S1. PROOFS

**Proof of Lemma 1:** Consider any two firms with \( x > x' \in [0, 1] \) and so \( v(x, \cdot) > v(x', \cdot) > 0 \). If \( w' = w(x', \theta, G) \) solves (6) for firm \( x' \), then optimality implies

\[
S1) \quad w' + \lambda(\cdot)[1 - F(W(w'))]v(x', \cdot) \leq \omega + \lambda(\cdot)[1 - F(W(\omega, \cdot))]v(x', \cdot)
\]

for all \( \omega < w' \) satisfying \( W(\omega, \theta, G) \geq W(\omega', \theta, G) \). As this inequality implies \( [1 - F(W(w'))] < [1 - F(W(\omega, \cdot))] \) for all such \( \omega \), then \( v(x', \cdot) < v(x, \cdot) \) and (S1) further imply

\[
w' + \lambda(\cdot)[1 - F(W(w'))]v(x, \cdot) < \omega + \lambda(\cdot)[1 - F(W(\omega, \cdot))]v(x, \theta, G)
\]

(with strict inequality) for all such \( \omega \). Thus if wage \( w' \) is optimal for firm \( x' \), firm \( x > x' \) sets a no lower wage. This completes the proof of Lemma 1. Q.E.D.

**Proof of Lemma 2:** We consider each part in turn.

(i) The distribution of posted wages is continuous (no mass points) and has connected support.

The proofs are by contradiction. Suppose there is a mass of firms that optimally post wage \( w'' \). Equation (6) implies a firm in this mass point is strictly better off by paying a marginally higher wage \( w' > w'' \), as this causes its quit rate to fall by a discrete amount. Wage \( w'' \) is therefore not optimal, which is the required contradiction.

Suppose the support is not connected; that is, there exist two equilibrium wages \( w', w'' \) with \( w' > w'' \), where no mass points imply \( F(W(w', \cdot), \cdot) = F(W(w'', \cdot), \cdot) \). Equation (6) implies that announcing \( w' \) is not optimal, which is the required contradiction.

(ii) Equilibrium wage strategies \( w(x, \theta, G) \) are strictly increasing in \( x \in [0, 1] \), where the lowest wage paid is \( w(0, \theta, G) = R(\theta, G) = b \).

Distribution function \( G(\cdot) \) must have a connected support (the startup entry distribution \( I_0 \) is uniform and so is connected). Hence equilibrium wage strategies must be strictly increasing in \( x \) because there can be no mass points.

We next prove \( w(0, \theta, G) = R(\theta, G) \) using a contradiction argument. First note that posting \( w(0, \theta, G) < R(\theta, G) \) cannot be optimal since all
workers quit into unemployment, which yields zero profit. Suppose instead \( w(0, \theta, G) > R(\theta, G) \). No mass points in \( F(\cdot) \) and (6) imply posting wage \( w' = R(\theta, G) \) strictly dominates posting wage \( w(0, \theta, G) > R(\theta, G) \), which contradicts \( w(0, \theta, G) \) an equilibrium wage offer.

We now show \( w(0, \theta, G) = b \). Let \( w(\theta, G) = w(0, \theta, G) \) denote the lowest wage paid in the market. As \( x = 0 \) is an absorbing state, then, conditional on survival, this firm forever posts wage \( w(\theta, G) \). Thus the value of being employed at firm \( x = 0 \), denoted \( W(\theta, G) \), is given by

\[
(S2) \quad rW(\theta, G) = w(\theta, G) + \delta(\theta)[V_u(\cdot) - W]
\]

\[
+ \lambda(\cdot) \int_{w}^{W} [W' - W] dF(W', \cdot)
\]

\[
+ \alpha \int_{\theta}^{\theta'} [W(\theta', \cdot) - W(\theta, \cdot)] dH(\theta' | \theta) + \frac{\partial W}{\partial t},
\]

where the term \( \partial W / \partial t \) describes the expected capital gain through the dynamic evolution of \( G \).

