Econometrica Supplementary Material

SUPPLEMENT TO "TRADING AND INFORMATION DIFFUSION IN OVER-THE-COUNTER MARKETS"

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APPENDIX C: PROOFS OMITTED FROM THE PRINTED VERSION

Proof of Proposition 3

Case 1: Circulant Networks

WE PROVIDE HERE AN OUTLINE of the proof. Further details of each step are available on request.

Step 1: We show that for circulant networks, each $\bar{z}^i_{ij} > 0$ for any $\rho \in (0, 1)$.

1. First, we show that $\bar{z}^i_{ij} > 0$ in the limit $\rho \to 0$. Clearly, when $\rho = 0$, the equilibrium V is a diagonal matrix as the signals of others are uninformative in this pure private value case. The starting point is to show that for diminishingly small ρ, the off-diagonal elements of V which are corresponding to first neighbors are diminishing at a slower rate than the rest of the off-diagonal elements. In particular, we conjecture and verify that there are constants a_0 and a_1 such that

$$\lim_{\rho\to 0}\left(\frac{V - a_0 I}{\rho}\right) = a_1 A,$$

where I and A are the identity matrix and the adjacency matrix, respectively. For this, we calculate the matrices $\bar{Y}$ and $\bar{Z}$ which correspond to a starting matrix $V^0 = a_0 I + a_1 A$ for a given a_0 and a_1 in problem (A.2), obtain the resulting new matrix $V^1 = (\bar{Y} + \bar{Z}V^0)$, observe that each nonzero element in $\bar{Z}$, $\bar{z}^i_{ij} > 0$ is positive, and verify that there are indeed a_0 and a_1 values for which $\lim_{\rho\to 0}(\frac{V^1 - a_0 I}{\rho}) = a_1 A$.

2. Given that all $\bar{z}^i_{ij}$ are positive in this limit, let us counterfactually assume that there is $\rho \in (0, 1)$ for which at least one $\bar{z}^i_{ij} < 0$. By continuity, there must be a ρ_0 for which all $\bar{z}^i_{ij} \geq 0$ but at least one of them is zero. But this implies that, for these parameters, dealer i finds the expectation of one of her neighbors uninformative. Let $\{i_k\}_{k=1,\dots,m^i}$ be the set of i's neighbors and, without loss of generality, suppose that the index of this neighbor is m^i. The only way this holds is that there is a linear combination of s^i and $\{e^{i_k}\}_{k=1,\dots,(m^i-1)}$ which replicates $e^{i_{m^i}}$, that is, that there is an arbitrary vector $[\lambda_0, \lambda_1 \cdots \lambda_{m^i-1}]$, such that

$$\lambda_0 s^i + \lambda_1 e^{i_1} + \cdots + \lambda_{m^i-1} e^{i_{(m^i-1)}} = e^{i_{m^i}}. \tag{C.1}$$

(a) Note that if the network is circulant, there must be an equilibrium where V is also circulant. To see this, note that problem (A.2) maps circulant networks into circulant net-

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works. Also, given that we prove the properties of $\mathbb{V}^{n\times n}$ vector-by-vector in the proof of Proposition 1, repeating those steps proves the existence of a circulant V fixed point. Furthermore, in this equilibrium, the rows corresponding to the expectation of agents i and j have to have the structure of $v_{i(i+l)} = v_{i(i-l)} = v_{j(j+l)} = v_{j(j-l)}$ for every $l \geq 0$ as long as $n \geq i-l, i+l, j-l, j+l \geq 1$. That is, the weight of each signal in the equilibrium expectation of a given dealer can depend only on whether that signal belongs to a first neighbor, or a second neighbor, etc., of the given dealer. This is coming from the symmetry across dealers in circulant networks and the symmetric informational content of their expectations in this equilibrium.

(b) However, given this symmetric structure of the equilibrium V matrix, there are no v_{ij} and $[\lambda_0, \lambda_1, \dots, \lambda_{m^i-1}]$ values which can solve the equations (C.1) unless all v_{ij} are the same. For instance, let us spell out the implied equation system for the first agent in a $(7,4)$ circulant network with $\bar{k}$ being the second neighbor. If the row of V corresponding to the expectation of the first neighbor of 1 has the structure of $v_1\ v_0\ v_1\ v_2\ v_3\ v_3\ v_2$, then his second neighbor must have the structure of $v_2\ v_1\ v_0\ v_1\ v_2\ v_3\ v_3$. Thus, we need

$$\lambda_0 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + (\lambda_1 + \lambda_2) \begin{pmatrix} v_1 \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_3 \\ v_2 \end{pmatrix} = (1 - \lambda_3) \begin{pmatrix} v_2 \\ v_1 \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_3 \end{pmatrix}$$

to hold for some scalars. It is easy to check that this implies that all $v-s$ are identical. However, it is also easy to check that a V with identical elements cannot be a fixed point.

This is a contradiction which concludes step 1.

Step 2: We show that $\bar{z}^i_{ij} < 1$ for any $\rho \in (0,1)$.

For this, note that by using forward induction on the fixed-point equation $V = \bar{Y} + \bar{Z}V$, we obtain that the equilibrium matrix V must satisfy

$$V = \bar{Y} \lim_{u\to\infty} \sum_0^u (\bar{Z})^u + \lim_{u\to\infty} (\bar{Z})^{u+1} V.$$

As $\rho \in (0,1)$, the diagonal of $\bar{Y}$ must be strictly positive, as s^i must contain residual information on the private value element of θ^i relative to the guesses of others. We know from Proposition 1 that V exists. From the fact that all elements of $\bar{Z}$ are nonnegative and from the fact that the Neumann series $\lim_{u\to\infty} \sum_0^u (\bar{Z})^u$ converges if and only if $\lim_{u\to\infty} (\bar{Z})^{u+1} = 0$ (see Meyer (2000, p. 618)), we must have that indeed $\lim_{u\to\infty} (\bar{Z})^{u+1} = 0$. As $\bar{Z}$ must be symmetric for a circulant network, and all elements are nonnegative, if any elements were larger than 1, then there were some elements of $\lim_{u\to\infty} (\bar{Z})^{u+1}$ which would not diminish (as the elements $(\bar{z}^i_{ij})^{u+1}$ will be a component in some elements of the matrix $(\bar{Z})^{u+1}$ for any i and j).

Step 3: Now, we search for equilibria such that beliefs are symmetric, that is,

$$z^i_{ij} = z^j_{ji}$$

for any pair ij that has a link in network g.

