SUPPLEMENT TO “CONSUMER SEARCH AND PRICE COMPETITION”  
(Econometrica, Vol. 86, No. 4, July 2018, 1257–1281)

MICHAEL CHOI  
Department of Economics, University of California, Irvine

ANOVIA YIFAN DAI  
Department of Economics, Hong Kong Baptist University

KYUNGMIN KIM  
Department of Economics, University of Miami

A. DISTRIBUTIONS OF EFFECTIVE VALUES

IN THIS SUPPLEMENT, we provide three examples in which $H_i(w_i)$ can be explicitly calculated.

(1) Uniform: suppose $V_i$ and $Z_i$ are uniform over $[0, 1]$ (i.e., $F_i(v) = G_i(v) = v$). Provided that $s \leq 1/2$ (which guarantees $z_i^* \in [0, 1]$), $z_i^* = 1 - \sqrt{2s}$. It is then straightforward to show that $H_i(w_i)$ is given as follows:

$$H_i(w_i) = \begin{cases} 
\frac{w_i^2}{2} & \text{if } w_i \in [0, z_i^*), \\
w_i - z_i^* + \frac{(z_i^*)^2}{2} & \text{if } w_i \in [z_i^*, 1), \\
2w_i - \frac{w_i^2}{2} - z_i^* + \frac{(z_i^*)^2}{2} - \frac{1}{2} & \text{if } w_i \in [1, 1 + z_i^*].
\end{cases}$$

Notice that, whereas $H_i$ is continuous, the density function $h_i$ has an upward jump at $z_i^*$. Therefore, $H_i$ is not globally log-concave. Nevertheless, it is easy to show that both $H_i$ and $1 - H_i$ are log-concave above $z_i^*$.

(2) Exponential: suppose $V_i$ and $Z_i$ are exponential distributions with parameters $\lambda_1$ and $\lambda_2$, respectively (i.e., $F_i(v) = 1 - e^{-\lambda_1 v}$ and $G_i(z_i) = 1 - e^{-\lambda_2 z_i}$). Provided that $s < 1/\lambda_2$ (which ensures that $z_i^* > 0$), then $z_i^* = -\log(\lambda_2 s) / \lambda_2$. For any $w_i \geq 0$,

$$H_i(w_i) = 1 - e^{-\lambda_2 \min[w_i, z_i^*]} - \frac{\lambda_2 e^{(\lambda_1 - \lambda_2) \min[w_i, z_i^*]} - 1}{e^{\lambda_1 w_i} (\lambda_1 - \lambda_2)} + (1 - e^{-\lambda_1 (\max[w_i, z_i^*] - z_i^*)}) e^{-\lambda_2 z_i^*}.$$  

Similarly to the uniform example, $H_i$ is not globally log-concave, because $h_i$ has a upward jump at $z_i^*$, but both $H_i$ and $1 - H_i$ are log-concave above $z_i^*$.
(3) Gumbel: suppose that \( V_i \) and \(-Z_i\) are standard Gumbel distributions (i.e., \( F_i(v_i) = e^{-e^{-v_i}} \) and \( G_i(z_i) = 1 - e^{-e^{z_i}} \)). For any \( w_i \in (-\infty, \infty) \),

\[
H_i(w_i) = \frac{1 + e^{-w_i - e^{z_i}} (1 + e^{w_i})}{1 + e^{w_i}}.
\]

Since both \( f_i \) and \( g_i \) are log-concave, \( 1 - H_i \) is log-concave by Proposition 2. Given the solution for \( H_i \) above, we have

\[
\frac{h_i(w_i)}{H_i(w_i)} = \frac{e^{z_i - w_i} - 1}{1 + e^{w_i + e^{z_i}(1 + e^{w_i})}} + \frac{1}{1 + e^{w_i}}.
\]

The first term falls in \( w_i \) whenever \( w_i \geq z_i^* \), while the second term constantly falls in \( w_i \). Therefore, \( H_i(w_i) \) is log-concave above \( z_i^* \).

