION 3: *Assuming that wealth follows the diffusion process* (3) *with death rate δ_t, population growth η_t, and wealth density of newborn agents g_{Bt}, the average wealth in the top*

[18] In a different context, Martin (2013) and Schmidt (2016) derive similar expressions for the risk-free rate and the equity premium in terms of the higher-order cumulants of (log) consumption growth. In these expressions, the shape of the utility function, as summarized by the relative risk aversion γ, plays a similar role as the shape of the wealth distribution, as summarized by ζ.

percentile $w_{\mathcal{P}t}$ follows the law of motion:[19]

$$\frac{\mathrm{d}w_{\mathcal{P}t}}{w_{\mathcal{P}t}} = \underbrace{\mathrm{E}^{g_t(\cdot)\cdot}\left[\mu_t(w)|w \geq q_t\right]\mathrm{d}t}_{\textit{Within}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{pw_{\mathcal{P}t}}\nu_t(q_t)^2\,\mathrm{d}t}_{\textit{Between}}$$
$$+ \underbrace{\left(\frac{1}{pw_{\mathcal{P}t}}\int_{q_t}^{\infty}(w-q_t)g_{Bt}(w)\,\mathrm{d}w - \frac{w_{\mathcal{P}t}-q_t}{w_{\mathcal{P}t}}\right)(\delta_t+\eta_t)\,\mathrm{d}t}_{\textit{Demography}}. \tag{9}$$

This proposition generalizes Proposition 2 to the case with demographic changes. The demography term depends critically on the wealth density among newborn individuals. When newborn individuals are not wealthy enough to be born into the top percentile (i.e., $g_{Bt}(w) = 0$ for $w \geq q_t$), the demography term is equal to $-(w_{\mathcal{P}t} - q_t)/w_{\mathcal{P}t} \cdot (\delta_t + \eta_t)\,\mathrm{d}t$. In contrast, when newborn individuals are as wealthy as their parents (i.e., $g_{Bt}(w) = g_t(w)$), the demography term is zero. I now examine a particular model of inheritance, which constitutes an intermediary case between these two extremes.

Inheritance Model. I assume that each individual has $k \in \mathbb{N}^*$ children, where k is a random variable. Children are born exactly when their parent die. To be consistent with a population growth rate of η_t, the average number of children per parent must equal $\mathrm{E}_k[k] = (\eta_t + \delta_t)/\delta_t$, where E_k denotes the average with respect to k. Individuals bequest a proportion $\chi \in (0,1]$ of their wealth at death, which is a random variable, and which is shared equally among their children.[20]

Under these assumptions, the demography term becomes:[21]

$$\text{Demography} = \left(\frac{1}{pw_{\mathcal{P}t}}\mathrm{E}_{k,\chi}\left[\int_{\frac{k}{\chi}q_t}^{\infty}\frac{k}{\mathrm{E}_k[k]}\left(\frac{\chi}{k}w - q_t\right)g_t(w)\,\mathrm{d}w\right] - \frac{w_{\mathcal{P}t}-q_t}{w_{\mathcal{P}t}}\right)(\delta_t+\eta_t)\,\mathrm{d}t,$$

where $\mathrm{E}_{k,\chi}$ denotes the joint expectation with respect to k and χ.

This formula allows us to derive a few properties about the demography term. First, the demography term is strictly negative in this model. This is the case even when individuals can fully pass their wealth to their children (i.e., $\chi = 100\%$), since the parents' wealth must still be shared across multiple children.

Second, the demography term decreases in the death rate and in the population growth rate. It increases with the inheritance rate χ (in the sense of first order stochastic dominance). Finally, the demography term increases after a mean-preserving spread in the distribution of the

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[19]More precisely, following the accounting framework 2, the demography term can be seen as the sum of a birth term equal to $\frac{1}{pw_{\mathcal{P}t}}\left(\int_{q_t}^{\infty}(w-q_t)g_{Bt}(w)\,\mathrm{d}w\right)(\delta_t+\eta_t)\,\mathrm{d}t$, a death term equal to $-(w_{\mathcal{P}t}-q_t)/w_{\mathcal{P}t}\cdot\delta_t\,\mathrm{d}t$, and a population growth term equal to $-(w_{\mathcal{P}t}-q_t)/w_{\mathcal{P}t}\cdot\eta_t\,\mathrm{d}t$.

[20]Menchik (1980) provides evidence of equal sharing among children. Cowell (1998) studies a related model of inheritance and its effect on the Pareto exponent of the wealth distribution.

[21]Formally, we have

$$g_{Bt}(w) = \mathrm{E}_{k,\chi}\left[\frac{k}{\mathrm{E}_k[k]}g_t\left(\frac{k}{\chi}w\right)\frac{k}{\chi}\right].$$

Plugging this expression into (9) gives the result.

inheritance rate χ or in the number of children k—this is because only children who inherit enough enter the top percentile.[22]

Stationary Case. The demography term takes a particularly simple form in the stationary case, when the wealth distribution has a Pareto tail. As above, assume that $\mu_t(w) = \mu(w)$, $\nu_t(w) = \nu(w)$, $\delta_t = \delta$, and $\eta_t = \eta$ with $\mu(w) \to \mu < \eta + \delta$ and $\nu(w) \to \nu$ as $w \to \infty$. Under these assumptions, the stationary distribution, if it exists, has a Pareto tail with Pareto exponent $\zeta > 1$.

Taking the limit $p \to 0$ in the expression for the demography term gives:[23]

$$0 = \underbrace{\mu \, \mathrm{d}t}_{\text{Within}} + \underbrace{\frac{\zeta - 1}{2} \nu^2 \, \mathrm{d}t}_{\text{Between}} + \underbrace{\frac{1}{\zeta} \left(\mathrm{E}_{k,\chi} \left[\frac{k}{\mathrm{E}_k[k]} \left(\frac{\chi}{k} \right)^{\zeta} \right] - 1 \right) (\delta + \eta) \, \mathrm{d}t}_{\text{Demography}} . \tag{10}$$

The effect of the shape of the wealth distribution on the between and demography term is simply summarized by its Pareto exponent, ζ. The demography term increases with the Pareto exponent ζ (i.e., it decreases with the level of wealth inequality). Intuitively, as when wealth inequality increases, wealth at the percentile threshold becomes smaller relative to the average wealth in the top percentile, which magnifies the negative effect of death and population growth.

This expression is useful to obtain a back-of-the-envelope approximation for the demography term, even outside the steady state. In the U.S., the Pareto exponent of the wealth distribution is approximately $\zeta = 1.5$, the death rate is 2%, and the population growth rate is 1%. With an inheritance rate $\chi = 50\%$ (which is roughly the top marginal estate tax rate in the U.S.), the formula predicts a demography term close to -1.5%.[24] As shown below, this is approximately equal to the average demography term measured for the Forbes 400 wealth share.

4. APPLICATION TO FORBES 400 WEALTH SHARE

In this section, I use the accounting framework developed in Section 2 to decompose the growth of the Forbes 400 wealth share. I interpret the results through the lens of the analytical framework developed in Section 3. In particular, I show that the closed-form expressions obtained in the previous section captures well the magnitude of the between and demography terms measured in the data.

Section 4.1 presents the data, Section 4.2 discusses the accounting decomposition's results, and Section 4.3 discusses the robustness of the results with respect to measurement error.

4.1. *Data*

I focus on the list of the wealthiest 400 American households constructed by *Forbes Magazine* annually since 1983.[25] The list is created by a dedicated staff of the magazine, based on a

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[22] Formally, this comes from the fact that $\int_{\frac{k}{\chi} q_t}^{\infty} k \left(\frac{\chi}{k} w - q_t \right) g_t(w) \, \mathrm{d}w$ is convex in χ and k.

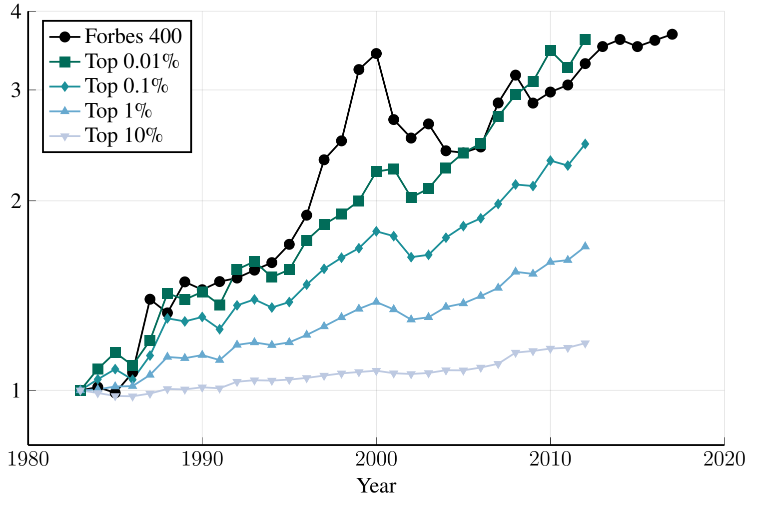
[23] More precisely, the demography term is the sum of a birth term equal to $\mathrm{E}_{k,\chi} \left[\frac{k}{\mathrm{E}_k[k]} \left(\frac{\chi}{k} \right)^{\zeta} \right] ((\delta + \eta)/\zeta) \, \mathrm{d}t$, a death term equal to $-(\delta/\zeta) \, \mathrm{d}t$, and a population growth term equal to $-(\eta/\zeta) \, \mathrm{d}t$.

[24] For this computation, I assume each household gives birth to either one or two households with equal probability; that is, $\mathrm{P}(k = 1) = \mathrm{P}(k = 2) = 0.5$.

[25] This dataset has been examined by economists in the past. For instance, Kaplan and Rauh (2013a), Kaplan and Rauh (2013b), and Capehart (2014) examine characteristics of households in the Forbes 400. Saez and Zucman (2016) compare their estimates for the top 0.01% and the ones implied by *Forbes*. Gârleanu and Panageas (2017) stress the growth of self-made billionaires over the long run compared to pre-existing billionaires.

mix of public and private information.[26] *Forbes* nominatively identifies the 400 wealthiest individuals in the U.S., allowing me to track the wealth of individuals from one year to the next, which is key for the accounting decomposition.[27] By contrast, other data sources used to track the level of wealth inequality in the U.S. rely on repeated cross-sections.[28] The Forbes 400 list includes 1,518 distinct households between 1983 and 2017. I remove from the entire sample all households that were later removed by *Forbes* due to posterior corrections or revisions in methodology (73 households).

FIGURE 2.—Cumulative Growth of Top Wealth Percentiles in the U.S. [a]



[a]The figure plots the cumulative growth of the Forbes 400 wealth share and of the wealth share of the top 10%, 1%, 0.1%, and 0.01% from Saez and Zucman (2016).

I obtain the number of U.S. households from the U.S. Census Bureau and the total household wealth from the Financial Accounts of the United States. I focus on the percentile group that includes the entirety of households in the Forbes 400 list in 2017. Because a percentile includes a constant fraction of the total population, this top percentile only includes 264 households in 1983. With a slight abuse of language, I use the term "Forbes 400 percentile" to refer to this top percentile and "Forbes 400 wealth share" to refer to its share of aggregate wealth.

While the Forbes 400 percentile accounts for a small percentage of the total U.S. population (3% of the top 0.01%), it accounts for a substantial share of total U.S. wealth (approximately

[26]*Forbes Magazine* reports that "we pored over hundreds of Securities Exchange Commission documents, court records, probate records, federal financial disclosures and Web and print stories. We took into account all assets: stakes in public and private companies, real estate, art, yachts, planes, ranches, vineyards, jewelry, car collections and more. We also factored in debt. Of course, we don't pretend to know what is listed on each billionaire's private balance sheet, although some candidates do provide paperwork to that effect."

[27]I extend the construction from Capehart (2014) to the 2012–2017 period. In certain cases, *Forbes* does not report the wealth of individuals who exit the Forbes 400. For these cases, I use a Kaplan and Meier (1958) estimator to obtain an estimate for the average wealth conditioning on exiting the Forbes 400. Appendix D discusses the accuracy of this imputation method.

[28]The three main datasets on the wealth distribution in the U.S. are the Survey of Consumer Finances, estate tax returns (see Kopczuk and Saez, 2004), and income tax returns (see Saez and Zucman, 2016), which all correspond to repeated cross-sections.

