Edmond Malinvaud's Contributions to Microeconomics*

Guy Laroque[†] **Bernard Salanié**[‡]

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Introduction

Edmond Malinvaud left no field of economics untouched, as the articles in the special issue of *Annals of Economics and Statistics* (June 2017) attest: they cover areas as diverse as econometric theory, macroeconomics, growth... We focus here on his main contributions to microeconomic theory. As in other areas of economics, Malinvaud's approach was always to use rigorous theory to analyze policy-oriented topics: planning and mechanism design, decision-making under uncertainty, and the role of insurance markets in ensuring an efficient functioning of the economy. We end our paper with his famous *Lectures*, which trained generations of microeconomists in France and abroad.

1 Planning and incentives

Neoclassical economists described market socialism early, from Walras to Pareto and Barone's famous 1908 paper. From the First World War and the October Revolution to the end of the Cold War, the study of socialism was fairly active among economists. In the 1920s, a heated Socialist Calculation Debate opposed von Mises and Hayek to Lange and others. Fred Taylor[1], president of the *American Economic Association* in 1929, gave his presidential lecture on "The Guidance of Production in a Socialist State". In his presentation, Taylor (1929) laid down an informal account of a tâtonnement process operated by a central planner. A

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[†]UCL and Sciences Po, guy.laroque@sciencespo.fr.

[‡]Department of Economics, Columbia University; e-mail: bsalanie@columbia.edu.

[1]No connection with the leader of the Efficiency Movement, who died in 1915.

more formal representation was given in Lange (1936); but the Second World War interrupted the research in the field.

Malinvaud restarted the studies in this area with two papers. The first one, Malinvaud (1967), was published in a book. The second, Malinvaud (1968b), is in French, in the *Canadian Journal of Economics/Revue Canadienne d'Économique*. Later on, a sizable part of the literature on planning appeared in the *Review of Economic Studies*, which became a natural habitat for the research on planning and incentives. The two papers of Malinvaud mentioned above lay down a model of a planned economy and describe its evolution over time according to a tâtonnement process. They study the convergence of the process, including the speed at which it converges[2].

We give below a short description of the model for a standard exchange economy, using as our main source the paper in French. There are a finite number of agents $i = 1, \ldots, I$, and a finite number of goods $h = 1, \ldots, H$. The prices of the goods are given by a vector p in $\mathcal{R}^H_+$. The total quantity of goods that are available is described by a vector ω in $\mathcal{R}^H_+$, and the planner endows each agent i with a fraction R_i of the aggregate endowment ($R_i \geq 0$, and $\sum_{i=1}^I R_i = 1$). This allows the planner some degree of control over the distribution of final allocations. Malinvaud described two planning procedures. In the first one (primal), the planner quotes a price vector to the consumers, and they answer with the quantities they will demand at these prices. In the second one (dual), the planner quotes to each agent i his vector of consumptions, and each agent reports how much he is ready to pay for an extra unit— i.e. his marginal rates of substitution (MRS) at this point.

1. **Primal**: Prices are normalized to be in the simplex, $\sum_{h=1}^H p_h = 1$. At stage s, the plan announces a (normalized) price vector p^s. Each agent i computes her demand x_i^s which maximizes her utility $U_i(x)$ over her budget set $p \cdot x = R_i p^s \cdot \omega$. Then prices are revised. For commodity h, $h = 1, \ldots, H$,

$$\frac{p_h^{s+1}}{p_h^s} - 1 = a \left[\frac{\sum_{i=1}^{I} x_{ih}^s}{\omega_h} - 1 \right],$$

   where a is a positive fixed number; it needs to be chosen small enough that the dynamic process will converge.

2. **Dual**: The last good, H, is chosen as the numéraire. Given a prospective consumption vector x_i^s assigned to agent i at stage s the MRS for commodi-

[2]One mathematical question, solved by Henry (1972), was the existence of a solution to differential equations with a multivalued right-hand side.

ties 1 to $(H-1)$ with respect to the numéraire are

$$\pi_{ih}^s = \frac{U'_{ih}(x_i^s)}{U'_{iH}(x_i^s)}$$

Define the "average MRS" by

$$\pi_{.h}^{s+1} = \sum_{i=1}^{I} R_i \pi_{ih}^s.$$

For the first $(H-1)$ goods, the consumption of agent i is revised according to

$$x_{ih}^{s+1} - x_{ih}^s = bR_i(\pi_{ih}^s - \pi_{.h})$$

where b is a positive fixed number, again chosen small enough. The intuition is simple: if agent i values good h more than the average agent, then the planner should naturally increase i's consumption of good h. For the budget constraint to hold, the increase should be proportional to R_i. The concavity of the utility functions yields convergence. The consumption of numéraire is also modified so that the budget constraint holds in the long run:

$$x_{iH}^s - x_{iH}^{s-1} = -\sum_{h=1}^{H-1} \pi_{.h}^{s-1}(x_{ih}^s - x_{ih}^{s-1}) - c\sum_{h=1}^{H} \pi_{.h}^{s-1}(x_{ih}^{s-1} - R_i\omega_h),$$

where $\pi_{.H}$ is always equal to 1 and c is a small positive fixed number.

