

## Supplement to “Rising skill premium and the dynamics of optimal capital and labor taxation”

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In this Online Appendix, we first describe our computation method. Then we develop and resolve simplified versions of our Ramsey and Mirrlees tax problems, respectively. Finally, we report the decomposition of the skill premium based on the capital-skill complementary effect and the relative quantity effect in the model.

### APPENDIX A: COMPUTATION METHOD

We compute the transitional dynamics of optimal taxation by using the following procedure:

1. Given the sequence  $\{q_t\}$  with  $q_0 = 1$ , we compute the steady state of the optimal allocation for the Ramsey problem and that of the constrained efficient allocation for the Mirrlees problem at each  $q_t$ . We then use these steady-state allocations as an initial guess for computing the transitional dynamics.
2. In the case of the Ramsey problem, we solve for the system of nonlinear equations (21)–(26) plus the implementability conditions (18), the resource constraints (13), the restriction (20), and the FOC with respect to  $\varphi$ . In the case of the Mirrlees problem, we solve for the system of nonlinear equations (41)–(45) plus the equality of the IC constraint (39) and the resource constraints (13). We substitute the resulting allocations into the IC constraints (39) and (40) to make sure that the incentive compatibility constraints for skilled and unskilled households are not violated.

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## APPENDIX B: SIMPLIFIED MODELS

## B.1 Ramsey problem

The Lagrangian for the Ramsey problem is as follows:

$$\max_{\{c_{it}, N_{st}, N_{ut}, K_{et+1}\}} \sum_{t=0}^{\infty} \beta^t \sum_{i \in \{s, u\}} \psi^i \left[ c_{it} - \chi \frac{N_{it}^2}{2} \right]$$

subject to the implementability conditions

$$\begin{aligned} & + \Psi^s \sum_{t=0}^{\infty} \beta^t \left[ c_{st} - \chi N_{st}^2 - (A_{s0} + T) \right], \\ & + \Psi^u \sum_{t=0}^{\infty} \beta^t \left[ c_{ut} - \chi N_{ut}^2 - (A_{u0} + T) \right], \end{aligned}$$

the resource constraints

$$+ \sum_{t=0}^{\infty} \beta^t \Gamma_t \left[ \mu N_{ut} + (1 - \mu) K_{et}^\lambda N_{st}^{1-\lambda} + \frac{K_{et}}{q_t} - \frac{K_{et+1}}{q_t} - C_t - G_t \right],$$

and the restriction that all households face the same labor marginal tax rate at any point in time

$$+ \sum_{t=0}^{\infty} \beta^t Y_t [N_{st} - N_{ut} \xi_t],$$

where we have used the FOCs of households to derive the implementability conditions and the last constraint invokes (31).<sup>1</sup>

The resulting FOCs are:

$$\psi^i + \Psi^i = \Gamma_t, \quad i = \{s, u\}, \quad (\text{B.1})$$

$$\Psi^s + \Psi^u = 0, \quad (\text{B.2})$$

$$\chi(\psi^u + 2\Psi^u)N_{ut} = \Gamma_t \mu - Y_t \xi_t, \quad (\text{B.3})$$

$$\chi(\psi^s + 2\Psi^s)N_{st} = \Gamma_t \mu \xi_t + Y_t \left[ 1 - N_{ut} \frac{\partial \xi_t}{\partial N_{st}} \right], \quad (\text{B.4})$$

$$\frac{\Gamma_t}{q_t} = \beta \left[ \Gamma_{t+1} \left( \frac{1}{q_{t+1}} + r_{et+1} \right) - Y_{t+1} N_{ut+1} \frac{\partial \xi_{t+1}}{\partial K_{et+1}} \right], \quad (\text{B.5})$$

where we have used (29) in the derivation of (B.4).

We obtain from (B.1)–(B.2):

$$\Psi^s = \frac{\psi^u - \psi^s}{2}; \quad \Psi^u = \frac{\psi^s - \psi^u}{2}; \quad \Gamma_t = \frac{\psi^s + \psi^u}{2} \equiv \Gamma. \quad (\text{B.6})$$

<sup>1</sup>We do not use the “market” weights  $\{\varphi^i\}_{i=\{s,u\}}$  here because the utility function takes the quasilinear form.