3% in 2017). Figure 2 plots the cumulative growth of the Forbes 400 wealth share since 1983 as well as the cumulative growth of the wealth share of the top 0.01%, 0.01%, 1%, and 10% from Saez and Zucman (2016). As discussed in their paper, most of the increase of wealth inequality during the period is concentrated in the top 0.01%. Moreover, the growth of the top 0.01% closely tracks the growth of the Forbes 400 wealth share, which suggests that understanding the growth of the Forbes 400 wealth share can shed light on the overall rise in top wealth inequality in the U.S.

4.2. *Results*

I now decompose the growth of the share of wealth owned by Forbes 400 percentile. More precisely, as discussed in Section 2, I apply the accounting framework (2) on wealth normalized by the average wealth in the economy. To obtain measures of the within and between terms which are not impacted by within-family transfers, I classify as birth (i.e., $\mathcal{B}$) any entry in the top percentile following the death of a family member (i.e., due to inheritance). Conversely, I classify as death (i.e., $\mathcal{D}$) any exit from the top percentile that is linked to an in-vivo transfer.

Table I reports the results. The first row in Ia shows the geometric mean of each term over the entire time period. I find that the 3.9% yearly growth of the Forbes 400 wealth share during the time period is the sum of a within term equal to 3.0%, a between term equal to 2.5%, and a demography term equal to -1.5%. Put differently, the between term (i.e., which measures the contribution of changes in relative rankings among households) had a large effect on the growth of the top wealth share: without it, the growth of the top wealth share would have been 1.4% instead of 3.9%.

Figure 3 plots the result of the accounting decomposition every year as well as the cumulative sum of each term over time. Yearly fluctuations in the Forbes 400 wealth share are almost entirely driven by the within term. As discussed in Gomez (2016), these large fluctuations reflect the fact that top households tend to be more exposed to stock market returns compared to the rest of the economy. By comparison, fluctuations in the between or demography terms tend to be much smaller. Appendix Figure D.1 plots the autocorrelogram of each term. While the within term is serially uncorrelated, the between term is positively correlated over time.

To focus on the low-frequency fluctuations of each term, Table Ia also reports the geometric mean of the within, between, and demography terms across three time periods of equal duration since 1983, which roughly correspond to distinct business cycles. The first period covers 1983–1994, which includes the 1990–1991 recession. The second period covers 1994–2005, which includes the 2001 recession, and the third period covers 2005–2017, which includes the 2007–2009 recession. The main finding is that the between term exhibits important fluctuations over time. In particular, the between term has been steeply decreasing over time: it decreases from 3.3% in the first part of the sample (1983–1994) to 1.5% in the third part of the sample (2005–2017). In contrast, the demography term has remained relatively stable. I now study each term in more detail.

4.2.1. *Within Term*

I first study briefly the within term. As defined in the accounting framework (2), this term is defined annualy as the growth of normalized wealth for households in the top percentile at the beginning of each year (and that do not die during the year). Because individual wealth is normalized by the average wealth in the economy, this term corresponds approximately to the difference between the growth of the average wealth of households initially in the top percentile and the growth of the average wealth in the economy; that is, using the notations from Section

TABLE I

DECOMPOSING THE GROWTH OF THE FORBES 400 WEALTH SHARE (1983-2017)[a]

(a) Summary

	Growth of Top Percentile Wealth Share			
	Total	Within	Between	Demography
All Years	3.9	3.0	2.5	−1.5
1983-1994	4.3	2.5	3.3	−1.4
1994-2005	3.7	2.9	2.7	−1.8
2005-2017	3.7	3.6	1.5	−1.4

(b) Within

	Within		
	Total	Average Wealth Growth in Top Percentile	−Growth of U.S. Wealth per Capita
All Years	3.0	5.7	−2.6
1983-1994	2.5	4.8	−2.2
1994-2005	2.9	7.1	−4.1
2005-2017	3.6	5.2	−1.5

(c) Between

	Between						
	Total	Inflow			Outflow		
		Total	$n_{\mathcal{I}}$	$\frac{w_{\mathcal{I},1}-q_1}{w_{\mathcal{P}_0,0}}$	**Total**	$n_{\mathcal{O}}$	$\frac{q_1-w_{\mathcal{O},1}}{w_{\mathcal{P}_0,0}}$
All Years	2.5	1.9	12.3	15	0.6	9.7	6
1983-1994	3.3	2.4	15.4	15	0.9	12.5	7
1994-2005	2.7	2.2	12.8	16	0.5	10.2	5
2005-2017	1.5	1.2	9.0	13	0.4	6.8	5

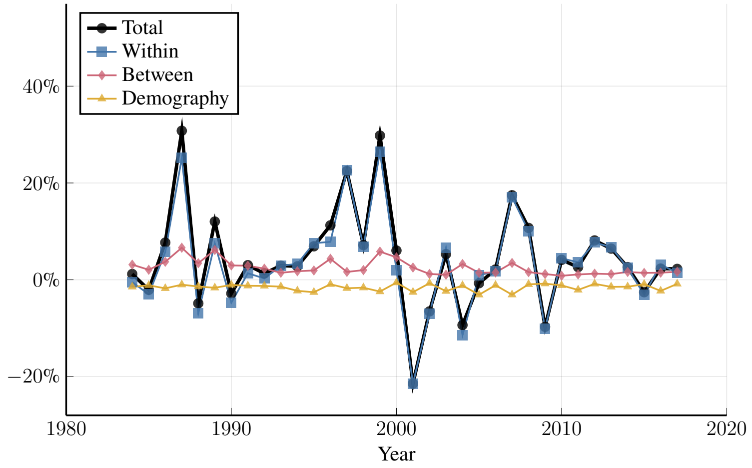
(d) Demography

	Demography									
	Total	Birth			Death			Population Growth		
		Total	$n_{\mathcal{B}}$	$\frac{w_{\mathcal{B},1}-q_1}{w_{\mathcal{P}_0,0}}$	**Total**	$n_{\mathcal{D}}$	$\frac{q_1-\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}}w_{\mathcal{D},0}}{w_{\mathcal{P}_0,0}}$	**Total**	$1-n_{\mathcal{P}_0}$	$\frac{q_1-\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}}w_{\mathcal{P}_0,0}}{w_{\mathcal{P}_0,0}}$
All Years	−1.5	0.4	0.8	66	−1.1	2.2	−51	−0.8	1.2	−68
1983-1994	−1.4	0.4	1.2	76	−1.0	2.7	−38	−0.7	1.3	−59
1994-2005	−1.8	0.4	0.9	42	−1.2	2.1	−57	−1.0	1.4	−72
2005-2017	−1.4	0.4	0.5	77	−1.1	1.8	−59	−0.7	0.9	−73

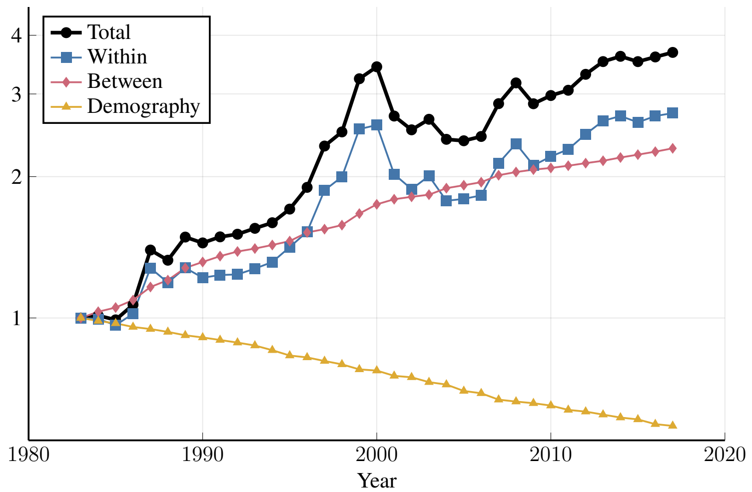
[a] Table I reports the result of the accounting framework (2) for the Forbes 400 wealth share. Table Ib-Ic-Id report detailed statistics on the within, between, and demography terms, respectively. All terms in percentage. Data are from *Forbes*, the Bureau of Economic Analysis, and the Financial Accounts of the United States.

FIGURE 3.—Decomposing the Growth of the Forbes 400 Wealth Share[a]

(a) Annual Growth Rates



(b) Cumulative Growth Rates



[a]Figure 3a plots the annual growth rate of the Forbes 400 wealth share as well as the within, between, and demography terms defined in the accounting framework (2). Figure 3b plots the same series cumulated over time; that is, $\prod_{s=1984}^{t}(1+x_s)$ for each series x_t. Data are from *Forbes* and the Financial Accounts of the United States.

2:[29]

$$\text{Within} = \frac{w_{\mathcal{P}_0 \setminus \mathcal{D},1} / w_{\Omega_1,1} - w_{\mathcal{P}_0 \setminus \mathcal{D},0} / w_{\Omega_0,0}}{w_{\mathcal{P}_0 \setminus \mathcal{D},0} / w_{\Omega_0,0}} \tag{11}$$

[29]Note that this approximation would be exact with the logarithmic decomposition discussed in Appendix B.

$$\approx \underbrace{\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1} - w_{\mathcal{P}_0\setminus\mathcal{D},0}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}}}_{\text{Average Wealth Growth in Top Percentile}} - \underbrace{\frac{w_{\Omega_1,1} - w_{\Omega_0,0}}{w_{\Omega_0,0}}}_{\text{Growth of U.S. Wealth per capita}}. \tag{12}$$

I report the result of this decomposition in Table Ib. The geometric mean of the average wealth growth of households initially in the top percentile is 5.7% while the geometric mean of the growth of the average wealth in the economy is 2.6% (both numbers are deflated for inflation using the PCE price index). Note that the difference between these two terms is indeed very close to the geometric mean of the within term, which is 3.0%.

Because the first term in this decomposition corresponds to the growth the average wealth for a *fixed* subset of the population (the households initially in the top percentile), one can further decompose it using the budget constraint of all households in this subset. One simple decomposition is to write the average wealth growth of these households as the difference between their average portfolio return and their average consumption rate (i.e., their consumption as a fraction of their wealth).[30] Since households in Forbes 400 hold the market in average (Gomez, 2016), I proxy their average return using the value-weighted stock market return from CRSP, which has a geometric mean of 8.4% during the time period. This implies that the average consumption rate of households in Forbes 400 is $8.4\% - 5.7\% = 2.7\%$ in average.[31]

4.2.2. *Between Term*

I now study the between term. Following the accounting framework (2), the between term is the sum of an inflow and an outflow terms. Each term is constructed as the product of two quantities: the intensity of individuals getting in (resp. out of) the top percentile and the average contribution of each entrant (resp. exit) to the average wealth in the top percentile. I report each term separately in Table Ic. Note that, in average, a little more than 10% of households in Forbes 400 enter or exit the top percentile annually due to changes in wealth rankings.

Variance. I now compare the between term to the closed-form expression obtained in the case of the diffusion (log-normal) model presented in Section 3. This model makes a certain number of assumptions that are not strictly satisfied in the data (mainly, that wealth follows a diffusion, that there is a continuum of agents, and that the time period is infinitesimally small), and so it is interesting to see how good of an approximation it really is.

This model-implied between term is $1/2 \cdot g_t(q_t)q_t^2/(pw_{\mathcal{P}t}) \cdot \nu_t(q_t)^2$, where $g_t(q_t)$ denotes the wealth density at the percentile threshold, q_t denotes the wealth at the percentile threshold, $w_{\mathcal{P}t}$ denotes the average wealth in the top, and $\nu_t(q_t)$ denotes the yearly standard deviation of wealth growth at the percentile threshold (see Proposition 2).

To construct this model-implied between term, I first estimate the yearly variance of (log) wealth growth at the percentile threshold, $\nu_t(q_t)^2$ using local regression techniques. More precisely, I regress the (log) wealth growth and its square on log wealth around the percentile threshold.[32] I obtain an estimate for the yearly standard deviation by combining the local estimates for the first and second moments. As shown in Table IIa, I obtain an average standard deviation equal to 0.27. As shown in Figure 4a, the estimate exhibits important fluctuations at the business cycle frequency, with a particularly large spike during the dot-com bubble.

---

[30] This residual also includes taxes paid minus labor income as a proportion of wealth.

