Econometrica Supplementary Material

SUPPLEMENT TO "ASSORTATIVE MATCHING WITH LARGE FIRMS"
(*Econometrica*, Vol. 86, No. 1, January 2018, 85–132)

JAN EECKHOUT
Department of Economics, UPF-ICREA-GSE and University College London

PHILIPP KIRCHER
Department of Economics, University of Edinburgh

ADDITIONAL FIGURES

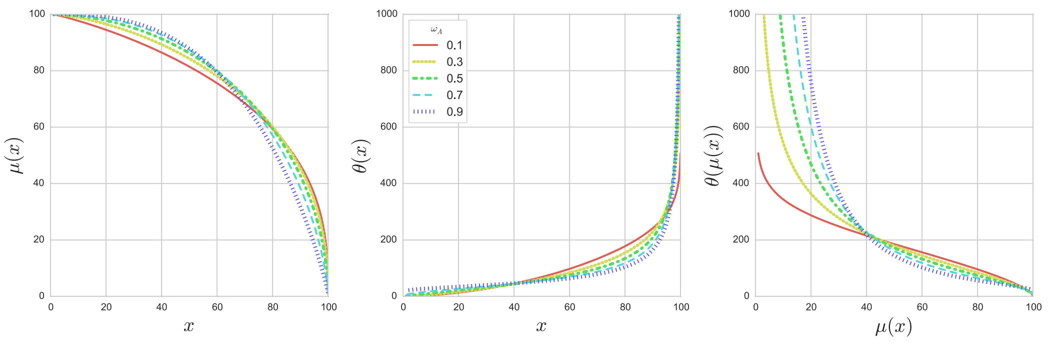

FIGURE S1.—For different values of ω_A, NAM Allocation $\mu(x)$ and intensity/size θ. Simulation with both H_f, H_w uniform on [0, 1], $\omega_\theta = 0.5$, and $\sigma_A = 1.1$.

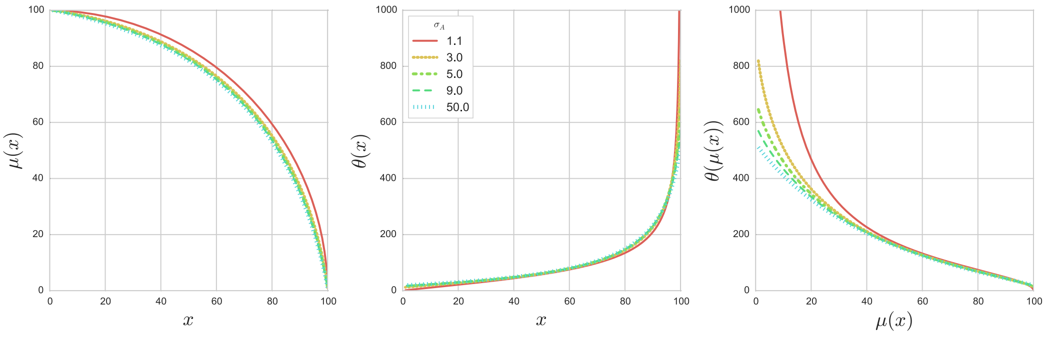

FIGURE S2.—For different values of σ_A, NAM Allocation $\mu(x)$ and intensity/size θ. Simulation with both H_f, H_w uniform on [0, 1], $\omega_A = 0.5$, and $\omega_\theta = 0.5$.

