Econometrica Supplementary Material

SUPPLEMENT TO "OPTIMAL TAXATION, MARRIAGE, HOME PRODUCTION, AND FAMILY LABOR SUPPLY"

(*Econometrica*, Vol. 87, No. 1, January 2019, 291–326)

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In these online Appendices, we (i) provide formal proofs of Proposition 1–5; (ii) describe the iterative algorithm and solution approximation methods for calculating the marriage market equilibrium; (iii) describe our empirical tax and transfer schedule implementation; (iv) describe the set of targeted estimation moments; (v) present additional tables and simulation results.

APPENDIX A: PROOF OF PROPOSITION 1

WE ASSUME that the distribution $G_{ij}(\mathbf{w}, \mathbf{X}, \boldsymbol{\epsilon})$ is absolutely continuous and twice continuously differentiable. The individual utility functions $u^i(\ell^i, q^i, Q; \mathbf{X}^i)$ and $u^j(\ell^j, q^j, Q; \mathbf{X}^j)$ are assumed increasing and concave in ℓ, q, and Q, and with $\lim_{q^i \to 0} u^i(\ell^i, q^i, Q; \mathbf{X}^i) = \lim_{q^j \to 0} u^j(\ell^j, q^j, Q; \mathbf{X}^j) = -\infty$. To proceed, we define the *excess demand function* as

$$ED_{ij}(\boldsymbol{\lambda}) = \mu_{ij}^d(\boldsymbol{\lambda}^i) - \mu_{ij}^s(\boldsymbol{\lambda}^j), \quad \forall i = 1, \dots, I, j = 1, \dots, J.$$

Here and in what follows, we suppress the dependence of the excess demand functions (and other objects) on the tax system T. Equilibrium existence is synonymous with the excess demand for all types being equal to zero at some vector $\boldsymbol{\lambda}^* \in [0, 1]^{I \times J}$, that is, $ED_{ij}(\boldsymbol{\lambda}^*) = 0$, $\forall i, j$. Equilibrium uniqueness implies that there is a single vector that achieves this.[1] Under our regularity conditions, we have that: (i) $U_{ij}^i(\lambda_{ij})$ *and* $U_{ij}^j(\lambda_{ij})$ *are continuously differentiable in* λ_{ij}; (ii) $\partial U_{ij}^i(\lambda_{ij})/\partial \lambda = -\lambda_{ij}/(1 - \lambda_{ij}) \cdot \partial U_{ij}^j(\lambda_{ij})/\partial \lambda < 0$; (iii) $\lim_{\lambda_{ij} \to 0} ED_{ij}(\lambda_{ij}, \boldsymbol{\lambda}_{-ij}) > 0$; and (iv) $\lim_{\lambda_{ij} \to 1} ED_{ij}(\lambda_{ij}, \boldsymbol{\lambda}_{-ij}) < 0$.

A.1. *Properties of the Excess Demand Functions*

We now state further properties of the excess demand functions. We have

$$\partial ED_{ij}(\boldsymbol{\lambda})/\partial \lambda_{ij} < 0, \tag{1a}$$

$$\partial ED_{ik}(\boldsymbol{\lambda})/\partial \lambda_{ij} > 0; \quad \text{if } k \neq j, \tag{1b}$$

$$\partial ED_{kj}(\boldsymbol{\lambda})/\partial \lambda_{ij} > 0; \quad \text{if } k \neq i, \tag{1c}$$

$$\partial ED_{kl}(\boldsymbol{\lambda})/\partial \lambda_{ij} = 0; \quad \text{if } k \neq i, l \neq j, \tag{1d}$$

where equation (1d) follows the Type-I extreme value distribution's IIA property.

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[1] Reformulating their matching model as a demand system, Galichon, Kominers, and Weber (2018) also used the properties of the excess demand function to provide a proof of existence and uniqueness with a more general heterogeneity structure. A proof using the marriage matching function and Type-I extreme value errors is presented in Galichon, Kominers, and Weber (2014).

 https://doi.org/10.3982/ECTA14528

A.2. *Equilibrium Existence*

We construct a continuous function $\boldsymbol{\Gamma} : [0,1]^{I\times J} \rightarrow [0,1]^{I\times J}$ such that any fixed-point $\boldsymbol{\lambda}^*$ is an equilibrium of the marriage market. Brouwer's fixed-point theorem then implies existence. Letting $\psi > 0$, we define

$$\boldsymbol{\Gamma}(\boldsymbol{\lambda}) = \psi \cdot \mathbf{ED}(\boldsymbol{\lambda}) + \boldsymbol{\lambda}.$$

Notice that $\boldsymbol{\lambda}^*$ is a fixed point, $\boldsymbol{\lambda}^* = \psi \cdot \mathbf{ED}(\boldsymbol{\lambda}^*) + \boldsymbol{\lambda}^*$, if and only if excess demand is identically zero, that is, $\mathbf{ED}(\boldsymbol{\lambda}^*) = 0$. The following lemmas establish that we can choose ψ small enough so that the range of $\boldsymbol{\Gamma}$ is $[0,1]^{I\times J}$.

LEMMA 1: *The excess demand functions are continuously differentiable with* $\mathbf{ED}(\mathbf{0}_{I\times J}) \succcurlyeq \mathbf{0}_{I\times J}$ *and* $\mathbf{ED}(\mathbf{1}_{I\times J}) \preccurlyeq \mathbf{0}_{I\times J}$.

PROOF OF LEMMA 1: The continuous differentiability follows directly from the regularity conditions described above. $\mathbf{ED}(\mathbf{0}_{I\times J}) \succcurlyeq \mathbf{0}_{I\times J}$ and $\mathbf{ED}(\mathbf{1}_{I\times J}) \preccurlyeq \mathbf{0}_{I\times J}$ follow from our regularity conditions along with equations (1a)–(1d). Intuitively, there is no supply when $\boldsymbol{\lambda} = \mathbf{0}_{I\times J}$, and no demand when $\boldsymbol{\lambda} = \mathbf{1}_{I\times J}$. *Q.E.D.*

LEMMA 2: *Let* $0 < \psi \leq (\sup_{i,j,\boldsymbol{\lambda}} |\frac{\partial ED_{ij}(\boldsymbol{\lambda})}{\partial \lambda_{ij}}|)^{-1}$, *then* $\mathbf{0}_{I\times J} \preccurlyeq \boldsymbol{\Gamma}(\boldsymbol{\lambda}) \preccurlyeq \mathbf{1}_{I\times J}$.

PROOF OF LEMMA 2: Such a ψ exists by the extreme value theorem because $\mathbf{ED}(\boldsymbol{\lambda})$ is continuously differentiable on $[0,1]^{I\times J}$. Now, we have that $\psi \cdot \partial ED_{ij}(\boldsymbol{\lambda})/\partial \lambda_{ij} + 1 \geq 0$. This combined with equations (1b)–(1d) being nonnegative implies that $\partial \boldsymbol{\Gamma}(\boldsymbol{\lambda})/\partial \boldsymbol{\lambda} \succcurlyeq \mathbf{0}_{I\times J}$. Consequentially,

$$\boldsymbol{\Gamma}(\mathbf{0}_{I\times J}) \preccurlyeq \boldsymbol{\Gamma}(\boldsymbol{\lambda}) \preccurlyeq \boldsymbol{\Gamma}(\mathbf{1}_{I\times J}).$$

Finally, by Lemma 1, $\mathbf{0}_{I\times J} \preccurlyeq \boldsymbol{\Gamma}(\mathbf{0}_{I\times J})$ and $\boldsymbol{\Gamma}(\mathbf{1}_{I\times J}) \preccurlyeq \mathbf{1}_{I\times J}$. *Q.E.D.*

Thus, from Lemma 2, Brouwer's conditions are satisfied and an equilibrium exists.

A.3. *Equilibrium Uniqueness*

Suppose the equilibrium is not unique. Consider any distinct vectors of Pareto weights $\boldsymbol{\lambda}^* \neq \boldsymbol{\lambda}'$ with $\mathbf{ED}(\boldsymbol{\lambda}^*) = \mathbf{ED}(\boldsymbol{\lambda}') = \mathbf{0}$. Then let $B^* = \{\langle i,j\rangle | \lambda^*_{i,j} < \lambda'_{i,j}\}$ denote the pairings where $\boldsymbol{\lambda}^*$ is *strictly* less than $\boldsymbol{\lambda}'$. As the labeling of $\boldsymbol{\lambda}^*$ and $\boldsymbol{\lambda}'$ is arbitrary, without loss of generality, we take B^* to be nonempty. Defining $U^i_{i0}(\lambda_{i0}) = U^i_{i0}$ the following holds:

$$\begin{aligned}
\sum_{\langle i,j\rangle \in B^*} \mu^d_{ij}(\boldsymbol{\lambda}^*) &= \sum_i m_i \Pr\Big[\max_{\{j:\langle i,j\rangle\in B^*\}} \big\{U^i_{ij}(\lambda^*_{ij}) + \theta^i_{ij}\big\} > \max_{\{j:\langle i,j\rangle\notin B^* \vee j=0\}} \big\{U^i_{ij}(\lambda^*_{ij}) + \theta^i_{ij}\big\}\Big] \\
&> \sum_i m_i \Pr\Big[\max_{\{j:\langle i,j\rangle\in B^*\}} \big\{U^i_{ij}(\lambda'_{ij}) + \theta^i_{ij}\big\} > \max_{\{j:\langle i,j\rangle\notin B^* \vee j=0\}} \big\{U^i_{ij}(\lambda'_{ij}) + \theta^i_{ij}\big\}\Big] \\
&= \sum_{\langle i,j\rangle \in B^*} \mu^d_{ij}(\boldsymbol{\lambda}').
\end{aligned}$$

The outside inequality is strict because $U^i_{ij}(\lambda_{ij})$ is strictly *decreasing* in λ_{ij}, and because θ^i_{ij} has full support. Thus, the measure of type-i men who would choose type-j women (the demand) from the set B^* is strictly higher under $\boldsymbol{\lambda}^*$ compared to $\boldsymbol{\lambda}'$. By the same arguments, the measure of type-j females who would choose type-i males (the supply) from the set B^* is strictly lower under $\boldsymbol{\lambda}^*$ compared to $\boldsymbol{\lambda}'$. It therefore follows that

$$\sum_{\langle i,j\rangle\in B^*} ED_{ij}(\boldsymbol{\lambda}^*) > \sum_{\langle i,j\rangle\in B^*} ED_{ij}(\boldsymbol{\lambda}'),$$

which is a contradiction. Hence, the equilibrium must be unique.

