Supplement to "Uncertainty-driven business cycles: Assessing the markup channel"

(Quantitative Economics, Vol. 12, No. 2, May 2021, 587-623)

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Appendix A: Theoretical model

A.1 Model equations

The model equations after imposing a symmetric equilibrium are given by:

1. Production function:

$$Y_{t} = Y^{\text{norm}} \Big[\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \big(Z_{t} \big(N_{t} - N^{o} \big) \big)^{\frac{\psi-1}{\psi}} \Big]^{\frac{\psi}{\psi-1}} - \Phi$$
(A.1)

2. Firm FOC for renting N_t :

$$\Xi_{p,t} \frac{W_t}{P_t} = MPL_t, \tag{A.2}$$

3. Definition marginal product of labor

$$MPL_{t} = Y^{\text{norm}} \Big[\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \Big(Z_{t} \big(N_{t} - N^{o} \big) \Big)^{\frac{\psi-1}{\psi}} \Big]^{\frac{\psi}{\psi-1}-1} \\ \times \frac{(1-\alpha) \Big(Z_{t} \big(N_{t} - N^{o} \big) \Big)^{\frac{\psi-1}{\psi}}}{N_{t} - N^{o}},$$
(A.3)

which, in the presence of no overhead labor and fixed costs, simplifies to

$$MPL_t = (1 - \alpha)(Z_t)^{\frac{\psi - 1}{\psi}} \left(\frac{Y_t}{N_t}\right)^{\frac{1}{\psi}}$$

4. Firm profits:

$$D_t = Y_t - N_t \frac{W_t}{P_t} - I_t - \frac{\phi_P}{2} (\Pi_t - \bar{\Pi})^2 Y_t$$
(A.4)

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5. Firm FOC for renting K_t :

$$\Xi_{p,t}R_t^K = MPK_t, \tag{A.5}$$

6. Definition marginal product of capital

$$MPK_{t} = Y^{\text{norm}} \Big[\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \Big(Z_{t} \big(N_{t} - N^{o} \big) \Big)^{\frac{\psi-1}{\psi}} \Big]^{\frac{\psi}{\psi-1} - 1} \frac{\alpha K_{t}^{\frac{\psi-1}{\psi}}}{K_{t}}$$
(A.6)

which, in the presence of no fixed costs, simplifies to

$$MPK_t = \alpha \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\psi}}$$

7. Firm FOC for P_t :

$$\begin{split} \phi_p \bigg[\Pi^{-1} \frac{P_t}{P_{t-1}} - 1 \bigg] \Pi^{-1} \frac{P_t}{P_{t-1}} \\ &= \frac{\phi_P \theta_p}{2} \bigg(\Pi^{-1} \frac{P_t}{P_{t-1}} - 1 \bigg)^2 + (1 - \theta_p) + \theta_p \Xi_{p,t}^{-1} \\ &+ \phi_p \mathbb{E}_t \bigg\{ M_{t+1} \frac{Y_{t+1}}{Y_t} \bigg[\Pi^{-1} \frac{P_{t+1}}{P_t} - 1 \bigg] \bigg[\Pi^{-1} \frac{P_{t+1}}{P_t} \bigg] \bigg\}, \end{split}$$
(A.7)

where M_t is the stochastic discount factor defined below.

8. Firm FOC for capital:

$$q_t = E_t M_{t+1} \left(R_{t+1}^k + (1-\delta) q_{t+1} \right)$$
(A.8)

9. Firm FOC for investment:

$$1 = q_t \left(1 - \frac{\phi_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi_k \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) + \phi_k E_t M_{t+1} q_{t+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$
(A.9)

10. Definition value function:

$$V_t = \frac{\left(C_t^{\eta} (1 - N_t)^{1 - \eta}\right)^{1 - \sigma}}{1 - \sigma} + \beta \mathbb{E}_t V_{t+1}$$
(A.10)

11. Partial derivative of lifetime utility with respect to consumption:

$$V_{C,t} = \eta \frac{1}{C_t} \left(C_t^{\eta} (1 - N_t)^{1 - \eta} \right)^{1 - \sigma}$$
(A.11)

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12. FOC with respect to W:

$$0 = V_{N_{t}}(-\theta_{w})N_{t} + \frac{V_{C,t}}{1+\tau_{t}^{c}} \bigg[(1-\theta_{w}) \big(1-\tau_{t}^{n}\big)N_{t}\frac{W_{t}}{P_{t}} - \phi_{w} \bigg(\Pi^{-1}\frac{W_{t}}{W_{t-1}} - 1\bigg)\frac{W_{t}}{\Pi W_{t-1}}Y_{t} \bigg] \\ + \beta \frac{V_{C,t+1}}{1+\tau_{t+1}^{c}} \bigg[\phi_{w} \bigg(\Pi^{-1}\frac{W_{t+1}}{W_{t}} - 1\bigg)\Pi^{-1}\frac{W_{t+1}}{W_{t}}Y_{t+1} \bigg],$$
(A.12)

