

In Memoriam: Hugo F. Sonnenschein

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Hugo F. Sonnenschein left us this past July 15 at the age of four scores. He had a fulfilled intellectual and academic life. The multitude of his friends, disciples and collaborators will sorely miss him.

He was a native of New York, obtained his undergraduate degree at the university of Rochester and went on to obtain a Ph.D at Purdue university in 1964. He pursued his academic career at the universities of Minnesota (64-70), Massachusetts (70-73), Northwestern (73-76), Princeton (76-88 and 91-93), Pennsylvania (88-91) and, finally, Chicago (93-2021). He was Editor of *Econometrica* from 1977 to 1984 and President of the *Econometric Society* in 1989. In 1990 he was elected member of the *National Academy of Sciences* and in 2005 he became Distinguished Fellow of the *American Economic Association*.

Sonnenschein was one of the great economic theorists of his generation. His work was unerringly insightful and deep. He was outstanding at posing questions and elegant in approaching the answers. He touched many areas of economics, including applied theory issues in taxation, international trade and welfare economics. But the essential Sonnenschein was the theorist deeply concerned by foundational issues especially, but not only, in general equilibrium and social choice settings:

1.- Much of his research involves the interplay of individual and social behavior. More concretely, an all-encompassing question for him was: “to what an extent the classical axioms of ordered preferences (completeness, transitivity,...)

constrain, or are essential to constrain, individual or aggregate market behavior". In this vein, and in the context of classical equilibrium theory:

- In his earliest research he showed that the transitivity axiom was not necessary at all for the existence of a well-defined demand set. As this set does not need to be convex the existence of a general equilibrium (of Walras' type) did not follow from the methodology of fixed-point theorems applied to aggregate demand correspondences. The general result came later and again Sonnenschein, with Wayne Shafer, played a central role in its development by resuscitating the disaggregate methodology used in the original Arrow-Debreu proof for the existence of a price equilibrium.
- In the context of market demand the rationality requirement of underlying transitive preferences translates into symmetry conditions for Slutsky matrices, in itself a knife-edge property. A weaker and more appealing rationality condition is Samuelson's weak axiom (WA) of revealed preference (the "law of demand"). Sonnenschein, with collaborators, clarified the essential equivalence of the WA and the negative (quasi) - definiteness of the Slutsky matrix.
- The more transcendent contribution of Hugo emerged from the following observation: indeed, may be for the existence of demand or of an equilibrium, the full rationality of individual preferences is not necessary. Yet, if we have it, we could ask which additional properties we may hope for in the individual or, more crucially, the aggregate demand. For individuals it is the negative definiteness of Slutsky matrices (or, equivalently, the strong axiom of revealed preferences). For the aggregate we know that aggregate excess demand functions have to be homogeneous of degree zero and satisfy Walras' law.

Sonnenschein asked: is that all? He conjectured that the answer is yes and proved it for the case of two commodities. The proof was completed for the general case by R. Mantel and G. Debreu and it constitutes the celebrated Sonnenschein- Mantel – Debreu theorem. It has been variously interpreted as the death-knoll of equilibrium theory or as evidence that more information on individual characteristics (for example, on the distribution of initial endowments) is required if significant restrictions on the aggregate are to be obtained. In any case, the theorem inaugurated a new type of theoretical (“anything goes”) exercise that has been extensively resorted to: take a theoretical analysis that, from axioms on a basic model, establishes properties of a set of derived equations. Verify then the exhaustiveness of the theory by checking whether any set of equations formally similar to the derived ones, and satisfying all the properties already known, can be obtained from some particular instance of the basic model.

2.- Sonnenschein also examined the interplay of individual and social in the context of the social choice theory that emerged from the seminal work of Arrow. In this tradition, the methodological approach to the aggregation of individual into social preferences put more emphasis in the “nothing goes” than in the “anything goes”, the idea being that knowing that desirable, good postulates are incompatible has to force us to go deeper into the understanding and formulation of positive theories of aggregation. With Arrow and Sen as points of departure, Sonnenschein proved the limits of the impossibility results by shifting the focus of the social requirement from rationality (transitivity) to the capacity to take decisions (no Condorcet cycles, or acyclicity), even random decisions. Then some room appeared for possibility results, although the consideration of further desirable axioms devolved soon the impossibility. In this work he

collaborated with students, Salvador Barberà for the random outcome or the writer of these lines for acyclicity. As a natural development he also became interested, and published with a number of his students, in voting theory. His paper on *Voting by Committees* (with Barberà and Lin Zhou), emphasizing the significance of exploring the limits of non-manipulable voting systems, can be highlighted.