We also obtain<sup>2</sup>

$$Y_t = \frac{\mu \xi_t}{1 + \lambda} [\psi^u (1 - \tau_{L_t}) - \Gamma], \quad (\text{B.7})$$

$$Y_t = \frac{\mu}{\xi_t} [\Gamma - \psi^s (1 - \tau_{L_t})]. \quad (\text{B.8})$$

Putting (B.7) and (B.8) together yields equation (33).

Using (30), (31), (B.6), (B.7), and (33), we have from (B.5),

$$r_{et+1} = \frac{1}{\beta q_t} - \frac{1}{q_{t+1}} + \left( \frac{N_{st+1}}{K_{et+1}} \right) \frac{\lambda \mu \xi_{t+1}}{1 + \lambda} \left[ \frac{\psi^u - \psi^s}{\psi^s + \frac{\xi_{t+1}^2}{1 + \lambda} \psi^u} \right].$$

Substituting the above  $r_{et+1}$  in (32) and using (30) gives equation (34).

## B.2 Mirrlees problem

The Lagrangian for the Mirrlees problem is as follows:

$$\max_{\{c_{it}, N_{st}, N_{ut}, K_{et+1}\}} \sum_{t=0}^{\infty} \beta^t \sum_{i \in \{s, u\}} \psi^i \left[ c_{it} - \chi \frac{N_{it}^2}{2} \right]$$

subject to the IC constraint

$$+ \Lambda \sum_{t=0}^{\infty} \beta^t \left[ c_{st} - \chi \frac{N_{st}^2}{2} - c_{ut} + \chi \frac{1}{2} \left( \frac{N_{ut}}{\xi_t} \right)^2 \right],$$

and the resource constraints

$$+ \sum_{t=0}^{\infty} \beta^t \Gamma_t \left[ \mu N_{ut} + (1 - \mu) K_{et}^\lambda N_{st}^{1-\lambda} + \frac{K_{et}}{q_t} - \frac{K_{et+1}}{q_t} - C_t - G_t \right].$$

The resulting FOCs are

$$\psi^s + \Lambda = \Gamma_t; \quad (\text{B.9})$$

$$\psi^u - \Lambda = \Gamma_t,$$

$$-\psi^s \chi N_{st} + \Gamma_t w_{st} - \Lambda \left[ \chi N_{st} + \chi \frac{N_{ut}^2}{\xi_t^3} \frac{\partial \xi_t}{\partial N_{st}} \right] = 0, \quad (\text{B.10})$$

<sup>2</sup>Using (29), (30), and (B.6), we have from (B.4),

$$\psi^u \chi N_{st} = \Gamma w_{st} + Y_t \left[ 1 + \lambda \frac{N_{ut}}{N_{st}} \xi_t \right],$$

which by (31) and (29) gives (B.7). Using (B.6), we have from (B.3),

$$\psi^s \chi N_{ut} = \Gamma \mu - Y_t \xi_t,$$

which by (31) and (28) gives (B.8).

$$-\psi^u \chi N_{ut} + \Gamma_t w_{ut} + \Lambda \chi \frac{N_{ut}}{\xi_t^2} = 0, \quad (\text{B.11})$$

$$\frac{\Gamma_t}{q_t} = \beta \left[ \Gamma_{t+1} \left( \frac{1}{q_{t+1}} + r_{et+1} \right) - \Lambda \chi \frac{N_{ut+1}^2}{\xi_{t+1}^3} \frac{\partial \xi_{t+1}}{\partial K_{et+1}} \right]. \quad (\text{B.12})$$

From (B.9), we have

$$\Lambda = \frac{\psi^u - \psi^s}{2}; \quad \Gamma_t = \frac{\psi^u + \psi^s}{2} \equiv \Gamma. \quad (\text{B.13})$$

