Supplement to "Tax-and-transfer progressivity and business cycles"

(*Quantitative Economics*, Vol. 14, No. 4, November 2023, 1367–1400)

Youngsoo Jang
School of Economics, University of Queensland

Takeki Sunakawa
Graduate School of Economics, Hitotsubashi University

Minchul Yum
Department of Economics, University of Southampton and CEPR

This article contains the supplemental contents to the main paper. Section A contains all the proofs for the results in Section 2. Section B describes the details about the representative-agent model. Section C contains the description of the divisible labor model and quantitative results from the model. Section D describes aggregate data and Section E describes micro data. Section F presents the estimation of idiosyncratic productivity risk. Section G provides details about numerical methods used for the heterogeneous-agent models. Section H includes additional quantitative results from the model and Section I contains additional empirical results.

Appendix A: Proofs in Section 2

Proof of Proposition 1. Assume $T_i = 0$. Then we can rewrite

$$\underline{a}_i = zx_i.$$

Therefore,

$$N_i = 1 - \exp(-zx_i).$$

Given this, note that

$$\begin{aligned}\varepsilon_i \equiv \frac{\partial N_i}{\partial z}\frac{z}{N_i} &= x_i \exp(-zx_i)\frac{z}{1-\exp(-zx_i)}\\ &= \frac{zx_i \exp(-zx_i)}{1-\exp(-zx_i)}.\end{aligned}$$

Youngsoo Jang: youngsoo.jang@uq.edu.au
Takeki Sunakawa: takeki.sunakawa@gmail.com
Minchul Yum: m.yum@soton.ac.uk

Available at http://qeconomics.org. https://doi.org/10.3982/QE1568

For expositional convenience, assume that x is continuous for now:

$$\begin{aligned}
\varepsilon(x) &= \frac{zx\exp(-zx)}{1-\exp(-zx)}, \\
\frac{\partial\varepsilon(x)}{\partial x} &= \frac{\big[z\exp(-zx) - z^2x\exp(-zx)\big]\big[1-\exp(-zx)\big] - zx\exp(-zx)\big[z\exp(-zx)\big]}{\big[1-\exp(-zx)\big]^2} \\
&= \frac{\exp(-zx)z[1-zx]\big[1-\exp(-zx)\big] - z^2x\exp(-zx)\big[\exp(-zx)\big]}{\big[1-\exp(-zx)\big]^2} \\
&= \frac{z\exp(-zx)\big\{1-zx-\exp(-zx)\big\}}{\big[1-\exp(-zx)\big]^2}.
\end{aligned}$$

Since $\exp(-zx) < 1$ for all $z, x > 0$,

$$\begin{aligned}
\frac{\partial\varepsilon(x)}{\partial x} &= \frac{z\exp(-zx)\big(1-zx-\exp(-zx)\big)}{\big[1-\exp(-zx)\big]^2} < \frac{z\exp(-zx)(1-zx-1)}{\big[1-\exp(-zx)\big]^2} \\
&= \frac{z\exp(-zx)(-zx)}{\big[1-\exp(-zx)\big]^2} < 0.
\end{aligned}$$

$\square$

Proof of Proposition 2. Since

$$\begin{aligned}
\frac{\partial N_l}{\partial z} &= \exp(-\underline{a}_l)(1-\lambda), \\
\frac{\partial N_h}{\partial z} &= \exp(-\underline{a}_h)(1+\lambda)
\end{aligned}$$

we have

$$\begin{aligned}
\frac{\partial}{\partial\omega}\left(\frac{\partial N_l}{\partial z}\right) &= \exp(-\underline{a}_l)(1-\lambda)T\lambda > 0, \\
\frac{\partial}{\partial\omega}\left(\frac{\partial N_h}{\partial z}\right) &= -\exp(-\underline{a}_h)(1+\lambda)T\lambda < 0.
\end{aligned}$$

Also, note that

$$\begin{aligned}
\frac{\partial N_l}{\partial\omega} &= -\exp(-\underline{a}_l)T\lambda < 0 \\
\frac{\partial N_h}{\partial\omega} &= \exp(-\underline{a}_l)T\lambda > 0.
\end{aligned}$$

$\square$

Proof of Proposition 3. Since

$$\begin{aligned}
\varepsilon &\equiv \frac{\partial N}{\partial z}\frac{z}{N} \\
&= \left(\pi_l\frac{\partial N_l}{\partial z} + \pi_h\frac{\partial N_h}{\partial z}\right)\frac{z}{\pi_l N_l + \pi_h N_h}
\end{aligned}$$

the aggregate labor supply elasticity is given by

$$\varepsilon = z\frac{\exp(-\underline{a}_l)(1-\lambda)+\exp(-\underline{a}_h)(1+\lambda)}{2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)},$$

where

$$\begin{aligned}\underline{a}_l &= z(1-\lambda)-T-T\omega\lambda,\\ \underline{a}_h &= z(1+\lambda)-T+T\omega\lambda.\end{aligned}$$

Then we have

$$\begin{aligned}\frac{\partial\varepsilon}{\partial\omega} &= z\Big(\big[\exp(-\underline{a}_l)(1-\lambda)(-1)(-T\lambda)+\exp(-\underline{a}_h)(1+\lambda)(-1)T\lambda\big]\\ &\quad\times\big[2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big]\\ &\quad-\big[\exp(-\underline{a}_l)(1-\lambda)+\exp(-\underline{a}_h)(1+\lambda)\big]\\ &\quad\times\big[-\exp(-\underline{a}_l)(-1)(-T\lambda)-\exp(-\underline{a}_h)(-1)T\lambda\big]\Big)\\ &\quad/\big[2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big]^2\\ &= zT\lambda\Big(\big[\exp(-\underline{a}_l)(1-\lambda)-\exp(-\underline{a}_h)(1+\lambda)\big]\big[2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big]\\ &\quad+\big[\exp(-\underline{a}_l)(1-\lambda)+\exp(-\underline{a}_h)(1+\lambda)\big]\big[\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big]\Big)\\ &\quad/\big[2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big]^2.\end{aligned}$$

The sign of $\frac{\partial\varepsilon}{\partial\omega}$ is equal to that of the numerator, which can be rewritten as

$$\begin{aligned}\text{Numerator} &= 2(1-\lambda)\exp(-\underline{a}_l)-(1-\lambda)\exp(-2\underline{a}_l)-(1-\lambda)\exp(-\underline{a}_h-\underline{a}_l)\\ &\quad-2(1+\lambda)\exp(-\underline{a}_h)+(1+\lambda)\exp(-\underline{a}_h-\underline{a}_l)+(1+\lambda)\exp(-2\underline{a}_h)\\ &\quad+(1-\lambda)\exp(-2\underline{a}_l)-(1-\lambda)\exp(-\underline{a}_h-\underline{a}_l)\\ &\quad+(1+\lambda)\exp(-\underline{a}_h-\underline{a}_l)-(1+\lambda)\exp(-2\underline{a}_h)\\ &= 2\big[(1-\lambda)\exp(-\underline{a}_l)-(1+\lambda)\exp(-\underline{a}_h)+2\lambda\exp(-\underline{a}_h-\underline{a}_l)\big].\end{aligned}$$

Letting $\theta=\frac{(1-\lambda)}{(1+\lambda)}$, we can rewrite

$$\begin{aligned}&2(1+\lambda)\left[\frac{(1-\lambda)}{(1+\lambda)}\exp(-\underline{a}_l)-\exp(-\underline{a}_h)+\frac{2\lambda}{(1+\lambda)}\exp(-\underline{a}_h-\underline{a}_l)\right]\\ &\quad=2(1+\lambda)\big[\theta\exp(-\underline{a}_l)+(1-\theta)\exp(-\underline{a}_h-\underline{a}_l)-\exp(-\underline{a}_h)\big].\end{aligned}$$