[31] One alternative decomposition could be to write the average wealth growth of households in a given subset as the sum of a re-evaluation gain and the product of their income multiplied by their saving rate. The key point is that this would identify the actual saving rate of these households rather than a "synthetic" version biased by composition changes (Saez and Zucman, 2016).

[32] I use a local polynomial of degree one, with a triangular kernel with a bandwidth of one.

TABLE II

MODEL-IMPLIED BETWEEN TERM IN THE FORBES 400 [a]

(a) Moments

	Dispersion of Wealth Growth			Shape of the Wealth Distribution		
	Std. Dev.	Skewness	Excess Kurt.	$q_t/w_{\mathcal{P}t}$	$g_t(q_t)q_t/p$	$g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$
All Years	0.27	−0.34	6.58	0.34	1.29	0.45
1983-1994	0.27	−0.53	6.42	0.42	1.49	0.63
1994-2005	0.31	−0.26	7.70	0.30	1.23	0.38
2005-2017	0.25	−0.23	5.72	0.30	1.15	0.35

(b) Between (Model-Implied)

	Model Implied Between Term			
	Total	Due to Std. Dev.	Due to Skewness	Due to Excess Kurt.
All Years	2.0	1.7	−0.2	0.4
1983-1994	2.8	2.4	−0.2	0.5
1994-2005	1.9	1.8	−0.5	0.6
2005-2017	1.2	1.1	−0.1	0.2

[a] Table IIa shows the standard deviation, skewness, and excess kurtosis of log wealth growth at the percentile threshold using local regression techniques. It also shows summary statistics on the shape of the wealth distribution at the percentile threshold using local regression techniques. If the wealth distribution was exactly Pareto with exponent ζ, the second column would equal $1 - 1/\zeta$, the third column would equal ζ and the last column would equal $\zeta - 1$. Table IIb uses these moments as input in (8) to report the between term implied by the second, third, and fourth cumulants of log wealth growth, where all terms are percentage. Data are from *Forbes*.

I then estimate $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ every year. As discussed in Section 3, this term can be written as the product of $q_t/w_{\mathcal{P}t}$, the ratio between wealth at the percentile threshold and average wealth above the threshold, and $g_t(q_t)q_t/p$, the relative mass of households around the percentile threshold. The first term can be directly observed in the data, while the second term can be estimated by regressing the log number of households above a given wealth level on log wealth around the percentile threshold.[33] Table IIa reports the results of this estimation. The first term, $q_t/w_{\mathcal{P}t}$, averages to 0.34 while the second term, $g_t(q_t)q_t/w_{\mathcal{P}t}$, averages to 1.29. The product of these two terms, $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$, averages to 0.45—if the distribution had a Pareto tail, this would correspond to a Pareto exponent ζ of 1.45, which is consistent with existing studies.[34] Figure 4a plots this term every year: note that it decreases slowly over time, indicating a steady rise in wealth concentration during the time period.

Finally, I combine the estimates for the dispersion of wealth growth and for the shape of the wealth distribution to construct a model-implied between term $1/2 \cdot g_t(q_t)q_t^2/(pw_{\mathcal{P}t}) \cdot \nu_t(q_t)^2$. Table IIb shows that the model-implied between term is 1.7% in average, which tends to be lower than the actual between term, 2.5%. Figure 4b plots the model-implied between term and the actual between term every year: despite the difference in levels, the model-implied term tracks well the dynamics of the between term and, in particular, its decline over time.

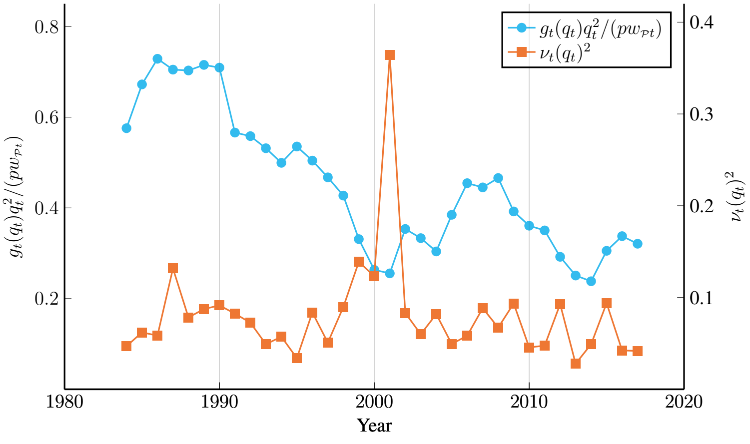
One good property of the model-implied between term is that one can decompose its change over time into a term due to the change in the variance of wealth growth and a term due to a

[33] Indeed, we have $\partial_{\ln q} \ln \left(\int_q^\infty g_t(w)\, \mathrm{d}w \right) = g_t(q_t)q_t/p$. I use a kernel with a bandwidth of one around the percentile threshold.

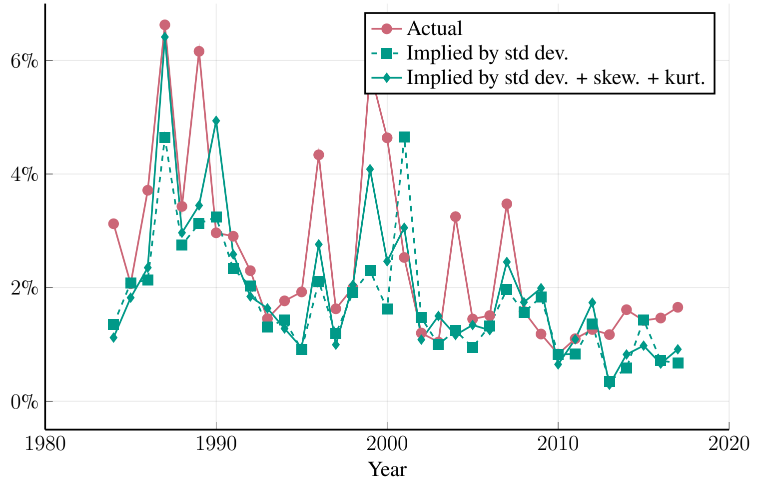
[34] See, for instance, Klass et al. (2006).

FIGURE 4.—Between Term in the Forbes 400 through the Lens of the Statistical Model[a]

(a) Estimates of $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ and $\nu_t(q_t)^2$



(b) Between Term



[a]Figure 4a plots the time series of $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ and the cross-sectional variance of log wealth growth at the percentile threshold $\nu_t(q_t)^2$. Figure 4b plots the between term, the one implied by the variance of log wealth growth (log-normal model), and the one implied by variance, skewness, and excess kurtosis of log wealth growth (jump-diffusion model). Data are from *Forbes*.

change in wealth concentration:

$$\underbrace{\Delta\left(\frac{1}{2}\frac{g_t(q_t)q_t^2}{pw_{\mathcal{P}t}}\nu_t(q_t)^2\right)}_{-1.3\%} = \underbrace{\left\langle\frac{1}{2}\frac{g_t(q_t)q_t^2}{pw_{\mathcal{P}t}}\right\rangle\Delta\left(\nu_t(q_t)^2\right)}_{-0.3\%} + \underbrace{\langle\nu_t(q_t)^2\rangle\Delta\left(\frac{1}{2}\frac{g_t(q_t)q_t^2}{pw_{\mathcal{P}t}}\right)}_{-1.0\%},$$

where Δx (resp. $\langle x \rangle$) denotes the difference (resp. average) of variable x between the 2005-2017 and 1983-1984.[35] I obtain that most of the decline in the between term is driven by a rise in wealth concentration, which made it harder for new fortunes to displace existing fortunes, rather than by a decline in idiosyncratic shocks.

What explains the dispersion of wealth growth for households around the percentile threshold? Since most households in the Forbes 400 tend to disproportionately invest their own firms, one simple hypothesis is that it is driven by the cross-sectional dispersion of stock market returns. To test this idea, I regress the yearly variance of household-level wealth growth on the equal-weighted variance of firm-level returns, computed using stock-level returns from the Center for Research in Security Prices (CRSP). Table III shows that the resulting $R^2 \approx 0.73$ is high, suggesting the yearly fluctuations in the variance of firm returns explain well the yearly fluctuations in the variance of wealth growth. The coefficient of 0.34 can be interpreted as saying that the average household in the Forbes 400 holds $\sqrt{0.34} \approx 60\%$ of its wealth in their firm and the rest in a diversified portfolio of firms. Finally, the estimate for the intercept is close to zero, meaning that the level of portfolio concentration, identified purely from time-series variation, also accounts for the level of the variance of wealth growth. This suggests that the variance of wealth growth is almost entirely driven by the variance of firm-level returns. I will use this observation to construct a model-implied between term for the 20th century in Section 5.

TABLE III

DISPERSION OF WEALTH GROWTH AND DISPERSION OF FIRM-LEVEL RETURNS[a]

	Variance of Wealth Growth
	(1)
Variance of Firm Returns	0.34
	(0.05)
Constant	–0.01
	(0.01)
R^2	0.73
Period	1983-2017
N	34

[a] The table shows the result of regressing the cross-sectional variance of (log) wealth growth $\nu_t(q_t)^2$ for the Forbes 400 on the cross-sectional variance of firm-level (log) returns. Estimation is done via OLS. Standard errors are in parentheses and are estimated using Newey-West with three lags. Data are from *Forbes* and CRSP.

Higher-Order Cumulants. As discussed in Section 3, when the process for wealth follows a jump-diffusion process, the between term depends not only on the variance of wealth growth but also on all of its higher-order cumulants. I now evaluate the contribution of the third and fourth cumulant for the between term.

To simplify the analysis, I focus on the expression for the model-implied between term in the case in which the distribution has a Pareto tail with Pareto exponent ζ and in which higher-order cumulants are constant in the right tail of the wealth distribution. In this case, Equation (8) gives that the term due to skewness is $^1/_{3!}(\zeta^2 - 1) \cdot \text{skewness} \cdot \text{sd}^3$, while the term due to kurtosis is $^1/_{4!}(\zeta^3 - 1) \cdot \text{excess kurtosis} \cdot \text{sd}^4$. I use local polynomial regressions to estimate the third and fourth moments of wealth growth at the percentile threshold. I estimate the Pareto

[35] The quantification of each term is done using Table IIb.

exponent ζ as one plus the estimated $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ to be consistent with the variance term estimated above.

As shown in Table IIa, I find that the average estimated skewness is negative at -0.34 (i.e., more downward realizations compared to the log-normal distribution), while the average excess kurtosis is positive at 6.6 (i.e., more extreme realizations compared to the log-normal distribution). After plugging these estimates into Equation (8), I find that the skewness decreases the between term by 0.2%, while the excess kurtosis increases the between term by 0.4% annually (see Table IIb). These results suggest that, overall, the effect of higher-order cumulants on the between term is relatively small. As discussed in Section 3.1, one reason is that top wealth inequality is so high that, at the yearly horizon, most of the entry in the top percentile is driven by households close to the top percentile threshold rather than by households away from the top percentile threshold with extremely high wealth realizations.

Figure 4b compares the between term implied by the diffusion model (i.e., due to the variance of wealth growth) to the one implied by the jump-diffusion model (i.e., including higher-order cumulants). Both series remain close over time, suggesting higher-order cumulants do not matter much for the dynamics of the between term either. One exception is 2001, when the term due to variance drastically overestimates the actual between term, in contrast with the term including higher-order cumulants. This reflects the fact that the distribution of wealth shocks was extremely left-skewed during the burst of the dot-com bubble, which limited the effect of this dispersion in wealth growth on the growth of the top percentile wealth share.

4.2.3. *Demography Term*

I now study the demography term. Following the accounting framework (2), the demography term is the sum of three terms due to birth, death, and population growth. Each term is constructed as the product of two quantities: the intensity of individuals entering (resp. exiting) the top percentile and the average contribution of each entrant (resp. exit) to the average wealth in the top percentile. I report each term separately in Table Id. Note that the death rate of households in the Forbes 400, $n_{\mathcal{D}}$, averages to 2.2% while the rate of population growth, $1 - n_{\mathcal{P}_0}$, averages to 1.2%.