These two procedures share a number of desirable properties. In particular, any stationary point of these dynamic processes is an equilibrium. Such a point is a local attractor of the process. There is a trade off in the choice of the adjustment speeds a, b or c: a large change requires high enough speeds, but they may destabilize the process and jeopardize convergence.

Malinvaud adapted the above analysis to economies with Leontief production functions, constant returns to scale, and several inputs and outputs. In his 1971 paper, he discussed in more detail a model with two consumers, one private good and one public good. He provided a diagrammatic representation of the procedures, and he discussed the limits to redistribution and the revelation of preferences. The contrast between the role of prices and quantities in the two above procedures has generated a number of papers, see e.g. Weitzman (1974) and Freixas (1980). Are some economies better monitored through prices, and others with quantitative planning? In the remainder of this section, we focus on issues related to the exchange of information between the center and the agents, as originally approached by Malinvaud.

1. **Public goods.** The extension to models with public goods was undertaken simultaneously by Drèze and de la Vallée Poussin (1971) and Malinvaud

(1971). This generated a lot of attention in the profession: planning procedures are of particular interest in such a case since, as is well known, the pure non-cooperative competitive mechanism generally leads to inefficient situations in the presence of public goods. Although the solutions proposed by these two procedures to the problem of production and exchange of private goods are very different, they both adopt the same rules for the revision of the public goods quantities. This class of models became to be known as the Malinvaud and Drèze-de la Vallée Poussin procedures, or (MDP) procedures.

The MDP procedures enjoy two interesting properties. During the procedure a social surplus appears which can be redistributed among consumers so as to permit an increase in all utility levels—i.e. the procedure is monotonic. Moreover, under usual convexity assumptions for small enough speeds the procedure converges to an efficient (or Pareto optimal) allocation.

2. **Neutrality.** Knowing that an efficient situation will obtain if the MDP procedure is implemented, a further question arises, which was stated and solved in Champsaur (1976). What efficient situations can be reached in such a way? Champsaur gave conditions under which the following answer holds. Every efficient situation which is preferred or equivalent to the initial situation can be reached by the use of the MDP procedure with a suitable choice of the distribution of the social surplus appearing during the procedure. This choice can be made before the beginning of the procedure and can be kept constant along the procedure. Therefore any negotiation taking place before the beginning of the procedure can legitimately be concentrated upon the choice of a distribution of this surplus. Nobody can object to the rules followed in the revision of the public goods quantities since these rules do not have by themselves any distributive implications. We can say that they are *neutral*.

3. **Information and myopia.** Until 1972 and apart from brief discussions in Drèze and de la Vallée Poussin (1971), Malinvaud (1971) and Malinvaud (1972c), the literature on planning or iterative planning dedicated little attention to the important question of how the center could elicit information from the agents.

   In the MDP procedure, the planner asks the agents to report their marginal rates of substitution (MRS). If the agents report their true MRS, the procedure has all desirable properties: monotonicity, convergence to a Pareto optimal allocation, and neutrality. However it is generally the case that, by sending a message different from his true MRS, an agent can expect an improvement of his situation. Does such a misrepresentation generate inefficiency of the final outcome of the MDP procedure? To study this question, a simplifying assumption was adopted, which proved interesting and fruitful: the agents are *myopic* in the sense that they only look at the immediate consequence of their message at each instant, ignoring the subsequent effects. The ensuing “local” or “instantaneous” noncooperative game was studied by Drèze and de la Vallée Poussin (1971) who showed that a maximin behavior implies the revelation of the true MRS.

Malinvaud (1972c) conjectured that noncooperative and myopic behaviors should not prevent the procedure from yielding efficient outcomes. Roberts (1979b) and Roberts (1979a) proposed a rigorous formulation for the latter idea. He considered the Nash equilibrium of the local game and showed that, although the agents do not report their true MRS in such a case, the procedure keeps nice properties: monotonicity, and convergence to a Pareto optimal allocation. Roberts (1979b) emphasizes the importance of the myopia assumption in getting these positive results: *Of course, if agents adopt more sophisticated behavior, then the desirable properties shown here may no longer obtain... if agents are able to predict the full impact of their strategic choices on the final allocation and if they adopt strategies which are intertemporally consistent, the Nash equilibrium under the MDP procedure would generally not correspond to correct revelation. Furthermore, although the characterization of the Nash equilibria is an open question, it would appear very unlikely that they would be Pareto optimal.*

4. **Incentive compatibility and revelation in planning procedures.** Particularly since the publication of Hurwicz (1973)'s fundamental paper, the myopia assumption appeared to be overly restrictive; and following Roberts' work, the search for *incentive compatible* planning mechanisms has been active. The aim was to better understand the trade-off between the redistributive power of the planner and the length of the agents' horizon in a non cooperative Nash equilibrium setup.