Jan Eeckhout: jan.eeckhout@upf.edu
Philipp Kircher: philipp.kircher@ed.ac.uk

 https://doi.org/10.3982/ECTA14450

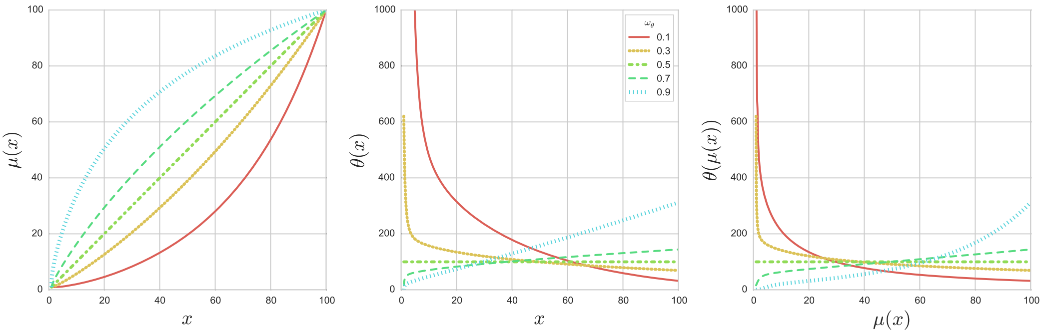

FIGURE S3.—For different values of ω_θ, PAM Allocation $\mu(x)$ and intensity/size θ. Simulation with both H_f, H_w uniform on [0, 1], $\omega_A = 0.5$, and $\sigma_A = 0.5$.

CONDITIONS FOR SORTING WITH MULTIPLE SKILL INPUTS

Let there be two distinct skill inputs x_1 and x_2, each with a continuum of possible types $x_1, x_2 \in \mathbb{R}_+$. Then we analyze the allocation for a general technology $f(x_1, \theta_1, x_2, \theta_2, y)$ and wages $w_1(x_1)$ and $w_2(x_2)$.

Then the first-order conditions are

$$\begin{aligned} f_{x_1}(x_1, \theta_1, x_2, \theta_2, y) - \theta_1(x_1)w_1'(x_1) &= 0, \\ f_{\theta_1}(x_1, \theta_1, x_2, \theta_2, y) - w_1(x_1) &= 0, \\ f_{x_2}(x_1, \theta_1, x_2, \theta_2, y) - \theta_2(x_2)w_2'(x_2) &= 0, \\ f_{\theta_2}(x_1, \theta_1, x_2, \theta_2, y) - w_2(x_2) &= 0. \end{aligned}$$

The corresponding Hessian is

$$\mathbf{H} = \begin{pmatrix} f_{x_1x_1} - \theta_1 w_1'' & f_{x_1\theta_1} - w_1' & f_{x_1x_2} & f_{x_1\theta_2} \\ f_{x_1\theta_1} - w_1' & f_{\theta_1\theta_1} & f_{x_2\theta_1} & f_{\theta_1\theta_2} \\ f_{x_1x_2} & f_{x_2\theta_1} & f_{x_2x_2} - \theta w_2'' & f_{x_2\theta_2} - w_2' \\ f_{x_1\theta_2} & f_{\theta_1\theta_2} & f_{x_2\theta_2} - w_2' & f_{\theta_2\theta_2} \end{pmatrix},$$

and must be negative semi-definite.

We denote the equilibrium allocations by $y = \mu_1(x_1)$ and $y = \mu_2(x_2)$. Substituting $x_2 = \mu_2^{-1}(y) = \mu_2^{-1}[\mu_1(x_1)]$ and $x_1 = \mu_1^{-1}(y) = \mu_1^{-1}[\mu_2(x_2)]$, we can therefore write the FOC's evaluated at the equilibrium allocation as

$$\begin{aligned} f_{x_1}\big(x_1, \theta_1, \mu_2^{-1}\big[\mu_1(x_1)\big], \theta_2, \mu_1(x_1)\big) - \theta_1(x_1)w_1'(x_1) &= 0, \\ f_{\theta_1}\big(x_1, \theta_1, \mu_2^{-1}\big[\mu_1(x_1)\big], \theta_2, \mu_1(x_1)\big) - w_1(x_1) &= 0, \\ f_{x_2}\big(\mu_1^{-1}\big[\mu_2(x_2)\big], \theta_1, x_2, \theta_2, \mu_2(x_2)\big) - \theta_2(x_2)w_2'(x_2) &= 0, \\ f_{\theta_2}\big(\mu_1^{-1}\big[\mu_2(x_2)\big], \theta_1, x_2, \theta_2, \mu_2(x_2)\big) - w_2(x_2) &= 0. \end{aligned}$$