Death and Population Growth. I first compare the terms due to death and population growth measured in the data to the terms implied by the model presented in Section 3. Proposition 3 predicts that at short horizons, the terms due to death and population growth should be close to $-(w_{\mathcal{P}t} - q_t)/w_{\mathcal{P}t} \cdot \delta_t$ and $-(w_{\mathcal{P}t} - q_t)/w_{\mathcal{P}t} \cdot \eta_t$, respectively. Since the ratio between wealth at the threshold and wealth above the threshold, $q_t/w_{\mathcal{P}t}$, averages to 0.34 (Table IIa), this gives a model-implied death term of -1.5% and a model-implied population growth term of -0.8%. As reported in Table Id, these terms are -1.1% and -0.8% in the data, respectively. While it is a good fit, the model slightly overestimates the magnitude of the actual death term. This reflects the fact that, during the time period, households that died tended to be less wealthy than the average household in the Forbes 400 (whereas the model assumes a death rate independent of wealth).

Table Id reports the average of the terms due to death and population growth over three time periods. Note that, even though the death rate in the Forbes 400 percentile and the population growth rate in the U.S. have declined over time, the terms due to death and population growth have remained pretty much constant. This comes from the fact that the decline in the rate of population renewal has been counterbalanced by a rise in wealth inequality (i.e., a decrease in $q_t/w_{\mathcal{P}t}$), which magnified the negative effect of given death rate and population growth rate on the average wealth in the top percentile.

Birth. I now study the term due to birth. The inheritance model discussed in Section 3.2 gives a closed-form expression for the birth term equal to $\mathrm{E}_{k,\chi}\left[\frac{k}{\mathrm{E}_k[k]}\left(\frac{\chi}{k}\right)^{\zeta}\right](\delta+\eta)/\zeta$ (Equation (10)). To quantify this model-implied birth term, I use the estimate for ζ obtained for the between term; that is, $1+g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$; that is, $\zeta \approx 1.45$. I calibrate an inheritance rate $\chi = 0.5$ from the top marginal estate tax during the time period. Finally, I assume that the distribution for the number of children k is binomial, with probabilities $\mathrm{P}(k=1) = 1-\mathrm{P}(k=2) = 0.45$ chosen to match the rate of population growth.[36] After combining these parameter, I obtain an average birth term equal to 0.5%, which is very close to the actual average of the birth term measured in the data, 0.4%.

4.3. *Measurement Error*

The wealth of the richest individuals in the economy is inevitably measured with error. I now discuss the effect of this measurement error on the growth of the Forbes 400 wealth share, as well as its decomposition into a within, between, and demography terms.

The first concern is that *Forbes* may systematically overestimate the wealth of the top 400 households. For instance, Atkinson (2008) argues the magazine may give inflated values for wealth because debts are harder to track than assets. Along these lines, Raub et al. (2010) show that the wealth of deceased households reported on estate tax returns is approximately half of the wealth estimated by *Forbes*.[37] However, this type of measurement error in levels does not impact the growth of top wealth shares or its accounting decomposition.

A more important concern is that *Forbes* measures the wealth of top households with noise. If the measurement error is completely persistent, as noted in Luttmer (2002), this would lead *Forbes* to overestimate the level of top wealth shares. This type of error does not affect the growth of top wealth shares or its accounting decomposition. If, however, the measurement error has a transitory component, it may generate artificial entry and exit in the top percentile. This would lead me to underestimate the within term and overestimate the between term.

As discussed above, I removed from the sample every household that was later removed by *Forbes* due to methodological error. Still, to assess the importance of this transitory measurement error in the remaining sample, I measure the serial correlation of individual wealth growth in the remaining sample. The idea is that economic theory suggests that wealth is close to a random walk at large levels of wealth (see, e.g., Achdou et al., forthcoming). Therefore, the amount of negative serial correlation in wealth growth can be informative about the importance of transitory measurement errors. Table D.II in Appendix D shows that the serial correlation of wealth growth at the individual level is close to zero, suggesting that transitory measurement error do not play a big role in the between term.[38]

A final concern is that *Forbes*'s coverage may become more and more extensive over time, and therefore the magazine gradually discovers rich households that were not reported earlier. This would lead me to overestimate the between term as well as the growth of top wealth shares. To mitigate this effect, I dropped the first year in which the Forbes 400 list was published, which is 1982. Another reassuring fact is that I find that the actual between term is well approximated by the model-implied between term, which only uses the distribution of wealth growth among existing households. Still, the 0.4% average gap observed between the actual between term and the one implied by the statistical model may be due to this type of errors (see Table II).

[36]Indeed, remember that we must have $\mathrm{E}_k[k]\delta = \eta + \delta$.

[37]Alternatively, this may reflect the fact that these households under-report their wealth on these tax returns.

[38]In Appendix D, I show that the relative bias in the between term is well approximated by -2ρ, where ρ is the AR(1) coefficient of wealth growth. With an estimated ρ approximately equal to -0.01, this suggests a measurement error of only four basis points.

5. QUANTIFYING THE BETWEEN TERM IN THE ABSENCE OF PANEL DATA

The previous section shows that the diffusion (log-normal) model approximates well the level and the dynamics of the actual between term. This suggests that one can use this model to quantify the between term in settings where panel data are not available. I now use this idea to quantify the between term for the top 1%, 0.1%, and 0.01% of the wealth distribution over the 20th century, for which panel data are not available.

Methodology I first estimate the yearly standard deviation of wealth growth at each percentile threshold $\nu_t(q_t)$ by taking the product of the share of wealth invested in equity (using data from Kopczuk and Saez, 2004 for 1916–1962 and Saez and Zucman, 2016 for 1962–2012) and the cross-sectional standard deviation of firm-level returns. This is motivated by the fact that in the Forbes 400, the variance of wealth growth correlates well with the variance of returns (Table III). I scale this product so that the standard deviation of the top 0.01% matches the standard deviation of the Forbes 400 in 1983–2012. Table IV shows the estimated standard deviation for top percentiles. Over the time period, the standard deviation equals 0.14 for the top 1% and 0.21 for the top 0.01%. Note that the average standard deviation of wealth increases in the right tail of the wealth distribution, which reflects the fact that top percentiles tend to invest more in equity.

I then estimate the shape of the wealth distribution $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ at the top 1%, 0.1%, and 0.01% percentiles using cross-sectional data on the wealth distribution available from Kopczuk and Saez (2004) for 1916–1962 and Saez and Zucman (2016) for 1962–2012. As explained above, the ratio $q_t/w_{\mathcal{P}t}$ can be directly measured as the ration between wealth at the percentile threshold and the average wealth in the top percentile, while $g_t(q_t)q_t/p$ can be estimated by comparing the ratio between two log top percentiles to the ratio of their respective log quantiles.[39] Table IV shows the estimated $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ for top percentiles. The estimate does not vary much across top percentiles, reflecting that the wealth distribution is close to Pareto above the top 1%.

Results. Figure 5b plots the between term implied by a diffusion model, $1/2 \cdot g_t(q_t)q_t^2/(pw_{\mathcal{P}t}) \cdot \nu_t(q_t)^2$, for the top 1%, top 0.1%, and top 0.01% percentiles from 1916 to 2012. The between term roughly follows a U-shape over time for all top percentiles. The between term for the top 0.01% peaked at 2% during the Great Depression and then steadily decreased, reaching its minimum in 1945. The between term again increased starting in 1960 and reached its maximum at the height of the dot-com bubble. Overall, the between term was roughly twice as high in 1983–2012 as it had been for the rest of the century.

To better understand what drives the between term over time, Figure 5a plots separately the term due to the idiosyncratic variance of wealth $\nu_t(q_t)^2$ and the term due to the shape of the wealth distribution $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ for the top 0.01%. Relative fluctuations in $\nu_t(q_t)^2$ are much larger than the relative fluctuations in $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$.[40] In other words, most of the time-series fluctuations in the between term are driven by fluctuations in the idiosyncratic volatility of wealth growth rather than by changes in the shape of the wealth distribution.

I now examine the variation of the between term across top percentiles. According to Saez and Zucman (2016), the yearly growth rate of the top 0.01%'s wealth share in 1982–2012

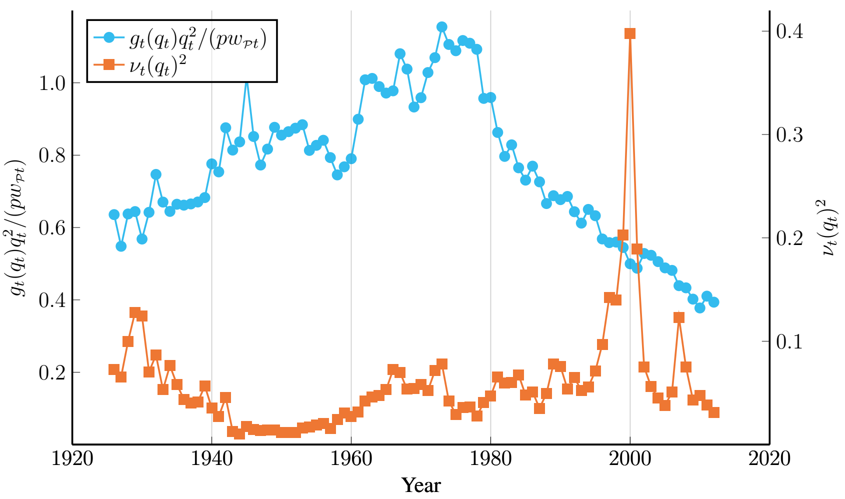
[39] This is a discretized version of the regression of the log top percentile on log quantiles performed in the previous section.

[40] More precisely, the ratio between the 90th and 10th quantile of the realizations of $\nu_t(q_t)^2$ is approximately eight, while it is only two for $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$. These large fluctuations in the idiosyncratic volatility of stock market returns have been examined by Campbell et al. (2001) and Herskovic et al. (2016). Theories to explain these fluctuations have been discussed in Brandt et al. (2009), Fink et al. (2010), and Herskovic et al. (2020).

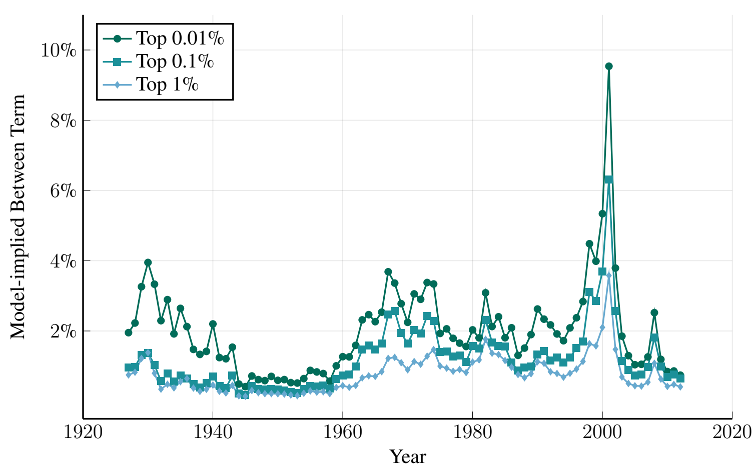
averaged to 4.3%, while the yearly growth rate of the top 1% averaged to 1.9%, a difference of 2.4% per year. Table IV suggests that the differences in the between term between the two percentiles can explain a 1.3% differential. This difference is almost fully driven by differences in the standard deviation of wealth growth across the two percentiles rather than by differences in the shape of the wealth distribution.

FIGURE 5.—Model-implied Between Term for Top Wealth Percentiles in the U.S. over the 20th Century[a]

(a) Estimates of $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ and $\nu_t(q_t)^2$ for the Top 0.01%



(b) Model-implied Between Term



[a]Figure 5a plots the estimated $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ and $\nu_t(q_t)^2$ over time for the top 0.01%. The shape of the wealth distribution $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ is estimated using cross-sectional data on the wealth distribution. The variance of log wealth growth $\nu_t(q_t)^2$ is proxied by multiplying the share of wealth invested in equity with the cross-sectional standard deviation of stock market returns. Figure 5b plots the diffusion term implied by the diffusion model, $1/2 \cdot g_t(q_t)q_t^2/(pw_{\mathcal{P}t}) \cdot \nu_t(q_t)^2$, for the top 1%, top 0.1%, and top 0.01%. Data are from Kopczuk and Saez (2004), Saez and Zucman (2016), and CRSP.