   Champsaur and Laroque (1982) and Laroque and Rochet (1983) investigated Roberts' conjectures by dropping the myopia assumption. Thus an agent's strategy is a function from $[0, T]$ into the set of possible MRS and he is supposed to consider the utility reached at date T, neglecting any event occuring after T. The agents are supposed to behave in a rather sophisticated manner: they make accurate forecasts concerning the announcements which will be made by the other agents on $[0, T]$ and use these expectations in order to compute the best (intertemporally consistent) strategy.

   In this set up, the MDP procedure still yields approximately efficient outcomes provided that it is operated long enough. Indeed, if the agents act according to their T horizon expectations while the procedure is kept running, the economy converges to an efficient allocation. This efficiency property is not surprising if the agents' horizon T is short relative to the length of the procedure's operation. In fact, a stronger result holds: the efficiency property is preserved, provided that the time horizon T goes to infinity. In such a case T can also be interpreted as the time length allotted to planning by the Center, T being known by every agent as part of the rules of the procedure. Thus, approximate efficiency is compatible with a fully intertemporally consistent behavior.

For T infinite, there does not exist a Nash equilibrium in intertemporal strategies, thus partially confirming Roberts' intuition. What then is the impact of the manipulation by the agents in this kind of procedure? There is indeed an important consequence: more manipulation narrows the influence of the policy tools controlled by the planner (for the MDP procedure, these policy tools are constituted by the distributional weights chosen by the central planner). For a given range of the policy tools, the power of the center, while remaining significant, is strongly reduced in a situation of myopic manipulation compared to the case of no manipulation at all. In the presence of intertemporal manipulation, if the horizon T is large enough, the planner loses any significant influence on the outcome of the procedure. More precisely, when T tends to infinity, whatever the distributional weights, every Nash equilibrium approaches: (i) a competitive equilibrium in an exchange economy, and (ii) a Lindahl equilibrium in an economy with public goods. This last point is related to Hurwicz (1973) which gives conditions under which these allocations are competitive if the Nash equilibria of a noncooperative game correspond to Pareto optimal allocations of, for example, an exchange economy.

The literature has made large advances in this area, both by moving away from planning procedures to consider general implementation problems, and by allowing the agents to possess private information about any aspect of the economy. Gul and Postlewaite (1992), McLean and Postlewaite (2002) and McLean and Postlewaite (2003) are important contributions in the field. Their work defines the "informational size" of each agent to be the extent to which he can modify the center's belief by misrepresenting his private information. They prove that if all agents become "informationally small" as the economy grows, there exists an incentive-compatible mechanism that implements efficient allocations.

2 The economics of risky prospects

Malinvaud worked on risk at two points in his career. It was the topic of his first international publication, when he was visiting the Cowles Commmission in Chicago in 1952; and he authored a series of important papers on this topic between 1969 and 1973.

2.1 The independence axiom

Malinvaud (1952) is really a very short note in *Econometrica.* Every economist knows that von Neumann and Morgenstern axiomatized expected utility in *The Theory of Games and Economic Behavior* (1947). But in fact their list of axioms left out what we now call the independence axiom, which drives the linearity in probabilities. Samuelson in particular, according to the account of Moscati (2016), had pointed out that something seemed to be missing[3], and while he did not state the independence axiom precisely he gave it its name. Malinvaud's note explained that von Neumann and Morgenstern had stated their assumptions on preferences over indifference classes of events; and they had endowed these indifference classes with a linear mixture operation by which, for two such classes u and v and any $\lambda \in [0,1]$,

$$\lambda u \oplus (1-\lambda)v$$

was implicitly assumed to be an equivalence class too. He then showed that this was tantamount to assuming that all lotteries formed by an event in u and an event in v must leave the decision-maker indifferent—a form of the independence axiom. Herstein and Milnor (1953) would soon build on a similar remark to extend expected utility to general mixture spaces.