We now take the total derivative of the first two FOCs with respect to x_1 and of the last two FOCs with respect to x_2 and obtain

$$f_{x_1x_1} + f_{x_1\theta_1}\theta_1' + f_{x_1x_2}\big[\mu_2^{-1}\big]'\mu_1' + f_{x_1\theta_2}\theta_2'\big[\mu_2^{-1}\big]'\mu_1' + f_{x_1y}\mu_1' - \theta_1'w_1' - \theta_1 w_1'' = 0,$$

$$f_{x_1\theta_1} + f_{\theta_1\theta_1}\theta_1' + f_{x_2\theta_1}\big[\mu_2^{-1}\big]'\mu_1' + f_{\theta_1\theta_2}\big[\mu_2^{-1}\big]'\mu_1' + f_{x_1y}\mu_1' - w_1' = 0,$$
$$f_{x_2}\big(\mu_1^{-1}\big[\mu_2(x_2)\big], \theta_1, x_2, \theta_2, \mu_2(x_2)\big) - \theta_2(x_2)w_2'(x_2) = 0,$$
$$f_{\theta_2}\big(\mu_1^{-1}\big[\mu_2(x_2)\big], \theta_1, x_2, \theta_2, \mu_2(x_2)\big) - w_2(x_2) = 0,$$

which implies

$$\begin{aligned}
f_{x_1x_1} - \theta_1 w_1'' &= \theta_1' w_1' - f_{x_1\theta_1}\theta_1' - \big[f_{x_1x_2}\big[\mu_2^{-1}\big]' + f_{x_1\theta_2}\theta_2'\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1',\\
f_{x_1\theta_1} - w_1' &= -f_{\theta_1\theta_1}\theta_1' - \big[f_{x_2\theta_1}\big[\mu_2^{-1}\big]' + f_{\theta_1\theta_2}\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1',\\
f_{x_2x_2} - \theta_2 w_2'' &= \theta_2' w_2' - f_{x_2\theta_2}\theta_2' - \big[f_{x_1x_2}\big[\mu_1^{-1}\big]' + f_{x_2\theta_1}\theta_1'\big[\mu_1^{-1}\big]' + f_{x_2y}\big]\mu_2',\\
f_{x_2\theta_2} - w_2' &= -f_{\theta_2\theta_2}\theta_2' - \big[f_{x_1\theta_2}\big[\mu_1^{-1}\big]' + f_{\theta_1\theta_2}\big[\mu_1^{-1}\big]' + f_{x_2y}\big]\mu_2',
\end{aligned}$$

and therefore the Hessian **H** can be written as

$$\begin{pmatrix}
\begin{array}{l}\theta_1' w_1' - f_{x_1\theta_1}\theta_1' - \big[f_{x_1x_2}\big[\mu_2^{-1}\big]' \\ \quad + f_{x_1\theta_2}\theta_2'\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1'\end{array} & \begin{array}{l}-f_{\theta_1\theta_1}\theta_1' - \big[f_{x_2\theta_1}\big[\mu_2^{-1}\big]' \\ \quad + f_{\theta_1\theta_2}\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1'\end{array} & f_{x_1x_2} & f_{x_1\theta_2}\\
\begin{array}{l}-f_{\theta_1\theta_1}\theta_1' - \big[f_{x_2\theta_1}\big[\mu_2^{-1}\big]' \\ \quad + f_{\theta_1\theta_2}\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1'\end{array} & f_{\theta_1\theta_1} & f_{x_2\theta_1} & f_{\theta_1\theta_2}\\
f_{x_1x_2} & f_{x_2\theta_1} & f_{x_2x_2} - \theta w_2'' & f_{x_2\theta_2} - w_2'\\
f_{x_1\theta_2} & f_{\theta_1\theta_2} & f_{x_2\theta_2} - w_2' & f_{\theta_2\theta_2}
\end{pmatrix},$$