The flow value of being unemployed and choosing home production is given by

\[
(S3) \quad rV_u = b + \lambda(\cdot) \int_{W}^{W'} [W' - V_u(\cdot)] dF(W', \cdot)
\]

\[
+ \alpha \int_{\theta}^{\theta'} [V_u(\theta', \cdot) - V_u(\theta, \cdot)] dH(\theta' | \theta) + \frac{\partial V_u}{\partial t},
\]

while free entry into entrepreneurship implies \( V_u(\cdot) \) is also given by

\[
(S4) \quad E(\theta, G) = \frac{\mu}{b} \int_{0}^{1} [v(x, \theta, G) + W(w(x, \cdot), \theta, G) - V_u(\cdot)] dx + \frac{\partial V_u}{\partial t},
\]

where at rate \( \mu/E \), the entrepreneur creates a new startup company, which, with one employee, generates expected profit \( v(x, \theta, G) \) that is sold to outside investors for its value, and he/she becomes the first employee with value \( W(w', \theta, G) \) on equilibrium wage \( w' = w(x, \cdot) \). Thus free entry implies
where it is assumed that $\mu/b$ is sufficiently small that $E < U$ along the equilibrium path. As the definition of the reservation wage implies $W(\theta, G) = V_u(\theta, G)$, (S2) and (S3) now imply $w(\theta, G) = b$.

(iii) Given any job offer $(w', \theta, G)$, each employee believes $x = \hat{x}(w', \theta, G)$, where $\hat{x} \in [0, 1]$ solves

$$w(\hat{x}, \theta, G) = w' \quad \text{when} \quad w' \in [b, w(1, \theta, G)],$$

$$\hat{x} = 0 \quad \text{when} \quad w' < b,$$

$$\hat{x} = 1 \quad \text{when} \quad w' > w(1, \theta, G).$$

It follows directly, as wages are fully revealing, that beliefs must be consistent with Bayes rule and that beliefs are monotonic;

(iv) That any employee on wage $w' \geq b$ quits if and only if the outside offer $w'' \geq w'$ was established in the text.

(v) That any employee on wage $w' < b$ quits into unemployment follows since workers believe the firm’s state $\hat{x} = 0$ and that the firm will forever post wage $w = b$ in the future, and so given $w' < b$, it is better to be unemployed.

This completes the proof of Lemma 2. Q.E.D.

PROOF OF PROPOSITION 1: We first show that (12) is necessary. Equation (11) implies the firm’s optimal wage $w$ satisfies the necessary first order condition

$$1 - v(x, \cdot) \frac{h(\hat{x}, \cdot)G'(\hat{x}) \partial \hat{x}}{G(\hat{x})} \frac{\partial \hat{x}}{\partial w} = 0,$$

where belief $\hat{x}(w, \cdot)$ solves $w = w(\hat{x}, \cdot)$. As Lemma 2 implies $\partial \hat{x}/\partial w = [1/\partial w/\partial x]$, (S5) implies that (12) is a necessary condition for equilibrium.

To show that (12) is sufficient, let $w(\cdot, \theta, G)$ denote the solution to the initial value problem defined in Proposition 1. As $G(0) = U > 0$, this solution is continuous and strictly increasing in $x$.

Now consider any firm $x \in (0, 1]$ and let

$$C(w, \theta, G) = w + v(x, \theta, G) \int_{\hat{x}(w, \theta, G)}^{1} \frac{h(z, \theta, G) dG(z)}{G(z)}$$

describe the minimand in (11). If the firm sets a lower wage $w' = w(x', \cdot) < w$ with $x' \in [0, x)$, its employees believe $\hat{x} = x' < x$. Hence

$$\frac{\partial C}{\partial w'}(w', \theta, G) = 1 - v(x, \theta, G) \frac{h(x', \theta, G) dG(x')}{G(x')} \frac{\partial \hat{x}}{\partial w'}$$

for such $w'$. But (S5) implies

$$1 - v(x', \cdot) \frac{h(x', \theta, G)G'(x') \partial \hat{x}}{G(x')} \frac{\partial \hat{x}}{\partial w'} = 0$$
at \( x' \) and combining yields
\[
\frac{\partial C}{\partial w'} = 1 - \frac{v(x, \theta, G)}{v(x', \theta, G)} < 0
\]
because values \( v(\cdot) \) are strictly increasing in \( x \). Thus for \( w' < w(x, \theta, G) \), an increase in \( w' \) strictly decreases \( C(\cdot) \). The same argument establishes that increasing \( w' \) when \( x' \in (x, 1] \) strictly increases \( C(\cdot) \). Finally note for wages \( w' > w(1, \theta, G) \), the worker's belief is fixed at \( \hat{\theta} = 1 \) and so higher wages strictly increase \( C, \) while wage \( w' < b \) does not satisfy the constraint \( W \geq V_u \). Hence given all other firms offer wages according to Proposition 1, the cost minimizing wage for any firm \( x \in [0, 1] \) is to offer \( w = w(x, \theta, G) \). This completes the proof of Proposition 1.