Since $\exp(-x)$ is convex, we know

$$\begin{aligned}\theta\exp(-\underline{a}_l)+(1-\theta)\exp\big(-(\underline{a}_h+\underline{a}_l)\big) &> \exp\big(-\{\theta\underline{a}_l+(1-\theta)(\underline{a}_h+\underline{a}_l)\}\big)\\ &= \exp\big(-\{(1-\theta)\underline{a}_h+\underline{a}_l\}\big).\end{aligned}$$

Applying this inequality, we have

$$\begin{aligned}\text{Numerator} &= 2(1+\lambda)\big[\theta\exp(-\underline{a}_l) + (1-\theta)\exp(-\underline{a}_h - \underline{a}_l) - \exp(-\underline{a}_h)\big] \\ &> 2(1+\lambda)\big[\exp\big(-\big\{(1-\theta)\underline{a}_h + \underline{a}_l\big\}\big) - \exp(-\underline{a}_h)\big] \geq 0\end{aligned}$$

if and only if

$$\begin{aligned}(1-\theta)\underline{a}_h + \underline{a}_l &\leq \underline{a}_h, \\ \underline{a}_l &\leq \theta\underline{a}_h, \\ (1+\lambda)\big[z(1-\lambda) - T - T\omega\lambda\big] &\leq (1-\lambda)\big[z(1+\lambda) - T + T\omega\lambda\big], \\ z(1+\lambda)(1-\lambda) - (1+\lambda)T - (1+\lambda)T\omega\lambda &\leq z(1+\lambda)(1-\lambda) - (1-\lambda)T + (1-\lambda)T\omega\lambda, \\ -(1+\lambda) - (1+\lambda)\omega\lambda &\leq -(1-\lambda) + (1-\lambda)\omega\lambda, \\ -1 &\leq \omega,\end{aligned}$$

which is always satisfied. □

Proof of Proposition 4. Note that

$$\begin{aligned}\chi_0 &= \frac{(1-\lambda)\big(1-\exp(-\underline{a}_l)\big) + (1+\lambda)\big(1-\exp(-\underline{a}_h)\big)}{2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)} \\ &= \frac{1-\lambda-\exp(-\underline{a}_l)+\lambda\exp(-\underline{a}_l)+1+\lambda-\exp(-\underline{a}_h)-\lambda\exp(-\underline{a}_h)}{2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)} \\ &= \frac{2-(1-\lambda)\exp(-\underline{a}_l)-(1+\lambda)\exp(-\underline{a}_h)}{2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)}.\end{aligned}$$

Therefore, we have

$$\begin{aligned}\frac{\partial\chi_0}{\partial z} &= \frac{\big[(1-\lambda)^2\exp(-\underline{a}_l)+(1+\lambda)^2\exp(-\underline{a}_h)\big]\big[2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big]}{\big(2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big)^2} \\ &\quad - \frac{\big[2-(1-\lambda)\exp(-\underline{a}_l)-(1+\lambda)\exp(-\underline{a}_h)\big]\big[\exp(-\underline{a}_l)(1-\lambda)+\exp(-\underline{a}_h)(1+\lambda)\big]}{\big(2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big)^2} \\ &= \frac{1}{\big(2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big)^2} \\ &\quad \times \left\{\begin{array}{c} 2(1-\lambda)^2\exp(-\underline{a}_l)+2(1+\lambda)^2\exp(-\underline{a}_h) \\ -(1-\lambda)^2\exp(-2\underline{a}_l)-(1+\lambda)^2\exp(-\underline{a}_h-\underline{a}_l) \\ -(1-\lambda)^2\exp(-\underline{a}_h-\underline{a}_l)-(1+\lambda)^2\exp(-2\underline{a}_h) \\ -2(1-\lambda)\exp(-\underline{a}_l)-2(1+\lambda)\exp(-\underline{a}_h) \\ +(1-\lambda)^2\exp(-2\underline{a}_l)+(1+\lambda)(1-\lambda)\exp(-\underline{a}_h-\underline{a}_l) \\ +(1+\lambda)(1-\lambda)\exp(-\underline{a}_h-\underline{a}_l)+(1+\lambda)^2\exp(-2\underline{a}_h) \end{array}\right\}\end{aligned}$$

$$= \frac{2\lambda(\lambda-1)\exp(-\underline{a}_l) + 2\lambda(\lambda+1)\exp(-\underline{a}_h) - 4\lambda^2\exp(-\underline{a}_h - \underline{a}_l)}{\big(2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big)^2}$$
$$= \frac{2\lambda\big\{(\lambda-1)\exp(-\underline{a}_l) + (\lambda+1)\exp(-\underline{a}_h) - 2\lambda\exp(-\underline{a}_h - \underline{a}_l)\big\}}{\big(2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big)^2} < 0. \qquad \square$$

Proof of Proposition 5. Define

$$\Phi(\omega) \equiv \log\left(\frac{\partial \chi_0}{\partial z}\right).$$

Since the log transformation preserves monotonicity, it suffices to show that $\Phi'(\omega) < 0$. As

$$\begin{aligned}\Phi(\omega) = {} & \log 2\lambda + \log\big\{(\lambda-1)\exp(-\underline{a}_l) + (\lambda+1)\exp(-\underline{a}_h) - 2\lambda\exp(-\underline{a}_h - \underline{a}_l)\big\} \\ & - 2\log\big(2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)\big)\end{aligned}$$

we have

$$\begin{aligned}\Phi'(\omega) = {} & \frac{-T\lambda(\lambda-1)\exp(-\underline{a}_l) + T\lambda(\lambda+1)\exp(-\underline{a}_h)}{(\lambda-1)\exp(-\underline{a}_l) + (\lambda+1)\exp(-\underline{a}_h) - 2\lambda\exp(-\underline{a}_h - \underline{a}_l)} \\ & - 2\frac{T\lambda\exp(-\underline{a}_l) - T\lambda\exp(-\underline{a}_h)}{2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)} \\ = {} & \frac{\underbrace{T\lambda(1-\lambda)\exp(-\underline{a}_l) + T\lambda(\lambda+1)\exp(-\underline{a}_h)}_{\text{positive}}}{\underbrace{(\lambda-1)\exp(-\underline{a}_l) + (\lambda+1)\exp(-\underline{a}_h) - 2\lambda\exp(-\underline{a}_h - \underline{a}_l)}_{\text{negative}}} \\ & - 2\frac{\underbrace{T\lambda\big[\exp(-\underline{a}_l) - \exp(-\underline{a}_h)\big]}_{\text{positive}}}{\underbrace{2-\exp(-\underline{a}_l)-\exp(-\underline{a}_h)}_{\text{positive}}} \\ < {} & 0. \qquad \square\end{aligned}$$

Appendix B: Representative-agent (RA) model

We first describe the environment of Model (RA). At the beginning of each period, the stand-in household holds the assets of that period k. The aggregate state variables are the aggregate capital K and the aggregate TFP shock z_k, with the latter following the same stochastic process as in the baseline model. Taking the real wage rate $w(K, z_k)$, the real interest rate $r(K, z_k)$, and the aggregate law of motion $\Gamma(K, z_k)$ as given, the dynamic decision problem of the representative household can be written as the following functional equation:

$$V(k, K, z_k) = \max_{\substack{k' \geq 0, c \geq 0 \\ n \in [0,1]}} \left\{ \log c - Bn + \beta \sum_{l=1}^{N_z} \pi_{kl}^z V\big(k', K', z_l'\big) \right\}$$

$$\text{subject to } c + k' \leq (1 - \tau_l) w(K, z_k) n + \big(1 + r(K, z_k)\big) k + T,$$

$$K' = \Gamma(K, z_k).$$

The household maximizes utility by choosing its optimal consumption c, the next period's capital k', and its labor supply n. The utility of the stand-in household is linear with respect to employment n due to the aggregation theory of Rogerson (1988). The budget constraint states that the sum of consumption c and the next period's capital k' should be less than or equal to the sum of net-of-tax labor income $(1 - \tau_l)w(K, z_k)n$, current capital k, capital income $r(K, z_k)k$, and government transfers T.