B. PROOF OF THE SECOND CLAIM IN PROPOSITION 2 (CONT’D)

Since

\[
(\log H_i'(w_i'))'' = \frac{(h_i''(w_i'))(w_i')H_i''(w_i') - h_i''(w_i')^2}{H_i''(w_i')^2},
\]

it suffices to show that \((h_i''(w_i'))(w_i')H_i''(w_i') - h_i''(w_i')^2 < 0\) for all \( w_i' \), provided that \( \sigma \) is sufficiently large. Integrate equation (2) by parts; we have

\[
H_i''(w_i') = \int_{w_i'}^{v_i'} G_i(w_i' - v_i') \, dF_i''(v_i')
\]

for \( w_i' < v_i' + z_i^* \). In this case, \( H_i'' \) is log-concave by Prékopa’s theorem. For \( w_i' \geq v_i' + z_i^* \), we have

\[
H_i''(w_i') = \int_{w_i'}^{v_i'} G_i(w_i' - v_i') \, dF_i''(v_i') + F_i''(w_i' - z_i^*).
\]

By straightforward calculus,

\[
\frac{h_i''(w_i')}{H_i''(w_i')} = \frac{\int_{w_i'}^{v_i'} g_i(w_i' - v_i') \, dF_i''(v_i') + (1 - G_i(z_i^*))f_i''(w_i' - z_i^*)}{\int_{w_i'}^{v_i'} G_i(w_i' - v_i') \, dF_i''(v_i') + F_i''(w_i' - z_i^*)}.
\]

Changing the variables with \( a = F_i''(v_i') \) and \( r = F_i''(w_i' - z_i^*) \), the above equation becomes

\[
\frac{h_i''((F_i')^{-1}(r) + z_i^*)}{H_i''((F_i')^{-1}(r) + z_i^*)} = \frac{\int_r^1 g_i((F_i')^{-1}(r) - (F_i')^{-1}(a) + z_i^*) \, da + (1 - G_i(z_i^*))f_i''((F_i')^{-1}(r))}{\int_r^1 G_i((F_i')^{-1}(r) - (F_i')^{-1}(a) + z_i^*) \, da + r}.
\]
Since $V_i^\sigma \equiv \sigma V_i$, we have $F_i^\sigma(v_i^\sigma) = F_i(v_i^\sigma / \sigma)$, $(F_i^\sigma)^{-1}(r) = \sigma F_i^{-1}(r)$, $f_i^\sigma((F_i^\sigma)^{-1}(r)) = f_i(F_i^{-1}(r)) / \sigma$, and $(f_i^\sigma)'(F_i^{-1}(r)) = f_i(F_i^{-1}(r)) / \sigma^2$. Arranging the terms in the right-hand side above yields

$$\frac{\sigma h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*))}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*) \, da + (1 - G_i(z_i^*)) f_i(F_i^{-1}(r))}{\int_r^1 G_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*) \, da + r}.$$  

Since $F_i^{-1}(r) - F_i^{-1}(a) \leq 0$, the denominator converges to $r$ as $\sigma$ explodes. Integrating $\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*) \, da$ in the numerator by parts yields

$$G_i(z_i^*) f_i(F_i^{-1}(r)) + \int_r^1 G_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*) \, df_i(F_i^{-1}(a)).$$

Again, since $F_i^{-1}(r) - F_i^{-1}(a) \leq 0$, the second term vanishes as $\sigma$ tends to infinity, and thus the numerator converges to $G_i(z_i^*) f_i(F_i^{-1}(r))$. Therefore,

$$\lim_{\sigma \to \infty} \frac{\sigma h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{f_i(F_i^{-1}(r))}{r}$$

Following a similar procedure, we have

$$\lim_{\sigma \to \infty} \frac{\sigma (h_i^\sigma)'((F_i^\sigma)^{-1}(r) + z_i^*)}{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} = \frac{(1 - G_i(z_i^*)) f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))}.$$

Altogether,

$$\lim_{\sigma \to \infty} \sigma \left[ \frac{(h_i^\sigma)'((F_i^\sigma)^{-1}(r) + z_i^*)}{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} - \frac{h_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)}{H_i^\sigma((F_i^\sigma)^{-1}(r) + z_i^*)} \right]
= \frac{(1 - G_i(z_i^*)) f_i'(F_i^{-1}(r))}{f_i(F_i^{-1}(r))} - \frac{f_i(F_i^{-1}(r))}{r}$$

$$= \frac{(1 - G_i(z_i^*)) \left[ f_i'(F_i^{-1}(r)) \frac{1}{f_i(F_i^{-1}(r))} - \frac{f_i(F_i^{-1}(r))}{r} \right] - G_i(z_i^*) f_i(F_i^{-1}(r))}{r} < 0. \quad (8)$$

Provided $s_i$ is not too large, then $G_i(z_i^*)$ and $1 - G_i(z_i^*)$ are in $(0, 1)$, so the sign of the expression is determined by both terms.\(^1\) The square bracket term is weakly negative because $F$ is log-concave; thus the entire expression is weakly negative. The strict inequality (8) holds for each $r \in [0, 1]$ because $f_i(F_i^{-1}(r)) / r > 0$ when $r \in [0, 1)$ and