APPENDIX B: MARRIAGE MARKET NUMERICAL ALGORITHM

In this Appendix, we describe the iterative algorithm and the solution approximation method that we use to calculate the market clearing vector of Pareto weights. The algorithm is based on that presented in Galichon, Kominers, and Weber (2014, 2018). We first note that using the conditional choice probabilities from equation (5) we are able to write the quasi-demand equation of type-i men for type-j spouses as

$$\sigma_\theta \times \left[\ln \mu_{ij}^d(T, \boldsymbol{\lambda}^i) - \ln \mu_{i0}^d(T, \boldsymbol{\lambda}^i)\right] = U_{ij}^i(T, \lambda_{ij}) - U_{i0}^i(T). \tag{2}$$

Similarly, the conditional choice probabilities for females from equation (6) allows us to express the quasi-supply equation of type-j women to the $\langle i, j\rangle$ submarket as

$$\sigma_\theta \times \left[\ln \mu_{ij}^s(T, \boldsymbol{\lambda}^j) - \ln \mu_{0j}^s(T, \boldsymbol{\lambda}^j)\right] = U_{ij}^j(T, \lambda_{ij}) - U_{0j}^j(T). \tag{3}$$

The algorithm proceeds as follows:

1. Provide an initial guess of the measure of both single males $0 < \mu_{i0}^d < m_i$ for $i = 1, \dots, I$, and single females $0 < \mu_{0j}^s < f_j$ for $j = 1, \dots, J$.

2. Taking the difference of the quasi-demand (equation (2)) and the quasi-supply (equation (3)) functions for each $\langle i, j\rangle$ submarriage market and imposing the market clearing condition $\mu_{ij}^d(T, \boldsymbol{\lambda}^i) = \mu_{ij}^s(T, \boldsymbol{\lambda}^j)$ we obtain

$$\sigma_\theta \times \left[\ln \mu_{0j}^s - \ln \mu_{i0}^d\right] = U_{ij}^i(T, \lambda_{ij}) - U_{i0}^i(T) - \left[U_{ij}^j(T, \lambda_{ij}) - U_{0j}^j(T)\right], \tag{4}$$

which given the single measures μ_{i0}^d and μ_{0j}^s (and the tax schedule T) are only a function of the Pareto weight for that submarriage-market λ_{ij}. Given our assumptions on the utility functions, there exists a unique solution to equation (4). This step therefore requires solving for the root of $I \times J$ univariate equations.

3. From *Step 2*, we have a matrix of Pareto weights $\boldsymbol{\lambda}$ given the single measures μ_{i0}^d and μ_{0j}^s from *Step 1*. These measures can be updated by calculating the conditional choice probabilities (equations (5) and (6)). The algorithm returns to Step 2 and repeats until the vector of single measures for both males and females has converged.

In practice, we are able to implement this algorithm by first evaluating the expected utilities $U_{ij}^i(T, \lambda)$ and $U_{ij}^j(T, \lambda)$ for each marital match combination $\langle i, j\rangle$ on a fixed grid of Pareto weights $\lambda \in \boldsymbol{\lambda}^{\text{grid}}$ with $\inf[\boldsymbol{\lambda}^{\text{grid}}] \gtrapprox 0$ and $\sup[\boldsymbol{\lambda}^{\text{grid}}] \lessapprox 1$. We may then replace $U_{ij}^i(T, \lambda)$ and $U_{ij}^j(T, \lambda)$ with an approximating parametric function so that no expected values are actually evaluated within the iterative algorithm.

Note that calculating the expected values within a match are (by many orders of magnitude) the most computationally expensive part of the algorithm. While our empirical exercise incorporates market variation in taxes and transfers, in an application where each market only differs by the population vectors and/or the demographic transition functions, the computational cost in calculating the equilibrium for all markets is *approximately independent* of the number of markets K considered. This follows given that the initial evaluation of expected values on $\boldsymbol{\lambda}^{\text{grid}}$ is independent of market in this case.

APPENDIX C: EMPIRICAL TAX AND TRANSFER SCHEDULE IMPLEMENTATION

In this Appendix, we describe our implementation of the empirical tax and transfer schedules for our estimation exercise. Since some program rules will vary by U.S. state, here we are explicit in indexing the respective parameters by market.[2] Our measure of taxes includes both state and federal Earned Income Tax Credit (EITC) programs, and we also account for the Food Stamps Program and the Temporary Assistance for Needy Families (TANF) program. It does not include other transfers (e.g., Medicaid) and non-income taxes such as sales and excises taxes.[3]

Consider (a married or single) household ι in market k, with household earnings $E_{\iota k} = \mathbf{h}_w^{\iota k} \cdot \mathbf{w}_{\iota k}$ and demographic characteristics $\mathbf{X}_{\iota k}$. As before, the demographic conditioning vector comprises marital status and children. The total net tax liability for such a household is given by $T_{\iota k} = \widetilde{T}_{\iota k} - Y_{\text{TANF}}^{\iota k} - Y_{\text{FSP}}^{\iota k}$, where $\widetilde{T}_{\iota k}$ is the (potentially negative) tax liability from income taxes and the EITC, $Y_{\text{TANF}}^{\iota k}$ and $Y_{\text{FSP}}^{\iota k}$ are the respective (non-negative) amounts of TANF and Food Stamps.

Income taxes and EITC. Our measure of income taxes $\widetilde{T}_{\iota k}$ includes both federal and state income taxes, as well as federal and state EITC. In addition to market, the tax schedules that we calculate may also vary with marital status and with children. These schedules are calculated prior to estimation with the National Bureau of Economic Research TAXSIM calculator, as described in Feenberg and Coutts (1993). We assume joint filing status for married couples. For singles with children, we assume head-of-household filing status.

Food Stamp Program. Food Stamps are available to low-income households both with and without children.[4] For the purposes of determining the entitlement amount, net household earnings are defined as

$$N_{\text{FSP}}^{\iota k} = \max\{0, E_{\iota k} + Y_{\text{TANF}}^{\iota k} - D_{\text{FSP}}[\mathbf{X}_{\iota k}]\},$$

where $Y_{\text{TANF}}^{\iota k}$ is the dollar amount of TANF benefit received by this household (see below), and $D_{\text{FSP}}[\mathbf{X}_{\iota k}]$ is the standard deduction, which may vary with household type. The dollar

[2]Since we define a market as a Census Bureau-designated division, we apply the state tax rules that correspond to the most populous state within a defined market.

[3]While Medicaid is the largest U.S. means-tested program in terms of overall expenditure, the bulk of this expenditure (67% in 2006) goes on the disabled and aged population. Neither of these groups are part of our analysis. Furthermore, very few structural labor supply models actually incorporate in-kind transfers such as Medicaid, as quantifying the value to recipients is much more complicated. See the recent survey by Chan and Moffitt (2018). In the case of Medicaid, there is both a transfer and an insurance component, and we have experimented with incorporating the transfer value in the budget constraint. We construct this value using data from the full-year consolidated Household Component data files of the Medical Expenditure Survey, together with the 2006 state Medicaid rules from Ross, Cox, and Marks (2007) to determine eligibility, and find that incorporating this transfer value has very little impact on either the initial estimation results, or our subsequent optimality simulations.