2.2 First-order certainty equivalence

Certainty equivalence was a very early result in expected utility theory: it appears (minus the name) in Theil (1954). Let the utility function be quadratic and the relationship between instruments (decision variables) and outcomes have additive errors. Assume moreover that the first moments of these errors are zero and that their second moments do not depend on the instruments. Then the presence of the errors does not change the agents' optimal decisions. To see this, denote x the instruments, and y the outcomes. We write the "outcome equation" as

$$y = f(x) + u$$

where $Eu = 0$ and the distribution of u does not depend on x. With utility index

$$a'y - \frac{1}{2}y'\Sigma y,$$

[3]Samuelson was reluctant to accept the axiom at first; Moscati (2016) tells the story of how he finally came to accept it, and how his correspondence with Savage on this topic inspired the Sure-Thing principle.

the expected utility from decision x is

$$
\begin{aligned}
& E\left(a'\left(f(x)+u\right)-\frac{1}{2}\left(f(x)+u\right)'\Sigma\left(f(x)+u\right)\right) \\
& = E\left(a'f(x)-\frac{1}{2}f(x)'\Sigma f(x)\right) \\
& - f(x)'\Sigma\, Eu \\
& - \frac{1}{2}E\left(u'\Sigma u\right).
\end{aligned}
$$

The second line on the right-hand side is zero, and the third line does not depend on x. Therefore maximizing expected utility boils down to maximizing the first line, which is simply the objective function when $u \equiv 0$.

Simon (1956) and Theil (1957) had extended this result to dynamic decision-making, and Theil (1964) to stochastic models of outcomes. He had shown how to handle stochastic a, B and Σ. But the theory of certainty equivalence remained founded on highly restrictive specifications, most prominently quadratic utility and an additive outcome equation. Malinvaud (1969a,b) showed that these assumptions can be dispensed with: when uncertainty is "small", then to a first-order approximation, the optimal decisions are unchanged by the presence of errors.

To see this in our simple example, denote $v(x, y)$ the utility function. Parameterize errors as σu, where we will fix the distribution of u and take expansions as a function of the scale of the uncertainty σ. We continue to assume that the first two moments of u are independent of the decisions x. Our outcome equation now is $y = g(x, \sigma u)$, so that the agent chooses x to maximize $Ev(x, g(x, \sigma u))$. We will assume that all relevant functions are smooth.

First expand g to a second order[4] in σu:

$$g(x, \sigma u) = g(x, 0) + g_2(x, 0)\sigma u + \frac{1}{2}g_{22}(x, 0)\sigma^2 u^2 + o(\sigma^2 u^2).$$

Then plug it into the objective function

$$
\begin{aligned}
v[x, g(x, \sigma u)] = v[x, g(x, 0)] + v_2[x, g(x, 0)] & \left(g_2(x, 0)\sigma u + \frac{1}{2}g_{22}(x, 0)\sigma^2 u^2\right) \\
& + \tfrac{1}{2}v_{22}[x, g(x, 0)]\left(g_2(x, 0)\sigma u + \tfrac{1}{2}g_{22}(x, 0)\sigma^2 u^2\right)^2 + o(\sigma^2 u^2).
\end{aligned}
$$

and impose $Eu = 0$ to obtain

$$Ev(x, g(x, \sigma u)) = v(x, g(x, 0)) + \frac{\sigma^2}{2}(v_{22}g_2^2 + v_2 g_{22})Eu^2 + o(\sigma^2).$$

[4] We denote f_i the derivative of any function f with respect to its ith argument.

Denote $V_0(x) \equiv v(x, g(x, 0))$ the objective function that the agent would maximize in the absence of uncertainty, and x_0 the associated optimal decisions. Define

$$\delta(x) = v_{22}(x, g(x, 0))g_2^2(x, 0) + v_2(x, g(x, 0))g_{22}(x, 0).$$

Simple calculations show that the optimal decisions are

$$x(\sigma) = x_0 - \sigma^2 \frac{\delta'(x_0)}{2V_0''(x_0)} Eu^2 + o(\sigma^2). \tag{1}$$

The absence of a term in σ in this expression demonstrates "first-order certainty equivalence". To cite Malinvaud (1969b, p. 715):

> [...] the optimal initial decision changes little with the degree of uncertainty as long as this latter is small. Decisions taken on the basis of models in which the random disturbances are neglected should be close to optimal as long as these disturbances have zero expected values and the differentiability conditions are satisfied.

Malinvaud proved this result in much greater generality than is done here, with dynamic decision problems and possibly state-dependent preferences. The expansion (1) also shows that the leading effect of increased uncertainty on optimal decisions has the sign of $\delta'(x_0)$, since V_0 is by definition maximal at x_0. This sounds promising, but unfortunately δ combines first and second derivatives of the utility index v with first derivatives of the outcome equation g—so that the sign of $\delta'(x_0)$ depends on derivatives up to the *third* order.