\(^1\)If $s_i$ is large so that $G_i(z_i^*) = 0$, then $W_i = V_i + z_i^*$ and $H_i$ has the same shape as $F_i$, and thus is log-concave.
\( f'_i(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0 \) when \( r = 1 \).\(^2\) Altogether, for each \( r \in [0,1] \) there is a \( \tilde{\sigma} \), such that if \( \sigma > \tilde{\sigma} \), then \( (\log H_i^\sigma(w_i^\sigma))(\sigma) > 0 \), \( h_i^\sigma(w_i^\sigma) < 0 \) where \( w_i^\sigma = F_i^{-1}(r) + z_i^* \). Since \([0,1]\) is a compact convex set and \((\log H_i^\sigma(w_i^\sigma))''\) is continuous in \( r \), there exists \( \tilde{\sigma} = \max_{\sigma \in [0,1]} \tilde{\sigma}_r < \infty \) such that if \( \sigma > \tilde{\sigma} \), then \((h_i^\sigma)' / h_i^\sigma = h_i^\sigma / H_i^\sigma < 0 \) for all \( r \in [0,1] \), or equivalently \( H_i^\sigma(w) \) is log-concave for all \( w_i^\sigma > v_i^\sigma + z_i^* \). Finally, if \( f_i(v_i) = 0 \), then the ratio \( h_i^\sigma(w_i^\sigma) / H_i^\sigma(w_i^\sigma) \) is continuous at \( v_i^\sigma + z_i^* \). Since this ratio is decreasing for \( w_i < v_i^\sigma + z_i^* \) and decreasing for \( w_i > v_i^\sigma + z_i^* \) when \( \sigma \) is large, it is globally decreasing when \( \sigma \) is large, or equivalently, \( H_i^\sigma(w_i^\sigma) \) is globally log-concave.

C. Example of a Mixed-Strategy Equilibrium

Now we assume \( F_i \) is degenerate and characterize a symmetric mixed-strategy equilibrium. Assume there are two symmetric sellers and \( u_0 = c_i = v_i = 0 \). Assume \( Z_i \) is exponentially distributed with parameter \( \lambda \), namely, \( G_i(z) = 1 - e^{-\lambda z} \). Assume \( s < 1/\lambda \) so that \( z^* > 0 \). Below, we characterize the distribution of prices and show that it has decreasing density.

Let \( Q_i = \min(Z_i, z^*) - P_i \), and let \( \Gamma_i \) and \( \gamma_i \) be its distribution function and density function, respectively. Note that the equilibrium price \( P_i \) is ex ante random in a mixed-strategy equilibrium. Moreover, in a symmetric equilibrium, the distribution of \( P_i \) has no mass point, for if it has a mass point, then a seller can get an upward jump in demand by moving the location of the mass point slightly to the left. Since the density of \( P_i \) exists (its c.d.f. is atomless), the density \( \gamma_i \) also exists.

First, we derive the demand function in a mixed-strategy equilibrium. By the eventual purchase theorem, consumers buy from seller 1 if \( \min(z^*, Z_i) - P_1 > \max(Q_2, 0) \). Therefore, no consumer will buy from seller 1 if \( P_1 > z^* \). For all \( P_1 > z^* \), consumers buy from seller 1 when \( z^* - P_1 > Q_2 \) and \( Z_i - P_1 > \max(Q_2, 0) \). Therefore, for all \( P_1 < z^* \), seller 1’s demand and its derivative are given by

\[
D_i(p_1) = \int_q^{z^* - p_1} \left(1 - G(p_1 + \max(q, 0))\right) d\Gamma_2(q) = \int_q^{z^* - p_1} e^{-\lambda(p_1 + \max(q, 0))} d\Gamma_2(q),
\]

\[
D'_i(p_1) = -e^{-\lambda z^*} \gamma_2(z^* - p_1) - \lambda \int_q^{z^* - p_1} e^{-\lambda(p_1 + \max(q, 0))} d\Gamma_2(q).
\]

Therefore, the first-order necessary condition with respect to \( p_1 \) is

\[
\frac{1}{p_1} = \frac{-D'_i(p_1)}{D_i(p_1)} = \frac{e^{-\lambda z^*} \gamma_2(z^* - p_1)}{D_i(p_1)} + \lambda.
\]