[4]Food Stamp parameters for 2006 are obtained from U.S. Department of Agriculture, Food and Nutrition Service (Wolkwitz, 2007).

amount of Food Stamp entitlement is then given by

$$Y_{\mathrm{FSP}}^{\iota k} = \max\{0, Y_{\mathrm{FSP}}^{\max}[\mathbf{X}_{\iota k}] - \tau_{\mathrm{FSP}} \times N_{\mathrm{FSP}}^{\iota k}\},$$

where $Y_{\mathrm{FSP}}^{\max}[\mathbf{X}_{\iota k}]$ is the maximum food stamp benefit amount for a household of a given size and $\tau_{\mathrm{FSP}} = 0.3$ is the phase-out rate.[5]

TANF. Financial support to families with children is provided by TANF.[6] Given the static framework we are considering, we are not able to incorporate certain features of the TANF program, notably the time limits in benefit eligibility (see Chan, 2013). For the purposes of entitlement calculation, we define net household earnings as

$$N_{\mathrm{TANF}}^{\iota k} = \max\{0, (1 - R_{\mathrm{TANF}}^{k}) \times (E_{\iota k} - D_{\mathrm{TANF}}^{k}[\mathbf{X}_{\iota k}])\},$$

where the *dollar* earnings disregard $D_{\mathrm{TANF}}^{k}[\mathbf{X}_{\iota k}]$ varies by market and household characteristics. The market-level *percent* disregard is given by R_{TANF}^{k}. The dollar amount of TANF entitlement is then given by

$$Y_{\mathrm{TANF}}^{\iota k} = \min\{Y_{\mathrm{TANF}}^{\max}[\mathbf{X}_{\iota k}], \max\{0, r_{\mathrm{TANF}}^{k} \times (Y_{\mathrm{TANF}}^{\widehat{\max}}[\mathbf{X}_{\iota k}] - N_{\mathrm{TANF}}^{\iota k})\}\}.$$

Here, $Y_{\mathrm{TANF}}^{\max}[\mathbf{X}_{\iota k}]$ defines the maximum TANF receipt in market k for a household with characteristics $\mathbf{X}_{\iota k}$, while $Y_{\mathrm{TANF}}^{\widehat{\max}}[\mathbf{X}_{\iota k}]$ defines what is typically referred to as the *payment standard*. The ratio r_{TANF}^{k} is used in some markets to adjust the total TANF amount.[7]

APPENDIX D: IDENTIFICATION

D.1. *Proof of Proposition 2*

Consider a given market $k \leq K$. From the conditional choice probabilities (equations (5) and (6)) and imposing market clearing $\mu_{ij}^{d}(T, \boldsymbol{\lambda}^{i}) = \mu_{ij}^{j}(T, \boldsymbol{\lambda}^{j}) = \mu_{ij}(T, \boldsymbol{\lambda})$, we have that

$$\ln \mu_{ij}(T, \boldsymbol{\lambda}) - \ln \mu_{i0}(T, \boldsymbol{\lambda}^{i}) = [U_{ij}^{i}(T, \lambda_{ij}) - U_{i0}^{i}(T)]/\sigma_{\theta}, \tag{5a}$$

$$\ln \mu_{ij}(T, \boldsymbol{\lambda}) - \ln \mu_{0j}(T, \boldsymbol{\lambda}^{j}) = [U_{ij}^{j}(T, \lambda_{ij}) - U_{0j}^{j}(T)]/\sigma_{\theta}. \tag{5b}$$

The left-hand side of equations (5a) and (5b) are obtained from the empirical marriage matching function and is therefore identified. Now consider variation in this object as we vary population vectors. Importantly, variation in population vectors has no impact on the value of the single state and only affects the value in marriage through its influence on the Pareto weight λ_{ij}. That is, such variation serves as a *distribution factor* (see Bourguignon,

[5] In practice, the Food Stamp Program also has a gross-earnings and net-earnings income test. These require that earnings are below some threshold related to the federal poverty level for eligibility (see, e.g., Chan, 2013). For some families, these rules would mean that there may be a discontinuous fall in entitlement (to zero) as earnings increase. We also assume a zero excess shelter deduction in our calculations and do not consider asset tests. Incorporating asset tests (even in a dynamic model) is challenging as the definition of *countable* assets does not correspond to the usual assets measure in life-cycle models.

[6] We obtain TANF parameters from 2006 from the Urban Institute's Welfare Rules Data Book; see Rowe and Murphy (2006).

[7] As in the case of Food Stamps, we do not consider the similar gross and net income eligibility rules that exist for TANF, as well as the corresponding asset tests. We also do not consider eligibility time limits.

Browning, and Chiappori, 2009). From a marginal perturbation in, for example, the male population vector we obtain

$$\sum_{i'} \frac{\partial\left[\ln \mu_{ij}(T, \boldsymbol{\lambda}) - \ln \mu_{i0}\left(T, \boldsymbol{\lambda}^{i}\right)\right]}{\partial m_{i'}} \, \mathrm{d}m_{i'} = \frac{1}{\sigma_\theta} \frac{\partial U_{ij}^{i}(T, \lambda_{ij})}{\partial \lambda_{ij}} \sum_{i'} \frac{\partial \lambda_{ij}}{\partial m_{i'}} \, \mathrm{d}m_{i'}, \tag{6a}$$

$$\sum_{i'} \frac{\partial\left[\ln \mu_{ij}(T, \boldsymbol{\lambda}) - \ln \mu_{0j}\left(T, \boldsymbol{\lambda}^{j}\right)\right]}{\partial m_{i'}} \, \mathrm{d}m_{i'} = \frac{1}{\sigma_\theta} \frac{\partial U_{ij}^{j}(T, \lambda_{ij})}{\partial \lambda_{ij}} \sum_{i'} \frac{\partial \lambda_{ij}}{\partial m_{i'}} \, \mathrm{d}m_{i'}. \tag{6b}$$

Taking the ratio of the partial derivatives in equations (6a) and (6b), we define

$$\pi_{ij} = \frac{\partial U_{ij}^{i}(T, \lambda_{ij})/\partial \lambda_{ij}}{\partial U_{ij}^{j}(T, \lambda_{ij})/\partial \lambda_{ij}}.$$

We proceed by combining the definition of z_{ij} with equation (3) from the main text which requires that $(1 - \lambda_{ij}) \cdot \partial U_{ij}^{i}(T, \lambda_{ij})/\partial \lambda_{ij} + \lambda_{ij} \cdot \partial U_{ij}^{j}(T, \lambda_{ij})/\partial \lambda_{ij} = 0$. It immediately follows that $\lambda_{ij} = \pi_{ij}/(\pi_{ij} - 1)$, which establishes identification.

D.2. *Proof of Proposition 3*

The identification proof will proceed in two steps. First, we demonstrate identification of the time allocation problem for single individuals. Second, we show how we use the household time allocation patterns to identify the home production technology for married couples. The following assumptions are used in the proof of identification in this section. While some of them are easily relaxed, for reasons of clarity and ease of exposition, and because they relate directly to the empirical and optimal design analysis, these assumptions are maintained here. We also only consider identification of the model without the fixed cost of labor force participation, as it adds nothing to the analysis.

ASSUMPTION ID-1: *The state-specific errors, $\epsilon_{\mathbf{a}^i}$ are distributed Type-I extreme value with location parameter zero and an unknown scale parameter, σ_ϵ.*

ASSUMPTION ID-2: *The systematic utility function is additively separable in leisure, ℓ^i, private consumption, q^i, and home goods, Q^i. That is,*

$$u^i\left(\ell^i, q^i, Q^i, \mathbf{X}^i\right) = u_q^i\left(q^i, \mathbf{X}^i\right) + u_\ell^i\left(\ell^i, \mathbf{X}^i\right) + u_Q^i\left(Q^i, \mathbf{X}^i\right).$$

ASSUMPTION ID-3: *There is a known private consumption level $\widehat{q}$ such that $\partial u_q^i(\widehat{q}, \mathbf{X}^i)/\partial q = 1$.*

ASSUMPTION ID-4: *$u_Q^i(Q^i, \mathbf{X}^i)$ is monotonically increasing in Q, that is, $\partial u_Q^i(Q^i, \mathbf{X}^i)/\partial Q > 0$.*

ASSUMPTION ID-5: *There exist an element of $\mathbf{X}^i$, $\mathbf{X}_r^i$, such that $\mathbf{X}_r^i$ affects $\zeta_{i0}(\mathbf{X}^i)$ but not $u_Q^i(Q^i, \mathbf{X})$. Also there exists an $\mathbf{X}_*^i$ such that $\zeta_{i0}(\mathbf{X}_*^i) = 1$.*

ASSUMPTION ID-6: *The support of Q is the same for both single individuals and married couples.*

ASSUMPTION ID-7: *Conditional on work hours* h_w^i, *the tax schedule* T *is differentiable in earnings, with* $\partial T(w^i h_w^i, y^i; \mathbf{X}^i)/\partial w h_w \neq 1$.

ASSUMPTION ID-8: *The utility of function of the private good,* $u_q^i(q^i, \mathbf{X}^i)$, *is monotonically increasing and quasi-concave in* q^i.