3.- Sonnenschein also dwelt with another foundational issue for an extension of the Walrasian model to contexts where the state of the world is not directly observable but it can be partly inferred from prices. With R. Anderson he gave conditions for the existence of a suitably defined rational expectations equilibrium.

4.- Beyond the Walrasian price taking model, Sonnenschein also devoted attention and much dedication to the theory of monopoly and oligopoly. On one hand, with J. Roberts, he observed that very simple settings could generate non-convex valued reaction functions and therefore serious existence problems. On the other hand, he undertook with W. Novshek a very sophisticated and elegant analysis of to what extent Walrasian price taking equilibrium could be considered a limit of Cournot- Nash oligopoly equilibrium when the efficiency size of the individual firms becomes small relative to the size of the market, with especial consideration to the role of free entry. They uncovered a surprising relevance for existence issues of conditions usually associated with stability.

5.- While not at the center of his published work Sonnenschein kept all his career a keen interest in game theory. His students know well how prominently it figured in his lectures. The consequence is that he nurtured at Princeton a notable cohort of game theorists. An important and influential paper on the Coase conjecture (on pricing

of durable goods), written with F. Gul and R. Wilson, wears witness to this aspect of Sonnenschein creativity.

Sonnenschein was a great scholar but he was also great in two related dimensions, in university administration and in the mentoring of students. His administrative career was brilliant: Dean at the University of Pennsylvania (1988-91), Provost at Princeton (91-93) and President of the University of Chicago (93-2000). At Chicago he promoted a major renewal of its distinctive undergraduate program, maintaining its spirit and essential traits but allowing for a richer undergraduate experience. It was controversial in its moment but it is regarded now as a success. It is witness in any case of his managerial courage. He also served in the boards of a variety of institutions (board of trustees of the University of Rochester, his alma mater, from 1992; board of directors of the Mathematical Sciences Research Institute, from 2006). His university leadership was not limited to the US. He made it a point of advising and encouraging serious academic adventures around the world. To mention two: he was from 2008 to the time of his passing the chair of the Scientific Council of the *Barcelona Graduate School of Economics*, and a member of the Advisory Board of the *King Fahd University* from 2006.

Students are important to scholars. But in the case of Sonnenschein the extent of his mentoring inclination cannot be overstated. It was phenomenal. He was an exemplary teacher. His graduate seminars were designed to convey where the research frontier was and were full of stimulus and open questions. Papers flew from the seminars. He had many official students but it was remarkable how he never refrained from helping and interacting with younger scholars that, often, become his adopted students. In fact, if one was lucky enough to write a paper with him, as was the case of the author of these lines, he was adopted for life. I quote from the obituary of the university of Chicago

(<https://news.uchicago.edu>) some sentences of one of his students, Phil Reny, also his colleague at Chicago: “his teaching style was elegant, gentle and encouraging”, “aside from his brilliance, which was immense, his great asset was his ability to see value in his students ‘ ideas, which he nurtured better than anyone I know” “he would never miss an opportunity to tell us that what we were doing was important work, which had a tremendous impact on us”. As an unofficial student the author of these lines can only concur and would like add a personal note: sharing with him the BBVA Economics, Finance and Management Award of 2009 added enormously to the honor.

The adjectives one can adorn Sonnenschein with are many: thoughtful, penetrating, setter of high standards, serious, warm. But there is one that towers the lot: generous. His students, his collaborators, the institutions he has helped with his advice can attest to it. You only needed to ask Hugo once. If he accepted a responsibility he would deliver much beyond what could be reasonably expected. In words from the obituary of the Barcelona GSE: “R. Marimon: I was impressed by how much time and energy Hugo gave to the Barcelona GSE”, “T. Garcia-Milà: Hugo contributed greatly to the Barcelona GSE. His wise advice at all times, his continuous support, his energy and enthusiasm will always be remembered and immensely missed”

(<https://www.barcelonagse.eu/news/memoriam-hugo-sonnenschein-1940-2021>).

On behalf of the Econometric Society community we conclude sending our deeply felt sympathy and condolences to the companion of all his life, Elizabeth Gunn Sonnenschein, and to his beloved daughters and grandchildren.