The system (21) becomes

$$\frac{y^i}{\left(1-\sum_{k\in g^i} z^i_{ik}\frac{2-z^i_{ik}}{4-\left(z^i_{ik}\right)^2}\right)}=\bar{y}^i,$$

$$z^i_{ij}\frac{\dfrac{2-z_{ij}}{4-z^2_{ij}}}{\left(1-\sum_{k\in g^i} z^i_{ik}\frac{2-z^i_{ik}}{4-\left(z^i_{ik}\right)^2}\right)}=\bar{z}^i_{ij},$$

for any $i\in\{1,2,\dots,n\}$. Working out the equation for z^i_{ij}, we obtain

$$\frac{z^i_{ij}}{2+z^i_{ij}}=\bar{z}^i_{ij}\left(1-\sum_{k\in g^i}\frac{z^i_{ik}}{2+z^i_{ik}}\right),$$

and summing up for all $j\in g^i$,

$$\sum_{j\in g^i}\frac{z^i_{ij}}{2+z^i_{ij}}=\sum_{j\in g^i}\bar{z}^i_{ij}\left(1-\sum_{k\in g^i}\frac{z^i_{ik}}{2+z^i_{ik}}\right).$$

Denote

$$S^i\equiv\sum_{k\in g^i}\frac{z^i_{ik}}{2+z^i_{ik}}.$$

Substituting above and summing again for $j\in g^i$,

$$S^i\left(1+\sum_{j\in g^i}\bar{z}^i_{ij}\right)=\sum_{j\in g^i}\bar{z}^i_{ij}$$

or

$$S^i=\frac{\sum_{j\in g^i}\bar{z}^i_{ij}}{\left(1+\sum_{j\in g^i}\bar{z}^i_{ij}\right)}.$$

We can now obtain

$$z^i_{ij}=\frac{2\bar{z}^i_{ij}\left(1-S^i\right)}{1-\bar{z}^i_{ij}\left(1-S^i\right)} \tag{C.2}$$

and

$$y^i=\bar{y}^i\left(1-S^i\right).$$

Finally, the following logic shows that $z^i_{ij} \leq 2$. As $\bar{z}^i_{ij} < 1$, $2\bar{z}^i_{ij} < (1 + \sum_{j \in g^i} \bar{z}^i_{ij})$ implying that $2\bar{z}^i_{ij}(1 - S^i) < 1$ or $2\bar{z}^i_{ij}(1 - S^i) < 2(1 - \bar{z}^i_{ij}(1 - S^i))$, which gives the result by (C.2).

Case 2: Star Networks

We give the closed-form solutions for the star network in Appendix B.2. One can check by straightforward algebra that the resulting z^i_{ij} are indeed in the $[0, 2]$ interval.

Proof of Proposition 7

Observe that by symmetry across periphery dealers, $V = (I - \bar{Z})^{-1}\bar{Y}$, for star network has the elements of

$$\begin{aligned} v_{11} &= \bar{y}_C \frac{1}{1 - (n-1)\bar{z}_C \bar{z}_P}, \\ v_{i1} &= \bar{y}_C \frac{\bar{z}_P}{1 - (n-1)\bar{z}_C \bar{z}_P}, \\ v_{ii} &= \bar{y}_P \frac{1 - (n-2)\bar{z}_C \bar{z}_P}{1 - (n-1)\bar{z}_C \bar{z}_P}, \\ v_{1i} &= \bar{y}_P \frac{\bar{z}_C}{1 - (n-1)\bar{z}_C \bar{z}_P}, \\ v_{ij} &= \bar{y}_P \frac{\bar{z}_C \bar{z}_P}{1 - (n-1)\bar{z}_C \bar{z}_P}, \end{aligned}$$

where $\bar{y}_C$, $\bar{y}_P$ are the weights on the private signal and $\bar{z}_C$, $\bar{z}_P$ are the weights on the others' guesses in the central and periphery agents' guessing function, respectively. As maximizing $E(-(\theta - e^i)^2)$ is equivalent with maximizing

$$2\,\mathrm{tr}(V \Sigma_{\theta \mathbf{s}}) - \mathrm{tr}\big(V \Sigma V^\top\big),$$

where $\Sigma_{ii} = 1 + \sigma^2$, $\Sigma_{ij} = \rho$, $[\Sigma_{\theta s}]_{ii} = 1$, $[\Sigma_{\theta s}]_{ij} = \rho$, we calculate the expressions for the components of this objective function:

$$\begin{aligned} \big[V \Sigma V^\top\big]_{11} &= \big(1 + \sigma^2\big) v_{11}^2 + \big(1 + \sigma^2\big)(n-1) v_{1i}^2 + \rho 2(n-1) v_{1i} v_{11} + \rho(n-1)(n-2) v_{1i}^2 \\ &= \frac{\big((1 + \sigma^2)\bar{y}_C^2 + \big((1 + \sigma^2) + \rho(n-2)\big)(n-1)\bar{y}_P^2 \bar{z}_C^2 + \rho 2(n-1)\bar{y}_C \bar{y}_P \bar{z}_C\big)}{\big(1 - (n-1)\bar{z}_C \bar{z}_P\big)^2} \end{aligned}$$

and

$$\begin{aligned} &\big[V \Sigma V^\top\big]_{ii} \\ &\quad = \big((n-2)(n-3) v_{ij}^2 + (n-2) 2(v_{i,1} + v_{i,i}) v_{ij} + 2 v_{i,1} v_{i,i}\big)\rho \\ &\qquad + \big(\sigma^2 + 1\big)\big(v_{ii}^2 + (n-2) v_{ij}^2 + v_{i,1}^2\big) \\ &\quad = \frac{\Big(\bar{y}_C + \bar{z}_C \bar{y}_P (n-2)\Big(1 - \frac{(n-1)}{2}\bar{z}_C \bar{z}_P\Big)\Big) 2\bar{z}_P \bar{y}_P \rho + \big(\sigma^2 + 1\big)\big(\big(1 - (n-2)\bar{z}_C \bar{z}_P\big)^2 \bar{y}_P^2 + (n-2)\bar{y}_P^2 \bar{z}_P^2 \bar{z}_C^2 + \bar{y}_C^2 \bar{z}_P^2\big)}{\big(1 - (n-1)\bar{z}_C \bar{z}_P\big)^2} \end{aligned}$$

and

$$\operatorname{tr}(V\Sigma V^{\top}) = [V\Sigma V^{\top}]_{11} + (n-1)[V\Sigma V^{\top}]_{ii}.$$

Also,

$$\begin{aligned}\operatorname{tr}(V\Sigma_{\theta s}) &= v_{11} + (n-1)v_{ii} + \rho(n-1)(v_{1i}+v_{i1}) + \rho(n-1)(n-2)v_{ij} \\ &= \frac{\bar{y}_C + \rho(n-1)\bar{y}_P\bar{z}_C}{(1-(n-1)\bar{z}_C\bar{z}_P)} + (n-1)\frac{\bar{y}_P(1-(n-2)\bar{z}_C\bar{z}_P(1-\rho)) + \rho\bar{y}_C\bar{z}_P}{(1-(n-1)\bar{z}_C\bar{z}_P)}.\end{aligned}$$

This implies that

$$\lim_{\delta\to 0}\frac{\partial U(\bar{z}_C+\delta, \bar{z}_P+\delta, \bar{y}_C-\delta, \bar{y}_P-\delta)}{\partial\delta} = -\frac{f(\bar{z}_P, \bar{z}_C, \bar{y}_C, \bar{y}_P; n, \rho, \sigma)}{(-1+(n-1)\bar{z}_C\bar{z}_P)^3},$$

where $f(\cdot)$ is a polynomial. Then we substitute in the analytical expressions for the decentralized optimum $\bar{z}_C^*$, $\bar{z}_P^*$, $\bar{y}_C^*$, $\bar{y}_P^*$ given in closed form in Appendix B.2 and rewrite $\lim_{\delta\to 0}\frac{\partial U(\bar{z}_C^*+\delta, \bar{z}_P^*+\delta, \bar{y}_C^*-\delta, \bar{y}_P^*-\delta)}{\partial\delta}$ as a fraction. Both the numerator and the denominator are polynomials of σ^2 of order 9. A careful inspection reveals that each of the coefficients is positive for any $\rho \in (0,1)$ and $n \geq 3$. (Details on the resulting expressions in these calculations are available from the authors on request.)