TABLE IV

MODEL-IMPLIED BETWEEN TERM OVER THE 20TH CENTURY [a]

	Top 1%	Top 0.1%	Top 0.01%
$g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$	0.61	0.65	0.76
$\nu_t(q_t)^2$	0.03	0.04	0.06
Between Term (Model-Implied)	0.8%	1.2%	2.0%

[a] Table IV uses the diffusion (log-normal) model to quantify the between term over the 20th century for the top 1%, 0.1%, and 0.01% between 1926 and 2012. The shape of the wealth distribution $g_t(q_t)q_t^2/(pw_{\mathcal{P}t})$ is estimated using cross-sectional data on the wealth distribution. The variance of log wealth growth $\nu_t(q_t)^2$ is proxied by multiplying the share of wealth invested in equity with the cross-sectional standard deviation of stock market returns. Given these two estimates, the yearly model-implied between term is obtained as $1/2 \cdot g_t(q_t)q_t^2/(pw_{\mathcal{P}t}) \cdot \nu_t(q_t)^2$. Data are from Kopczuk and Saez (2004), Saez and Zucman (2016), and CRSP.

6. IMPLICATIONS FOR WEALTH MOBILITY

In this section, I discuss the implications of the accounting decomposition for the relation between wealth inequality and mobility. More precisely, I focus on a downward concept of mobility: how long, on average, does a rich household remain in a top percentile? The advantage of this notion of "downward" mobility is that it only depends on the wealth dynamics of individuals in the right tail of the wealth distribution, which is easier to model.[41]

For the remainder of this section, I assume that the law of motion of wealth is given by a diffusive process with constant drift and idiosyncratic volatility; that is,

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu\,\mathrm{d}t + \nu\,\mathrm{d}B_{it}, \tag{13}$$

with a reflecting boundary at a wealth level normalized to one. Moreover, I assume that individuals die with death rate $\delta > 0$, population grows at rate $\eta \geq 0$, and newborn agents are born with initial wealth $w = 1$. Under these assumptions, the stationary density of wealth is given by $g(w) = \zeta w^{-\zeta-1}$ for $w \geq 1$, where ζ is the positive solution of the quadratic equation $\zeta\mu + {}^1\!/_2\zeta(\zeta-1)\nu^2 - (\delta+\eta) = 0$.[42]

Consider a wealth level $\underline{w} > 1$ and denote $T_{\underline{w}}(w)$ the average time an individual with initial wealth $w \geq \underline{w}$ remains above $\underline{w}$:

$$T_{\underline{w}}(w) = \mathrm{E}_t[\inf\{\tau \text{ s.t. } w_{it+\tau} \leq \underline{w} \text{ or } i \text{ dies}\}|w_{it} = w].$$

The following lemma gives a closed-form formula for this average first passage time:

LEMMA 4—Average First Passage Time: *When wealth follows the law of motion* (13), *the average first passage time is*[43]

$$T_{\underline{w}}(w) = \frac{1}{\delta}\left(1 - \left(\frac{w}{\underline{w}}\right)^{-\xi}\right), \tag{14}$$

where $-\xi$ *denotes the negative solution of the quadratic equation* $\xi\mu + {}^1\!/_2\xi(\xi-1)\nu^2 - \delta = 0$.

[41] In particular, compared to a notion of "upward" mobility, we can abstract from the role of labor income or government programs, which matter most for households below the top 1%.

[42] See, for instance, Gabaix et al. (2016).

[43] The average first passage time of a Brownian motion is a classic result; for instance, see Karlin and Taylor (1981).

Naturally, the average first passage time increases with wealth: it equals 0 at the percentile threshold and converges to $1/\delta$ as wealth tends to infinity (i.e., the average time before death). More interestingly, the exponent ξ governs the speed at which the average first passage time converges to $1/\delta$ as wealth tends to infinity.

The average first passage time increases with the average wealth growth of individuals μ and decreases with the idiosyncratic volatility ν (through change in ξ). Intuitively, the higher the dispersion of wealth shocks, the more likely are negative shocks.

Still, an increase in idiosyncratic volatility does not necessarily mean that wealth mobility decreases. Indeed, an increase in idiosyncratic volatility increases wealth inequality, which increases the typical distance between a household in the top percentile and the percentile threshold. To take this effect into account, I now examine the average first passage time for an *average* household in a top percentile. Consider a given top percentile $p \in (0, 1)$ and denote q the wealth of the last person in the top percentile at time $t = 1$ (i.e., the top $p-$quantile). Denote T the average first passage time for an average household in a top percentile:

$$T = \mathrm{E}^g \left[T_q(w) | w \geq q \right], \tag{15}$$

where E^g denotes the expectation with respect to the stationary wealth density g. The next proposition gives a simple closed form formula for the average passage time for an average household in the top percentile p.

PROPOSITION 5—Average First Passage Time for an Average Household: *The average time an average household in the top percentile* p *remains in the top is*

$$T = \frac{1}{\delta}\left(1 + \frac{\zeta}{\xi}\right)^{-1}. \tag{16}$$

Interestingly, T does not depend on the top percentile p. This reflects the "scale-free" property of Pareto distributions. More precisely, T only depends on the ratio between ζ and ξ, which is intuitive: ξ controls the speed at which the first passage time decays as wealth tends to infinity while ζ controls the speed at which the density of wealth decays as wealth tends to infinity.

T increases with the average wealth growth of households, μ. This is because a higher average wealth growth of top households increases both the average first passage time at a given wealth level (i.e., decreases ξ) and the level of wealth inequality (i.e., decreases ζ). These two forces combine to decrease mobility.

In contrast, the effect of idiosyncratic volatility ν on T is more ambiguous. On the one hand, a rise in idiosyncratic volatility decreases the average first passage time from a given wealth level (i.e., it decreases ξ). On the other, it also increases the level of inequality in the long run (i.e., it decreases ζ). Still, for typical parameters, this second force does not fully compensate the first one, and mobility increases.

A Stylized Calibration. I now plug the results of the accounting framework for the Forbes 400 percentile in the expression for T in (16) to evaluate the effect of the observed rise in top wealth shares on long-run wealth mobility. I calibrate the death rate δ and population growth rate η to match the respective rates reported in Table Id; that is, $\delta = 2.2\%$ and $\eta = 1.2\%$. For the initial (pre-1980) steady state, I pick $\nu = 0.10$ to match the average idiosyncratic volatility for the top 0.01% in 1960–1980 and $\mu = 1.5\%$ to match an initial Pareto exponent of the stationary wealth distribution $\zeta = 1.8$ (from Section 5). These parameters give $\xi = 3.3$. Using Equation (16), this implies that the average time in the top percentile is $T = 30$ years.

I consider a permanent increase in the drift and volatility of wealth consistent with the results of the accounting framework for the Forbes 400 in Table Ia, that is, a permanent change to $\mu = 3\%$ and $\nu = 0.27$. In this new steady state, the Pareto exponent ζ is approximately equal to 1.1, while ξ is approximately equal to 0.7. Using Equation (5), this implies that the average time in the top percentile is $T = 18$ years. Even though the new steady state has a higher level of wealth inequality, it also has a higher level of wealth mobility.[44]

7. CONCLUSION

This paper develops an accounting framework to better understand the dynamics of top wealth shares. The growth of a top percentile wealth share can be decomposed into three distinct terms: a within term (the growth of the top wealth share absent any composition change), a between term (which accounts for the flow of individuals in and out of the top percentile), and a demography term (which accounts for the entry and exit of individuals in the economy). This accounting framework can be seen as an "integrated" version of the Kolmogorov forward equation. Using this connection, I obtain simple analytical expressions for the within, between, and demography terms over short periods of time. These closed-form expressions are useful to understand the mechanical forces behind top wealth shares. They can also help researchers to quantify the between and demography terms in the absence of panel data.

After applying this framework on the growth of the Forbes 400 wealth share, I find that the between term has played a key role in the recent rise in top wealth inequality: more than half of the recent rise in top wealth inequality is due to the rise of new fortunes in top percentiles.

My findings have direct implications for our understanding of top wealth inequality. While the existing literature has focused on factors driving the average growth of existing fortunes (i.e., their average return on capital or their average saving rate), this paper stresses the importance of understanding better the growth rate of new fortunes (i.e., the heterogeneity in growth rates within wealthy households).

While this paper is purely descriptive, the moments identified by the accounting framework could be used to discipline existing models of wealth inequality. Finally, the methodology developed in this paper should be useful in other settings. As a first step in this direction, I also use the accounting framework to decompose the rise in billionaires in China and Russia (Appendix E) as well as the rise in top income shares in the U.S. (Appendix F). Another interesting application would be to examine the rise in firm concentration (Autor et al., 2017; Hartman-Glaser et al., 2019; Gutiérrez and Philippon, 2019). I leave these topics for future research.

APPENDIX A: PROOFS

PROOF OF PROPOSITION 1: I first decompose the average wealth of individuals in the top percentile at time $t = 1$ as follows:

$$\begin{aligned} w_{\mathcal{P}_1,1} &= n_{\mathcal{P}_0\setminus\mathcal{D}} w_{\mathcal{P}_0\setminus\mathcal{D},1} - n_{\mathcal{O}} w_{\mathcal{O},1} + n_{\mathcal{B}} w_{\mathcal{B},1} + n_{\mathcal{I}} w_{\mathcal{I},1} \\ &= \frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}} \left(n_{\mathcal{P}_0} w_{\mathcal{P}_0,0} - n_{\mathcal{D}} w_{\mathcal{D},0} \right) - n_{\mathcal{O}} w_{\mathcal{O},1} + n_{\mathcal{B}} w_{\mathcal{B},1} + n_{\mathcal{I}} w_{\mathcal{I},1}. \end{aligned}$$

[44] This relates to the empirical findings of Kopczuk et al. (2010), who find that even though labor inequality increased at the end of the 20th century, labor mobility remained more or less constant.

I now add and subtract q_1, the wealth of the last person in the top percentile at time $t=1$:[45]

$$w_{\mathcal{P}_1,1} = \frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}} w_{\mathcal{P}_0,0} - (1-n_{\mathcal{P}_0})\left(\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}} w_{\mathcal{P}_0,0} - q_1\right) - n_{\mathcal{D}}\left(\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}} w_{\mathcal{D},0} - q_1\right)$$
$$+ n_{\mathcal{O}}(q_1 - w_{\mathcal{O},1}) + n_{\mathcal{B}}(w_{\mathcal{B},1} - q_1) + n_{\mathcal{I}}(w_{\mathcal{I},1} - q_1).$$

Subtracting and dividing by the average wealth in the top percentile at time $t=0$ gives Equation (2). *Q.E.D.*

PROOF OF PROPOSITION 2: *Step 1— Existence, smoothness, and upper bounds for the wealth density.* I first show that there is a weak solution of the stochastic differential equation (SDE) and that its density is smooth. Let $\omega_{it} = \ln w_{it}$ denote log wealth. Using Ito's lemma, the process for log wealth ω_{it} solves the following SDE:

$$\mathrm{d}\omega_{it} = \left(\mu_t\left(e^{\omega_{it}}\right) - \frac{1}{2}\nu_t\left(e^{\omega_{it}}\right)^2\right)\mathrm{d}t + \nu_t\left(e^{\omega_{it}}\right)\mathrm{d}B_{it}.$$

As shown in Rogers (1985), the solution of this SDE process has a weak solution and its transition density between s and t, denoted by $\pi_{s\to t}(\omega_s,\omega_t)$, is $\mathcal{C}^\infty$ in t,ω_s,ω_t, positive everywhere, and satisfies the Kolmogorov forward equation.