D.2.1. *Step 1: The Identification Using the Singles Problem*

Consider the problem of a single type-i male. Let $\mathcal{A}^i = \{1, \dots, \overline{A}^i\}$ be an index representation set of time allocation alternatives, with $\widehat{u}^i(a)$ denoting the systematic part of utility associated with alternative $a \in \mathcal{A}^i$ (where the dependence on conditioning variables is suppressed for notational compactness). Without loss of generality, let $a = 1$ be the choice where the individual does not work and has the lowest level of home hours. Under Assumption ID-1, well-known results imply that the following holds:

$$\log\left[\frac{\mathbb{P}(a)}{\mathbb{P}(1)}\right] = \frac{\widehat{u}^i(a) - \widehat{u}^i(1)}{\sigma_\epsilon}, \tag{7}$$

where the conditional choice probabilities $\mathbb{P}(\cdot)$ should be understood as being conditional on $[y^i, w^i, \mathbf{X}^i, T]$. Taking the partial derivative of equation (7) with respect to w^i and using Assumption ID-2 yields

$$\frac{\partial \log\big[\mathbb{P}(a)/\mathbb{P}(1)\big]}{\partial w} = \frac{1}{\sigma_\epsilon} \cdot \frac{\partial u_q^i\big(q^i(a); \mathbf{X}^i\big)}{\partial q} \cdot \left[1 - \frac{\partial T\big(w^i h_w^i(a), y^i; \mathbf{X}^i\big)}{\partial w h_w}\right] \cdot h_w^i(a), \tag{8}$$

where $q^i(a)$ and $h_w^i(a)$ are the respective private consumption and market work hours associated with the allocation a. The conditional choice probabilities and the marginal tax rates are known, and hence, given Assumptions ID-3 and ID-7, the scale coefficient for the state-specific errors σ_ϵ is identified. Hence, the marginal utility of private consumption is identified. Integrating equation (8) and combining with equation (7) implies that the sum $u_\ell^i(\ell^i; \mathbf{X}^i) + u_Q^i(Q^i; \mathbf{X}^i)$ is identified up to a normalizing constant. Then for each level of feasible home hours, both $u_\ell^i(\ell^i; \mathbf{X}^i)$ and $u_Q^i(Q^i; \mathbf{X}^i)$ are identified by varying the level of market hours and fixing either home time or leisure. Under Assumption ID-5, the home efficiency parameter $\zeta_{i0}(\mathbf{X}^i)$ is identified by comparing $u_Q^i(Q^i(a); \mathbf{X}^i)$ across different values of $\mathbf{X}^i$.

D.2.2. *Step 2: Identification of Marriage Home Production Function*

In Step 1, we show that the subutilities are identified up to a normalizing constant. Without loss of generality, we set the location normalization to be zero in what follows. Consider a $\langle i, j\rangle$ household with the time allocation set $\mathcal{A}_{ij} = \{1, \dots, \overline{A}\}$, $\overline{A} = \overline{A}^i \times \overline{A}^j$, and let $\widehat{u}^{ij}(a) = (1 - \lambda_{ij}) \times \widehat{u}^i(a) + \lambda_{ij} \times \widehat{u}^j(a)$ denote the systematic part of household utility associated with $a \in \mathcal{A}_{ij}$. Let $\varepsilon_a^{ij} = (1 - \lambda_{ij}) \times \epsilon_a^i + \lambda_{ij} \times \epsilon_a^j$, and define $\mathcal{G}_{ij}^a(\cdot)$ to be the joint cumulative distribution function of $[\varepsilon_a^{ij} - \varepsilon_1^{ij}, \dots, \varepsilon_a^{ij} - \varepsilon_{a-1}^{ij}, \varepsilon_{a+1}^{ij} - \varepsilon_a^{ij}, \dots, \varepsilon_a^{ij} - \varepsilon_A^{ij}]$. For each $a \in \{1, \dots, \overline{A} - 1\}$, define

$$\mathbb{P}(a) = \mathcal{Q}_j\big(\widehat{\mathbf{u}}^{ij}\big) \equiv \mathcal{G}_{ij}^a\big(\widehat{u}_a^{ij} - \widehat{u}_1^{ij}, \dots, \widehat{u}_a^{ij} - \widehat{u}_{a-1}^{ij}, \widehat{u}_a^{ij} - \widehat{u}_{a+1}^{ij}, \dots, \widehat{u}_a^{ij} - \widehat{u}_A^{ij}\big),$$

with $\widehat{\mathbf{u}}^{ij} = [\widehat{u}_1^{ij} - \widehat{u}_{\overline{A}}^{ij}, \dots, \widehat{u}_{\overline{A}-1}^{ij} - \widehat{u}_{\overline{A}}^{ij}]^\intercal$ defining the $(\overline{A} - 1)$ vector of utility differences, and let $\mathcal{Q}(\widehat{\mathbf{u}}^{ij}) = [\mathcal{Q}_1(\widehat{\mathbf{u}}^{ij}), \dots, \mathcal{Q}_{A-1}(\widehat{\mathbf{u}}^{ij})]^\intercal$ define a $(\overline{A} - 1)$ dimensional vector function. Then, by Proposition 1 of Hotz and Miller (1993), the inverse of $\mathcal{Q}(\widehat{\mathbf{u}}^{ij})$ exists.[8] Given that the distribution of $\boldsymbol{\epsilon}$ is known and λ_{ij} is identified, the inverse of $\mathcal{Q}(\widehat{\mathbf{u}}^{ij})$ is known. Hence, the vector $\widehat{\mathbf{u}}^{ij} = \mathcal{Q}^{-1}(\mathbb{P}(1), \dots, \mathbb{P}(\overline{A}-1))$ is identified. Define

$$\begin{aligned}\Delta_{ij}(a) = \widehat{\mathbf{u}}^{ij}_{[a]} &- (1-\lambda_{ij}) \times \big[u^i_\ell\big(\ell^i\big(a^i\big); \mathbf{X}^i\big) + u^i_q\big(\big(1 - s_{ij}(a; \lambda_{ij})\big) \cdot q(a); \mathbf{X}^i\big)\big] \\ &- \lambda_{ij} \times \big[u^j_\ell\big(\ell^j\big(a^j\big); \mathbf{X}^j\big) + u^i_q\big(s_{ij}(a; \lambda_{ij}) \cdot q(a); \mathbf{X}^j\big)\big].\end{aligned}$$

The arguments from Step 1 imply that $u^i_q(q^i; \mathbf{X}^i)$ and $u^j_q(q^j; \mathbf{X}^j)$ are known. From Proposition 2, we have that λ_{ij} are identified. These, together with Assumption ID-2 and Assumption ID-4, imply that $s_{ij}(a; \lambda_{ij})$ is also known. Thus, identification of $\Delta_{ij}(a)$ follows. Finally, the definition of $\widehat{u}^{ij}(a)$ and Assumption ID-2 imply

$$\Delta_{ij}(a) = (1-\lambda_{ij}) \times u^i_Q\big(\widetilde{Q}_{ij}\big(h^i_Q(a), h^j_Q(a); \mathbf{X}\big), \mathbf{X}^i\big) + \lambda_{ij} \times u^j_Q\big(\widetilde{Q}_{ij}\big(h^i_Q(a), h^j_Q(a); \mathbf{X}\big), \mathbf{X}^j\big).$$

The subutility function of the public good does not depend on $\mathbf{w}$. Therefore, once we observe different values of these two variables, both $u^j_Q(\widetilde{Q}_{ij}(h^i_Q(a), h^j_Q(a); \mathbf{X}), \mathbf{X}^j)$ and $u^i_Q(\widetilde{Q}_{ij}(h^i_Q(a), h^j_Q(a); \mathbf{X}), \mathbf{X}^i)$ are identified. Finally, under Assumption ID-4 the inverse of u^i_Q and u^j_Q exist, and hence $\widetilde{Q}_{ij}(h^i_Q(a^i), h^j_Q(a^j); \mathbf{X})$ is identified.

APPENDIX E: ESTIMATION MOMENTS

In this Appendix, we describe the set of targeted estimation moments. Recall that there are nine markets ($K = 9$) and three education groups/types for both men ($I = 3$) and women ($J = 3$) in our empirical application. The first set of moments (denoted G1) relate to the marriage market. Within each market, we describe the number of single men and women by own education, and married households by joint education ($K \times [I + J + I \times J]$ moments). The second set of moments (G2) describe labor supply patterns. By market, gender, marital status, and own education, we describe mean conditional work hours and employment rates ($K \times 4 \times [I + J]$ moments); aggregating over markets, we describe the fraction of individuals in nonemployment/part-time/full-time status by gender, marital status, the presence of children, and own/joint education level (for singles/couples, resp.) ($6 \times [I + J] + 12 \times I \times J$ moments); the mean and standard deviation of conditional work hours is described by gender, marital status, and own education, while mean conditional hours for married men and women are also described by joint education levels ($8 \times [I + J] + 2 \times I \times J$ moments). The third set of moments (G3) describe accepted wages. The mean and standard deviation of accepted log-wages are described by gender, marital status, and own education ($4 \times [I + J]$ moments). The fourth set of moments (G4) describe earnings, with the mean and standard deviation calculated using the same set of conditioning variables as for wages (again, $4 \times [I + J]$ moments). The fifth set of moments (G5) relate to home time. Similar to labor supply, we describe the fraction of individuals with low/medium/high unconditional home hours by gender, marital status, children, and own/joint education level (for singles/couples respectively) ($6 \times [I + J] + 12 \times I \times J$ moments); the mean and standard deviation of unconditional home hours is described by gender, marital status, and own education, while mean unconditional home hours for married men and women are also described by joint education levels ($8 \times [I + J] + 2 \times I \times J$ moments). In total, we have 765 moments.

[8]Notice that ε^{ij}_a is not i.i.d. However, independence is not required for the Hotz and Miller (1993) proposition.

APPENDIX F: PROOF OF PROPOSITION 4

In this Appendix, we derive the contribution of the marital shocks within each match to the social-welfare function. We proceed in two steps. First, we characterize the *distribution* of martial preference shocks within a particular match, recognizing the nonrandom selection into a given pairing. Second, given this distribution, we obtain the adjustment term using our specification of the utility transformation function.