Proof of Proposition 8

The first part comes by the observation that as $z_V \to 1 - \frac{1}{n-1}$, $t_V \to \infty$, while t_{CN} is finite for these parameters. The second part comes from taking the limit $\rho \to 1$ of the ratio of the corresponding closed-form expressions we report in Appendices B.1 and B.3. In particular,

$$\begin{aligned}\lim_{\rho\to 1}\frac{t_{CN}}{t_V} &= \frac{2n-3}{n-1} > 1, \\ \lim_{\rho\to 1}\frac{-\frac{\beta_{CN}}{2}E(p_{ij}^2)}{-\frac{\beta_V}{2}E(p_V^2)} &= \left(\frac{2n-3}{n-1}\right)^2 > 1, \\ \lim_{\rho\to 1}\frac{\frac{n(n-1)}{2}E(q_{CN}(\theta^i - p_{CN}))}{E(q_V(\theta^i - p_V))} &= \frac{2n-3}{(n-1)^2} < 1, \\ \lim_{\rho\to 1}\frac{\frac{n(n-1)}{2}E(q_{CN}\theta^i) + \frac{\beta_{CN}}{2}E(p_{ij}^2)}{E(q_V\theta^i) + \frac{\beta_V}{2}E(p_V^2)} &= \frac{3-8n+4n^2}{3(n-1)^2} > 1.\end{aligned}$$

Proof of Proposition 9

The statements come with simple algebra from the closed-form expressions we report in Appendix B.2.

The first part comes by the observation that as $z_V \to 1 - \frac{1}{n-1}$, $t_V \to \infty$, while t_C and t_P are finite for these parameters. The second part comes from taking the limit $\rho \to 1$ of the ratio of the corresponding closed-form expressions we report in Appendices B.3 and B.2. In particular,

$$\lim_{\rho\to 1} \frac{t_V}{t_C} = (n-1)\frac{z_C + z_P - z_C z_P}{(2-z_P)\big((n-1)z_V - (n-2)\big)} = \infty,$$

$$\lim_{\rho\to 1} \frac{t_V}{t_P} = (n-1)\frac{z_C + z_P - z_C z_P}{(2-z_C)\big((n-1)z_V - (n-2)\big)} = \frac{n-1}{n} < 1.$$

Proof of Proposition 10

Formally, we define the price-discovery game as follows. In round 0, each dealer i chooses a bidding strategy $B^i_{ij}(s^i; \{\pi^j_{ij,\tau}\}_{j\in g^i})$ that describes the counter-offers that traders at desk i should make in round $\tau+1$, conditional on the bids they received in round $\tau \geq 0$, such that

$$B^i_{ij}\big(s^i; \{\pi^j_{ij,\tau}\}_{j\in g^i}\big) = \pi^i_{ij,\tau+1}, \tag{C.3}$$

for each $j \in g^i$. If there exists a price and quantity vector $\{\bar{p}^i_{ij}, \bar{q}^j_{ij}\}_{ij\in g}$ with

$$\bar{p}^i_{ij} = \bar{p}^j_{ij},$$

$$\bar{q}^i_{ij} + \bar{q}^j_{ij} + \beta_{ij}\bar{p}^i_{ij} = 0,$$

and

$$\lim_{\tau\to\infty} \pi^i_{ij,\tau} = \big(\bar{p}^i_{ij}, \bar{q}^i_{ij}\big),$$

for every $ij \in g$ and for any random starting vector $\{\pi^i_{ij,0}\}_{ij\in g}$, then trade takes place.

The payoff for a dealer i is the expected profit $E[\sum_{j\in g^i} \bar{q}^i_{ij}(\theta^i - \bar{p}^i_{ij})]$, provided $\{\bar{p}^i_{ij}, \bar{q}^j_{ij}\}_{ij\in g}$ exist, and minus infinity otherwise. Thus, taking each other dealer's bidding strategy as given, dealer i solves

$$\max_{\{B^i_{ij}(s^i;\{\pi^j_{ij,\tau}\}_{j\in g^i})\}_{j\in g^i}} E\bigg[\sum_{j\in g^i} \bar{q}^i_{ij}\big(\theta^i - \bar{p}^i_{ij}\big)\Big| s^i\bigg].$$

Starting from an equilibrium in the OTC game, we construct a bidding strategy for dealer i as follows. When a trader at desk i receives a bid $\pi^j_{ij,\tau} = \{p^j_{ij,\tau}, q^j_{ij,\tau}\}$ from each of his counterparties $j \in g^i$, she transforms $p^j_{ij,\tau}$ to

$$e^j_\tau = \frac{p^j_{ij,\tau}\big(t^i_{ij} + t^j_{ij} - \beta_{ij}\big) - t^i_{ij}e^i_{\tau-1}}{t^j_{ij}}$$

for each $j \in g^i$. Then, she updates her expectation about the asset value to be

$$e^i_{\tau+1} = \bar{y}^i s^i + \bar{\mathbf{z}}_{g^i}\mathbf{e}_{g^i,\tau}. \tag{C.4}$$

Finally, she constructs the counter-offer $\pi^i_{ij,\tau+1}$ with elements

$$p^i_{ij,\tau+1} = \frac{t^i_{ij} e^i_{\tau+1} + t^j_{ij} e^j_{\tau}}{t^i_{ij} + t^j_{ij} - \beta_{ij}},$$

$$q^i_{ij,\tau+1} = t^i_{ij}\left(e^i_{\tau+1} - p^i_{ij,\tau+1}\right).$$

First, we show that if bidding functions are defined as above, the OTC price-discovery process converges to the equilibrium prices and quantities in the OTC game. To see this, we write (C.4) in matrix form as

$$\mathbf{e}_{\tau+1} = \bar{Y}\mathbf{s} + \bar{Z}\mathbf{e}_{\tau},$$

where $\mathbf{e}_{\tau+1} = (e^i_{\tau+1})_{i=1,\dots,n}$ and $\bar{Y}$, $\bar{Z}$ are constructed from $\bar{y}^i$ and $\bar{\mathbf{z}}_{g^i}$, respectively. Note that, starting from any random vector $\mathbf{e}_0$, we will have

$$\mathbf{e}_{\tau+1} = \left(I + \bar{Z} + \cdots + (\bar{Z})^{\tau}\right)\bar{Y}\mathbf{s} + (\bar{Z})^{\tau+1}\mathbf{e}_0.$$

In step 2 of the proof of Proposition 3, we show that the fact that all elements of $\bar{Z}$ are positive together with the existence of equilibrium in the conditional guessing game imply that $\lim_{u\to\infty}(\bar{Z})^{u+1} = 0$, which in turn implies that $(I - \bar{Z})$ is nonsingular (see Meyer (2000, p. 618)), $(I - \bar{Z})^{-1} \geq 0$, and

$$(I - \bar{Z})^{-1} = \sum_{\tau=1}^{\infty}(\bar{Z})^{\tau}$$

(see Meyer (2000, pp. 620 and 618)).