Using (46) and (B.13), we have equation (47) from (B.11). Using (30), (46), and (B.13), we have from (B.10),

$$1 - \tau_s(t) = \frac{\Gamma}{\psi^s + \Lambda} + \frac{\Lambda \lambda (1 - \tau_u(t)) w_{ut} (N_{ut}/N_{st})}{(\psi^s + \Lambda) \xi_t^2 w_{st}} = 1 + \frac{(\psi^u - \psi^s) \lambda (1 - \tau_u(t)) N_{ut}}{(\psi^u + \psi^s) \xi_t^3 N_{st}},$$

which implies  $\tau_s(t) < 0$ . From (46), we have  $\frac{N_{ut}}{N_{st}} = \frac{(1 - \tau_u(t))}{(1 - \tau_s(t))} \frac{1}{\xi_t}$ . Substituting this result in the above equation leads to

$$[1 - \tau_s(t)]^2 - [1 - \tau_s(t)] - \frac{(\psi^u - \psi^s) \lambda (1 - \tau_u(t))^2}{(\psi^u + \psi^s) \xi_t^4} = 0,$$

which in turn leads to equation (48).<sup>3</sup>

Using (30) and (B.13), we have from (B.12),

$$r_{et+1} = \frac{1}{\beta q_t} - \frac{1}{q_{t+1}} + \frac{\lambda \chi (\psi^u - \psi^s)}{(\psi^u + \psi^s)} \frac{1}{\xi_{t+1}^2} \frac{N_{ut+1}^2}{N_{st+1}} \frac{N_{st+1}}{K_{et+1}}.$$

Substituting the above  $r_{et+1}$  in (32) and using (28), (30), (46), and (47) gives equation (49).

## APPENDIX C: EVOLUTION OF SKILL PREMIUM

In the face of increasing  $q_t$  (ESTP), we report how  $\ln \xi$  in (11) evolves over time under optimal taxation.

### C.1 Ramsey taxation

The right-hand panel of Figure C.1 reports the evolution of  $K_e/N_s$  and  $N_u/N_s$  over time in the face of an increasing  $q_t$ . According to (11), while  $K_e/N_s$  is the sole allocation that determines the capital-skill complementarity effect of the skill premium,  $N_u/N_s$  is the sole allocation that determines the relative quantity effect of the skill premium. The evolutions of  $K_e/N_s$  and of  $N_u/N_s$  shown in the right-hand panel of Figure C.1 explain why the skill premium shown in the left-hand panel of Figure C.1 is on the rise. Simply put, the capital-skill complementarity effect dominates the relative quantity effect.

<sup>3</sup>The other root of the quadratic equation is ruled out since it implies that  $\tau_s(t) \geq 0$ .

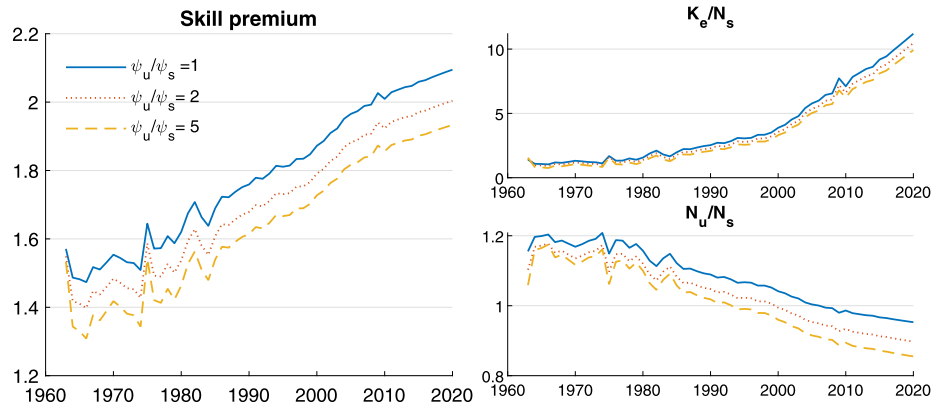


FIGURE C.1. Log skill premium and allocations.

### C.2 *Mirrleesian taxation*

The figure for Mirrleesian taxation resembles Figure C.1 for Ramsey taxation.

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