As discussed in Friedman (1964) (Section 9.6), the fact that $\mu_t(\exp(.))$ and $\nu_t(\exp(.))$ are bounded gives upper bounds on the derivatives of the transition density: there exist constants A, d such that for all $\omega_0,\omega,t \in [0,T]$ and any integer $n \geq 0$,

$$|\partial_t \pi_{0\to t}(\omega_0,\omega)| \leq \frac{A}{t^{3/2}} e^{-d\frac{(\omega-\omega_0)^2}{t}}, \tag{17}$$

$$|\partial_\omega^n \pi_{0\to t}(\omega_0,\omega)| \leq \frac{A}{\sqrt{t^{1+n}}} e^{-d\frac{(\omega-\omega_0)^2}{t}}. \tag{18}$$

Denote γ_t the density of log wealth, which is given by $\gamma_0(\omega) = e^\omega g_0(e^\omega)$ and

$$\gamma_t(\omega) = \int_{\mathbb{R}} \gamma_0(\omega_0)\pi_{0\to t}(\omega_0,\omega)\,\mathrm{d}\omega_0,$$

for any time $t>0$. Since $\pi_{0\to t}$ is positive everywhere, γ_t is positive everywhere. Moreover, the dominated convergence theorem implies that γ_t inherit the smoothness of $\pi_{0\to t}$; that is, $\gamma_t \in \mathcal{C}^\infty$.[46]

Next, I prove that the wealth distribution has finite mean at any time $t \geq 0$. We have, for $t \geq 0$ and $x \geq 0$,

$$\begin{aligned}\int_0^x e^\omega \gamma_t(\omega)\,\mathrm{d}\omega &= \int_0^x e^\omega\left(\int_{\mathbb{R}} \gamma_0(\omega_0)\pi_{0\to t}(\omega_0,\omega)\,\mathrm{d}\omega_0\right)\mathrm{d}\omega \\ &\leq \int_{\mathbb{R}} \gamma_0(\omega_0)\left(\int_0^x e^\omega \frac{A}{\sqrt{t}} e^{-d\frac{(\omega-\omega_0)^2}{t}}\,\mathrm{d}\omega\right)\mathrm{d}\omega_0 \\ &\leq \left(\int_{\mathbb{R}} \gamma_0(\omega_0)e^{\omega_0}\,\mathrm{d}\omega_0\right)\left(\int_{\mathbb{R}} e^u \frac{A}{\sqrt{t}} e^{-d\frac{u^2}{t}}\,\mathrm{d}u\right),\end{aligned} \tag{19}$$

[45]This uses the fact that $1 = n_{\mathcal{P}_0} - n_{\mathcal{D}} - n_{\mathcal{O}} + n_{\mathcal{B}} + n_{\mathcal{I}}$.

[46]Equation (18) gives an upper bound for the space derivative of the transition density.

where the second line uses the upper bound (18) with $n=0$. This implies that $\int_0^\infty e^\omega \gamma_t(\omega)\,\mathrm{d}\omega < \infty$. Therefore, one can conclude that the density of wealth, $g_t(w) = \gamma_t(\ln w)/w$, exists, is in $\mathcal{C}^\infty$, is positive everywhere, and has finite mean.

Step 2—Law of motion of the average wealth in the top percentile. I now relate the law of motion of the average wealth in the top percentile, $\mathrm{d}w_{\mathcal{P}t}$, to the law of motion of the wealth density, $\mathrm{d}g_t$. Since g_t is positive everywhere, the function $q \to \int_q^\infty g_t(w)\,\mathrm{d}w$ is strictly decreasing, so there is a unique quantile q_t associated with the top percentile p, which is defined implicitly by

$$p = \int_{q_t}^\infty g_t(w)\,\mathrm{d}w. \tag{20}$$

Differentiating (20) with respect to time gives the law of motion of the top quantile:[47]

$$0 = \int_{q_t}^\infty \mathrm{d}g_t(w)\,\mathrm{d}w - g_t(q_t)\,\mathrm{d}q_t \tag{21}$$

We can use this expression to obtain the law of motion of the total wealth in the top percentile:

$$\begin{aligned}\mathrm{d}\left(\int_{q_t}^\infty w g_t(w)\,\mathrm{d}w\right) &= \int_{q_t}^\infty w\,\mathrm{d}g_t(w)\,\mathrm{d}w - q_t g_t(q_t)\,\mathrm{d}q_t \\ &= \int_{q_t}^\infty (w - q_t)\,\mathrm{d}g_t(w)\,\mathrm{d}w. \end{aligned}\tag{22}$$

This equation formalizes the core intuition of the accounting framework (2): individuals entering the top percentile increase total wealth in the top percentile by the difference between their wealth and the wealth of the individual they displace, q_t.

Step 3—Integrating by parts the Kolmogorov forward Equation. I now use the Kolmogorov forward equation to substitute $\mathrm{d}g_t$ by the drift and volatility of wealth in (22):

$$\mathrm{d}g_t(w) = -\partial_w(\mu_t(w) w g_t(w))\,\mathrm{d}t + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w))\,\mathrm{d}t.$$

Plugging it into (22) gives

$$\mathrm{d}\left(\int_{q_t}^\infty w g_t(w)\,\mathrm{d}w\right) = \int_{q_t}^\infty (w - q_t)\left(-\partial_w(\mu_t(w) w g_t(w))\,\mathrm{d}t + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w))\,\mathrm{d}t\right)\mathrm{d}w.$$

Integrating by parts gives:

$$\begin{aligned}\mathrm{d}\left(\int_{q_t}^\infty w g_t(w)\,\mathrm{d}w\right) &= \Big[(w - q_t)\,\mu_t(w) w g_t(w)\Big]_{q_t}^\infty \mathrm{d}t - \frac{1}{2}\Big[(w - q_t)\partial_w(\nu_t^2(w) w^2 g_t(w))\Big]_{q_t}^\infty \mathrm{d}t \\ &\quad + \left(\int_{q_t}^\infty \mu_t(w) w g_t(w)\,\mathrm{d}w\right)\mathrm{d}t + \frac{1}{2}\nu_t(q_t)^2 q_t^2 g_t(q_t)\,\mathrm{d}t. \end{aligned}\tag{23}$$

[47]The dominated convergence theorem ensures that we can differentiate with respect to time under the integral — (17) implies that, over a finite time period, $\partial_t g_t(w)$ (as well as $w\partial_t g_t(w)$) is uniformly dominated by an integrable function.

Note that $w^2 g_t(w) \to 0$ and $w\partial_w w^2 g_t(w) \to 0$ as $w \to \infty$ since the mean of the wealth distribution is finite and $\mu_t(.)$ and $\nu_t(.)$ are bounded. Therefore, Equation (23) simplifies to:

$$\mathrm{d}\left(\int_{q_t}^{\infty} w g_t(w)\,\mathrm{d}w\right) = \left(\int_{q_t}^{\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w\right)\mathrm{d}t + \frac{1}{2}\nu_t(q_t)^2 q_t^2 g_t(q_t)\,\mathrm{d}t. \tag{24}$$

Dividing (24) by $\int_{q_t}^{\infty} w g_t(w)\,\mathrm{d}w$ gives (4).

Step 4— Limit of the accounting framework as the time period tends to zero. I now show that, as the time period tends to zero, the within and between terms defined in the accounting framework (1) converge to the respective terms in (4). Between t and $t+\Delta t$, the within term in (1) corresponds to

$$\text{Within} = \frac{1}{pw_{\mathcal{P}t}} \int_{\log q_t}^{\infty} \left(\int_{\mathbb{R}} e^{\omega'} \pi_{t\to t+\Delta t}(\omega, \omega')\,\mathrm{d}\omega'\right) \gamma_t(\omega)\,\mathrm{d}\omega - 1. \tag{25}$$

Differentiating this expression with respect to Δt at $\Delta t = 0$ gives the within term in (4). The expression for the derivative of the between term is obtained as a residual. *Q.E.D.*

PROOF OF PROPOSITION 3: I follow the steps of the proof of Proposition 2, with the key difference that the composition of individuals in the economy now changes over time.

Step 1— Existence, smoothness, and upper bounds for the wealth density. Let $\omega_{it} = \ln(w_{it})$ be log wealth. Applying Ito's lemma, the law of motion of ω_{it} is given by

$$\mathrm{d}\omega_{it} = \left(\mu_t\left(e^{\omega_{it}}\right) - \frac{1}{2}\nu_t\left(e^{\omega_{it}}\right)^2\right)\mathrm{d}t + \nu_t\left(e^{\omega_{it}}\right)\mathrm{d}B_{it}.$$

Denote $\pi_{s\to t}(\omega_s, \omega_t)$ the transition density of log wealth between s and t and denote $\gamma_{Bt}(\omega) = e^{\omega} g_{Bt}(e^{\omega})$ the density of arriving agents at time t. We have $\gamma_0(\omega) = e^{\omega} g_0(e^{\omega})$ and

$$\gamma_t(\omega) = \int_0^t (\delta_s + \eta_s) e^{-\int_s^t (\delta_u + \eta_u)\,\mathrm{d}u} \left(\int_{\mathbb{R}} \gamma_{Bs}(\omega') \pi_{s\to t}(\omega', \omega)\,\mathrm{d}\omega'\right)\mathrm{d}s$$
$$+ e^{-\int_0^t (\delta_u + \eta_u)\,\mathrm{d}u} \int_{\mathbb{R}} \gamma_0(\omega_0) \pi_{0\to t}(\omega_0, \omega)\,\mathrm{d}\omega_0$$

for $t \geq 0$. Following the same steps as in the proof of Proposition 2, one can show that the density γ_t is positive everywhere, infinitely differentiable, with $\int_{\mathbb{R}} e^{\omega} \gamma_t(\omega)\,\mathrm{d}\omega < \infty$.

Step 2—Law of motion of average wealth in the top percentile. This step is similar to the proof of Proposition 2, and Equation (22) is unchanged.

Step 3-Integrating by parts the Kolmogorov forward Equation. In the presence of demographic changes, the Kolmogorov forward equation is

$$\mathrm{d}g_t(w) = -\partial_w(\mu_t(w) w g_t(w))\,\mathrm{d}t + \frac{1}{2}\partial_w^2(\nu_t^2(w) w^2 g_t(w))\,\mathrm{d}t + (\delta_t + \eta_t)\left(g_{Bt}(w) - g_t(w)\right)\mathrm{d}t.$$

Plugging this equation into (22) and integrating by parts gives

$$\mathrm{d}\left(\int_{q_t}^{\infty} w g_t(w)\,\mathrm{d}w\right) = \left(\int_{q_t}^{\infty} \mu_t(w) w g_t(w)\,\mathrm{d}w\right)\mathrm{d}t + \frac{1}{2}\nu_t(q_t)^2 q_t^2 g_t(q_t)\,\mathrm{d}t$$
$$+ \left(\int_{q_t}^{\infty} (w - q_t)(g_{Bt}(w) - g_t(w))\,\mathrm{d}w\right)(\delta_t + \eta_t)\,\mathrm{d}t$$

Dividing by $\int_{q_t}^{\infty} w g_t(w)\,\mathrm{d}w$ gives (9).

Step 4— Limit of the accounting framework as the time period tends to zero. I now show that, as the time period tends to zero, the within, between, and demography terms defined in the accounting framework (2) converge to the respective terms in (9). Between t and $t+\Delta t$, the terms defined in the accounting framework correspond to

$$\text{Within} = \frac{1}{p w_{\mathcal{P}t}} \int_{\log q_t}^{\infty} \left(\int_{\mathbb{R}} e^{\omega'} \pi_{t\to t+\Delta t}(\omega,\omega')\,\mathrm{d}\omega' \right) \gamma_t(\omega)\,\mathrm{d}\omega - 1,$$

$$\text{Birth} = \int_t^{t+\Delta t} e^{-\int_s^{t+\Delta t}(\delta_u+\eta_u)\,\mathrm{d}u}(\delta_s+\eta_s)$$
$$\times \left(\frac{1}{p w_{\mathcal{P}t}} \int_{\mathbb{R}} \left(\int_{\mathbb{R}} \max\left(e^{\omega'} - q_{t+\Delta t}, 0\right) \pi_{s\to t+\Delta t}(\omega,\omega')\,\mathrm{d}\omega' \right) \gamma_{Bs}(\omega)\,\mathrm{d}\omega \right) \mathrm{d}s,$$

$$\text{Death} = -\left(1 - e^{-\int_t^{t+\Delta t}\delta_s\,\mathrm{d}s}\right) e^{-\int_t^{t+\Delta t}\eta_s\,\mathrm{d}s} \frac{(1+\text{Within}) w_{\mathcal{P}t} - q_{t+\Delta t}}{w_{\mathcal{P}t}},$$

$$\text{Pop. Growth} = -\left(1 - e^{-\int_t^{t+\Delta t}\eta_s\,\mathrm{d}s}\right) \frac{(1+\text{Within}) w_{\mathcal{P}t} - q_{t+\Delta t}}{w_{\mathcal{P}t}}.$$

Differentiating with respect to Δt at $\Delta t = 0$ gives the within, birth, death, and population growth terms defined in (9). The between term is obtained as a residual. *Q.E.D.*