Thus, we have that

$$\lim_{\tau\to\infty}\mathbf{e}_{\tau+1} = (I - \bar{Z})^{-1}\bar{Y}\mathbf{s},$$

or the equilibrium expectations in the OTC game. But then, by definition, $\{\bar{p}^i_{ij}, \bar{q}^i_{ij}\}_{ij\in g}$ exist and coincide with the equilibrium of the OTC game.

The last step is to show that dealer i would not want to change her bidding strategy unilaterally. Note that for any such deviation to be meaningful, it has to imply alternative limit price and quantity vectors. If there is no convergence, dealer i receives a payoff of minus infinity. However, by construction, if a modified bidding strategy converges to different price and quantity vectors, then these vectors are also fixed points of generalized demand curves in the OTC game. However, the other dealers' bidding strategies are constructed based on their equilibrium demand functions in the OTC game. This implies that if dealer i wants to deviate from the equilibrium bidding strategies in the price-discovery game, he wants to deviate from his generalized demand curve in the original OTC game as well. But this is a contradiction.

APPENDIX D: DETAILS ON CALIBRATED EXAMPLE

D.1. *Baseline*

We have calibrated our model by finding the equilibrium and calculating the matched moments for a grid of parameter values. We have targeted values in Table D.I. We kept refining the grid to the point where the match was sufficiently close. The code and a detailed explanation are available on Peter Kondor's website.

TABLE D.I

MATCHED AND IMPLIED MOMENTS IN THE MODEL AND IN THE DATA[a]

Moments	Model at ($\rho = 0.014$, $\sigma = 0.1584$, $(-\beta)\sigma_\theta^2 = 7.3835$)	Data
average spread (%)	0.742	0.742
relative price dispersion, core (%)	371.3	371.3
total volume ($M)	277,676	277,676

[a]The data moments are from Hollifield, Neklyudov, and Spatt (2017, Tables 2, 3, and 10). We match 71% of total volume as this is the fraction of fully identified chains. For model, the average customer spread for a given dealer is $\frac{1}{|g^i|}\Sigma_{j\in g^i}\frac{2\beta}{t^i_{ij}}$, which we average over the whole sample. Relative price dispersion for core dealers is ratio of expected price dispersion to the absolute mean of prices in those transactions where one of the counterparties is a core dealer.

D.2. *Market Distress*

To model the effect of market distress, we drop the most connected node from our network. In Figure 3, this is dealer 1. As dealers 22–23, 25–29, 43, and 69 are connected to the rest of the market only through dealer 1, we drop those dealers, too. We have emphasized in the main text that, under our calibrated parameters, the first-order determinant of price impact, intermediation volume, and expected profit is dealer centrality. Consistently with this observation, the main channel through which our treatment affects the new equilibrium is that all the dealers who were connected to dealer 1 now have fewer trading partners. Therefore, these dealers face larger price impact, trade less, and earn smaller expected profit. Averaging over dealers implies larger average price impact and less volume. Price dispersion also increases as the disruption of information flows leads to less convergence in posterior expectations which is the main determinant of price dispersion by expression (14).

D.3. *Robustness*

We searched over the parameter range used in Figure 4 and observed that the connection between degree centrality and expected profit, gross volume, intermediation, information precision, and average price impact are qualitatively the same as illustrated in Figure 5. However, especially for larger correlation across dealers' values, ρ, degree centrality does not suppress all the other network characteristics to the same extent. Figure D.1, a variant of Figure 5 with $\rho = 0.8$, illustrates this observation.

APPENDIX E: TRADING IN SEGMENTED MARKETS

E.1. *General Setup*

Our framework can provide insights about trade in segmented markets as well. Markets are segmented when investors, such as hedge funds and asset management firms, trade in some markets but not in others. Although segmented, markets can be connected, in the sense agents are able to trade in multiple venues at the same time. To study the implications in segmented markets, we extend our model in the following way.

We consider an economy in which there are N trading posts connected in a network g. At each trading post, I, there exist n^I risk-neutral dealers. The entire set of dealers is denoted $\mathcal{N} = \bigcup_{I=N}^{N} I$. Each dealer $i \in I$ can trade with other dealers in his own trading post and with dealers at any trading post J that is connected with the trading post I by

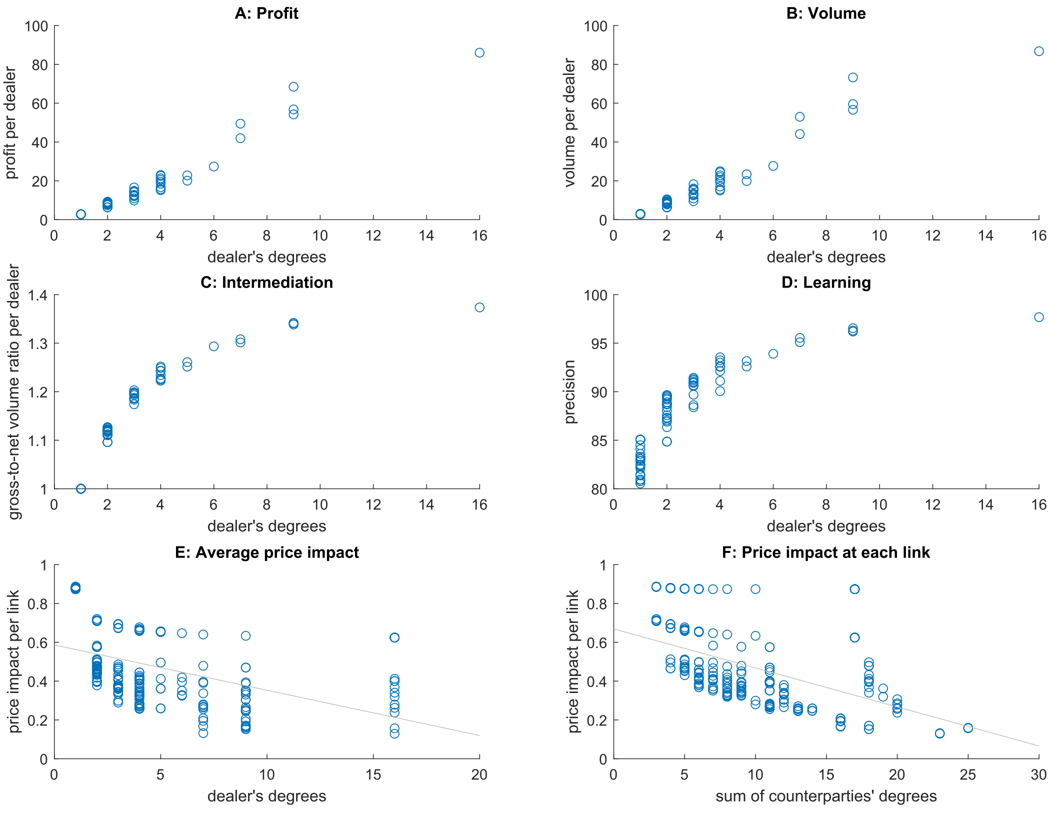

FIGURE D.1.—Panels A–D show each dealer's expected profit, gross volume, intermediation, and posterior information precision (as percentage of precision under fully revealing prices) against the number of the dealer's trading partners. Panels E and F show the price impact a dealer faces at a given link against the number of her trading partners, and against the sum of the trading partners of the two counterparties at the given link, respectively. Parameter values are $\rho = 0.8$, $\sigma = 0.1584$, $\beta = -1$, $\sigma_\theta^2 = 7.3835$. We added a least-squares line to Panels E and F.

a link IJ. Essentially, the link IJ represents a market in which dealers at trading posts I and J can trade with each other. However, they have access to trade in other markets at the same time. Let g^I denote the set of trading posts that are linked with I in the network g, and $m^I \equiv |g^I|$ represent the number of I's links.