Consider the first step. For brevity of notation, here we let U_j denote the expected utility of a given individual from choice/spousal type j. Associated with each alternative j is an extreme value error θ_j that has scale parameter σ_θ. We now characterize the distribution of θ_j conditional on j being chosen. Letting $p_j = (\sum_k \exp[(U_k - U_j)/\sigma_\theta])^{-1}$ denote the associated conditional choice probability. It follows that

$$\begin{aligned}
\Pr[\theta_j < x | j = \arg\max_k U_k + \theta_k] &= \frac{1}{\sigma_\theta p_j} \int_{-\infty}^{x} \prod_{k \neq j} \exp\bigl(-e^{-\frac{\theta_j + U_j - U_k}{\sigma_\theta}}\bigr) \exp\bigl(-e^{-\frac{\theta_j}{\sigma_\theta}}\bigr) e^{-\frac{\theta_j}{\sigma_\theta}} \,\mathrm{d}\theta_j \\
&= \frac{1}{\sigma_\theta p_j} \int_{-\infty}^{x} \exp\Bigl(-e^{-\frac{\theta_j}{\sigma_\theta}} \sum_k e^{-\frac{U_j - U_k}{\sigma_\theta}}\Bigr) e^{-\frac{\theta_j}{\sigma_\theta}} \,\mathrm{d}\theta_j \\
&= \frac{1}{\sigma_\theta p_j} \int_{-\infty}^{x} \exp\bigl(-e^{-\frac{\theta_j}{\sigma_\theta}} p_j^{-1}\bigr) e^{-\frac{\theta_j}{\sigma_\theta}} \,\mathrm{d}\theta_j \\
&= \exp\bigl(-e^{-\frac{\theta_j + \sigma_\theta \log p_j}{\sigma_\theta}}\bigr).
\end{aligned}$$

Hence, the distribution of the idiosyncratic payoff conditional on j being optimal is extreme value with the scale parameter σ_θ and the shifted location parameter $-\sigma_\theta \log p_j$.

Marital payoff adjustment term: $\delta < 0$. Using the utility transformation function (equation (13)) and letting $\mathbf{Z}_j$ denote the entire vector of post-marriage realizations in alternative j (wages, preference shocks, demographics), it follows that the contribution to social-welfare of an individual in this marital pairing may be written in the form

$$\begin{aligned}
&\int_{\theta_j} \int_{\mathbf{Z}_j} Y\bigl[v_j(\mathbf{Z}_j) + \theta_j\bigr] \,\mathrm{d}G_j(\mathbf{Z}_j) \,\mathrm{d}H_j(\theta_j) \\
&\quad = \int_{\theta_j} \exp(\delta\theta_j) \,\mathrm{d}H_j(\theta_j) \int_{\mathbf{Z}_j} \frac{\exp\bigl[\delta v(\mathbf{Z}_j)\bigr]}{\delta} \,\mathrm{d}G_j(\mathbf{Z}_j) - \frac{1}{\delta},
\end{aligned}$$

where we have suppressed the dependence on the tax system T.

We now complete our proof in the $\delta < 0$ case by providing an analytic characterization of the integral term over the idiosyncratic marital payoff. Using the result that $\theta_j|_{j=\arg\max_k U_k + \theta_k} \sim EV(-\sigma_\theta \log p_j, \sigma_\theta)$ from above, we have

$$\begin{aligned}
\int_{\theta_j} \exp(\delta\theta_j) \,\mathrm{d}H_j(\theta_j) &= \frac{1}{\sigma_\theta} \int_{\theta_j} \exp(\delta\theta_j) \exp\bigl(-[\theta_j + \sigma_\theta \log p_j]/\sigma_\theta\bigr) e^{-\exp(-[\theta_j + \sigma_\theta \log p_j]/\sigma_\theta)} \,\mathrm{d}\theta_j \\
&= \exp(-\delta\sigma_\theta \log p_j) \int_0^\infty t^{-\delta\sigma_\theta} \exp(-t) \,\mathrm{d}t \\
&= p_j^{-\delta\sigma_\theta} \Gamma(1 - \delta\sigma_\theta).
\end{aligned}$$

The second equality performs the change of variable $t = \exp(-[\theta_j + \sigma_\theta \log p_j]/\sigma_\theta)$, and the third equality uses the definition of the Gamma function. Since we are considering cases where $\delta < 0$, this integral will converge.

Marital payoff adjustment term: $\delta = 0$**.** Here, the contribution to social-welfare of a given individual in a given marital pairing is simply given by

$$\begin{aligned}\int_{\theta_j}\int_{\mathbf{Z}_j} Y\big[v_j(\mathbf{Z}_j) + \theta_j\big]\,\mathrm{d}G_j(\mathbf{Z}_j)\,\mathrm{d}H_j(\theta_j) &= \int_{\theta_j} \theta_j\,\mathrm{d}H_j(\theta_j) + \int_{\mathbf{Z}_j} v(\mathbf{Z}_j)\,\mathrm{d}G_j(\mathbf{Z}_j)\\ &= \sigma_\theta\gamma - \sigma_\theta \log p_j + \int_{\mathbf{Z}_j} v(\mathbf{Z}_j)\,\mathrm{d}G_j(\mathbf{Z}_j),\end{aligned}$$

with the second equality using the above result for the distribution of marital shocks within a match and then just applying the well-known result for the expected value of the extreme value distribution with a nonzero location parameter.

APPENDIX G: PROOF OF PROPOSITION 5

In Proposition 4, we present an expression for the contribution to social welfare in alternative marriage market positions. Note that in the $\delta = 0$ case we may write $\mathcal{W}^i_{ij}(T)$ in the familiar log-sum form

$$\mathcal{W}^i_{ij}(T, \boldsymbol{\lambda}^i) = \sigma_\theta\gamma + \sigma_\theta \log\left(\exp\big[U^i_{i0}(T)/\sigma_\theta\big] + \sum_{h=1}^{J} \exp\big[U^i_{ih}(T, \lambda_{ih})/\sigma_\theta\big]\right) = \mathcal{W}^i(T, \boldsymbol{\lambda}^i),$$

which is independent of the match pairing j. Letting $\mathcal{W}^j(T, \boldsymbol{\lambda}^j)$ be defined similarly, it then follows that the overall social-welfare function may be written as

$$\sum_i m_i \cdot \mathcal{W}^i(T, \boldsymbol{\lambda}^i) + \sum_j f_j \cdot \mathcal{W}^j(T, \boldsymbol{\lambda}^j).$$

Thus, relative to the form of the social-welfare function when marriage positions are fixed, we have the *type* specific expected values appearing rather than the *match* specific expected values. Differentiating with respect to τ, we obtain

$$\begin{aligned}\frac{\partial SWF(T)}{\partial\tau} &= \sum_i m_i\left[p^i_{i0}(T, \boldsymbol{\lambda}^i)\frac{\partial U^i_{i0}(T)}{\partial\tau} + \sum_j p^i_{ij}(T, \boldsymbol{\lambda}^i)\frac{\partial U^i_{ij}(T, \lambda_{ij})}{\partial\tau}\right]\\ &\quad + \sum_j m_j\left[p^j_{0j}(T, \boldsymbol{\lambda}^j)\frac{\partial U^j_{0j}(T)}{\partial\tau} + \sum_i p^j_{ij}(T, \boldsymbol{\lambda}^j)\frac{\partial U^j_{ij}(T, \lambda_{ij})}{\partial\tau}\right]\\ &\quad + \sum_i m_i \sum_j p^i_{ij}(T, \boldsymbol{\lambda}^i)\frac{\partial U^i_{ij}(T, \lambda_{ij})}{\partial\lambda_{ij}}\frac{\partial\lambda_{ij}}{\partial\tau}\\ &\quad + \sum_j m_j \sum_i p^j_{ij}(T, \boldsymbol{\lambda}^j)\frac{\partial U^j_{ij}(T, \lambda_{ij})}{\partial\lambda_{ij}}\frac{\partial\lambda_{ij}}{\partial\tau}.\end{aligned}$$

Finally, collecting terms and using equations (3), (5), (6), and (7), completes the proof.