Government then collects taxes on labor earnings $\tau_l wn$ to finance transfers T and government spending G. We keep the same assumptions on the firm side as in the heterogeneous-agent models. The resulting first-order conditions for K and L are the same as those presented in equations (3) and (4) in the main paper.

A recursive competitive equilibrium is a collection of factor prices $r(K, z_k)$, $w(K, z_k)$, household decision rules $g_k(k, K, z_k)$, $g_n(k, K, z_k)$, government policy variables τ_l, G, T, the household value function $V(k, K, z_k)$, the aggregate labor $L(K, z_k)$, and the aggregate law of motion for aggregate capital $\Gamma(K, z_k)$ such that:

1. Given factor prices $r(K, z_k)$, $w(K, z_k)$ and government policy τ_l, G, T, the value function $V(k, K, z)$ solves the household's decision problem, and the associated decision rules are

$$k'^* = g_k(k, K, z_k),$$

$$n^* = g_n(k, K, z_k).$$

2. Prices $r(K, z_k)$, $w(K, z_k)$ are competitively determined following equations (3) and (4) in the main paper.

3. Government balances its budget:

$$G + T = \tau_l w(K, z_k) L(K, z_k).$$

4. Consistency is satisfied: for all K,

$$K' = \Gamma(K, z_k) = g_k(K, K, z_k),$$

$$L(K, z_k) = g_n(K, K, z_k).$$

It is straightforward to calibrate the parameters of Model (RA) using the steady-state equilibrium equations. First, β is directly obtained by

$$\beta = (1 + r)^{-1}.$$

Then, given the targets of $T/Y = 0.044$, $L = 0.782$, and $\tau_l = 0.1111$, B is obtained by

$$B = \frac{(1-\tau_l)(1-\alpha)}{\left(1 - \delta\frac{K}{Y} - \frac{G}{Y}\right)L},$$

where

$$\frac{K}{Y} = \frac{\alpha}{r+\delta},$$
$$\frac{G}{Y} = \tau(1-\alpha) - \frac{T}{Y}.$$

Finally, since $Y/K = (K/L)^{\alpha-1}$, we can obtain K/L. This in turn gives us K, and thus Y. We can then obtain T using the calibration target ratio $T/Y = 0.044$. The resulting calibrated values are $\beta = 0.9901$, $B = 1.0164$, and $T = 0.1277$.

Appendix C: Heterogeneous-agent models without labor supply indivisibility

Indivisible labor supply is a key feature of our analysis. We illustrate this point by considering a heterogeneous-agent model with divisible labor. The economic environment in this model is mostly identical to the heterogeneous-agent models in the main text, and includes features such as idiosyncratic shocks, progressive taxation, and firm technology. However, one exception is that households can adjust their hours in a fully flexible way under the following period utility function with constant Frisch elasticity γ:

$$U(c,h) = \log c - \xi\frac{h^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}.$$

We consider two different values of $\gamma \in \{1, 2\}$. To illustrate the role of transfers for business cycle fluctuations in this alternative environment, we consider two cases: zero transfers and flat transfers. In the latter case, we target the same moment (4.4% of output) that is used in the main text. For each specification, we also calibrate ξ and β to target the full-time employment rate of 78.2% and the real interest rate of 1% where full-time is defined as hours greater than 0.2.

The results are summarized in Table A1, with two findings in particular being worth highlighting. First, the models without indivisible labor supply have difficulty in generating a sufficiently high volatility of hours worked, echoing the performance of representative-agent real business cycle models (Kydland and Prescott (1982)). Even with a relatively large value of $\gamma = 2$, the volatility of aggregate hours is considerably smaller than in the data. Moreover, these divisible labor models generate average labor productivity that is almost perfectly correlated with output, given that labor supply responses are nearly homogeneous across households, and thus do not vary negatively with individual productivity. This is in sharp contrast to our baseline models with labor

TABLE A1. Results from models without indivisibility.

	$\gamma = 1$		$\gamma = 2$		Indivisible	
$T/Y =$	0.00	0.044	0.00	0.044	(HA-N)	(HA-F)
σ_Y	1.15	1.16	1.27	1.28	1.48	1.46
σ_C/σ_Y	0.32	0.30	0.31	0.29	0.28	0.27
σ_I/σ_Y	2.77	2.77	2.79	2.79	2.99	2.99
σ_L/σ_Y	0.34	0.35	0.45	0.47	0.64	0.62
σ_H/σ_Y	0.28	0.30	0.36	0.39	0.51	0.60
$\sigma_{Y/H}/\sigma_Y$	0.73	0.70	0.65	0.62	0.54	0.57
$\text{Cor}(Y, C)$	0.91	0.91	0.91	0.90	0.85	0.84
$\text{Cor}(Y, I)$	0.99	0.99	0.99	0.99	0.99	0.99
$\text{Cor}(Y, L)$	0.98	0.98	0.98	0.98	0.96	0.96
$\text{Cor}(Y, H)$	0.98	0.97	0.98	0.99	0.95	0.87
$\text{Cor}(Y, Y/H)$	1.00	1.00	0.99	0.99	0.95	0.85
$\text{Cor}(H, Y/H)$	0.96	0.97	0.96	0.96	0.81	0.81

Note: Each model specification is calibrated to generate the same interest rate and the full-time employment rate.

supply indivisibility (recall Proposition 1 in Section 2). The second notable observation is that the presence of transfers appears almost irrelevant to the cyclicality of average labor productivity, although it does moderately raise the volatility of hours.

APPENDIX D: AGGREGATE DATA

The business cycle statistics are based on the aggregate time-series data from U.S. Bureau of Economic Analysis (BEA), National Income and Product Accounts (NIPA) tables covering the period from 1961Q1 to 2016Q4. For output, we use the "Real Gross Domestic Product (millions of chained 2012 dollars)" entry in Table 1.1.6. As for consumption, we use expenditures on non-durable goods and services, as reported in Table 2.3.5 (Personal Consumption Expenditure). Investment is constructed as the sum of expenditures on durable goods (Table 2.3.5) and private fixed investments (Table 5.3.5). The real values of consumption and investment are calculated using the price index for Gross Domestic Product from Table 1.1.4. Data on total hours worked are obtained from Cociuba, Prescott, and Ueberfeldt (2018). We modified all of the raw time series into per capita series by dividing the raw data by the quarterly population reported by Cociuba, Prescott, and Ueberfeldt (2018). Figure A1 plots the cyclical component of the real GDP per capita.

A target statistic regarding the size of income-security transfers is based on the aggregate data obtained also from the BEA NIPA Tables. Specifically, we use data from Table 3.12 (Government Social Benefits) on the Supplemental Nutrition Assistance Program (SNAP), Supplemental Security Income, Temporary Disability Insurance, and medical care (Medicaid, General Medical Assistance, and state child healthcare programs). Note that we do not include large programs such as Medicare, unemployment insurance, and veterans' benefits.