Let \( \pi^* \) be the equilibrium profit for the sellers in a symmetric equilibrium. Since seller 1 is indifferent between offering any prices in the support of \( P_i \) in equilibrium, \( \pi^* = p_1 D_i(p_1) \) for every \( p_1 \) in the support of \( P_i \). Using \( D_i(p_1) = \pi^*/p_1 \), the first-order

\(^2\)For \( r \in (0,1) \), the strict inequality (8) is true as \( f_i(F_i^{-1}(r)) > 0 \) within the support. Since \( f_i(F_i^{-1}(r))/r \) falls in \( r \) by log-concavity of \( F_i \), \( f_i(F_i^{-1}(r))/r > 0 \) at \( r = 0 \), and thus the strict inequality (8) also holds for \( r = 0 \). For \( r = 1 \), since \( f_i \) has unbounded upper support, \( f_i(F_i^{-1}(r)) \) falls in \( r \) when \( r \) is large. Therefore \( f_i(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0 \) for some \( r \in (0,1) \). Since \( f_i(F_i^{-1}(r))/f_i(F_i^{-1}(r)) \) falls in \( r \) by the log-concavity of \( f_i \), \( f_i(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0 \) when \( r = 1 \) and thus the inequality (8) holds when \( r = 1 \).
condition can be rewritten as
\[ \gamma_1(z^* - p_1) = \frac{\pi^*}{p_1} \left( \frac{1}{p_1} - \lambda \right) e^{\lambda z^*}. \]  
(9)

The first-order condition implies \( p_1 \leq 1/\lambda \) in equilibrium. Since \( p_1 \geq 0 \), the support of \( P_1 \) is a subset of the interval \([0, \min\{z^*, 1/\lambda\}]\). From equation (9), it is clear that the density \( \gamma_i \) of \( Q_i \) is monotonically increasing (because the right-hand side falls in \( p_1 \)).

Now we use the density of \( Q_i \) (i.e., \( \gamma_i \)) and that of \( Z_i \) to solve for the distribution of \( P_i \), by exploiting the equation \( Q_i = \min\{Z_i, z^*\} - P_i \). This is generally a hard problem because one must solve a complex differential equation. Below, we show that the problem is especially tractable when \( Z_i \) is exponentially distributed. Let \( B(p) \) be the distribution function of \( P_i \) in a symmetric equilibrium. The c.d.f. and p.d.f. of \( Q_i \) can be written as
\[ \Gamma_i(q) = \int_0^{\infty} \left[ 1 - B(\min\{z, z^*\} - q) \right] \lambda e^{-\lambda z} dz, \]
\[ \gamma_i(q) \equiv \Gamma'_i(q) = \int_0^{\infty} b(\min\{z, z^*\} - q) \lambda e^{-\lambda z} dz. \]

Substitute the equation for \( \gamma_i \) into the first-order condition (9); then
\[ \pi^* p \left( \frac{1}{p} - \lambda \right) e^{\lambda (2z^* - p)} = \int_0^{\infty} b(\min\{z - z^* + p, p\}) \lambda e^{-\lambda z} dz \]
\[ = \int_{-z^*}^{0} b(y + p) \lambda e^{-\lambda(y + z^*)} dy + b(p) e^{-\lambda z^*}. \]

The last line uses a change of variable \( y = z - z^* \). Now multiply both sides by \( e^{\lambda(z^* - p)} \), and let \( \tau(p) \equiv b(p) e^{-\lambda p} \) and \( T(p) \equiv \int_0^p \tau(y) dy \). Then we can rewrite the above equation as
\[ \frac{\pi^*}{p} \left( \frac{1}{p} - \lambda \right) e^{\lambda (2z^* - p)} = \lambda \int_{-z^*}^{0} \tau(y + p) dy + \tau(p). \]

Notice that, since \( p \geq 0 \) in equilibrium, the density \( b(q) = \tau(q) = 0 \) for all \( q < 0 \). Together with \( p \leq z^* \), we have \( \tau(y + p) = 0 \) for all \( y \in (-z^*, -p) \). In light of this, the lower support of the integral term can be replaced by \(-p\). Therefore, the equation above becomes
\[ \frac{\pi^*}{p} \left( \frac{1}{p} - \lambda \right) e^{\lambda (2z^* - p)} = \lambda \int_{-p}^{0} \tau(y + p) dy + \tau(p) = \lambda T(p) + \tau(p). \]  
(10)