As before, dealers trade a risky asset in zero net supply, and all trades take place at the same time. Each dealer is uncertain about the value of the asset. In particular, a dealer's value for the asset is given by θ^i, which is a random variable normally distributed with mean 0 and variance σ_θ^2. Moreover, we consider that values are interdependent across all dealers. In particular, $\mathcal{V}(\theta^i, \theta^j) = \rho\sigma_\theta^2$ for any two agents $i, j \in \mathcal{N}$. Each dealer receives a private signal, $s^i = \theta^i + \varepsilon^i$, where $\varepsilon^i \sim$ i.i.d. $N(0, \sigma_\varepsilon^2)$ and $\mathcal{V}(\theta^j, \varepsilon^i) = 0$, for all i and j.

A dealer $i \in I$ seeks to maximize her final wealth

$$\sum_{J \in g^I} q_{IJ}^i(\theta^i - p_{IJ}),$$

where q_{IJ}^i is the quantity traded by dealer i in market IJ, at a price p_{IJ}. Similarly to the OTC model, the trading strategy of the dealer i with signal s^i is a generalized demand function $\mathbf{Q}^i : R^{m^i} \to R^{m^i}$ which maps the vector of prices, $\mathbf{p}_{g^I} = (p_{IJ})_{J\in g^I}$, that prevail in the markets in which dealer i participates in network g into a vector of quantities she wishes to trade

$$\mathbf{Q}^i(s^i; \mathbf{p}_{g^I}) = \big(Q^i_{IJ}(s^i; \mathbf{p}_{g^I})\big)_{J\in g^I},$$

where $Q^i_{IJ}(s^i; \mathbf{p}_{g^I})$ represents her demand function in market IJ.

Apart from trading with each other, dealers also serve a price-sensitive customer base. In particular, we assume that for each market IJ, the customer base generates a downward-sloping demand

$$D_{IJ}(p_{IJ}) = \beta_{IJ} p_{IJ}, \tag{E.1}$$

with an arbitrary constant $\beta_{IJ} < 0$. The exogenous demand (E.1) ensures the existence of the equilibrium when agents are risk-neutral, and facilitates comparisons with the OTC model.

The expected payoff for dealer $i \in I$ corresponding to the strategy profile $\{\mathbf{Q}^i(s^i; \mathbf{p}_{g^I})\}_{i\in\mathcal{N}}$ is

$$E\left[\sum_{J\in g^I} Q^i_{IJ}(s^i; \mathbf{p}_{g^I})(\theta^i - p_{IJ})\Big| s^i\right],$$

where p_{IJ} are the prices for which all markets clear. That is, prices satisfy

$$\sum_{i\in I} Q^i_{IJ}(s^i; \mathbf{p}_{g^I}) + \sum_{j\in J} Q^j_{IJ}(s^j; \mathbf{p}_{g^J}) + \beta_{IJ} p_{IJ} = 0, \quad \forall IJ \in g. \tag{E.2}$$

E.2. *Equilibrium Concept*

As in the OTC game, we use the concept of Bayesian Nash equilibrium. For completeness, we reproduce it below.

DEFINITION 3: A Linear Bayesian Nash equilibrium of the segmented market game is a vector of linear generalized demand functions $\{\mathbf{Q}^i(s^i; \mathbf{p}_{g^I})\}_{i\in\mathcal{N}}$ such that $\mathbf{Q}^i(s^i; \mathbf{p}_{g^I})$ solves the problem

$$\max_{(Q^i_{IJ})_{J\in g^I}} E\left\{\left[\sum_{J\in g^I} Q^i_{IJ}(s^i; \mathbf{p}_{g^I})(\theta^i - p_{IJ})\right]\Big| s^i\right\}, \tag{E.3}$$

for each dealer i, where the prices p_{IJ} satisfy (E.2).

A dealer i chooses a demand function in each market IJ, in order to maximize her expected profits, given her information, s^i, and given the demand functions chosen by the other dealers.

E.3. *The Equilibrium*

In this section, we outline the steps for deriving the equilibrium in the segmented market game for any network structure. First, we derive the equilibrium strategies as a function of posterior beliefs. Second, we construct posterior beliefs that are consistent with dealers' optimal choices. In the OTC game, we used the conditional guessing game as an intermediate step in constructing beliefs. Here, we employ the same line of reasoning, although we do not explicitly introduce the conditional guessing game structure that would correspond to the segmented market game.

E.3.1. *Derivation of Demand Functions*

We conjecture an equilibrium in demand functions, where the demand function of dealer i in market IJ is given by

$$Q^i_{IJ}(s^i; \mathbf{p}_{g^I}) = t^I_{IJ}\left(y^I_{IJ}s^i + \sum_{K\in g^I} z^I_{IJ,IK}p_{IK} - p_{IJ}\right) \tag{E.4}$$

for any $i \in I$ and J. As evident in the notation, we consider that all dealers at trading post I are symmetric in their trading strategy, and weigh in same way the signal they receive and the prices they trade at. Nevertheless, they end up trading different quantities, as they have different realizations of the signal.

We solve the optimization problem (E.3) pointwise. That is, for each realization of the vector of signals, $\mathbf{s}$, we solve for the optimal quantity q^i_{IJ} that each dealer $i \in I$ demands in market IJ. Given the conjecture (E.4) and the market clearing conditions (E.2), the residual inverse demand function of dealer i in market IJ is

$$p_{IJ} = -\frac{t^I_{IJ}y^I_{IJ}\sum_{k\in I,k\neq i} s^k + t^J_{IJ}y^J_{IJ}\sum_{k\in J} s_k + (N_I-1)\sum_{L\in g^I,L\neq J} t^I_{IJ}z^I_{IJ,IL}p_{IL} + N_J\sum_{L\in g^J,L\neq I} t^J_{IJ}z^J_{IJ,JL}p_{JL} + q^i_{IJ}}{(N_I-1)t^I_{IJ}(z^I_{IJ,IJ}-1) + N_J t^J_{IJ}(z^J_{IJ,IJ}-1) + \beta_{IJ}}. \tag{E.5}$$