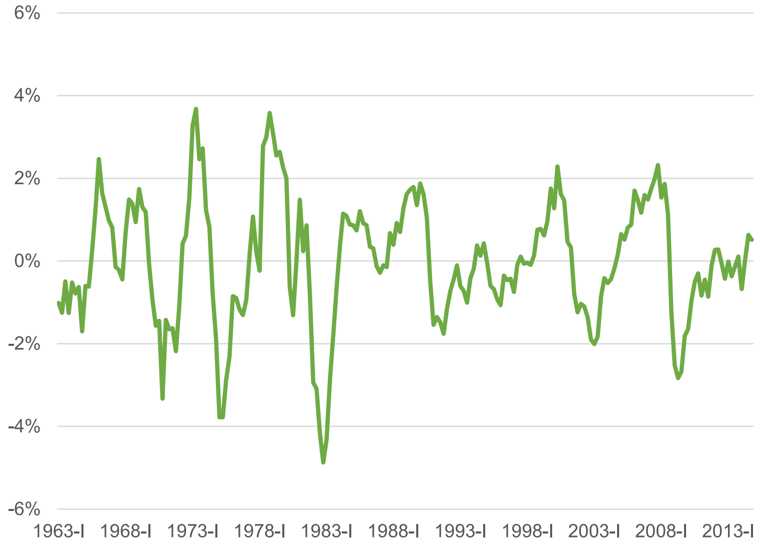

FIGURE A1. Cyclical component of real GDP per capita. *Note*: A quarterly series of real GDP per capita is detrended using HP filter with a smoothing parameter of 1600.

APPENDIX E: MICRO DATA

For the transfer-related statistics obtained at the micro level, we use data from the Survey of Income and Program Participation (SIPP). This data set is representative of the noninstitutionalized U.S. population and has a monthly survey period. The SIPP covers a wide range of information on income, wealth, and participation in various transfer programs. We choose samples from the first wave to the ninth wave of the SIPP, covering the years 2001 to 2003. The original data set is composed of a main module and several topical modules. While the main module contains monthly information on income and transfers, variables such as wealth are reported quarterly in the topical modules. We combine both modules on a quarterly basis.

We construct all variables at the household level. Data sets in the SIPP contain not only household variables but also individual variables so, in order to generate a household variable from its corresponding individual variable, we take the following steps. First, we identify households by their sample unit identifier (SSUID) and their sample household address identifier (SHHADID). Second, we add up the values of the variable in question for all members of the same household. The government transfers used to infer the degree of progressivity are based on a broad range of transfer programs including Supplemental Security Income (SSI), Temporary Assistant for Needy Family (TANF), the Supplemental Nutrition Assistance Program (SNAP), the Supplemental Nutrition Program for Women, Infants, and Children (WIC), childcare subsidies and Medicaid. We do not include age-dependent programs such as Social Security and Medicare. We also construct a broad household income variable: it consists of labor income, income from financial investments, and property income. We consider households whose head is aged between 23 and 65, and the results we presented are almost the same as for alternative age ranges around these limits. Finally, we convert the nominal values of all these variables to 2001 U.S. dollars using the CPI-U.

The empirical analysis in Section 6 is based on the PSID data. We choose samples for the period of 1969–2010. To avoid the oversampling of low-income household heads, we exclude households listed in the Survey of Economic Opportunity. We also drop the samples whose wage is below one half of the minimum wage. The nominal values are again converted into 2001 U.S. dollars using the CPI-U.

Appendix F: Estimation of idiosyncratic productivity risk

We estimate the persistence of idiosyncratic productivity risk in the U.S. using the PSID data, following Heathcote, Storesletten, and Violante (2010). Our measure of productivity is defined as a worker's hourly wage relative to other individuals. We consider household heads between the ages of 18 and 70, and whose wages were observed for at least four consecutive periods.[1] To focus on full-time workers, we drop the samples whose annual hours worked was less than 1000.

We run the ordinary least squares regression on the logarithm of the productivity (hourly wages) on a dummy for male, a cubic polynomial in potential experience (age minus years of education minus five), a time dummy, and a time dummy interacted with a college education dummy. We take its residual $x_{i,j}$ as an idiosyncratic productivity variable that contains a wide range of individual abilities valued by the labor market. This stochastic process is composed of the summation of a persistent process $\eta_{i,j}$ and a transitory process $\nu_{i,j}$ as described by

$$x_{i,j} = \eta_{i,j} + \nu_{i,j}, \nu_{i,j} \sim N(0, \sigma_\nu^2),$$
$$\eta'_{i,j} = \rho_\eta \eta_{i,j-1} + \epsilon'_{i,j}, \epsilon'_{i,j} \sim N(0, \sigma_\epsilon^2).$$

We use a minimum distance estimator to estimate the parameters of the process. This method is used to find parameters that minimize the distance between the empirical and theoretical moments. We take the covariance matrix of the residual $x_{i,j}$ as our moments, and denote θ by the vector $(\rho_\eta, \sigma_v, \sigma_\epsilon)$. We then let $m_{j,j+n}(\theta)$ be the covariance of the labor productivity between age j and $j+n$ individuals, and define $\hat{m}_{j,j+n}$ as the empirical counterpart of $m_{j,j+n}(\theta)$. We use the following moment conditions:

$$E[\hat{m}_{j,j+n} - m_{j,j+n}(\theta)] = 0,$$

where

$$\hat{m}_{j,j+n} = \frac{1}{N_{j,j+n}} \sum_{i=1}^{N_{j,j+n}} x_{i,j} \cdot x_{i,j+n}.$$

[1] We use a somewhat less restricted age range in order to obtain a large number of samples. Note that we impose stricter restrictions on wages and hours, which would naturally remove irrelevant samples such as retirees. Thus, a change in the age band leads to only relatively small changes in the estimated persistence of idiosyncratic shocks.

The moments can be represented by as an upper triangle matrix:

$$\bar{m}(\theta) = \begin{bmatrix} m_{0,0}(\theta) & m_{0,1}(\theta) & \cdots & \cdots & m_{0,J-1}(\theta) & m_{0,J}(\theta) \\ 0 & m_{1,1}(\theta) & \cdots & \cdots & m_{1,J-1}(\theta) & m_{1,J}(\theta) \\ 0 & 0 & m_{2,2}(\theta) & \cdots & m_{2,J-1}(\theta) & m_{2,J}(\theta) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & m_{J-1,J-1}(\theta) & m_{J-1,J}(\theta) \\ 0 & 0 & 0 & \cdots & 0 & m_{J,J}(\theta) \end{bmatrix}.$$

We denote a vector of $\bar{M}(\theta)$ by vectorizing $\bar{m}(\theta)$ with length $(J+1)(J+2)/2$. To estimate parameters θ, we solve

$$\min_{\theta} \left[\hat{\bar{M}} - \bar{M}(\theta)\right]' W \left[\hat{\bar{M}} - \bar{M}(\theta)\right],$$

where the weighting matrix W is set to be an identity matrix.[2]

Appendix G: Numerical methods used for the heterogeneous-agent models

G.1 *Solving for the equilibrium with aggregate risk*

The models which aggregate risk are solved with the following two steps. First, we solve for the individual policy functions given the forecasting rules (*the inner loop*). Then we update the forecasting rules by simulating the economy using those individual policy functions (*the outer loop*). We iterate the two steps until the forecasting rules converge, that is, when the difference between the old forecasting rule used in the inner loop and the new forecasting rule generated in the outer loop becomes small enough.