This was a discouraging conclusion at the time. There are of course special cases in which one can sign $\delta'(x_0)$. Malinvaud showed that it is always positive in a generalization of the standard safe/risky asset choice problem. To see how this fits into the framework of Malinvaud (1969b), let $0 \leq x \leq W$ be the amount invested in the risky asset, whose return is $R(x, \sigma u)$; the $(W - X)$ invested in the safe asset returns $S(W - x)$. If the utility index is a function of the sum of the returns, we have

$$v(x, g(x, \sigma u)) \equiv U\left(S(W - x) + R(x, \sigma u)\right).$$

We assume that $R_1(0, 0) > S'(W)$ and $R_1(W, 0) < S'(0)$, so that $0 < x_0 < W$. It is easy to see that

$$\delta(x) = U''\left(y(x)\right) R_2^2(x, 0) + U'\left(y(x)\right) R_{22}(x, 0),$$

with $y(x) = S(W - x) + R(x, 0)$. Taking the first derivative $\delta'(x)$ involves both derivatives of R_2 and R_{22} and derivatives of y. But for $\sigma = 0$, x_0 maximizes $S(W-x)+R(x, 0)$ in x; hence $y(x)$ is highest at $x = x_0$. It follows that $y'(x_0) = 0$ and when we compute $\delta'(x_0)$, the only non-zero terms are

$$2U''\left(y(x_0)\right) R_{21}R_2 + U'\left(y(x_0)\right) R_{221}.$$

which is positive for all concave U under very weak assumptions on the function R (including $R(x,s) \equiv \alpha(x)\beta(s)$ for increasing α and concave β.) Therefore risk discourages risky investments.

Unfortunately, this sensible result does not extend to more general utility functions of the form $U\left(S(W-x), R(x,\sigma u)\right)$. Think for instance of a subsistence farmer deciding how much to allow to a risky crop. Malinvaud shows that even if U is additive in its two arguments, more risk may very well *increase* investment in the risky asset if the marginal utility of the risky product decreases fast enough.

There is now a large literature that explores the comparative statics of increased risk, building on higher-order derivatives of the utility function such as prudence and temperance. The book by Gollier (2001) presents many such results.

2.3 Individual risks and collective risks

Malinvaud's two papers Malinvaud (1972a, 1973) provide the foundations of the theory of insurance in a market economy. They distinguish individual risks, which may be insurable, from collective risks which affect the whole economy and cannot be "smoothed out". It is easier to start from Malinvaud (1973), which was the Walras-Bowley lecture at the American winter meeting of the Econometric Society in 1971.

Consider an exchange economy with I consumers and H goods. We assume that the consumers are identical: they have the same preferences, the same endowments, and they face identical risks materialized by individual states $s_i = 1, \ldots, S$ for each consumer i. It is important to distinguish *individual states* s_i and the *social state* $\omega = (s_1, \ldots, s_I)$. We denote $\pi_I(\omega)$ the probability of social state ω and $s_i(\omega)$ the individual state for consumer i in that social state. Each consumer i has expected utility preferences $\sum_\omega \pi_I(\omega) u_{s_i(\omega)}(x_{i\omega})$ and an endowment $e(s_i)$ that only depend on his/her individual state. We assume that u is strictly concave.

We also define the *aggregate state* r as a particular (vector) statistic of the social state: $r_s(\omega)$ is the proportion of consumers in individual state s at ω. There are of course many social states ω with the same aggregate state r. Malinvaud assumes that any two social states that share the same aggregate state have the same probability. This is a natural interpretation of the notion that risks are individual; it holds for instance if the individual states $(s_1, \ldots, s_I)$ are exchangeable. It is important to note at this stage that we are not requiring that risks be independent and identically distributed (iid). Exchangeability is a much weaker assumption; it is essentially equivalent to imposing that the individual states be iid conditional on a collective state t. A very simple example would be a model where each s_i in $\{0, 1\}$ is drawn independently from a Bernoulli distribution with parameter $P(t)$, conditional on a random draw of a collective t.

Exchangeability is strong enough that a law of large numbers holds: as the number of consumers grows, the probability distribution $\bar{\pi}_I$ of aggregate states r converges to a distribution $\bar{\pi}_\infty$ on $[0,1]^\infty$. Take the Bernoulli example above. It is easy to see that *conditional on* t, $\bar{\pi}_\infty$ is a Dirac mass at $(r_0 = 1-P(t), r_1 = P(t))$. But the unconditional distribution of $r = (r_0, r_1)$ puts probabilities given by P on these mass points. Malinvaud's analysis focuses on *individual risks*: cases in which the limit distribution $\bar{\pi}_\infty$ is concentrated at one point ρ_∞. In our Bernoulli example, this rules out any dependence of the function P on a collective risk t; and $\rho_\infty = (1-P, P)$.