PROOF OF LEMMA 4: I first express the average time $T_{\underline{w}}(w_{it})$ by backward induction between t and $t+\Delta t$, where Δt is a short time period:

$$T_{\underline{w}}(w_{it}) = \delta\Delta t \times 0 + (1-\delta\Delta t) \times (\Delta t + \mathrm{E}_t[T_{\underline{w}}(w_{it+\Delta t})]).$$

Rearranging terms gives

$$0 = (1-\delta\Delta t)(\Delta t + \mathrm{E}_t[T_{\underline{w}}(w_{it+\Delta t}) - T_{\underline{w}}(w_{it})]) - T_{\underline{w}}(w_{it})\delta\Delta t.$$

Keeping only terms at the first order in Δt gives the following expression for $T_{\underline{w}}(w_{it})$:

$$0 = \mathrm{d}t + \mathrm{E}_t[\mathrm{d}T_{\underline{w}}(w_{it})] - T_{\underline{w}}(w_{it})\delta\,\mathrm{d}t.$$

Applying Ito's lemma gives the ordinary differential equation (ODE) satisfied by $T_{\underline{w}}$:

$$1 + T'_{\underline{w}}(w) w\mu + \frac{1}{2} T''_{\underline{w}}(w) w^2\nu^2 - T_{\underline{w}}(w)\delta = 0.$$

The solution of this ODE has the general form:

$$T_{\underline{w}}(w) = c_1 w^{\lambda_1} + c_2 w^{\lambda_2} + \frac{1}{\delta}, \tag{26}$$

where λ_1 and λ_2 are, respectively, the positive and negative roots of the quadratic equation $\mu\lambda + {}^1\!/\!_2\lambda(\lambda-1)\nu^2 - \delta$.[48] We have the following limit conditions:

$$T_{\underline{w}}(\underline{w}) = 0,$$

[48]This function is convex, converges to infinity as x converges to infinity, and equals $-\delta$ in zero; therefore there are exactly two zeros for this function, one negative and one positive.

$$\lim_{w\to+\infty} T_{\underline{w}}(w) = \frac{1}{\delta}.$$

This imposes $c_1 = 0$ and $c_2 = -1/(\delta q^{\lambda_2})$. Therefore, the solution of the ODE is

$$T_{\underline{w}}(w) = \frac{1}{\delta}\left(1 - \left(\frac{w}{\underline{w}}\right)^{\lambda_2}\right).$$

which gives (14) after defining $\xi = -\lambda_2$. *Q.E.D.*

PROOF OF PROPOSITION 5: Since the wealth distribution is Pareto, (15) becomes:

$$\begin{aligned} T &= \frac{1}{\delta}\left(1 - \frac{\int_q^\infty (w/q)^{-\xi} w^{-\zeta-1}\,\mathrm{d}w}{\int_q^\infty w^{-\zeta-1}\,\mathrm{d}w}\right) \\ &= \frac{1}{\delta}\left(1 - \frac{\zeta}{\zeta-\xi}\right), \end{aligned}$$

which gives (16).

The derivative of T with respect to the idiosyncratic variance of wealth, ν^2, is

$$\begin{aligned} \partial_{\nu^2} T &= -\frac{1}{\delta}\left(1 + \frac{\zeta}{\xi}\right)^{-2} \partial_{\nu^2}\left(\frac{\zeta}{\xi}\right) \\ &= \frac{1}{\delta}\left(1 + \frac{\zeta}{\xi}\right)^{-2} \frac{\zeta}{\xi}\left(\frac{\partial_{\nu^2}\xi}{\xi} - \frac{\partial_{\nu^2}\zeta}{\zeta}\right). \end{aligned}$$

As ν^2 increases, T decreases as long as the relative decrease of ξ is higher than the relative decrease of ζ. *Q.E.D.*

APPENDIX B: DECOMPOSING LOGARITHMIC GROWTH

Proposition 1 gives an accounting decomposition for the *arithmetic* growth of the average wealth in the top percentile. As a corollary, I now derive a decomposition for the *logarithmic* growth of the average wealth in the top percentile. Starting from Proposition 1, I first write the gross growth of the average wealth in a top percentile as a product of three terms:

$$\begin{aligned} \frac{w_{\mathcal{P}_1,1}}{w_{\mathcal{P}_0,0}} &= \left(\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}}\right)\left(1 + \frac{n_{\mathcal{I}}(w_{\mathcal{I},1} - q_1) + n_{\mathcal{O}}(q_1 - w_{\mathcal{O},1})}{n_{\mathcal{P}_0\setminus\mathcal{D}} w_{\mathcal{P}_0\setminus\mathcal{D},1} + n_{\mathcal{B}} w_{\mathcal{B},1} + (n_{\mathcal{I}} - n_{\mathcal{O}})q_1}\right) \\ &\times \left(1 + \frac{n_{\mathcal{B}}(w_{\mathcal{B},1} - q_1) - n_{\mathcal{D}}\left(\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}} w_{\mathcal{D},0} - q_1\right) - (1 - n_{\mathcal{P}_0})\left(\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}} w_{\mathcal{P}_0,0} - q_1\right)}{\frac{w_{\mathcal{P}_0\setminus\mathcal{D},1}}{w_{\mathcal{P}_0\setminus\mathcal{D},0}} w_{\mathcal{P}_0,0}}\right) \end{aligned}$$

Taking logarithms gives an accounting framework for the *logarithmic* growth of the average wealth in a top percentile:

$$\log\left(\frac{w_{\mathcal{P}_1,1}}{w_{\mathcal{P}_0,0}}\right) = \log\left(\frac{w_{\mathcal{P}_0\backslash\mathcal{D},1}}{w_{\mathcal{P}_0\backslash\mathcal{D},0}}\right) + \log\left(1 + \frac{n_{\mathcal{I}}(w_{\mathcal{I},1} - q_1) + n_{\mathcal{O}}(q_1 - w_{\mathcal{O},1})}{n_{\mathcal{P}_0\backslash\mathcal{D}} w_{\mathcal{P}_0\backslash\mathcal{D},1} + n_{\mathcal{B}} w_{\mathcal{B},1} + (n_{\mathcal{I}} - n_{\mathcal{O}}) q_1}\right)$$

$$+ \log\left(1 + \frac{n_{\mathcal{B}}(w_{\mathcal{B},1} - q_1) - n_{\mathcal{D}}\left(\frac{w_{\mathcal{P}_0\backslash\mathcal{D},1}}{w_{\mathcal{P}_0\backslash\mathcal{D},0}} w_{\mathcal{D},0} - q_1\right) - (1 - n_{\mathcal{P}_0})\left(\frac{w_{\mathcal{P}_0\backslash\mathcal{D},1}}{w_{\mathcal{P}_0\backslash\mathcal{D},0}} w_{\mathcal{P}_0,0} - q_1\right)}{\frac{w_{\mathcal{P}_0\backslash\mathcal{D},1}}{w_{\mathcal{P}_0\backslash\mathcal{D},0}} w_{\mathcal{P}_0,0}}\right).$$

The first term (*within*) is the logarithmic growth of the average wealth of individuals initially in the top percentile. The second term (*between*) is the log ratio between the average wealth in the top at time $t = 1$ and its counterfactual value after setting the wealth of individuals in $\mathcal{I}$ and $\mathcal{O}$ to q_1. The third term (*demography*) is the log ratio between this last value and the counterfactual average wealth in the top at time $t = 1$ in the absence of any composition change.

While this logarithmic decomposition looks slightly more complex, it has two useful properties. First, since logarithmic growth rates are time additive, one can exactly aggregate the results of the accounting framework across successive time periods to decompose the cumulative growth of the average wealth in the top percentile into the sum of a cumulative within, between, and demography terms. Second, normalizing wealth by the average wealth in the economy in the logarithmic decomposition (e.g., to decompose top wealth shares) is exactly equivalent to adjusting the within term by the logarithmic growth of the average wealth in the economy. These two properties only hold approximately in the arithmetic decomposition.[49]

Still, over a short horizon (e.g. a year), both decompositions give very similar results quantitatively. Due to its relative simplicity, I focus on the arithmetic decomposition in my empirical applications.

APPENDIX C: EXTENSIONS

Proposition 2, which gives the dynamics of the average wealth in a top percentile in terms of the dynamics of individual wealth, assumes that wealth follows a Markov diffusion. I now consider three extensions that allow for more general wealth dynamics.

C.1. *Type Heterogeneity*

The main text assumes that the drift and volatility of wealth only depend on the wealth level. In reality, individuals with similar wealth may have different wealth dynamics due to heterogeneous portfolios or heterogeneous consumption rates. To model this heterogeneity in a parsimonious way, I assume that individuals are split in $J \geq 1$ groups. The law of motion of the wealth for individual i in group $1 \leq j \leq J$ is

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_{jt}(w_{it})\,\mathrm{d}t + \nu_{jt}(w_{it})\,\mathrm{d}B_{it}, \tag{27}$$

where $\mu_{jt}(.)$ and $\nu_{jt}(.)$ are group specific drift and volatility that satisfy the regularity conditions of Section 3. Individuals transition between the different groups $1 \leq j \leq J$ with an arbitrary Markov transition matrix. Denote $\mathbf{g}_t = (g_{1t}, \ldots, g_{Jt})$ the joint density of wealth w and type j at time t.

[49] In particular, the geometric mean of the arithmetic growth of the average wealth in the top percentile is only approximately equal to the sum of the geometric mean of the within, between, and demography terms.

PROPOSITION 6: *Assume that individual wealth follows the diffusion process (27). The average wealth in the top percentile $w_{\mathcal{P}t}$ follows the law of motion:*

$$\frac{\mathrm{d}w_{\mathcal{P}t}}{w_{\mathcal{P}t}} = \underbrace{\mathrm{E}^{\mathbf{g}_t(\cdot)\cdot}\left[\mu_{jt}(w) \mid w \geq q_t\right]\mathrm{d}t}_{\textit{Within}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{pw_{\mathcal{P}t}}\mathrm{E}^{\mathbf{g}_t}\left[\nu_{jt}(w)^2 \mid w = q_t\right]\mathrm{d}t}_{\textit{Between}},$$

where $\mathrm{E}^{\mathbf{g}_t(\cdot)\cdot}$ denotes the wealth-weighted expectation with respect to the wealth density $\mathbf{g}_t$.

The expressions for the within and between terms are essentially the same as in the baseline model. The within term simply depends on the wealth-weighted *average* drift of individuals in the top percentile, while the between term depends on the *average* idiosyncratic volatility at the top percentile threshold. Note that drift heterogeneity does not appear in the between term: this comes from the fact that, during a short period of time Δt, the cross-sectional variance of wealth growth due to heterogeneous drifts is in Δt^2, rather than Δt.

C.2. *Aggregate Shocks.*

The main text assumes that wealth shocks are purely idiosyncratic. In reality, individuals are also differently exposed to aggregate shocks.[50]. To model this heterogeneity in a parsimonious way, I assume that the law of motion of the wealth for individual i in group $1 \leq j \leq J$ is

$$\frac{\mathrm{d}w_{it}}{w_{it}} = \mu_{jt}(w_{it})\,\mathrm{d}t + \sigma_{jt}(w_{it})\,\mathrm{d}Z_t + \nu_{jt}(w_{it})\,\mathrm{d}B_{it}, \tag{28}$$

where Z_t is an aggregate Brownian motion and, for any $1 \leq j \leq J$, $\mu_{jt}(.), \sigma_{jt}(.), \nu_{jt}(.)$ satisfy the regularity conditions given in Section 3.[51]

In this case, the distribution of wealth is stochastic. While the mathematics involved in keeping track of a stochastic distribution are more involved (see, e.g., Carmona and Delarue (2018)), I still derive informally the law of motion of the average wealth in the top percentile. As above, denote $\mathbf{g}_t$ the joint density of wealth and type at time t.