G.1.1 *Inner loop* In the inner loop, we solve for the following value functions: $V(a, x_i, K, z_k)$, $V^E(a, x_i, K, z_k)$, and $V^N(a, x_i, K, z_k)$. These value functions are stored on a non-evenly spaced grid for a and an evenly-spaced grid for K, with the number of grid points being $n_a = 400$ and $n_K = 40$, respectively. Unlike Chang and Kim (2006, 2007) and Takahashi (2014), we discretize the stochastic processes for x_i and z_k by using the Rouwenhorst (1995) method. We find that the approximation of continuous AR(1) processes with our estimate featuring very high persistence is considerably better with the Rouwenhorst method given the same number of grid points.[3] Our baseline results are based on $n_x = 10$ and $n_z = 5$, both of which replicate the true parameters of the continuous AR(1) processes very precisely.

To obtain $V(a, x_i, K, z_k) = \max[V^E(a, x_i, K, z_k), V^N(a, x_i, K, z_k)]$, we solve the following problems:

$$V^E(a, x_i, K, z_k) = \max_{\substack{a' \geq \underline{a}, \\ c \geq 0}} \left\{ \log c - B\bar{n} + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V\left(a', x_j', \hat{K}', z_l'\right) \right\} \tag{A1}$$

[2]Using the identity matrix has been common in the literature since Altonji and Segal (1996) show that the optimal weighting matrix generate severe small sample biases.

[3]Specifically, we use the simulated data from the methods of Rouwenhorst and Tauchen, and estimate the persistence and the standard deviation of the error terms in the AR(1) processes for both aggregate productivity shocks and idiosyncratic shocks (results available upon request).

subject to

$$c + a' \leq \tau(e, \bar{e})e\big(\hat{w}(K, z_k)\big) + \big(1 + \hat{r}(K, z_k)\big)a + T\big(\hat{w}(K, z_k), \hat{r}(K, z_k)\big)$$

and

$$\begin{aligned} V^N(a, x_i, K, z_k) &= \max_{\substack{a' > \underline{a}, \\ c > 0}} \left\{ \log c + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V\big(a', x_j', \hat{K}', z_l'\big) \right\}, \\ c + a' &\leq \big(1 + \hat{r}(K, z_k)\big)a + T\big(\hat{r}(K, z_k)\big). \end{aligned} \tag{A2}$$

To evaluate the functional value of the expected value function on $(a', \hat{K}')$, which are not on the grid points, we use the piecewise-linear interpolation. By solving these problems, we obtain the individual policy function for work $g_n(a, x_i, K, z_k)$ by comparing $V^E(a, x_i, K, z_k)$ with $V^N(a, x_i, K, z_k)$. We also obtain conditional policy functions for the optimal $a' : g_a^E(a, x_i, K, z_k)$ as the maximizer of the problem (A1) and $g_a^N(a, x_i, K, z_k)$ as the maximizer of the problem (A2).

G.1.2 *Outer loop* In the outer loop, we simulate the model economy based on the information obtained in the inner loop. We note that a key step is to find the market-clearing prices in each period during the simulation. Although this is computationally burdensome, we find that the results without the market-clearing step are substantially misleading, as is consistent with Takahashi (2014) and Chang and Kim (2014).

The measure of households $\mu(a, x_i)$ is approximated by a nonevenly spaced grid on a that is finer than that used in the inner loop (Rios-Rull (1999)) and has 4000 grid points. The variable K is then constructed by aggregating individual asset holdings over the measure of households: $\int_a \sum_{i=1}^{N_x} a\mu(da, x_i)$. Following Takahashi (2014), we use a bisection method to obtain the equilibrium factor prices in each simulation period as follows:

1. Set an initial range of (w_L, w_H) and calculate the aggregate labor demand $L^d = (1-\alpha)^{\frac{1}{\alpha}}(z_k/w)^{\frac{1}{\alpha}}K$ implied by the firm's FOC for each w. Note that r is obtained by using the relationship $r = z_k^{\frac{1}{\alpha}}\alpha(\frac{w}{1-\alpha})^{\frac{\alpha-1}{\alpha}} - \delta$, implied jointly by (3) and (4) in the main paper.

2. Calculate the aggregate efficiency unit of labor supply L^s at each w and make sure that the excess labor demand $(L^d - L^s)$ is positive at w_L and it is negative at w_H.

3. Compute $\tilde{w} = \frac{w_L + w_H}{2}$ and obtain $L^d - L^s$ at $\tilde{w}$. If $L^d - L^s > 0$, set $w_L = \tilde{w}$; otherwise, set $w_H = \tilde{w}$.

4. Continue updating (w_L, w_H) until $|w_L - w_H|$ is small enough.

Taking the measure of households $\mu(a, x_i)$, the aggregate state (K, z_k), and factor prices w and r as given, we compute the aggregate efficiency unit of labor supply $L^s(K, z_k)$. Specifically, we solve (A1) and (A2) given the expected value function in the next period using interpolation. Note that we use the valued function obtained in the inner loop and the forecasting rule (5) in the main paper for $\hat{K}' = \Gamma(K, z_k)$, which is not on the grid points of K. Then the individual household decision rules are given by

$$n = g_n(a, x_i, K, z_k) = \begin{cases} \bar{n} & \text{if } V^E(a, x_i, K, z_k) > V^N(a, x_i, K, z_k), \\ 0 & \text{otherwise.} \end{cases}$$

By having $n = g_n(a, x_i, K, z_k)$ for each grid point (a, x_i) on μ at hand, the aggregate efficiency unit of labor supply is obtained by $L^s(K, z_k) = \int_a \sum_{i=1}^{N_x} x_i g_n(a, x_i, K, z_k)\mu(da, x_i)$. After finding the market-clearing prices, we update the measure of households in the next period by using

$$a' = g_a(a, x_i, K, z_k) = \begin{cases} g^E(a, x_i, K, z_k) & \text{if } V^E(a, x_i, K, z_k) > V^N(a, x_i, K, z_k), \\ g^N(a, x_i, K, z_k) & \text{otherwise,} \end{cases}$$

and the stochastic process for x_i. We simulate the economy for 10,000 periods, as in Khan and Thomas (2008).

Finally, the coefficients $(a_0, a_1, a_2, b_0, b_1, b_2)$ in the forecasting rules

$$\log K' = a_0 + a_1 \log K + a_2 \log z, \tag{A3}$$

$$\log w = b_0 + b_1 \log K + a_2 \log z, \tag{A4}$$

are updated by ordinary least squares with the simulated sequence of $\{K', w, K, z\}$. Our parametric assumptions regarding the forecasting rules are the same as those made in Chang and Kim (2007, 2014) and Takahashi (2014, 2020). We repeat the whole procedure for the inner and outer loops until the coefficients in the forecasting rules converge.

As is clear in the forecasting rules (A3) and (A4), households predict prices and the future distributions of capital based only on the mean capital stock instead of the entire distribution. Therefore, it is important to check whether the equilibrium forecast rules are precise or not. We summarize the results regarding the accuracy of the forecasting rules for the future mean capital stock K' and for the wage w in Table A2. It is clear that all R^2 values are very high in all specifications. We also check the accuracy statistic proposed by Den Haan (2010). Since our dependent variables are logarithmic, we multiply the statistics by 100 to interpret them as percentage errors. We find that the mean errors are sufficiently small (considerably less than 0.1% for all cases) and the maximum errors are also reasonably small (not exceeding 0.8% for all cases).

G.2 *Impulse response functions*

There is no generally accepted way to calculate conditional impulse responses in nonlinear models. To compute impulse response functions in this paper, we follow the simulation-based procedure developed by Koop, Pesaran, and Potter (1996) (see also Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)):

TABLE A2. Estimates and accuracy of forecasting rules.