This equation is a first-order differential equation. The general solution is
\[ T(p) = Ce^{-\lambda p} - \frac{\pi^*}{p} e^{\lambda (2z^* - p)} \left( \lambda \log(p) + \frac{1}{p} \right), \]
where \( C \) is a constant. By \( b(p) = \tau(p) e^{\lambda p} \) and equation (10), the density \( b(p) \) is
\[ b(p) = \frac{\pi^*}{p} \left( \frac{1}{p} - \lambda \right) e^{2\lambda z^*} - \lambda T(p) e^{\lambda p} = \pi^* e^{2\lambda z^*} \left( \frac{1}{p^2} + \lambda^2 \log(p) \right) - \lambda C. \]
The constant $C$ is chosen so that $\int_0^{\min(z^*;1/\lambda)} b(p) \, dp = 1$. The value of $\pi^*$ can be solved by substituting the solution of $b(p)$ into the seller’s profit function. One can easily show that the density $b(p)$ falls in $p$ by the equation above and $p \leq 1/\lambda$.

### D. UNOBSERVABLE PRICES AND SEARCH COSTS

Anderson and Renault (1999) studied a stationary search model with unobservable prices, and showed that $\frac{\partial p^*}{\partial s} > 0$ provided that $1 - G(z)$ is log-concave. We argue that this insight may not hold when search is non-stationary, due to the presence of a prior value $V$. Assume there is no outside option and sellers are symmetric. Below, we show $\frac{\partial p^*}{\partial s} < 0$ is possible if the density of $V$ is log-concave and increasing, even when $1 - G(z)$ is log-concave.

**CLAIM 1:** The equilibrium price $p^*$ falls in $s$ when (i) $s$ is sufficiently small and (ii) $f'(\bar{v})/f(\bar{v}) > \lim_{z \to z^*} g(z)/(1 - G(z))$.

Since we have assumed $f(v)$ is log-concave, it is single-peaked in $v$. Therefore, the second condition requires $f'(v) > 0$ for all $\bar{v}$, and the upper support $\bar{v}$ must be finite.

**PROOF:** Let $\tilde{W}_i \equiv \max_{j \neq i} W_j$; then the demand for seller $i$ is given by (5). When prices are unobservable, seller $i$ controls $p_i$ but not $p_i^e$, so the measure of marginal consumers is

$$-\frac{dD_i(p_i, p_i^e, p^*)}{dp_i} \bigg|_{p_i = p_i^e = p^*} = E \left[ \int_{\tilde{W}_i - z^*}^{\tilde{\bar{v}}} g(\tilde{W} - v_i) \, dF(v_i) \right]$$

$$= \int_{\tilde{W}_i}^{\tilde{\bar{v}} + z^*} \left[ \int_{w - z^*}^{w} g(w - v_i) \, dF(v_i) \right] dH(w)^{n-1}.$$

In a symmetric equilibrium, $p^*$ solves

$$p^* - c = -\left( n \frac{dD_i(p_i, p_i^e, p^*)}{dp_i} \bigg|_{p_i = p_i^e = p^*} \right)^{-1}.$$

Since the right-hand side does not depend on $p^*$, to show $\frac{\partial p^*}{\partial s} < 0$, it suffices to show the right-hand side falls in $s$, or equivalently the following derivative is positive:

$$\frac{d}{ds} \int_{\tilde{W}_i}^{\tilde{\bar{v}} + z^*} \left[ \int_{w - z^*}^{w} g(w - v_i) \, dF(v_i) \right] dH(w)^{n-1}$$

$$= \frac{dz^*}{ds} \int_{\tilde{W}_i}^{\tilde{\bar{v}} + z^*} [g(z^*) f(w - z^*)] dH(w)^{n-1}$$

$$+ \int_{\tilde{W}_i}^{\tilde{\bar{v}} + z^*} \left[ \int_{w - z^*}^{w} g(w - v_i) \, dF(v_i) \right] \left[ \frac{f'(w - z^*)}{h(w)} + \frac{(n - 2) f(w - z^*)}{H(w)} \right] dH(w)^{n-1}.$$

The last line uses $dH(w)/ds = f(w - z^*)$ and $dh(w)/ds = f'(w - z^*)$. Next, substitute $dz^*/ds = -1/[1 - G(z^*)]$ (by equation (1)) into the derivative and divide the entire ex-
expression by $\int_{w}^{\hat{v}+z^*} f(w - z^*) \, dH(w)^{n-1}$; then the expression above has the same sign as

$$\frac{-g(z^*)}{1 - G(z^*)} + \int_{\hat{v}}^{\hat{v}+z^*} \left[ \int_{w-z^*}^{\hat{v}} g(w - v_i) \, dF(v_i) \right] \left[ \frac{f'(w - z^*)}{h(w)} + \frac{(n - 2)f(w - z^*)}{H(w)} \right] dH(w)^{n-1}$$