Whether risks are only individual or not, since they are exchangeable the probability that any agent i is in a state s is simply

$$\rho_I(s) = \sum_r \bar{\pi}_I(r) r_s;$$

if risks are purely individual, it converges to ρ_∞ as the number of consumers grows.

An Arrow-Debreu equilibrium with complete markets, whuich Malinvaud also calls an $A-$equilibrium, is classically defined as a set of contingent prices $(p(\omega))$ and allocations $(x_{i\omega})$ such that each consumer i maximizes his expected utility $\sum_\omega \pi_I(\omega) u_{s_i(\omega)}(x_{i\omega})$ under the budget constraint

$$\sum_\omega p(\omega) \cdot (x_{i\omega} - e(s_i(\omega))) = 0$$

and market demand equals total endowments: $\sum_{i=1}^I (x_{i\omega} - e(s_i(\omega))) = 0$ for each social state ω. Attaining such equilibria seems to require a number of markets that grows exponentially with the number of consumers: HS^I markets, or at least $(H + S^I)$ in Arrow's securities-based implementation. Malinvaud shows that if risks are purely individual, then as I goes to infinity, actuarially fair insurance joined to non-contingent goods markets implements an Arrow-Debreu equilibrium. To cite Cass, Chichilnisky, and Wu (1996, p. 335),

> When there is anonymous individual risk common to like groups of individuals, pooling that risk by means of mutual insurance permits substantial economizing on market transactions compared to those required if dealing instead with the full complement of pure Arrow securities.

To see this, we explore the existence of an Arrow-Debreu equilibrium where prices are proportional to probabilities: $p(\omega) = \pi_I(\omega) q$ for some vector of non-contingent goods prices q. Note that if prices take this form, then agents' demands can only depend on their individual states. Take an agent i who maximizes $\sum_\omega \pi_I(\omega) u_{s_i(\omega)}(x_{i\omega})$ under $\sum_\omega \pi_I(\omega) q \cdot (x_{i\omega} - e(s_i(\omega))) = 0$. Suppose that for some given s, the values of the $x_{i\omega}$ for all ω such that $s_i(\omega)$ are not all equal; then replacing them by their expected value conditional on $s_i(\omega) = s$ does not modify net expenditure, and given the strict concavity of u it increases the objective function.

Recall that by definition, $\sum_{s_i(\omega)=s} \pi_I(\omega)$ equals $\rho_I(s)$. Therefore demands $x(s)$ for such a price system must maximize $\sum_s \rho_I(s) u(x(s))$ under $\sum_s \rho_I(s) q \cdot (x(s) - e(s)) = 0$. With state-independent utility, this gives state-independent demands, which we denote $x = X(q; \rho_I)$; note that since $\sum_s \rho_I(s) = 1$, these demands are defined by the following two conditions:

- their value at prices q equals that of the expected endowment $\sum_s \rho_I(s)e(s)$
- and $u'(x)$ is proportional to q.

Choose prices q to equal the marginal utility of the expected endowment: $q = u'(\sum_s \rho_I(s)e(s))$. Clearly, demands at these prices simply equal the expected endowment. Now net market demands depend on the realized proportions $r(s)$ of individual states: they equal

$$\sum_s r(s) \left(X(q; \rho_I) - e(s)\right) = X(q; \rho_I) - \sum_s r(s)e(s).$$

At these particular prices, this equals $\sum_s(\rho_I(s) - r(s))e(s)$, which is nonzero. But *let risks be purely individual*, so that both r and ρ_I converge to ρ_∞; then aggregate net demand goes to zero and the market clears in the limit. Malinvaud calls this allocation a $B-$equilibrium.

This (quasi) equilibrium is Pareto optimal, like all Arrow-Debreu equilibria. When a consumer is in state s, his net demand is $X - e(s)$, which has value $v(s) = q \cdot (X - e(s))$. By construction, $\sum_s \rho_I(s)v(s) = 0$: an insurance contract that gives the consumer contingent incomes $v(s)$ is actuarially fair, and it allows him to consume X independently of the state. If such a (public or private) insurance system can operate, then it only needs to be complemented by goods markets in each state. In particular, if private insurers can observe the realization of individual states and face no costs, then implementing this Arrow-Debreu equilibrium only requires $(S-1)$ insurance markets and $S(H-1)$ goods markets in the limit with pure individual risks.

Malinvaud (1972a) is more general in that it introduces production and it also allows consumers and firms to belong to different types (with population sizes that increase proportionally to the size of the economy). With heterogeneity in types, some of the symmetry of Malinvaud (1973) is lost; so are uniqueness of B-equilibrium and global stability of the tâtonnement process. The main results of Malinvaud (1973) still hold: every B-equilibrium is an A-equilibrium, and it can be implemented using (costless) insurance markets for individual risks along with contingent markets for collective risks.