APPENDIX H: ADDITIONAL TABLES AND RESULTS

H.1. *Parameter Estimates*

In Table H.1, we present our model estimates, together with the accompanying standard errors, and the sets of moments that have an important influence on each parameter. We obtain these sets following the approach of Andrews, Gentzkow, and Shapiro (2017). This defines the local sensitivity of the parameter estimates with respect to the moment vector as the $B \times M$ matrix $\mathbf{S_m} = [\mathbf{D_m^\intercal W D_m}]^{-1}\mathbf{D_m^\intercal W}$. Given the scale of our moments are not always comparable, we multiply each element $[\mathbf{S_m}]_{bm}$ by the standard deviation of

TABLE H.1

PARAMETER ESTIMATES[a]

	Estimate	Standard error	Sensitivity moments
Log-wage offers:			
Male, high school and below: mean	2.635	0.003	G2, G3, G4, G5
Male, high school and below: s.d.	0.407	0.002	G3, G4
Male, some college: mean	2.890	0.003	G3, G4, G5
Male, some college: s.d.	0.404	0.002	G3, G4
Male, college: mean	3.400	0.003	G3, G4
Male, college: s.d.	0.528	0.003	G3, G4
Female, high school and below: mean	2.070	0.007	G2, G3, G4, G5
Female, high school and below: s.d.	0.523	0.003	G3, G4
Female, some college: mean	2.484	0.003	G3, G4, G5
Female, some college: s.d.	0.457	0.002	G3, G4
Female, college: mean	2.944	0.002	G3, G4, G5
Female, college: s.d.	0.501	0.002	G3, G4
Preference parameters:			
Leisure scale	0.253	0.028	G2, G5
Home good scale	0.495	0.028	G2, G5
Leisure curvature, σ_ℓ	1.826	0.097	G2, G5
Home good curvature, σ_Q	0.079	0.024	G5
Fixed costs (kids)	99.778	1.431	G2, G5
Marital shock, s.d.	0.123	0.004	G1, G2, G5
State specific error, s.d.	0.284	0.004	G2, G5
Home production technology:			
Male production share	0.168	0.005	G2, G5
Single prod. (no children), high school and below	1.970	0.106	G2, G5
Single prod. (no children), some college	1.946	0.092	G2, G5
Single prod. (no children), college	2.608	0.127	G2, G5
Male prod. (children)	3.319	0.210	G2, G5
Female prod. (children), high school and below	3.686	0.243	G2, G5
Female prod. (children), some college	4.079	0.278	G2, G5
Female prod. (children), college	5.123	0.372	G2, G5
HH prod. (children) female, high school and below	1.926	0.091	G2, G5
HH prod. (children) female, some college	2.715	0.147	G2, G5
HH prod. (children) female, college	2.421	0.126	G2, G5
HH prod. (children) educ. homogamy, high school and below	1.831	0.042	G2, G5
HH prod. (children) educ. homogamy, some college	1.176	0.009	G1, G2, G5
HH prod. (children) educ. homogamy, college	1.694	0.042	G1, G2, G5

[a] All parameters estimated simultaneously using a moment-based estimation procedure as detailed in Section 4 from the main text. See Online Appendix E for a definition of the moment groups, and Footnote 30 for a description of the method used to calculate standard errors. All incomes are expressed in dollars per-week in average 2006 prices.

(a) Optimal marginal tax rates, $\delta = 0$ (b) Empirical marginal tax rates

FIGURE H.1.—Empirical and optimal marginal tax rates. In panel (a), we show the optimal marginal rate schedule calculated with $\delta = 0$. In panel (b), we show the empirical marginal tax rate schedule, which is obtained by calculating the average marginal tax rate in each of the earnings brackets from our tax simulations. Empirical tax rates are calculated using the 2006 ACS and the NBER TAXSIM calculator, and comprise combined federal and state taxes, and both employer and employee FICA rates. All incomes are in thousands of dollars per year, expressed in average 2006 prices. See Footnote 9 for a definition of *low*, *medium*, and *high* spousal earnings.

the mth moment, $\sqrt{[\boldsymbol{\Sigma}]_{mm}}$. For each parameter, we calculate the moment with maximum (absolute) sensitivity, and consider any moment whose sensitivity is at least 20% of the maximal as being important. As we consider sets of moments, we describe a set as being important if at least one moment from that set is important according to this criterion.

H.2. *Comparison With U.S. Tax System*

In Figure H.1, we contrast the unrestricted tax schedules from our optimal tax simulations with the actual 2006 U.S. tax system (corresponding to our estimation sample period). Since the marginal tax rates in the actual system depend on more characteristics than marital status and earnings, we present the average marginal tax rate as faced by the population within a specified range of earnings. To ease comparison with the results from our simulation exercise, we calculate the averages using the tax brackets from our simulation exercise, and similarly define low, medium, and high, spousal earnings levels.[9] Clearly, there are very important differences between the tax schedules. In particular, since the U.S. taxes married couples on their total household income, then with a broadly progressive schedule applied to household earnings, there exists *positive* tax jointness.

H.3. *Social-Welfare Weights*

The redistributive preference of the government is reflected by the parameter δ, which enters the utility transformation presented in equation (13). In Table H.2, we present the underlying average social-welfare weights for alternative values $\delta \in \{-1, 0\}$. Given the maintained symmetry of the tax schedule, we present these welfare weights as a function of the lowest and highest earnings of a couple. For example, at the optimum, the table

[9]Recall that *low* is the arithmetic average of the marginal tax rate for spousal earnings $\{z_2 | z_2 \in \mathcal{Z}, z_2 \leq \$25{,}000\}$. Similarly, *medium* and *high*, respectively, correspond to spousal earnings $\{z_2 | z_2 \in \mathcal{Z}, \$25,000 < z_2 \leq \$85,000\}$ and $\{z_2 | z_2 \in \mathcal{Z}, \$85,000 < z_2 < \$250,000\}$.

TABLE H.2

SOCIAL-WELFARE WEIGHTS UNDER OPTIMAL SYSTEM[a]

		Lowest earnings range ($000s)								
		0–12.5	12.5–25	25–37.5	37.5–60	60–85	85–110	110–150	150–190	190+
$\delta = 0$:	0–12.5	2.735	–	–	–	–	–	–	–	–
		[0.563]								
Highest	12.5–25	1.964	1.461	–	–	–	–	–	–	–
earnings		[5.090]	[4.361]							
range	25–37.5	1.543	1.183	0.985	–	–	–	–	–	–
($000s)		[7.968]	[8.751]	[3.927]						
	37.5–60	1.221	0.974	0.827	0.707	–	–	–	–	–
		[10.134]	[8.579]	[7.104]	[3.114]					
	60–85	0.948	0.786	0.681	0.591	0.501	–	–	–	–
		[7.114]	[5.268]	[4.321]	[3.848]	[1.267]				
	85–110	0.772	0.657	0.578	0.507	0.435	0.381	–	–	–
		[2.280]	[1.551]	[1.255]	[1.118]	[0.770]	[0.120]			
	110–150	0.638	0.555	0.495	0.441	0.383	0.338	0.301	–	–
		[2.501]	[1.568]	[1.207]	[1.046]	[0.711]	[0.222]	[0.103]		
	150–190	0.496	0.442	0.401	0.362	0.320	0.285	0.257	0.223	–
		[1.159]	[0.643]	[0.464]	[0.376]	[0.237]	[0.071]	[0.064]	[0.009]	
	190+	0.388	0.354	0.326	0.298	0.268	0.242	0.220	0.193	0.170
		[0.485]	[0.233]	[0.155]	[0.123]	[0.074]	[0.021]	[0.019]	[0.005]	[0.001]
$\delta = -1$:	0–12.5	2.938	–	–	–	–	–	–	–	–
		[1.780]								
Highest	12.5–25	2.117	1.523	–	–	–	–	–	–	–
earnings		[8.074]	[4.791]							
range	25–37.5	1.574	1.171	0.913	–	–	–	–	–	–
($000s)		[8.967]	[8.659]	[3.704]						
	37.5–60	1.181	0.904	0.719	0.574	–	–	–	–	–
		[9.451]	[7.791]	[6.513]	[2.865]					
	60–85	0.813	0.646	0.525	0.427	0.325	–	–	–	–
		[6.155]	[4.920]	[4.074]	[3.656]	[1.237]				
	85–110	0.588	0.481	0.400	0.331	0.257	0.206	–	–	–
		[1.846]	[1.458]	[1.209]	[1.088]	[0.770]	[0.122]			
	110–150	0.425	0.359	0.306	0.258	0.205	0.167	0.137	–	–
		[2.070]	[1.481]	[1.174]	[1.041]	[0.728]	[0.228]	[0.106]		
	150–190	0.281	0.247	0.218	0.188	0.153	0.127	0.106	0.085	–
		[1.036]	[0.622]	[0.470]	[0.391]	[0.251]	[0.076]	[0.068]	[0.010]	
	190+	0.181	0.165	0.150	0.132	0.110	0.093	0.078	0.064	0.051
		[0.449]	[0.236]	[0.166]	[0.134]	[0.082]	[0.024]	[0.021]	[0.006]	[0.001]

[a]The table presents average social-welfare weights and joint probability mass under the optimal system for alternative δ values. The probability mass is presented in brackets. Earnings are in dollars per week in 2006 prices. Welfare weights are obtained by increasing consumption in the respective joint earnings bracket (with fraction $s_{ij}(\lambda_{ij})$ of this increase in an $\langle i, j\rangle$ match accruing to the female) and calculating a derivative of the social-welfare function; weights are normalized so that the probability-mass-weighted sum under the optimal tax system is equal to unity.

shows that when $\delta = 0$ the government would value a dollar transfer to a single earner couple with annual earnings $37,500–$60,000 approximately 1.9 ($\approx 1.221/0.638$) times as much as would if annual earnings were $110,000–$150,000. When $\delta = -1$, these weights decline much more rapidly, implying a much stronger redistributive motive (in the context of the preceding example, the relative value is now 2.8).