Now take $s \to 0$ and therefore $z^* \to \tilde{z}$. Since (i) $h(w) = \int_{w-z^*}^{\hat{v}} g(w - v_i) \, dF(v_i)$ as $z^* \to \tilde{z}$, and (ii) $f'(\hat{v})/f(\hat{v}) \leq f'(v)/f(v)$ for all $v < \hat{v}$ by the log-concavity of $f$, the limit of the above expression is at least

$$\lim_{z^* \to \tilde{z}} \frac{-g(z^*)}{1 - G(z^*)} + \frac{f'(\hat{v})}{f(\hat{v})}.$$

Finally, if $f'(\hat{v})/f(\hat{v}) > \lim_{z^* \to \tilde{z}} g(z^*)/(1 - G(z^*))$, then the last line is clearly positive and thus $\partial p^*/\partial s < 0$ when $s$ is small.\(^4\) Q.E.D.

To put this result in context, note that Haan, Moraga-González, and Petrikaite (2017) showed that in a symmetric duopoly model with unobservable prices, if $F$ has full support and $1 - G$ is log-concave, then $\partial p^*/\partial s > 0$. Since Claim 1 allows $n = 2$ and log-concave $1 - G$, the sign of $\partial p^*/\partial s$ is reversed in Claim 1 precisely because $F$ has a bounded upper support and rising density. Indeed, when $\hat{v} < \infty$ and $f' > 0$, as $s$ rises, the upper support of $H(w)$, namely, $\hat{v} + z^*$, falls while the density $h(w)$ rises at all $w < \hat{v} + z^*$. As a result, the measure of marginal consumers rises as the other sellers’ search costs rise. By this logic, as the other sellers’ search costs rise, seller $i$ is willing to lower $p_i$ to attract more marginal consumers. On the other hand, as $s_i$ rises, seller $i$ has an incentive to raise $p_i$ to extract more surplus from the visiting consumers. The overall effect depends on the relative strength of the two effects. We focus on small $s$ because the first effect is relatively stronger when $s$ is small—indeed, the magnitude of the change in the upper support $\partial (\hat{v} + z^*)/\partial s = -1/(1 - G(z^*))$ is the largest when $s \approx 0$. When $s \approx 0$, the relative strength of these two effects depends on the ratio $f'/f$ and the hazard rate $g/(1 - G)$, respectively.

Finally, since $f'(v)/f(v)$ falls in $v$ and $g(z)/(1 - G(z))$ rises in $z$, our second sufficient condition ensures $f'/f > g/(1 - G)$ at all $v$ and $z$.

E. CONSUMER SURPLUS AND SEARCH COSTS

We present an example where consumer surplus rises with search costs. Consider a symmetric duopoly environment with no outside option. Assume the prior and match val-

\(^3\)Integrate equation (2) by parts and differentiate with respect to $w$; then $h(w) = \int_{w-z^*}^{\hat{v}} g(w - v_i) \, dF(v_i) + (1 - G(z^*))f(w - z^*)$. The second term vanishes as $z^* \to \tilde{z}$.

\(^4\)If $\tilde{z} = \infty$, then $\int_{\hat{v}}^{\hat{v}+z^*} f(w - z^*) \, dH(w)^{n-1}$ vanishes as $s \to 0$, and thus $\lim_{s \to 0} \partial p^*/\partial s = 0$. But by continuity, the inequality $\partial p^*/\partial s < 0$ remains valid for small but strictly positive $s$. 

ues are uniform random variables with $V \sim U[0, 3/4]$ and $Z \sim U[0, 1]$. Since there is no outside option and $p_1 = p_2 = p^*$ in a symmetric equilibrium, every consumer purchases the product that offers the highest effective value. By Corollary 1, a (representative) consumer’s expected payoff is equal to

$$CS = E[\max(W_1, W_2)] - p^*.$$  

First, consider the effects of $s$ on $p^*$. The equilibrium price is $p^* = 6/(9 + 32s)$ by direct calculation. This implies

$$\frac{dp^*}{ds} = \frac{-192}{(9 + 32s)^2}.$$  

The expected value of the first-order statistic $\max\{W_1, W_2\}$ can be written as

$$E[\max(W_1, W_2)] = 2 \int_0^1 \int_0^{3/4} (v + \min\{z, z^*\})H(v + \min\{z, z^*\}) \, dv \, dz.$$  