With production comes the question of the objective function of firms. As is well-known, in complete markets all shareholders agree that each firm should maximize its market value[5]. Malinvaud (1972a) shows that if risks are individual and shareholders hold the same beliefs, they will all agree to instruct firms to maximize expected profits. This follows from the proportionality of state prices and state probabilities. It needn't be the case if shareholders hold different beliefs, or if some risks are collective.

One may take issue with Malinvaud's results since they only hold in the infinite limit, when individual risks are completely smoothed out. For any finite approximation, r (the realized proportions of agents in each individual state) differs from ρ_I (the agent's expectations): actuarially fair insurance markets cannot clear the markets exactly. Cass, Chichilnisky, and Wu (1996) build on Section 5 of Malinvaud (1973) to solve this difficulty. Their idea is to reinterpret aggregate states r as collective states (like the t in our Bernoulli example); this also allows them to extend Malinvaud's results to heterogeneous consumers and any type of collective risk. They show that one can get exact market clearing with actuarially fair insurance. The price to pay is that the number of insurance markets necessary increases since agents must be able to insure against all combinations of collective and individual risks.

These results still assume that agents agree on the probability distribution of risks. Chichilnisky and Heal (1998) further extend Malinvaud's contribution to encompass "unknown risks" such as climate change, for which there is considerable uncertainty on the probability distribution of costs[6]. Imagine several large countries whose governments have differing degrees of pessimism about the impact of climate change. At some point in the future, this impact will be measured in every country by the distribution of damages in its population. Chichilnisky and Heal propose the creation of markets in which governments can "bet" on the costs of climate change by buying or selling "statistical securities" that pay off when the frequency of damages in a given country takes a particular value. At market equilibrium, the more optimistic governments would buy securities that pay off when climate change turns out to be serious; and the more optimistic governments would sell these securities. As in Malinvaud's work, each country would also need to allow for mutual insurance contracts between its citizens. While markets for statistical securities may seem exotic, they have in fact been implemented in various forms. Catastrophe bonds have been traded since 1994, and a Caribbean Catastrophe Risk Insurance Facility links governments of the region.

[5]Drèze (1974) showed that things are more complicated when markets are incomplete. Each shareholder would like the firm to use state prices that incorporate his/her marginal rates of substitution, which are only equal across states if markets are complete.

[6]We ignore here the important fact that agents' actions affect the distribution of the risks of climate change.

3 Lectures on Microeconomic Theory

No discussion of Malinvaud's contribution to microeconomics would be complete without a mention of his landmark textbook. This was originally published in French in 1968 as *Leçons de microéconomie*, with three more editions in 1971, 1975 and 1982. The first and fourth French editions were translated into English as *Lectures on Microeconomic Theory* (1972 and 1985). As the preface to the first edition indicates,

> The present edition is a rather extensive revision and adaptation of lectures first prepared as an introduction to the course given by Maurice Allais at Ensae, Paris[7]. It is addressed to students who have a good background in mathematics and have been introduced to economic phenomena and concepts. But their power of abstraction is not considered as high enough to allow them to take immediate full advantage of the most rigorous and condensed works such as Debreu's *Theory of Value*.

By modern standards, the tone was uncompromising: "The scope of these lectures is satisfactorily defined by the table of contents, without the need for further discussion here [...] the text should be useful to those [who] are not prepared to be content with less rigorous presentations, which are naturally easier but are also responsible for some confusion." Over the years, the content evolved somewhat. Chapter 6 on "imperfect competition and game situations" almost doubled in size, and Malinvaud added sections in other chapters: on the theory of social choice, on second-best tax policy, on temporary equilibrium, on overlapping generations models, and on "financial equilibrium." The most obvious change was the introduction of a last chapter on "information" that briefly discussed information structures, asymmetric information, and rational expectations equilibria. The style of the lectures remained the same. It may have been too terse for some students; but the clarity of the exposition made it illuminating for so many of us.

References

Barone, E. (1908): "Il Ministro della Produzione nello Stato Collettivista," *Giornale degli Economisti*, 2.

Cass, D., G. Chichilnisky, and H.-M. Wu (1996): "Individual Risk and Mutual Insurance," *Econometrica*, 64, 333–341.

Champsaur, P. (1976): "Neutrality of Planning Procedures in an Economy with Public Goods," *The Review of Economic Studies*, 43, 293–299.

[7] Maurice Allais was a notoriously demanding teacher; hence the need for an "introductory" class.

Champsaur, P., and G. Laroque (1982): “Strategic Behavior in Decentralized Planning Procedures,” *Econometrica*, 50, 325–344.