Model	Dependent Variable	Coefficient Const.	$\log K$	$\log z$	R^2	Den Haan (2010) Error Mean (%)	Max (%)
(HA-T)	$\log K'$	0.1193	0.9554	0.0940	0.99997	0.088	0.445
	$\log w$	−0.2689	0.4242	0.8037	0.99818	0.086	0.749
(HA-N)	$\log K'$	0.1528	0.9413	0.1170	0.99998	0.087	0.499
	$\log w$	−0.5117	0.5291	0.6683	0.99918	0.060	0.452
(HA-F)	$\log K'$	0.1489	0.9431	0.1154	0.99998	0.086	0.427
	$\log w$	−0.4557	0.5045	0.6826	0.99920	0.058	0.425

- Draw $i = 1, \dots, N_{\text{sim}}$ sets of exogenous random variables for aggregate TFP shocks, each of which have $t = 1, \dots, T_{\text{sim}}$ periods.[4]
- For each set of i, simulate two sequences, one is from the shock economy and the other is from the no-shock economy.
  1. In the shock economy, simulate all interested variables X_{it}^{shock} for $t = 1, \dots, T_{\text{shock}} - 1$ as normal (as we do in the outer loop). Then, in period T_{shock}, impose a disturbance on aggregate TFP so that it takes an extreme value (e.g., the lowest one z_1). Simulate the economy as normal for the rest of the periods $t = T_{\text{shock}} + 1, \dots, T_{\text{sim}}$.[5]
  2. In the no-shock economy, simulate all interested variables X_{it}^{noshock} for all the periods without any restrictions. The two economies are different only in terms of the imposition of the extreme shock in period T_{shock}.
- The effect of the disturbance on X is given by the average percentage (or percentage point) difference between the two sequences:

$$\hat{X}_t = 100 \times \frac{1}{N_{\text{sim}}} \sum_{i=1}^{N_{\text{sim}}} \log\left(X_{it}^{\text{shock}} / X_{it}^{\text{noshock}}\right) \quad \text{(percentage difference)},$$

$$\hat{X}_t = 100 \times \frac{1}{N_{\text{sim}}} \sum_{i=1}^{N_{\text{sim}}} \left(X_{it}^{\text{shock}} - X_{it}^{\text{noshock}}\right) \quad \text{(percentage point difference)}.$$

The results are based on $N_{\text{sim}} = 2000$ simulations with each simulation having two sequences of the variables of interest for $T_{\text{sim}} = 150$ periods. The responses are equal to zero before T_{shock} by construction. The disturbance then hits the economy at period $T_{\text{shock}} = 50$, which we label as the first period in our figures.

[4]We use a random sampling with Markov chains. That is, by taking as given the index for today's aggregate productivity i and the conditional distribution for tomorrow's productivity $\{\pi_{ij}^z\}_{j=1}^{N_z}$ (i.e., the ith row of the Markov chain), we draw a random variable $u \sim U[0, 1]$ to pick up tomorrow's shock index j. We do so by choosing the highest j satisfying the condition $u < \sum_{k=1}^{j} \pi_{ik}^z$.

[5]Note that the effect of the disturbance is *persistent* because we sample aggregate productivity using the conditional distribution of the Markov chain.

Appendix H: Additional model results

Figure A2 displays aggregate labor supply curves and arc elasticities from the model specifications considered in Section 3.2 of the main paper. Figure A3 shows how they

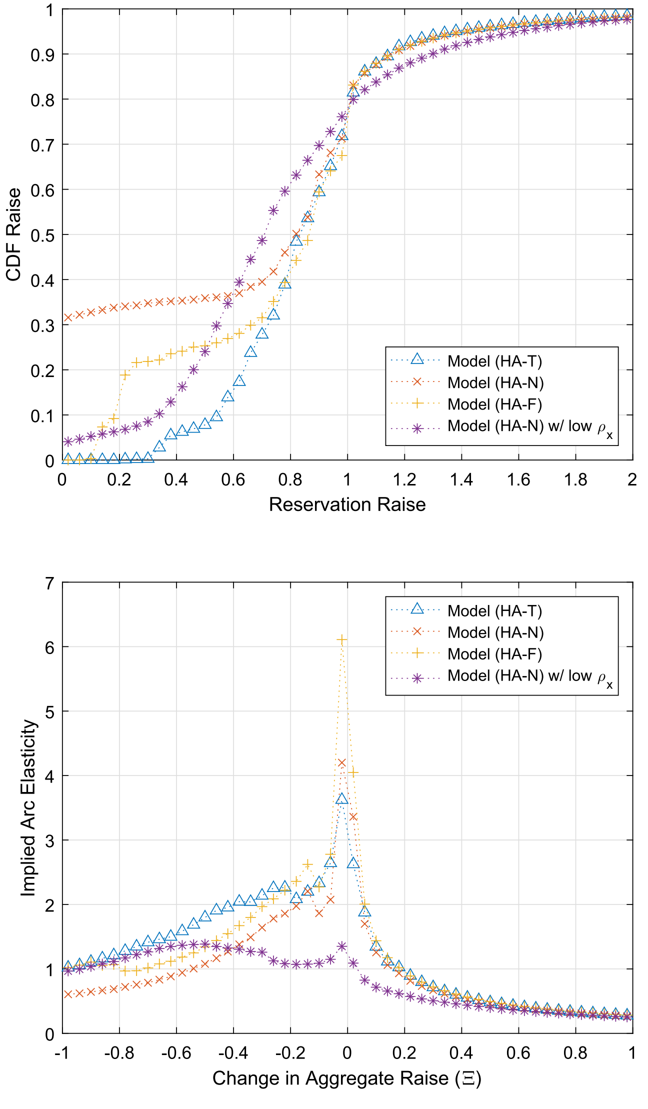

Figure A2. Aggregate labor supply elasticities and arc elasticities for different model specifications. *Note*: The arc elasticities (bottom panel) are computed, based on the reservation raise distribution that can be interpreted as a extensive-margin labor supply curve. The latter is smoothed as in Mui and Schoefer (2021). Specifically, we use moving averages with a window length of 5. The reservation raise value of ξ represents a gross percentage change in the agent's potential wage that would make the agent indifferent between working and nonworking, divided by 100. The bottom right panel is from a version of Model (HA-N) with $\rho_x = 0.929$ and $\sigma_x = 0.227$ (Chang and Kim (2007)).