Denote

$$I^J_i \equiv -\frac{t^I_{IJ}y^I_{IJ}\sum_{k\in I,k\neq i} s^k + t^J_{IJ}y^J_{IJ}\sum_{k\in J} s_k + (N_I-1)\sum_{L\in g^I,L\neq J} t^I_{IJ}z^I_{IJ,IL}p_{IL} + N_J\sum_{L\in g^J,L\neq I} t^J_{IJ}z^J_{IJ,JL}p_{JL}}{(N_I-1)t^I_{IJ}(z^I_{IJ,IJ}-1) + N_J t^J_{IJ}(z^J_{IJ,IJ}-1) + \beta_{IJ}} \tag{E.6}$$

and rewrite (E.5) as

$$p_{IJ} = I^J_i - \frac{1}{(N_I-1)t^I_{IJ}(z^I_{IJ,IJ}-1) + N_J t^J_{IJ}(z^J_{IJ,IJ}-1) + \beta_{IJ}} q^i_{IJ}. \tag{E.7}$$

The uncertainty that dealer i faces about the signals of others is reflected in the random intercept of the residual inverse demand, I^J_i, while her capacity to affect the price is reflected in the slope $-1/((N_I-1)t^I_{IJ}(z^I_{IJ,IJ}-1) + N_J t^J_{IJ}(z^J_{IJ,IJ}-1) + \beta_{IJ})$. In the segmented markets game, however, the random intercept I^J_i reflects not only the signals of the dealers at the trading post J, but also the signals of the other dealers at the trading post I.

Then, solving the optimization problem (E.3) is equivalent to finding the vector of quantities $\mathbf{q}^i = \mathbf{Q}^i(s^i; \mathbf{p}_{g^I})$ that solve

$$\max_{(q^i_{IJ})_{j\in g^I}} \sum_{J\in g^I} q^i_{IJ}\left(E(\theta^i|s^i, \mathbf{p}_{g^I}) - \left(I^J_i - \frac{q^i_{IJ}}{(N_I-1)t^I_{IJ}(z^I_{IJ,IJ}-1) + N_J t^J_{IJ}(z^J_{IJ,IJ}-1) + \beta_{IJ}}\right)\right).$$

From the first-order conditions, we derive the quantities q^i_{IJ} that dealer $i \in I$ trades in each market IJ, for each realization of $\mathbf{s}$, as

$$2\frac{1}{(N_I-1)t^I_{IJ}(z^I_{IJ,IJ}-1)+N_J t^J_{IJ}(z^J_{IJ,IJ}-1)+\beta_{IJ}} q^i_{IJ} = I^J_i - E(\theta^i|s^i, \mathbf{p}_{g^I}).$$

This implies that the optimal demand function

$$Q^i_{IJ}(s^i;\mathbf{p}_{g^i}) = -((N_I-1)t^I_{IJ}(z^I_{IJ,IJ}-1)+N_J t^J_{IJ}(z^J_{IJ,IJ}-1)+\beta_{IJ})(E(\theta^i|s^i,\mathbf{p}_{g^I})-p_{IJ}) \quad \text{(E.8)}$$

for each dealer i in market IJ.

Further, given our conjecture (E.4), equating coefficients in equation (E.8) implies that

$$E(\theta^i|s^i, \mathbf{p}_{g^I}s) = y^I_{IJ}s^i + \sum_{K\in g^I} z^I_{IJ,IK}p_{IK}.$$

However, the projection theorem implies that the belief of each dealer i can be described as a unique linear combination of her signal and the prices she observes. Thus, it must be that $y^I_{IJ} = y^I$, and $z^I_{IJ,JK} = z^I_{IK}$ for all I, J, and K. In other words, the posterior belief of a dealer i is given by

$$E(\theta^i|s^i, \mathbf{p}_{g^I}) = y^I s^i + \mathbf{z}_{g^I}\mathbf{p}_{g^I}, \quad \text{(E.9)}$$

where $\mathbf{z}_{g^I} = (z^I_{IJ})_{J\in g^I}$ is a *row* vector of size m^i. Then, we obtain that the trading intensity of dealer at trading post I satisfies

$$t^I_{IJ} = (N_I-1)t^I_{IJ}(1-z^I_{IJ}) + N_J t^J_{IJ}(1-z^J_{IJ}) - \beta_{IJ}. \quad \text{(E.10)}$$

If we further substitute this into the market clearing conditions (E.2), we obtain the price in market IJ as follows:

$$p_{IJ} = \frac{t^I_{IJ}\left(\sum_{i\in I} E(\theta^i|s^i,\mathbf{p}_{g^I})\right) + t^J_{IJ}\left(\sum_{j\in J} E(\theta^j|s^j,\mathbf{p}_{g^J})\right)}{N_I t^I_{IJ} + N_J t^J_{IJ} - \beta_{IJ}}. \quad \text{(E.11)}$$

From (E.10) and the analogous equation for t^J_{IJ}, it is straightforward to derive the trading intensity that dealers at trading post I and J have. This implies that we can obtain the price in each market IJ as

$$p_{IJ} = w^I_{IJ}\left(\sum_{i\in I} E(\theta^i|s^i,\mathbf{p}_{g^I})\right) + w^J_{IJ}\left(\sum_{j\in J} E(\theta^j|s^j,\mathbf{p}_{g^J})\right), \quad \text{(E.12)}$$

where

$$w^I_{IJ} \equiv \frac{z^J_{IJ}-2}{(N_J+N_I-1)z^I_{IJ}z^J_{IJ} - 2(N_I-1)z^I_{IJ} - 2(N_J-1)z^J_{IJ} - 4}.$$

This expression is useful to relate the belief of a dealer $i \in I$ to the beliefs of other dealers at the same trading post, and at trading posts that are connected to I.

E.3.2. *Derivation of Beliefs*

We follow the same solution method that we developed in Section 3.1. As before, the key idea is to reduce the dimensionality of the problem and use our conjecture about demand functions to derive a fixed point in beliefs, instead of the fixed point (E.8).

In the OTC game, we constructed each dealer's equilibrium belief as a linear combination of the beliefs of her neighbors in the network. For this, we introduced the conditional guessing game. The conditional guessing game was a useful intermediate step in making the derivations more transparent, as well as an informative benchmark about the role of market power for the diffusion of information.

In the segmented market game, it is less straightforward to formulate the corresponding conditional guessing game. Since there are multiple dealers at each trading post, it is not immediate how each dealer forms her guess. In particular, we would need to make additional assumptions about the linear combination of the guesses of dealers in the same trading post and dealers of the neighboring trading post, that each agent can condition her guess on.

Thus, in the segmented market game, we construct beliefs directly as linear combinations of signals. We conjecture that for each dealer $i \in I$, her belief is an affine combination of the signals of all dealers in the economy,

$$E(\theta^i | s^i, \mathbf{p}_{g^I}) = \bar{v}^I_{II} s^i + \sum_{K=1}^{N} v^I_{IK} S^K, \tag{E.13}$$

where $S^K = \sum_{k \in K} s^k$, $\forall K$. This further implies that

$$\sum_{i \in I} E(\theta^i | s^i, \mathbf{p}_{g^I}) = \bar{v}^I_{II} S^I + N_I \sum_{K=1}^{N} v^I_{IK} S^K.$$

Before we derive the fixed-point equation for beliefs, it is useful to write (E.12) in matrix form, for each trading post I. For this, we introduce some more notation. Unless specified otherwise, in the notation below we keep I fixed and vary $J \in \{1, \dots, N\}$. Let $\mathbf{p}^I$ be an N-column vector with elements p_{IJ} if $IJ \in g$, and 0 otherwise. Let $\mathbf{z}^I$ be an N-column vector with elements z^I_{IJ} if $IJ \in g$, and 0 otherwise. Similarly, let $\mathbf{w}^I$ be the N-column vector with elements w^I_{IJ} if $IJ \in g$, and 0 otherwise, and let W_I be a matrix with elements w^J_{IJ} on diagonal if IJ have a link, and 0 otherwise (all elements off-diagonal are 0, as well). Further, let $\mathbf{v}^I$ be the N-row vector with elements v^I_{IJ}, and $\bar{\mathbf{v}}^I$ be the N-row vector with elements $\bar{v}^I_{II}$ at position I and 0 otherwise. Let V be the square matrix with rows $\mathbf{v}^I$, and $\bar{V}$ be the matrix with rows $\bar{\mathbf{v}}^I$. Let $\mathbf{S}$ be the N-column vector with elements S^I. Let N be a square matrix with elements n^I on diagonal and 0 otherwise. Let $\mathbf{1}$ be the N-column vector of ones.