where the bottom right 2×2 matrix is given by

$$\begin{aligned}
&\begin{pmatrix} f_{x_2x_2} - \theta w_2'' & f_{x_2\theta_2} - w_2' \\ f_{x_2\theta_2} - w_2' & f_{\theta_2\theta_2} \end{pmatrix}\\
&\quad = \begin{pmatrix}
\begin{array}{l}\theta_2' w_2' - f_{x_2\theta_2}\theta_2' - \big[f_{x_1x_2}\big[\mu_1^{-1}\big]' \\ \quad + f_{x_2\theta_1}\theta_1'\big[\mu_1^{-1}\big]' + f_{x_2y}\big]\mu_2'\end{array} & \begin{array}{l}-f_{\theta_2\theta_2}\theta_2' - \big[f_{x_1\theta_2}\big[\mu_1^{-1}\big]' \\ \quad + f_{\theta_1\theta_2}\big[\mu_1^{-1}\big]' + f_{x_2y}\big]\mu_2'\end{array}\\
\begin{array}{l}-f_{\theta_2\theta_2}\theta_2' - \big[f_{x_1\theta_2}\big[\mu_1^{-1}\big]' \\ \quad + f_{\theta_1\theta_2}\big[\mu_1^{-1}\big]' + f_{x_2y}\big]\mu_2'\end{array} & f_{\theta_1\theta_2}
\end{pmatrix}.
\end{aligned}$$

For his matrix **H** to be negative definite, the kth-order leading principal minor must be negative when k is odd, and positive when k is even. That is, the following conditions need to be satisfied:

$$\theta_1' w_1' - f_{x_1\theta_1}\theta_1' - \big[f_{x_1x_2}\big[\mu_2^{-1}\big]' + f_{x_1\theta_2}\theta_2'\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1' < 0,$$

$$\begin{vmatrix}
\begin{array}{l}\theta_1' w_1' - f_{x_1\theta_1}\theta_1' - \big[f_{x_1x_2}\big[\mu_2^{-1}\big]' \\ \quad + f_{x_1\theta_2}\theta_2'\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1'\end{array} & \begin{array}{l}-f_{\theta_1\theta_1}\theta_1' - \big[f_{x_2\theta_1}\big[\mu_2^{-1}\big]' \\ \quad + f_{\theta_1\theta_2}\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1'\end{array}\\
\begin{array}{l}-f_{\theta_1\theta_1}\theta_1' - \big[f_{x_2\theta_1}\big[\mu_2^{-1}\big]' \\ \quad + f_{\theta_1\theta_2}\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1'\end{array} & f_{\theta_1\theta_1}
\end{vmatrix} > 0,$$

$$\begin{vmatrix} \theta_1' w_1' - f_{x_1\theta_1}\theta_1' - \big[f_{x_1x_2}\big[\mu_2^{-1}\big]' + f_{x_1\theta_2}\theta_2'\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1' & -f_{\theta_1\theta_1}\theta_1' - \big[f_{x_2\theta_1}\big[\mu_2^{-1}\big]' + f_{\theta_1\theta_2}\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1' & \theta_2' w_2' - f_{x_2\theta_2}\theta_2' - \big[f_{x_1x_2}\big[\mu_1^{-1}\big]' + f_{x_2\theta_1}\theta_1'\big[\mu_1^{-1}\big]' + f_{x_2y}\big]\mu_2' \\ -f_{\theta_1\theta_1}\theta_1' - \big[f_{x_2\theta_1}\big[\mu_2^{-1}\big]' + f_{\theta_1\theta_2}\big[\mu_2^{-1}\big]' + f_{x_1y}\big]\mu_1' & f_{\theta_1\theta_1} & -f_{\theta_2\theta_2}\theta_2' - \big[f_{x_1\theta_2}\big[\mu_1^{-1}\big]' + f_{\theta_1\theta_2}\big[\mu_1^{-1}\big]' + f_{x_2y}\big]\mu_2' \\ f_{x_1x_2} & f_{x_2\theta_1} & f_{x_2x_2} - \theta w_2'' \end{vmatrix} < 0,$$

$$|\mathbf{H}| > 0.$$

We are not able to distill the necessary and sufficient conditions for positive sorting from this set of conditions. The complication of this condition stems from the fact that there are now nontrivial complementarities between different skill inputs. This implies that the sorting conditions have complex interactions between the different skilled inputs.

In order to make progress, in the main text we focus on sufficient conditions.

Co-editor Gianluca Violante handled this manuscript.

Manuscript received 2 June, 2016; final version accepted 7 September, 2017; available online 19 September, 2017.