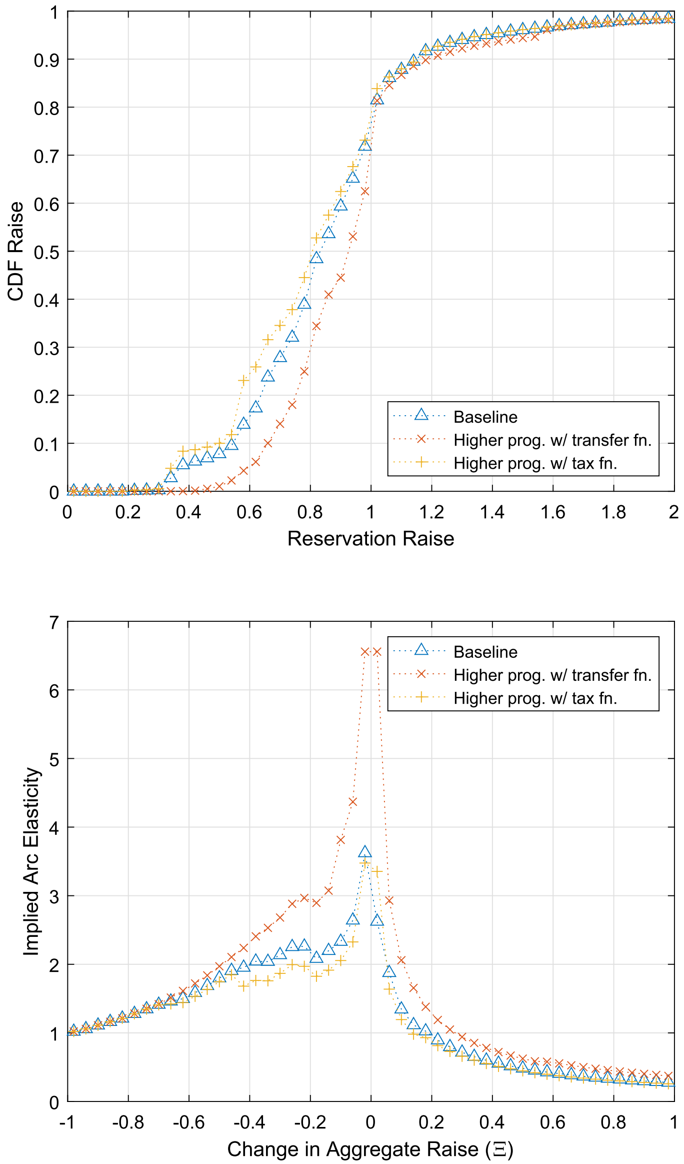

FIGURE A3. Aggregate labor supply elasticities and arc elasticities with higher progressivity. *Note*: We plot the reservation raise distribution or extensive-margin labor supply (top panel) and the corresponding arc elasticities (bottom panel) for two counterfactual exercises that increase progressivity in Section 5.3.

change from the counterfactual exercises conducted in Section 5.3 of the main paper.

Figure A4 plots the counterparts of Figure 3 in the main paper when a positive TFP shock is considered. Figure A5 plots percent changes in the market-clearing w_t and r_t following the negative TFP shock.

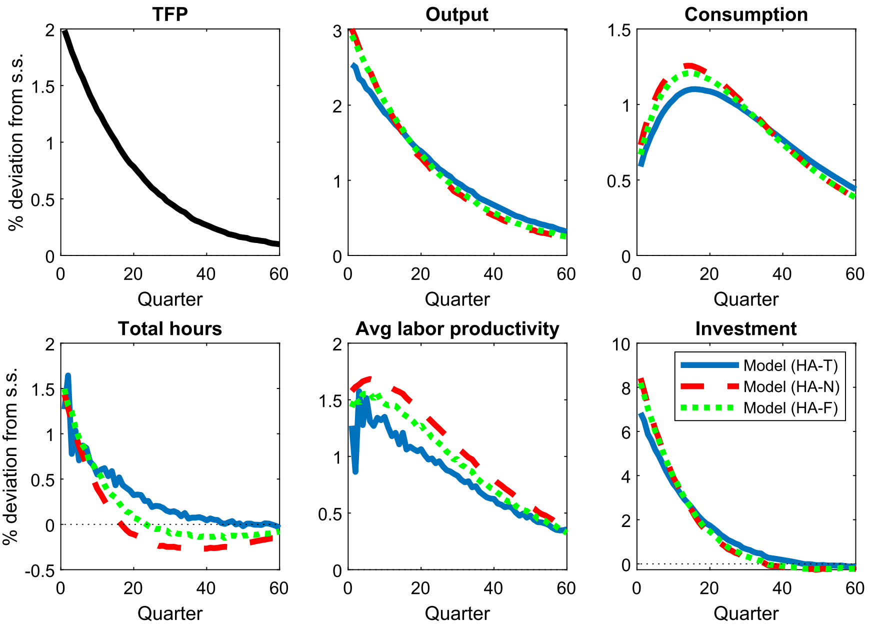

FIGURE A4. Impulse responses of macroeconomic aggregates with respect to positive TFP shocks. *Note*: TFP denotes total factor productivity. The figures display the IRFs of macroeconomic aggregates to a positive 2% TFP shock with persistence ρ_z.

Table A3 reports business cycle results for several alternative models recalibrated to match the same target statistics as in Table 1 in the main paper. First, we replace the progressive taxation system in equation (1) in the main paper with a linear taxation system while keeping the average tax constant. This is helpful for understanding how important the presence of progressive taxation is for business cycles while controlling for transfer progressivity. We find that its impact is very minimal for business cycle fluctuations. The

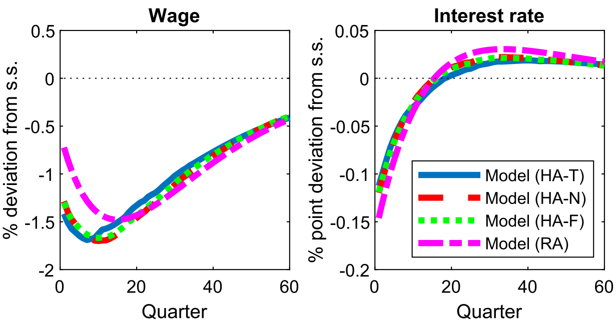

FIGURE A5. Impulse responses of equilibrium prices. *Note*: The figures display equilibrium market-clearing price responses, w_t and r_t, to a negative 2% TFP shock with persistence ρ_z. In heterogeneous-agent models, w_t captures the aggregate component of wages conditional on the worker selection in each period.

TABLE A3. Sensitivity checks.

	Baseline	Linear Taxation	$\underline{a}=0$	Gini Wage $=0.35$	Gini Wage $=0.37$
σ_Y	1.27	1.23	1.27	1.32	1.24
σ_C/σ_Y	0.27	0.28	0.27	0.26	0.27
σ_I/σ_Y	2.87	2.85	2.86	2.87	2.85
σ_L/σ_Y	0.50	0.47	0.50	0.53	0.48
σ_H/σ_Y	0.73	0.66	0.73	0.80	0.66
$\sigma_{Y/H}/\sigma_Y$	0.64	0.64	0.65	0.62	0.63
$\mathrm{Cor}(Y,C)$	0.85	0.87	0.84	0.84	0.85
$\mathrm{Cor}(Y,I)$	0.99	0.99	0.99	0.99	0.99
$\mathrm{Cor}(Y,L)$	0.92	0.91	0.92	0.94	0.92
$\mathrm{Cor}(Y,H)$	0.77	0.78	0.76	0.79	0.79
$\mathrm{Cor}(Y,Y/H)$	0.69	0.76	0.68	0.60	0.76
$\mathrm{Cor}(H,Y/H)$	0.07	0.18	0.04	−0.02	0.21

Note: Each alternative model is recalibrated to match the same target statistics as in the baseline model.

second sensitivity check concerns the borrowing limit. The third column in Table A3 reports the results from when we set $\underline{a}$ to zero, and these show that aggregate fluctuations are barely affected by this change. Next, we consider a change in target statistics regarding the variability of idiosyncratic shocks. Recall that the baseline model targets the Gini wage of 0.36. We find that, although its impact is not sizable, a higher wage variation tends to lower the cyclicality of average labor productivity and raise the relative volatility of hours.

TABLE A4. Probability of extensive margin adjustment, by wage quintile.