TABLE H.3

MARRIAGE MATCHING FUNCTIONS[a]

			Women		
			High school and below	Some college	College and above
(a) Unrestricted					
		–	0.117	0.090	0.077
Men	High school and below	0.136	0.156	0.061	0.039
	Some college	0.094	0.036	0.106	0.044
	College and above	0.055	0.028	0.059	0.187
(b) Independent					
		–	0.095	0.092	0.105
Men	High school and below	0.119	0.176	0.061	0.036
	Some college	0.099	0.038	0.104	0.038
	College and above	0.073	0.030	0.058	0.168
(c) Income splitting					
		–	0.104	0.102	0.115
Men	High school and below	0.137	0.167	0.055	0.032
	Some college	0.109	0.036	0.099	0.035
	College and above	0.074	0.031	0.059	0.165
(d) Income aggregation					
		–	0.165	0.135	0.136
Men	High school and below	0.198	0.118	0.046	0.029
	Some college	0.139	0.028	0.081	0.032
	College and above	0.099	0.027	0.053	0.150

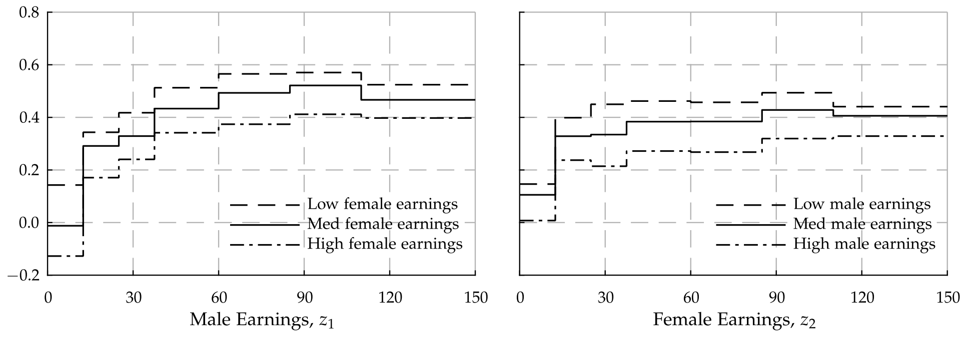
[a]The table shows marriage matching function under alternative tax schedule specifications. *Unrestricted* corresponds to the schedule described in Section 5.1. *Independent*, *Income splitting*, and *Income aggregation*, respectively, refer to independent individual taxation, and joint taxation with income splitting and aggregation; see Section 5.3 for details.

H.4. *Marriage Matching Patterns With Restricted Tax Schedules*

We describe marriage matching patterns when the form of jointness in the tax schedule is restricted. As described in the Section 5.3 from the main text, we consider (i) individual taxation; (ii) joint taxation with income splitting; (iii) joint taxation with income aggregation. Table H.3 presents the marriage matching functions. Relative to the unrestricted schedule, the greatest differences emerge when the tax schedule exhibits a strong non-neutrality with respect to marriage. Under joint taxation with income aggregation, there is a 15-percentage point lower marriage rate, and reduced assortative mating.

H.5. *Gender-Based Taxation*

In Section 5.4, we discussed results when we allowed the tax schedule for both single individuals and married couples to vary by gender. In Figure H.2, we present the marginal tax rate schedules for the husband (when we fix the value of his wife's earnings) and the marginal tax rate schedules for the wife (when we fix the value of her husband's earnings). Except at low earnings, married women typically have lower marginal tax rates than do men. For single individuals, we present the net-income schedule (rather than marginal tax rates) as there are important differences in out-of-work income; see Figure H.3. Here, we show the net-income schedule for single men and single women, together with the optimal schedule when the tax schedule for single individuals is gender neutral. At the optimum,



(a) Husband's marginal tax rates (b) Wife's marginal tax rates

FIGURE H.2.—Optimal tax schedule with gender-based taxation with $\delta = 0$. In panel (a), we show marginal tax rates for married men conditional on alternative values of female earnings. Panel (b) shows marginal tax rates for married women conditional on alternative values of female earnings. All incomes are in thousands of dollars per year, expressed in average 2006 prices. See Footnote 9 for a definition of *low*, *medium*, and *high* spousal earnings levels.

single women have both higher out-of-work income and lower marginal tax rates than do single men.

H.6. *Sensitivity to Node Choice*

Our main simulation results consider the choice of a tax schedule where the number of earnings nodes for each *individual* is exogenously set at $N = 10$ values. Here, we repeat our analysis from Section 5.2 when $\delta = 0$, but with $N = 18$ and $\mathcal{Z} = \{0, 12{,}500, 18{,}750, 25{,}000, 31{,}250, 37{,}500, 48{,}750, 60{,}000, 72{,}500, 85{,}000, 98{,}000, 110{,}000, 135{,}000, 150{,}000, 170{,}000, 190{,}000, 220{,}000, 250{,}000\}$.[10] This permits a considerably more detailed char-

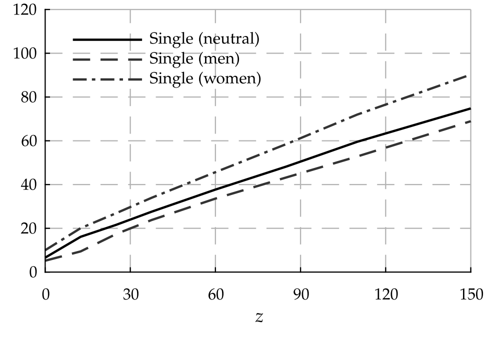
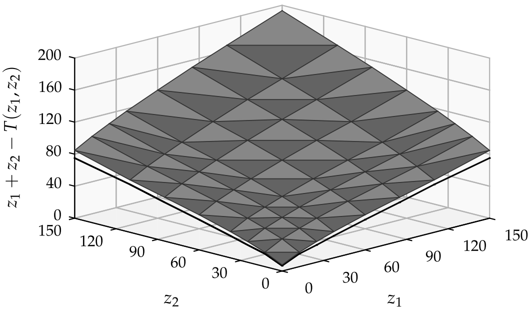
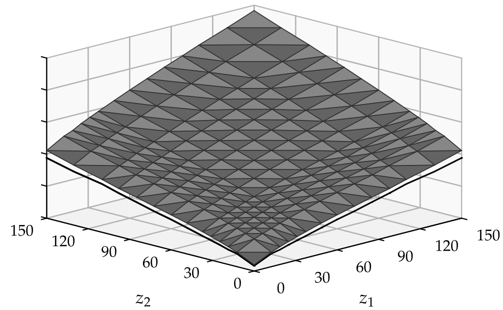


FIGURE H.3.—Net income schedule for single individual's with $\delta = 0$ under alternative tax specifications. The solid line shows the gender neutral net income schedule for single individuals when only taxes for married couples may be gender specific. The two broken lines are obtained when we allow a gendered tax schedule for both married couples and singles. The dashed (dash-dot) line shows the net income schedules for single men (women). All incomes are in thousands of dollars per year, expressed in average 2006 prices.

[10] Increasing the number of earnings nodes in the tax system requires a simultaneous increase in the number of wage integration nodes. If the distance between the earnings nodes becomes too narrow, the joint density in a triangle may become zero, in which case the welfare function and constraints will become locally flat as



(a) Net income function, $N = 10$



(b) Net income function, $N = 18$

FIGURE H.4.—Optimal tax schedule with $\delta = 0$ under alternative node specifications. In panel (a), we show net income as a function of labor earnings for single individuals (solid line) and couples (three-dimensional surface), with $N = 10$ earnings nodes (65 tax parameters). In panel (b), the same net income functions that are obtained with $N = 18$ earnings nodes (189 tax parameters) are shown. All incomes are in thousands of dollars per year, expressed in average 2006 prices.

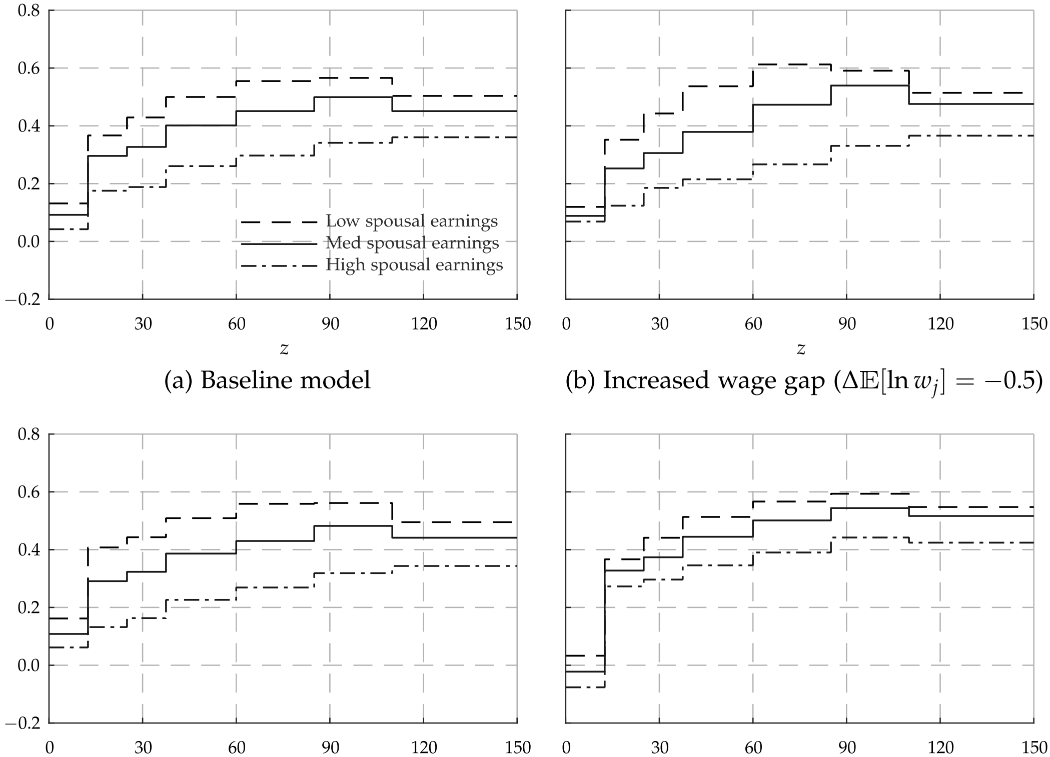
acterization of the tax schedule, now being represented by a total of 189 tax parameters (compared to 65 tax parameters in the $N = 10$ parametrization). In Figure H.4, we present the net-income schedules for singles and couples under this parametrization. For comparison, the original schedule from Figure 5 is reproduced alongside. The structure of taxes, including the implied degree of tax jointness, is clearly seen to be very similar in the two cases, with the surface in the $N = 18$ case essentially an interpolating polygon subdivision of the $N = 10$ case.