\[ Q.E.D. \]

S2. A [PARTIALLY POOLING] STATIONARY BAYESIAN EQUILIBRIA WITH MASS POINTS AND NON-MONOTONE BELIEFS

We construct a steady state example with \( \alpha, \gamma = 0 \) (no shocks) and \( \mu < \delta \), and homogenous firms \( p(x) = p \). Equilibrium implies that all firms make the same profit \( v(x) = \bar{v} \) and so hire at the same rate \( \bar{h} \), where \( c'(\bar{h}) = \bar{v}/\bar{p} \). With monotone beliefs, Proposition 1 establishes the equilibrium wage equation
\[
w(x) = b + \bar{h}\bar{v}\log\left[\frac{G(x)}{U}\right].
\]

We construct a stationary Bayesian equilibrium with a mass point as follows. Fix an \( x^c \in (0, 1) \) and define \( \bar{w} \equiv w(x^c) = b + \bar{h}\bar{v}\log[\frac{G(x^c)}{U}] \). Consider the set of equilibrium wage strategies
\[
w^*(x) = w(x) \quad \text{for} \quad x \in [0, x^c),
\]
\[
w^*(x) = \bar{w} \quad \text{for} \quad x \in [x^c, 1];
\]
that is, mass \( 1 - x^c \) of firms announce the same wage \( \bar{w} = w(x^c) \). Each firm’s steady state quit rate is then
\[
\hat{q}(x) = \int_x^{x^c} h(z, \theta, G) \frac{dG(z)}{G(z)} = -\bar{h}\log G(x) \quad \text{for} \quad x \in [0, x^c),
\]
\[
\hat{q}(x) = -\bar{h}\log G(x^c) \quad \text{for} \quad x \in [x^c, 1],
\]
since workers employed by firms in the mass point quit when indifferent. Steady state turnover arguments imply, for any \( x \leq x^c \), that \( G(x) \) must satisfy
\[
\delta[1 - G(x)] = \mu[1 - x] + \hat{q}(x)G(x)
\]
and so $G(x)$ is uniquely determined by the implicit function

\[(S6) \quad G(x) \left[ \delta - \overline{h} \log G(x) \right] = \delta - \mu[1 - x] \quad \text{for} \quad x \leq x^c.\]

It is easy to show that $x < 1$ implies $G(x) < 1$. Putting $x = 0$ in (3) implies that $\overline{v} > 0$ satisfies

\[(r + \delta)\overline{v} = \overline{p} - b - \overline{p}c(\overline{h}) + \overline{h}\overline{v}[1 + \log U],\]

with steady state unemployment $U = G(0) > 0$ given by the implicit function

\[U[\delta - \overline{h} \log U] = \delta - \mu.\]

In any such equilibrium, all firms $x \in [0, 1]$ make the same profit $\overline{v}$, but all firms with $x \geq x^c$ post the same wage $\overline{w}$ and have the same quit rate $\hat{q}(x^c) > 0$. This describes a stationary Bayesian equilibrium with the following beliefs:

**Non-Monotone Beliefs:** Given any job offer $w'$, each employee believes $x = \hat{x}(w')$, where $\hat{x}$ solves

\[
\begin{align*}
    w'(\hat{x}) &= w' \quad \text{when} \quad w' \in [b, \overline{w}), \\
    \hat{x} &\sim U[x^c, 1] \quad \text{when} \quad w' = \overline{w}, \\
    \hat{x} &= 0 \quad \text{when} \quad w' > \overline{w}, \\
    \hat{x} &= 0 \quad \text{when} \quad w' < b.
\end{align*}
\]

Should any firm in the mass point $x \in [x^c, 1]$ deviate to wage $w' > \overline{w}$, these beliefs imply workers expect wage $w = b$ in the entire future, which increases their quit rate to $\hat{q}(0) > \hat{q}(x^c)$. Equation (6) thus implies any such wage deviation is strictly profit reducing. As, by construction, all wages $w' \in [b, \overline{w}]$ generate equal value (while $w' < b$ generates zero profit because all quit into unemployment), a stationary Bayesian equilibrium exists with a mass point of firms offering $\overline{w}$.