Substituting our conjecture for beliefs (E.13) in the equation for the price (E.12), we obtain the vector of prices which dealers at each trading post I are trading as

$$\mathbf{p}^I = \mathbf{w}^I (\bar{\mathbf{v}}^I + n^I \mathbf{v}^I) \mathbf{S} + W^I (\bar{V} + \mathsf{N} V) \mathbf{S}.$$

We are now ready to formalize the result.

PROPOSITION E.1: *There exists an equilibrium in the segmented markets game if the following system of equations*:

$$\mathbf{v}^I = (\mathbf{z}^I)^\top (\mathbf{w}^I (\bar{\mathbf{v}}^I + n^I \mathbf{v}^I) + W^I (\bar{V} + \mathsf{N} V)) \mathbf{1}, \quad \forall I \tag{E.14}$$

and

$$\bar{v}^I_{II} = y^I, \quad \forall I$$

admits a solution in $\mathbf{v}^I$, *for each* I.

PROOF: As for the OTC game, the proof is constructive. Note that showing that equation (E.14) admits a solution is equivalent to showing that there exists a fixed point in $\mathbf{v}^I$. This is because the projection theorem implies that $\mathbf{z}^I$, and inherently, $\mathbf{w}^I$ are a function of $\mathbf{v}^I$.

Let $\mathbf{v}^I$ be a fixed point of (E.14) and $\bar{v}^I_{II} = y^I$, for each I. We construct an equilibrium for the segmented market game with beliefs given by (E.13), as follows. We choose conveniently $\mathbf{z}^I$ and $\mathbf{w}^I$ such that

$$E(\theta^i | s^i, \mathbf{p}_{g^I}) = y^I s^i + (\mathbf{z}^I)^\top (\mathbf{w}^I(\bar{\mathbf{v}}^I + n^I \mathbf{v}^I) + W^I(\bar{V} + \mathsf{N}V))\mathbf{S}$$

is satisfied. Then, it follows that the prices given by (E.11) and demand functions given by (E.8) are an equilibrium of the OTC game. *Q.E.D.*

The derivation we have outlined above also highlights the main technical difficulty of the segmented market game relative to the OTC game. That is, the signals of dealers in the same trading post obscure the (sum of) beliefs of the dealers in neighboring trading posts, such that a dealer can no longer invert the prices she observes and infer what are his neighbors' posteriors.

E.4. *Learning and Illiquidity in a Star Network*

In this section, we illustrate the effects of market integration on learning from prices and market liquidity in an example. In particular, we restrict ourselves to considering a star network, in which there are n_P dealers at each periphery trading post, and n_C dealers at the central trading post. In particular, we conduct the following numerical exercise. We consider an economy with nine agents. Keeping their information set fixed, we compare the following four market structures:

1. 8 trading posts connected in a star network, with one agent in each trading post ($N = 8$, $n_P = 1$, $n_C = 1$), that is, 8 trading venues. This is our baseline model with a star network.
2. 4 trading posts connected in a star network, with two agents in each periphery node and one agent in the central node ($N = 4$, $n_P = 2$, $n_C = 1$), that is, 4 trading venues.
3. 2 trading posts connected in a star network, with four agents in each periphery node and one agent in the central node ($N = 2$, $n_P = 4$, $n_C = 1$), that is, 2 trading venues.
4. A centralized market ($N = 1$, $n_P = 9$, $n_C = 0$), that is, a single trading venue.

We consider two directions. First, we investigate what drives the illiquidity that central and periphery agents face for changing degrees of market segmentation. We concentrate on (il)liquidity as this is a more commonly reported variable in the empirical literature, and we leave the analysis of welfare and expected profits to Appendix E.5. Second, to complement the analysis in Section 4, we also analyze how much dealers can learn from prices under these market structures.

The left and center panels in Figure E.1 show the average illiquidity that a periphery dealer, $\frac{1}{t_P}$, and a central dealer, $\frac{1}{t_C}$, face in each of the scenarios described above. We

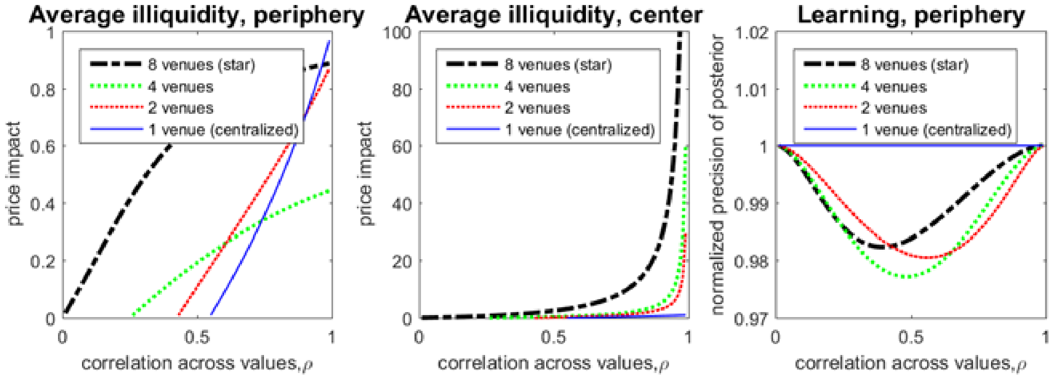

FIGURE E.1.—Illiquidity on segmented markets. We show our measure of illiquidity for central agents , $\frac{1}{t_C}$ (left panel), and for periphery agents, $\frac{1}{t_P}$ (right panel), when there are 8 trading venues (dotted), 4 trading venues (dashed), 2 trading venues (dash-dotted), and in the centralized market (solid) as a function of the correlation across values, ρ. Other parameter values are $\sigma_\theta^2 = 1$, $\sigma_\varepsilon^2 = 0.1$, $B = 1$.

also plot the average illiquidity that any agent in a centralized market, $\frac{1}{t_V}$, faces. For easy comparison, all the parameters are the same as in Section 5.1.

To highlight the intuition, we start with the extreme cases of market segmentation comparing illiquidity under a star network and in a centralized market.

E.4.1. *Extreme Cases of Market Segmentation With a Star Network*

In this part, we compare illiquidity of dealers in a centralized market and that of a periphery or central dealer in a star network.