H.7. *Perturbation Experiments*

In Section 5.6, we described the design implications of increasing the gender wage gap. Here, we present results from this experiment, and also presents additional perturbation comparative static exercises. In what follows, we define $\Delta T'(z) \equiv T'(z, Low) - T'(z, High)$ to be the difference in average marginal tax rates at earnings z, as the spousal earnings level is changed from *Low* to *High*. In our baseline model from Section 5.2 when $\delta = 0$, we have $\Delta T'(30{,}000) = 24.1\%$ and $\Delta T'(70{,}000) = 25.8\%$. The baseline marginal tax rate schedule is reproduced as Figure H.5(a).

Gender wage gap. We consider an exogenous increase in the gender wage gap by reducing the mean of the offered log wage distribution for women. Intuitively, the more dissimilar are spouses, the greater scope is there to achieve welfare gains from introducing some degree of jointness in the tax system. In Figure H.5(b), we consider a change in mean offered log wages of $\Delta\mathbb{E}[\ln w_j] = -0.5$ for all female types $j = 1, \dots, J$. This perturbation results in increased negative tax jointness, and we now obtain $\Delta T'(30{,}000) = 25.8\%$ and $\Delta T'(70{,}000) = 34.6\%$. There are also important changes in the tax schedule for single individuals (not shown), with marginal tax rates increasing by around 10-percentage points on average. This change partially offsets the impact that changes in the wage distribution have on within household inequality, but still, in the new equilibrium the wife's Pareto weight is everywhere lower. Relative to a system of independent taxation, the unrestricted schedule represents a welfare gain that is equivalent to almost 4% of tax revenue.

elements of $\boldsymbol{\beta}_T$ are varied. This is also why we do not attempt to endogenously determine the node positions together with the tax level parameter vector $\mathbf{t}_{N\times 1}$ and matrix $\mathbf{T}_{N\times N}$.



(a) Baseline model

(b) Increased wage gap ($\Delta\mathbb{E}[\ln w_j] = -0.5$)

(c) Reduced assortative mating ($\varrho = \sigma_\theta$)

(d) Zero home time efficiency ($\zeta = 0$)

FIGURE H.5.—Perturbations of optimal tax schedule with $\delta = 0$. The figure shows marginal tax rates under: (a) the baseline model; and perturbations where: (b) the gender wage gap is increased; (c) assortative mating is reduced through inclusion of an additive utility cost for educationally homogamous marriages; (d) the home time efficiency parameter is reduced to zero everywhere. Marginal tax rates are shown conditional on spousal earnings, z_2. See Footnote 9 for a definition of *low*, *medium*, and *high* spousal earnings.

When the income differences are increased further, there are even larger increases in tax jointness, and even larger welfare gains. For example, when $\Delta\mathbb{E}[\ln w_j] = -1$ we obtain a welfare gain equivalent to around 6.5%.

Assortative mating. Related to the above, we consider how the degree of *assortative mating* influences the design problem. Frankel (2014) considered a simple binary model to analyse taxation design when couples have correlated types. In the context of uncorrelated types (as in KKS (2009)) negative jointness is obtained, although this result is shown to be attenuated when the degree of exogenous assortative mating is increased. In our environment, we *endogenously* change the degree of assortative mating by augmenting the individual utility function to include the additive payoff $\overline{\theta}_{ij}$. In what follows, we set $\overline{\theta}_{ij} = -\varrho \times \mathbb{1}[i = j]$ so that a value $\varrho > 0$ reduces the utility in educationally homogamous marriages but does not have a direct impact on the time allocation problem. In Figure H.5(c), we show the impact that this modification has on the structure of marginal rates when $\delta = 0$. In the illustrations here, we set $\varrho = \sigma_\theta$ so the reduction in expected utility is equal in value to a one-standard-deviation idiosyncratic marital payoff. This reduction in correlation among types *increases* the degree of negative tax jointness, with $\Delta T'(30{,}000) = 28.0\%$ and $\Delta T'(70{,}000) = 29.0\%$. There are only very small changes in the tax schedule for single individuals. Under this parametrization, we obtain larger welfare gains from jointness: individual taxation implies a welfare loss that is equivalent to 2.6% of tax revenue.

Home production. It has long been recognized that home production activities provide an important economic benefit associated with marriage. The design problem faced by the social planner is also different in a model with home production versus a model without home production. First, it affects the degree of inequality both across and within households. Those households with low wages are able to substitute away from market work toward home activities (reducing between household inequality). Yet, the differences in home productivity across households may increase the extent of inequality. Second, as men and women differ in their home productivity, a model without home production has consequences for the economic value both within and outside marriage and therefore within-household inequality. Third, if time spent in home production is not taxed while the time spent in market activities is taxed, then the planner must consider how taxes distort relative factor input prices. Fourth, a model without home production implies different own-wage and cross-wage labor supply elasticities. Fifth, complementarity in the home production technology is a crucial determinant of the degree of assortative mating, and a model without home production would imply very different marital patterns.

Given the wide ranging and complex effects that home production has upon the design problem, we consider a quantitative exploration that involves changing the home time efficiency parameter vector $\boldsymbol{\zeta} = \{\zeta_{i0}, \zeta_{0j}, \zeta_{ij}\}_{i\leq I, j\leq J}$. In Figure H.5(d), we present results setting $\boldsymbol{\zeta} = \mathbf{0}$. Conditional on spousal earnings, the marginal tax rate structure is more progressive, and the degree of tax jointness is decreased. We now have $\Delta T'(30{,}000) = 14.5\%$ and $\Delta T'(70{,}000) = 17.6\%$. The same pattern is true for single individuals. Namely, marginal tax rates decrease at low earnings, but otherwise increase.

REFERENCES

ANDREWS, I., M. GENTZKOW, AND J. M. SHAPIRO (2017): "Measuring the Sensitivity of Parameter Estimates to Estimation Moments," *Quarterly Journal of Economics*, 132 (4), 1553–1592.[11]

BOURGUIGNON, F., M. BROWNING, AND P.-A. CHIAPPORI (2009): "Efficient Intra-Household Allocations and Distribution Factors: Implications and Identification," *Review of Economic Studies*, 76 (2), 503–528.[5,6]

CHAN, M. K. (2013): "A Dynamic Model of Welfare Reform," *Econometrica*, 81 (3), 941–1001.[5]

CHAN, M. K., AND R. A. MOFFITT (2018): "Welfare Reform and the Labor Market" National Bureau of Economic Research Working Paper 24385. [4]

FEENBERG, D., AND E. COUTTS (1993): "An Introduction to the TAXSIM Model," *Journal of Policy Analysis and Management*, 12 (1), 189–194.[4]

FRANKEL, A. (2014): "Taxation of Couples Under Assortative Mating," *American Economic Journal: Economic Policy*, 6 (3), 155–177.[17]

GALICHON, A., S. D. KOMINERS, AND S. WEBER (2014): "An Empirical Framework for Matching With Imperfectly Transferable Utility," Social Science Research Network Working Paper Series. [1,3]

——— (2018): "Costly Concessions: An Empirical Framework for Matching With Imperfectly Transferable Utility," Journal of Political Economy. (Forthcoming). [1,3]

HOTZ, V. J., AND R. A. MILLER (1993): "Conditional Choice Probabilities and the Estimation of Dynamic Models," *Review of Economic Studies*, 60 (3), 497–529.[8]

KLEVEN, H. J., C. T. KREINER, AND E. SAEZ (2009): "The Optimal Income Taxation of Couples," *Econometrica*, 77 (2), 537–560.[17]

ROSS, D. C., L. COX, AND C. MARKS (2007): "Resuming the Path to Health Coverage for Children and Parents: A 50 State Update on Eligibility Rules, Enrollment and Renewal Procedures, and Cost-Sharing Practices in Medicaid and SCHIP in 2006," Kaiser Family Foundation, Menlo Park, California. [4]

ROWE, G., AND M. MURPHY (2006): *Welfare Rules Databook: State TANF Policies as of July 2006*. Washington, DC: The Urban Institute. [5]

WOLKWITZ, K. (2007): "Characteristics of Food Stamp Households: Fiscal Year 2006," US Department of Agriculture. Report No. FSP-07-CHAR. [4]

Co-editor Liran Einav handled this manuscript.

Manuscript received 6 July, 2016; final version accepted 23 July, 2018; available online 10 August, 2018.