Chichilnisky, G., and G. Heal (1998): “Financial Markets and Unknown Risks,” in *Sustainability, Dynamic and Uncertainty*, ed. by G. Chichilnisky, G. Heal, and A. Vercelli. Kluwer.

Drèze, J. (1974): “Investment under Private Ownership: Optimality, Equilibrium and Stability,” in *Allocation under Uncertainty*, ed. by J. Drèze. McMillan.

Drèze, J., and D. de la Vallée Poussin (1971): “A Tatonnement Process for Public Goods,” *The Review of Economic Studies*, 38, 133–150.

Freixas, X. (1980): “Monopolistic Behavior, Prices and Quantities,” *The Review of Economic Studies*, 47(4), 747–753.

Gollier, C. (2001): *The Economics of Risk and Time*. MIT Press.

Gul, F., and A. Postlewaite (1992): “Asymptotic Efficiency in Large Economies with Asymmetric Information,” *Econometrica*, 60, 1273–1292.

Henry, C. (1972): “Differential Equations with Discontinuous Right-Hand Side for Planning Procedures,” *Journal of Economic Theory*, 4, 545–551.

Herstein, I., and J. Milnor (1953): “A Measurable Approach to Expected Utility,” *Econometrica*, 21, 291–297.

Hurwicz, L. (1973): “The Design of Mechanisms for Resource Allocation,” *American Economic Review*, 63, 1–30.

Lange, O. (1936): “On the Economic Theory of Socialism: Part One,” *The Review of Economic Studies*, 4(1), 53–71.

Laroque, G., and J.-C. Rochet (1983): “Myopic versus Intertemporal Manipulation in Decentralized Planning Procedures,” *The Review of Economic Studies*, 50, 187–195.

Malinvaud, E. (1952): “Note on von Neumann-Morgenstern’s Strong Independence Axiom,” *Econometrica*, 20(4), 679–679.

——— (1967): “Decentralized procedure for planning,” in *Activity analysis in the theory of growth and planning*, ed. by E. Malinvaud, and M. Bacharach. MacMillan, New York.

——— (1968a): *Leçons de microéconomie*. Dunod.

——— (1968b): "Notes for the Study of Planning Procedures," *The Canadian Journal of Economics/ Revue Canadienne d'Économique*, 1(1), 16–36.

——— (1969a): "Décision en face de l'aléatoire et situation certaine approximativement équivalente," *Cahiers du Séminaire d'Econométrie*, (11), 37–50.

——— (1969b): "First Order Certainty Equivalence," *Econometrica*, 37(4), 706–718.

——— (1971): "A Planning Approach to the Public Good Problem," *The Swedish Journal of Economics*, 73(1), 96–112.

——— (1972a): "The allocation of individual risks in large markets," *Journal of Economic Theory*, 4(2), 312–328.

——— (1972b): *Lectures on Microeconomic Theory*. North Holland.

——— (1972c): "Prices for Individual Consumption, Quantity Indicators for Collective Consumption," *The Review of Economic Studies*, 39, 385–405.

——— (1973): "Markets for an Exchange Economy with Individual Risks," *Econometrica*, 41(3), 383–410.

McLean, R., and A. Postlewaite (2002): "Informational Size and Incentive Compatibility," *Econometrica*, 70, 2421–2453.

——— (2003): "Informational Size and Incentive Compatibility with Aggregate Uncertainty," *Games and Economic Behavior*, 45, 410–433.

Moscati, I. (2016): "Retrospectives: How Economists Came to Accept Expected Utility Theory: The Case of Samuelson and Savage," *Journal of Economic Perspectives*, 30(2), 219–236.

Roberts, J. (1979a): "Incentives in Planning Procedures for the Provision of Public Goods," *Review of Economic Studies*, 46, 283–292.

——— (1979b): "Strategic Behavior in the MDP Planning Procedure," in *Aggregation and Revelation of Preferences*. North Holland, Amsterdam.

Simon, H. (1956): "Dynamic Programming under Uncertainty with a Quadratic Criterion Function," *Econometrica*, 24, 74–81.

Taylor, F. M. (1929): "The Guidance of Production in a Socialist State," *American Economic Review*, 19, 1–8.

Theil, H. (1954): "Econometric Models and Welfare Maximisation," *Weltwirtschaftliches Archiv*, 72, 60–83.

——— (1957): "A Note on Certainty Equivalence in Dynamic Planning," *Econometrica*, 25, 346–349.

——— (1964): *Optimal Decision Rules for Government and Industry*. North Holland.

Weitzman, M. L. (1974): "Prices vs. Quantities," *The Review of Economic Studies*, 41(4), 477–491.