The solid curve in Panel D and the curves in Panel F in Figure 2 illustrate that compared to any agent in a centralized market, the central agent in the star faces higher trading price impact in general, but the periphery agents tend to face smaller price impact when the correlation across values is sufficiently high. We partially prove this result. The following proposition states that if ρ is sufficiently large, illiquidity for the central agent is larger, while illiquidity for the periphery agents is lower than that for an agent in a centralized market and, when ρ is sufficiently small, illiquidity for any agent in a star network is larger than the illiquidity for any agent in a centralized market.

PROPOSITION E.2:
1. *When ρ is sufficiently small, such that z_V is sufficiently close to $1-\frac{1}{n-1}$, then illiquidity for any agent in a star network is larger than for any agent in a centralized market.*
2. *In the common value limit, when $\rho \to 1$,*
(a) *illiquidity for a central agent is higher in a star network than for any agent in a centralized market, and*
(b) *illiquidity for a periphery agent is lower in a star network than for any agent in a centralized market.*

PROOF: The first part comes by the observation that as $z_V \to 1-\frac{1}{n-1}$, $t_V \to \infty$, while t_C and t_P are finite for these parameters. The second part comes from taking the limit $\rho \to 1$ of the ratio of the corresponding closed-form expressions we report in Appendices B.3

and B.2. In particular,

$$\lim_{\rho\to 1}\frac{t_V}{t_C}=(n-1)\frac{z_C+z_P-z_Cz_P}{(2-z_P)\big((n-1)z_V-(n-2)\big)}=\infty,$$

$$\lim_{\rho\to 1}\frac{t_V}{t_P}=(n-1)\frac{z_C+z_P-z_Cz_P}{(2-z_C)\big((n-1)z_V-(n-2)\big)}=\frac{n-1}{n}<1.$$

Q.E.D.

Similarly to the comparison between the complete OTC network and the centralized market in Section 5.1.2, there are two main forces that drive the illiquidity ratios $\frac{t_V}{t_C}$ and $\frac{t_V}{t_P}$. First, the best response function (31) of a dealer in a centralized market is steeper and has a larger intercept than the best response function (26) of central and periphery dealers in the star OTC network. Simple algebra shows that if, counterfactually, the adverse selection parameters were equal, $z_P=z_C=z_V$, then $\frac{t_V}{t_C}|_{z_V=z_C=z_P}=\frac{t_V}{t_P}|_{z_V=z_C=z_P}>1$, that is, illiquidity for any agents in the OTC market would be higher than for any agent in the centralized market. This is the effect which dominates when ρ is small.

Second, parameters z_C, z_V, and z_P differ. As we stated in Proposition 9, central agents face less liquid markets than periphery agents, $\frac{1}{t_P}<\frac{1}{t_C}$, because periphery agents are more concerned about adverse selection ($z_C<z_P$). This implies that $\frac{t_V}{t_C}>\frac{t_V}{t_P}$ and difference is increasing for higher ρ. In fact, in the common value, the central agent faces an infinitely illiquid market in the sense that $t_C\to 0$, but consumers provide a relatively liquid trading environment for periphery agents. For periphery agents, this is sufficiently strong to reduce their price impact below the centralized market level as stated in the second part of the proposition.

E.4.2. *Intermediate Cases of Market Segmentation With a Star Network*

Interestingly, while the illiquidity a central agent faces is monotonic in segmentation, the illiquidity a periphery agent faces is not. We see in the left panel of Figure E.1 how the relative strength of the two forces identified in Section E.4.1 plays out in the four scenarios we consider. First, related to the effect of decentralization on best response functions, illiquidity for any agent decreases as the market structure approaches a centralized market. Second, the effect coming from the differing weights of z_C and z_P is weaker in more centralized markets. The reason is that as central dealers observe fewer prices in more centralized markets, they put a larger weight, z_C, in each price, implying a smaller difference between z_P and z_C. This is the reason why the illiquidity a periphery agent faces under the 2-trading-venues structure increases with ρ almost as fast as in centralized markets. With 4 venues, the effect of ρ is weaker.

Turning to the effect of segmentation on learning, note that, for the central dealer, prices are fully revealing under any of the segmented market structures in this exercise. This is because each price she observes is a weighted sum of her own signal and the sum of signals of the periphery dealers trading in each venue. Hence, the prices the central dealer observes represent a sufficient statistic for all the useful information in the economy. This would not be the case if there were more than one dealer at the central trading post.

In contrast, as it is shown in the right panel of Figure E.1, a periphery agent in a segmented market always learns less than the central agent, or any agent in a centralized market. Interestingly, for small correlation across values, ρ, a periphery agent in a more segmented market learns more, while for a sufficiently large correlation across values, the opposite is true. The intuition relies on the relative strength of opposing forces. The price

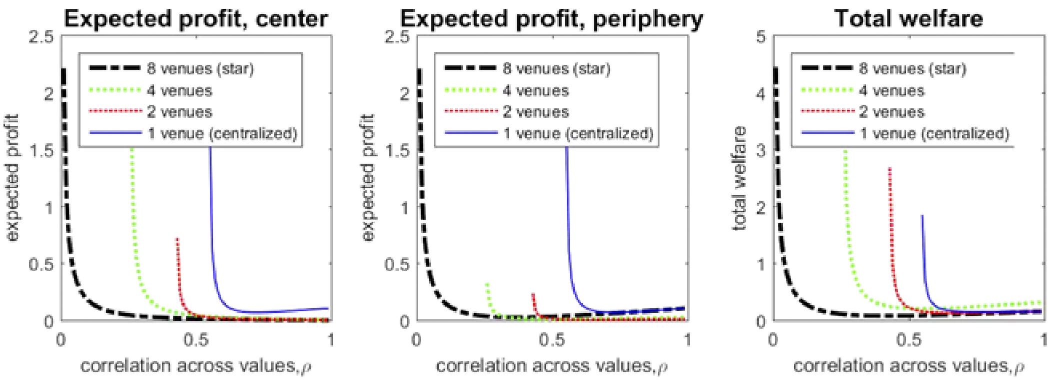

FIGURE E.2.—Chnages in expected profit for the center agent and the periphery agents in a star network, as well as welfare, as a function of market segmentation.

a periphery agent learns from is a weighted average of the sum of posteriors of periphery agents in the same trading post and the posterior of the central agent. The posterior of the central agent is more informative than any of the posteriors that periphery agents at the same trading post have. The more segmented the market is, the easier it is for a dealer at a periphery trading post to isolate the posterior of the central dealer (e.g., in the baseline star network, any price reveals the posterior of the central dealer perfectly). At the same time, the sum of the posteriors of periphery dealers at a periphery trading post is more informative in a less segmented market, as the noise in the signal, as well as the private value components, tend to cancel out. This latter effect helps learning more when the private value component is more important, that is, when ρ is small. This explains the pattern in the right panel of Figure E.1.

E.5. *Welfare and Expected Profit in the Star Network*

Finally, we illustrate with Figure E.2 how expected profit and welfare changes with market segmentation. We leave the detailed analysis for future research and highlight only two interesting observations. First, as trading intensities were not monotonic for the periphery in the degree of segmentation, expected profit is not monotonic either. Also, total welfare is also not monotonic in segmentation.

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