PROPOSITION 7: *Assuming that individual wealth follows the diffusion process (28), the average wealth in the top percentile $w_{\mathcal{P}t}$ follows the law of motion:*

$$\frac{\mathrm{d}w_{\mathcal{P}t}}{w_{\mathcal{P}t}} = \underbrace{\mathrm{E}^{\mathbf{g}_t(\cdot)\cdot}\left[\mu_{jt}(w) \mid w \geq q_t\right]\mathrm{d}t + \mathrm{E}^{\mathbf{g}_t(\cdot)\cdot}\left[\sigma_{jt}(w) \mid w \geq q_t\right]\mathrm{d}Z_t}_{\textit{Within}}$$
$$+ \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{pw_{\mathcal{P}t}}\left(\mathrm{E}^{\mathbf{g}_t}\left[\nu_{jt}(w)^2 \mid w = q_t\right] + \mathrm{Var}^{\mathbf{g}_t}\left[\sigma_{jt}(w) \mid w = q_t\right]\right)\mathrm{d}t}_{\textit{Between}},$$

where $\mathrm{E}^{\mathbf{g}_t(\cdot)\cdot}$ (resp. $\mathrm{Var}^{\mathbf{g}_t}$) denotes the wealth-weighted expectation (resp. variance) with respect to the density $\mathbf{g}_t$.

[50]See, for instance, Di Tella (2016) or Wolff (2002)

[51]At the cost of more cumbersome notations, these expressions can be easily extended to the case in which Z_t is a multi-dimensional Brownian to model (i.e., heterogeneous exposure to multiple aggregate shocks, such as industry-specific shocks).

Heterogeneous exposure to aggregate shocks affects both the within term and the between term. The within term now loads on the aggregate Brownian motion: the exposure of the average wealth is given by the wealth-weighted exposure of individuals in the top percentile. More interestingly, the between term now depends not only on the average idiosyncratic volatility of individuals at the percentile threshold but also on the variance of their exposures to aggregate risks. Intuitively, both terms contribute to the cross-sectional variance of wealth growth at the percentile threshold.

C.3. *Jump-Diffusion*

The main text assumes that wealth follows a diffusion model. I now consider the more general case in which the wealth process follows a jump-diffusion model.[52] Formally, I assume that wealth follows the law of motion:

$$\begin{aligned}\frac{\mathrm{d}w_{it}}{w_{it\text{-}}} &= \mu_t(w_{it\text{-}})\,\mathrm{d}t + \nu_t(w_{it\text{-}})\,\mathrm{d}B_{it} \\ &\quad + \left(e^{\phi_t(w_{it\text{-}})U_i} - 1\right)\mathrm{d}N_{it} - \mathrm{E}_U\left[e^{\phi_t(w_{it\text{-}})U_i} - 1\right]\lambda\,\mathrm{d}t,\end{aligned} \tag{29}$$

where N_{it} is a compound Poisson process with intensity λ and U_i is a bounded random variable with density f_U. The function ϕ_t allows the effective log sizes of jumps, $\phi_t(w)U_i$, to depend on wealth w and time t. Without loss of generality, I compensate the compound Poisson process by $\mathrm{E}_U\left[e^{\phi_t(w_{it\text{-}})U_i} - 1\right]\lambda$, where E_U denotes the expectation with respect to the random variable U, so that $\mu_t(w)$ still corresponds to the instantaneous expected growth rate of wealth.

I assume the same regularity conditions for μ_t and ν_t as in Section 3, with the additional assumptions that (i) ϕ_t is positive, (ii) $\phi_t(\exp(.))$ possesses bounded derivatives of all orders and (iii) the resulting wealth density is smooth enough, that is, $g_t \in \mathcal{C}^\infty$, and, over a finite time period, $w\partial_t g_t(w)$ is uniformly dominated by an integrable function.[53,54]

PROPOSITION 8: *Assuming that wealth follows the jump-diffusion process* (28)*, the average wealth in the top percentile $w_{\mathcal{P}t}$ follows the law of motion:*[55]

$$\begin{aligned}\frac{\mathrm{d}w_{\mathcal{P}t}}{w_{\mathcal{P}t}} &= \underbrace{\mathrm{E}^{g_t(\cdot)\cdot}\left[\mu_t(w)|w \geq q_t\right]\mathrm{d}t}_{\textit{Within}} + \underbrace{\frac{1}{2}\frac{g_t(q_t)q_t^2}{p w_{\mathcal{P}t}}\nu_t(q_t)^2\,\mathrm{d}t}_{\textit{Between (Diffusion Part)}} \\ &\quad + \underbrace{\frac{1}{p w_{\mathcal{P}t}}\mathrm{E}_U\left[\int_0^{q_t}(e^{\phi_t(w)U}w - q_t)^+ g_t(w)\,\mathrm{d}w + \int_{q_t}^{\infty}(q_t - e^{\phi_t(w)U}w)^+ g_t(w)\,\mathrm{d}w\right]\lambda\,\mathrm{d}t}_{\textit{Between (Jump Part)}}.\end{aligned} \tag{30}$$

In the presence of jumps, the between term is the sum of two terms: the first term is due to the diffusive part of the process (i.e., the Brownian motion B_{it}) while the second term is due

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[52] A large literature in finance argues for the presence of jumps in asset prices, which would imply jumps in individual wealth (e.g., Ait-Sahalia (2004), Barro and Ursúa (2012)).

[53] That is, for any finite time $T > 0$, there exists f_T such that $\int_{\mathbb{R}_+} f_T(w)\,\mathrm{d}w < \infty$, and for any $t < T$, $w\partial_t g_t(w) \leq f_T(w)$. As in the proof of Proposition 2, this will be needed to use the dominated convergence theorem to relate the law of motion of $w_{\mathcal{P}t}$ to the law of motion of the wealth density g_t.

[54] While I believe it to be a consequence of the other assumptions, I could not find a proof in the existing literature. The closest paper is Cass (2006), which examines the case of a time-homogeneous process.

[55] Here, and elsewhere, "x^+" denotes the positive part of x; that is, $x^+ = \max(x, 0)$.

to the jump part of the process (i.e., the compound Poisson process N_{it}). The jump part is the sum of an inflow term and an outflow term, which correspond, respectively, to the contribution of positive and negative jumps on the law of motion of the average wealth in the top percentile. In contrast with the term due to the diffusion part of the process, the jump term depends on the density of wealth and on the distribution of jump sizes *across the whole distribution*, not just at the percentile threshold. This reflects the fact that, during an infinitesimal period of time $\mathrm{d}t$, individuals can now enter the top percentile from any part of the wealth distribution.

Higher-Order Cumulants. When the distribution of jumps is regular enough, the between term can be rewritten as a Taylor expansion of the wealth density and jump sizes around the percentile threshold.

Formally, for $n \geq 2$, denote $\kappa_{nt}(w)$ the "instantaneous" cumulant of (log) wealth growth, defined as the n-th derivative of the instantaneous cumulant generating function:

$$\kappa_{nt}(w) = \partial_\tau \partial_\theta^n \log \mathrm{E}_t \left[e^{\theta \log\left(\frac{w_{it+\tau}}{w_{it\text{-}}}\right)} | w_{it\text{-}} = w \right] \Big|_{\theta=0,\tau=0}.$$

An application of Ito's lemma, combined with (29), gives the following expression for the instantaneous cumulants:

$$\kappa_{nt}(w) = \begin{cases} \nu_t(w)^2 + \lambda \phi_t(w)^2 \mathrm{E}_U\left[U^2\right] & \text{for } n = 2, \\ \lambda \phi_t(w)^n \mathrm{E}_U\left[U^n\right] & \text{for } n > 2. \end{cases} \tag{31}$$

The second cumulant (i.e., the instantaneous variance) is the sum of a term due to the diffusive part of the process, $\nu_t(w)^2$, and a term that depends on the second moment of the random variable U. For $n > 2$, the instantaneous cumulant of order n depends on the n-th moment of the random variable U. When the process follows a diffusion (i.e., $\lambda = 0$), all cumulants of order higher than two are zero.

Assuming that the function $v \to \int_{q_t e^{-v}}^{q_t} (e^v w - q_t) \frac{1}{\phi_t(w)} f_U\left(\frac{J}{\phi_t(w)}\right) g_t(w)\,\mathrm{d}w$ is analytic, the between term can be rewritten as a sum of all higher-order cumulants:

COROLLARY 9: *Assuming that wealth follows the jump-diffusion process (29), the average wealth in the top percentile $w_{\mathcal{P}t}$ follows the law of motion:*

$$\frac{\mathrm{d}w_{\mathcal{P}t}}{w_{\mathcal{P}t}} = \underbrace{\mathrm{E}^{g_t(\cdot)\cdot}\left[\mu_t(w) | w \geq q_t\right] \mathrm{d}t}_{\textit{Within}} + \underbrace{\sum_{n=2}^{\infty} \frac{1}{n!} \frac{q_t}{p w_{\mathcal{P}t}} \sum_{m=0}^{n-2} (-w\partial_w)^m \left(\kappa_{nt}(w) w g_t(w)\right)|_{w=q_t} \mathrm{d}t}_{\textit{Between}}. \tag{32}$$

This says that the between term can be re-expressed as an infinite series of terms, where the term of order n depends on the n-th cumulant of (log) wealth growth κ_{nt}.

The first term, $1/2 (g_t(q_t) q_t^2 / p w_{\mathcal{P}t}) \kappa_{2t}(q_t)$, captures the contribution of the variance of (log) wealth growth. It is similar to the term obtained for a diffusion process, with the only difference that $\kappa_{2t}(q_t)$ now reflects the total variance of (log) wealth growth at the percentile threshold, which also depends on the intensity and distribution of jumps, as seen in Equation (31).

When the wealth process follows a diffusion, cumulants of order $n > 2$ are zero, and therefore terms of order $n > 2$ are equal to zero.

Note that the term of order n also depends on the first $n - 2$ *derivatives* of the cumulant κ_{nt} and of the wealth density g_t around the percentile threshold. Intuitively, higher-order terms reflect the contribution of large wealth realizations, which depend on the wealth density as well as on the distribution of jump sizes far from the percentile threshold.

Stationary Case. As in the diffusion model, the between term takes a particularly simple form when the wealth distribution has a Pareto tail. Assume that $\mu_t(w) = \mu(w)$, $\nu_t(w) = \nu(w)$, and $\phi_t(w) = \phi(w)$, with $\mu(w) \to \mu < 0$, $\nu(w) \to \nu$, and $\phi(w) \to \phi$ as $w \to \infty$. Under these assumptions, the stationary distribution, if it exists, has a Pareto tail with Pareto exponent $\zeta > 1$.

For a distribution with a Pareto tail, we have, as $p \to 0$,

$$\frac{1}{pw_{\mathcal{P}}}\mathrm{E}_U\left[\int_0^q \left(e^{\phi(w)U}w - q\right)^+ g(w)\,\mathrm{d}w\right] \to \mathrm{E}_U\left[\left(\frac{e^{\zeta\phi U}-1}{\zeta} - \left(e^{\phi U}-1\right)\right)1_{U\geq 0}\right],$$

and

$$\frac{1}{pw_{\mathcal{P}}}\mathrm{E}_U\left[\int_q^\infty \left(q - e^{\phi(w)U}\right)^+ g(w)\,\mathrm{d}w\right] \to \mathrm{E}_U\left[\left(\frac{e^{\zeta\phi U}-1}{\zeta} - \left(e^{\phi U}-1\right)\right)1_{U\leq 0}\right].$$

Therefore, taking the limit $p \to 0$ in Proposition 8 gives a simple balance equation for top wealth shares:

$$0 = \underbrace{\mu\,\mathrm{d}t}_{\text{Within}} + \underbrace{\frac{\zeta-1}{2}\nu^2\,\mathrm{d}t + \mathrm{E}_U\left[\frac{1}{\zeta}\left(e^{\zeta\phi U}-1\right) - \left(e^{\phi U}-1\right)\right]\lambda\,\mathrm{d}t}_{\text{Between}}.$$

Using a Taylor series for $x \to e^{x\phi U}$ around $x = 0$ gives

$$0 = \underbrace{\mu\,\mathrm{d}t}_{\text{Within}} + \underbrace{\sum_{n=2}^{\infty}\frac{\zeta^{n-1}-1}{n!}\kappa_n\,\mathrm{d}t}_{\text{Between}}. \tag{33}$$

where κ_n denotes the limit of the cumulant of order n in the right tail of the wealth distribution; that is,

$$\kappa_n = \begin{cases} \nu^2 + \lambda\phi^2\mathrm{E}_U\left[U^2\right] & \text{for } n = 2, \\ \lambda\phi^n\mathrm{E}_U\left[U^n\right] & \text{for } n > 2. \end{cases}$$

Alternatively, one could obtain Equation (33) by taking the limit $p \to 0$ in Corollary 9.

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