	The Length of Tracking Time T								
	5 years			10 years			15 years		
Wage quintile		Switches			Switches			Switches	
in base year	All	Pos only	Neg only	All	Pos only	Neg only	All	Pos only	Neg only
1st	0.146	0.097	0.049	0.119	0.079	0.039	0.103	0.070	0.033
2nd	0.093	0.060	0.033	0.080	0.052	0.029	0.072	0.046	0.026
3rd	0.075	0.045	0.030	0.066	0.040	0.027	0.063	0.039	0.024
4th	0.069	0.037	0.032	0.061	0.033	0.028	0.055	0.030	0.025
5th	0.072	0.040	0.032	0.062	0.033	0.029	0.060	0.031	0.029
Base years	1969–1993 ($J=25$)			1969–1988 ($J=20$)			1969–1983 ($J=15$)		
Avg. no. obs in base years	1677			1189			834		
Total no. obs.	41,920			23,783			12,514		
Avg. age	40.2			41.0			41.5		

Note: The full-time employment threshold is set to 1500 annual hours. Numbers in parentheses show the number of base years. We use samples whose age is between 22 and 64 (inclusive) and who are heads and are not self-employed. "All" refers to the baseline estimates when using both positive and negative switches, whereas "pos only" and "neg only" use only positive ones (i.e., $E_{i,t}=1$ and $E_{i,t-1}=0$) and only negative ones (i.e., $E_{i,t}=0$ and $E_{i,t-1}=1$), respectively.

TABLE A5. Full-time employment changes in recessions excluding samples with unemployment spells, by wage quintile.

	Recession				
	1973–76	1980–83	1990–92	2000–02	2006–10
Wage quintile in peak year					
1st	−10.7	−4.7	−7.8	−5.2	−8.6
2nd	−5.5	−0.8	−4.9	−3.6	−8.8
3rd	−6.6	−3.4	−4.7	−1.6	−6.4
4th	−4.2	−6.1	−3.5	−4.0	−7.2
5th	−5.2	−5.2	−4.1	−1.8	−4.7
No. obs.	1547	1481	1765	2454	2365

Note: The full-time employment threshold is set to 1000 annual hours. The year ranges denote the peak and trough years of each recession. Reported values are percentage changes in the full-time employment rate by wage quintiles (in the peak year of each recession) following the same set of individuals. Those who experienced unemployment spells in either the peak year or the trough year are excluded. The results for the first recession is omitted because the unemployment information is available only since the 1976 wave (or the year of 1975).

APPENDIX I: ADDITIONAL EMPIRICAL RESULTS

We provide additional results presented in Section 6 for sensitivity checks. Specifically, Table A4 reports the counterpart of Table 7 in the main paper when we use 1500 hours as a full-time threshold value. Table A5, Table A6 and Table A7 show the counterparts of Table 8 in the main paper when we exclude samples with unemployment spells only (Table A5) or when we use 1500 hours as a full-time threshold value only (Table A6) or when we consider both (Table A7).

TABLE A6. Full-time employment changes in recessions, by wage quintile.

	Recession					
	1969–71	1973–76	1980–83	1990–92	2000–02	2006–10
Wage quintile in peak year						
1st	−7.3	−10.4	−11.1	−7.1	−8.3	−17.9
2nd	−7.0	−10.5	−10.6	−8.3	−8.9	−16.3
3rd	−5.8	−8.2	−6.3	−7.7	−6.7	−14.9
4th	−4.2	−4.7	−8.0	−7.2	−5.8	−11.1
5th	−1.0	−3.9	−5.2	−3.3	−2.1	−7.4
No. obs.	1655	1756	2007	2166	2924	2802

Note: The full-time employment threshold is set to 1500 annual hours. The year ranges denote the peak and trough years of each recession. Reported values are percentage changes in the full-time employment rate by wage quintiles (in the peak year of each recession) following the same set of individuals.

TABLE A7. Full-time employment changes in recessions excluding samples with unemployment spells, by wage quintile.

	Recession				
	1973–76	1980–83	1990–92	2000–02	2006–10
Wage quintile in peak year					
1st	−8.5	−3.0	−7.2	−7.0	−9.0
2nd	−4.7	−4.3	−5.7	−6.4	−11.1
3rd	−6.1	−5.2	−6.9	−4.8	−8.7
4th	−4.4	−6.2	−3.9	−5.5	−8.7
5th	−2.5	−6.1	−2.4	−2.7	−5.2
No. obs.	1547	1481	1765	2454	2365

Note: The full-time employment threshold is set to 1500 annual hours. The year ranges denote the peak and trough years of each recession. Reported values are percentage changes in the full-time employment rate by wage quintiles (in the peak year of each recession) following the same set of individuals. Those who experienced unemployment spells in either the peak year or the trough year are excluded. The results for the first recession is omitted because the unemployment information is available only since the 1976 wave (or the year of 1975).

REFERENCES

Altonji, Joseph G. and Lewis M. Segal (1996), "Small-sample bias in GMM estimation of covariance structures." *Journal of Business & Economic Statistics*, 14 (3), 353–366. [11]

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry (2018), "Really uncertain business cycles." *Econometrica*, 86 (3), 1031–1065. [13]

Chang, Yongsung and Sun-Bin Kim (2006), "From individual to aggregate labor supply: A quantitative analysis based on a heterogeneous agent macroeconomy." *International Economic Review*, 47 (1), 1–27. [11]

Chang, Yongsung and Sun-Bin Kim (2007), "Heterogeneity and aggregation: Implications for labor-market fluctuations." *American Economic Review*, 97 (5), 1939–1956. [11, 13, 15]

Chang, Yongsung and Sun-Bin Kim (2014), "Heterogeneity and aggregation: Implications for labor-market fluctuations: Reply." *The American Economic Review*, 104 (4), 1461–1466. [12, 13]

Cociuba, Simona E., Edward C. Prescott, and Alexander Ueberfeldt (2018), "US hours at work." *Economics Letters*, 169, 87–90. [8]

Den Haan, Wouter J. (2010), "Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents." *Journal of Economic Dynamics and Control*, 34 (1), 79–99. [13, 14]

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2010), "The macroeconomic implications of rising wage inequality in the United States." *Journal of Political Economy*, 118 (4), 681–722. [10]

Khan, Aubhik and Julia K. Thomas (2008), “Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics.” *Econometrica*, 76 (2), 395–436. [13]

Koop, Gary, M. Hashem Pesaran, and Simon M. Potter (1996), “Impulse response analysis in nonlinear multivariate models.” *Journal of Econometrics*, 74 (1), 119–147. [13]

Kydland, Finn E. and Edward C. Prescott (1982), “Time to build and aggregate fluctuations.” *Econometrica*, 50 (6), 1345–1370. [7]

Mui, Preston and Benjamin Schoefer (2021), “Reservation raises: The aggregate labor supply curve at the extensive margin.” NBER Working Papers No. 28770. [15]

Rios-Rull, Victor (1999), “Computation of equilibria in heterogeneous agent models.” In *Computational Methods for the Study of Dynamic Economies*, Oxford University Press. [12]

Rogerson, Richard (1988), “Indivisible labor, lotteries and equilibrium.” *Journal of Monetary Economics*, 21 (1), 3–16. [6]

Rouwenhorst, K. Geert (1995), “Asset pricing implications of equilibrium business cycle models.” In *Frontiers of Business Cycle Research* (Thomas F. Cooley, ed.), 294–330, Princeton University Press, Princeton, NJ. [11]

Takahashi, Shuhei (2014), “Heterogeneity and aggregation: Implications for labor-market fluctuations: Comment.” *American Economic Review*, 104 (4), 1446–1460. [11, 12, 13]

Takahashi, Shuhei (2020), “Time-varying wage risk, incomplete markets, and business cycles.” *Review of Economic Dynamics*, 37, 195–213. [13]

Co-editor Kjetil Storesletten handled this manuscript.

Manuscript received 5 March, 2020; final version accepted 2 June, 2023; available online 12 July, 2023.