Next, we consider the effect of $s$ on $E[\max(W_1, W_2)]$. By equation (1), $dz^*/ds = -1/(1 - z^*)$. This result and the equation above imply

$$\frac{dE[\max(W_1, W_2)]}{ds} = -2 \int_0^{3/4} \left[ H(v + z^*) + (v + z^*)h(v + z^*) \right] dv$$

$$- \frac{2}{1 - z^*} \int_0^1 \left[ \int_0^{3/4} (v + \min\{z, z^*\})H_z(v + \min\{z, z^*\}) \, dv \right] dz,$$  

(11)

where $H_z(w)$ is defined as

$$H_z(w) \equiv \frac{dH(w)}{dz^*} = -f(w - z^*)(1 - G(z^*)) = -\frac{4}{3}(1 - z^*) \quad \text{for} \ w \in [z^*, z^* + 4/3],$$

and otherwise 0.

Now we evaluate the effect of an increase in $s$ on $CS$ at $s = 0$. When $s = 0$, $z^* = 1$ by equation (1). By direct calculation, the density and distribution function of $W$ are

$$h(w) = \begin{cases} 4w/3 & \text{if } w \leq 3/4, \\ 1 & \text{if } 3/4 < w < 1, \\ 7/3 - 4w/3 & \text{if } 7/4 \geq w > 1, \end{cases}$$

$$H(w) = \begin{cases} 2w^2/3 & \text{if } w \leq 3/4, \\ w - 3/8 & \text{if } 3/4 < w < 1, \\ 7w/3 - 2w^2/3 - 25/24 & \text{if } 7/4 \geq w > 1. \end{cases}$$

This pricing formula is also provided by Haan, Moraga-González, and Petrikaite (2017). They showed that $p^* = 3\bar{z}^2\bar{v}/(3\bar{z}^2 + 3\bar{v}\bar{u} - \bar{v}^2)$, assuming the return to search is sufficiently high so that the consumers who visit seller 1 first will always visit seller 2 with a strictly positive probability. They showed that this assumption is satisfied when $s$ is sufficiently small and $\bar{z} > \bar{v}$. Both conditions are satisfied in our example.
Substitute the expressions for \( h, H, \) and \( H_z \) into equation (11); then

\[
\frac{dE}{ds} \left[ \max \{W_1, W_2\} \right]_{s=0} = -2 \left[ \int_0^1 \left[ H(v + 1) + (v + 1)h(v + 1) \right] dv \right]
\]

\[
+ \frac{8}{3} \int_0^1 \int_0^\frac{3}{4} (v + z) 1_{\{v+z>1\}} dv dz
\]

\[
= -2 \left[ \int_1^2 \frac{2}{3} - 2w^2 + \frac{14}{3} w - \frac{25}{24} dw \right] + \frac{8}{3} \left( \frac{45}{128} \right)
\]

\[
= -\frac{21}{16}.
\]

Altogether, a consumer’s expected surplus rises in \( s \) when \( s = 0 \) because

\[
\frac{dCS}{ds} \bigg|_{s=0} = \frac{dE}{ds} \left[ \max \{W_1, W_2\} \right]_{s=0} - \frac{dp^*}{ds} \bigg|_{s=0} = -\frac{21}{16} + \frac{192}{81} = \frac{457}{432} > 0.
\]

Intuitively, as \( s \) rises, each consumer pays a larger utility cost to visit sellers. On the other hand, they are better off because the equilibrium price \( p^* \) falls in \( s \). This example shows that the latter effect can dominate the former when \( s \) is small.

F. PRE-SEARCH INFORMATION: PROOF OF LEMMA 1

It suffices to show there exists \( a' \in (0, 1) \) such that \( \partial h(H^{-1}(a))/\partial \alpha < 0 \) if and only if \( a > a' \). Let \( \Phi \) denote the standard normal distribution function and \( \phi \) denote its density function. Since \( V \sim \mathcal{N}(0, \alpha^2) \) and \( Z \sim \mathcal{N}(0, 1 - \alpha^2) \), \( F(v) = \Phi(v/\alpha) \) and \( G(z) = \Phi(z/\sqrt{1-\alpha^2}) \). Inserting these into equation (2) and differentiating \( H(w) \) with respect to \( \alpha \) yield

\[
H_\alpha(w) \equiv \frac{\partial H(w)}{\partial \alpha} = - \left[ 1 - \Phi \left( \frac{z^*}{\sqrt{1-\alpha^2}} \right) \right] \left( \frac{w - z^*}{\alpha^2} \right) \phi \left( \frac{w - z^*}{\alpha} \right),
\]

where \( \partial z^*/\partial \alpha \) can be obtained from equation (1) by applying the implicit function theorem. Differentiating again with respect to \( w \) gives

\[
h_\alpha(w) \equiv \frac{\partial h(w)}{\partial \alpha} = - \left[ 1 - \Phi \left( \frac{z^*}{\sqrt{1-\alpha^2}} \right) \right] \left[ 1 - \left( \frac{w - z^*}{\alpha} \right)^2 \right] \frac{1}{\alpha^2} \phi \left( \frac{w - z^*}{\alpha} \right).
\]

Now observe that

\[
\frac{\partial h(H^{-1}(a))}{\partial \alpha} = h_\alpha(H^{-1}(a)) - H_\alpha(H^{-1}(a)) \frac{\partial h(H^{-1}(a))}{h(H^{-1}(a))}.
\]

Let \( w = H^{-1}(a) \) and apply \( H_\alpha(w) \) and \( h_\alpha(w) \) to the equation. Then,

\[
\frac{\partial h(H^{-1}(a))}{\partial \alpha} = -\frac{1}{\alpha^2} \left[ 1 - \Phi \left( \frac{z^*}{\sqrt{1-\alpha^2}} \right) \right] \phi \left( \frac{w - z^*}{\alpha} \right) \left[ 1 - \left( \frac{w - z^*}{\alpha^2} \right) - (w - z^*) \frac{h'(w)}{h(w)} \right].
\]
Since $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1 - \alpha^2)$, the density of $W = V + \min\{Z, z^*\}$ is

$$h(w) = \frac{1}{\sqrt{1 - \alpha^2}} \int_{-\infty}^{\infty} \phi \left( \frac{w - \min\{z, z^*\}}{\alpha} \right) \phi \left( \frac{z}{\sqrt{1 - \alpha^2}} \right) dz$$

$$= \frac{1}{\sqrt{1 - \alpha^2}} \int_{-\infty}^{\infty} \phi \left( \frac{w - z^*}{\alpha} + \max\{r, 0\} \right) \phi \left( \frac{z^* - \alpha r}{\sqrt{1 - \alpha^2}} \right) dr,$$

where the second line changes variable $r = (z^* - z)/\alpha$. Since $\partial \phi(x) / \partial x = -x \phi(x)$,

$$h'(w) = \frac{w - z^*}{\alpha^2} - \frac{\int_{-\infty}^{\infty} \max\{r, 0\} \phi \left( \frac{w - z^*}{\alpha} + \max\{r, 0\} \right) \phi \left( \frac{z^* - \alpha r}{\sqrt{1 - \alpha^2}} \right) dr}{\alpha \int_{-\infty}^{\infty} \phi \left( \frac{w - z^*}{\alpha} + \max\{r, 0\} \right) \phi \left( \frac{z^* - \alpha r}{\sqrt{1 - \alpha^2}} \right) dr}.$$

Applying this to the above equation leads to

$$\frac{\partial h(H^{-1}(a))}{\partial \alpha} = -1 + \frac{\left( w - z^* \right)^2}{\alpha} + \frac{\left( w - z^* \right) h'(w)}{h(w)}$$

$$= -1 + \frac{(z^* - w)}{\alpha} \int_{-\infty}^{\infty} \mathbb{1}_{[r \geq 0]} r \phi \left( \frac{w - z^*}{\alpha} + \max\{r, 0\} \right) \phi \left( \frac{z^* - \alpha r}{\sqrt{1 - \alpha^2}} \right) dr.$$

The last expression is clearly negative if $w > z^*$. In addition, it converges to $-\infty$ as $w$ tends to $-\infty$. For $w \leq z^*$, it decreases in $w$ because $(z^* - w)$ falls in $w$ and the density $\phi((w - z^*)/\alpha + \max\{r, 0\})$ is log-submodular in $(w, r)$. Therefore, there exists $w'$ less than $z^*$ such that the expression is positive if and only if $w < w'$. The desired result follows from the fact that $w = H^{-1}(a)$ is strictly increasing in $a$.

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Co-editor Dirk Bergemann handled this manuscript.

Manuscript received 7 November, 2016; final version accepted 25 February, 2018; available online 6 March, 2018.