13. Partial derivative of lifetime utility with respect to labor:

$$V_{N,t} = -(1-\eta) \frac{1}{1-N_t} \left(C_t^{\eta} (1-N_t)^{1-\eta} \right)^{1-\sigma}$$
(A.13)

14. Definition stochastic discount factor:

$$M_{t+1} \equiv \frac{\frac{\partial V_t}{\partial C_{t+1}}}{\frac{\partial V}{\partial C_t}} \frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \beta \frac{1+\tau_t^c}{1+\tau_{t+1}^c} \left(\frac{C_{t+1}^{\eta}(1-N_{t+1})^{1-\eta}}{C_t^{\eta}(1-N_t)^{1-\eta}}\right)^{1-\sigma} \left(\frac{C_t}{C_{t+1}}\right)$$
(A.14)

15. Euler equation

$$1 = R_t \mathbb{E}_t \{ M_{t+1} \Pi_{t+1}^{-1} \}$$
(A.15)

16. Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left(\left(\frac{\Pi_t}{\Pi}\right)^{\phi_{R\pi}} \left(\frac{Y_t}{Y_t^{\text{HP}}}\right)^{\phi_{Ry}}\right)^{1-\rho_R}.$$
(A.16)

17. Law of motion for capital:

$$K_{t+1} = K_t(1-\delta) + I_t \left(1 - \frac{\phi_k}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)$$
(A.17)

18. Definition of model-consistent HP-filter output gap:

$$\begin{aligned} Y_{t}^{\text{HP}}(1+6\times1600) + Y_{t-1}^{\text{HP}}(-4\times1600) + E_{t}Y_{t+1}^{\text{HP}}(-4\times1600) \\ &+ Y_{t-2}^{\text{HP}}\times1600 + E_{t}Y_{t+2}^{\text{HP}}1600 \\ &= Y_{t}(6\times1600) + Y_{t-1}(-4\times1600) + E_{t}Y_{t+1}(-4\times1600) \\ &+ Y_{t-1}1600 + E_{t}Y_{t+1}1600 \end{aligned} \tag{A.18}$$

19. Budget constraint household after imposing that $B_t/P_t = 0 \forall t$:¹

$$(1+\tau_t^c)C_t = (1-\tau_t^n)\frac{W_t}{P_t}N_t + C_t - \frac{\phi_w}{2}\left(\Pi^{-1}\frac{W_t}{W_{t-1}} - 1\right)^2 Y_t + T_t + D_t$$
(A.19)

20. Budget constraint government:

$$\tau_t^c C_t + \tau_t^n \frac{W_t}{P_t} N_t = G_t + T_t.$$
(A.20)

¹Note that for the purpose of model simulations, we set $\tau_t^c = \tau^c$ and $\tau_t^n = \tau^n$.

These 20 equations define the evolution of the following 20 variables: C_t , I_t , K_t , D_t , M_t , MPL_t , MPK_t , N_t , Π_t , q_t , R_t , R_t^K , T_t , V_t , $V_{C,t}$, $V_{N,t}$, $\frac{W_t}{P_t}$, $\Xi_{p,t}$, Y_t , Y_t^{HP} .

Finally, the exogenous processes for \hat{Z}_t , σ_t^z , \hat{G}_t , and σ_t^g are given by

$$\hat{Z}_t = \rho_z \hat{Z}_{t-1} + \sigma_t^z \varepsilon_t^z, \tag{A.21}$$

$$\hat{G}_{t} = \rho_{g}\hat{G}_{t-1} + \phi_{gy}\hat{Y}_{t-1} + \sigma_{t}^{g}\varepsilon_{t}^{g}, \qquad (A.22)$$

$$\sigma_t^z = (1 - \rho_{\sigma^z})\bar{\sigma}^z + \rho_{\sigma^z}\sigma_{t-1}^z + \eta_{\sigma^z}\varepsilon_t^{\sigma^z}, \qquad (A.23)$$

$$\sigma_t^g = (1 - \rho_{\sigma^g})\bar{\sigma}^g + \rho_{\sigma^g}\sigma_{t-1}^g + \eta_{\sigma^g}\varepsilon_t^{\sigma^g}.$$
(A.24)

A.2 Additional derivations for model calibration

A.2.1 *Frisch elasticity* This section shows how to compute the Frisch elasticity of labor supply for our model. The resulting expression will be used in steady-state computations to determine the weight of leisure in the Cobb–Douglas felicity function, that is, when determining η . As shown in, for example, Domeij and Floden (2006), the Frisch elasticity η^{λ} can be computed from

$$\eta^{\lambda} = \frac{U_N(C, N)}{\left(U_{NN}(C, N) - \frac{U_{CN}^2(C, N)}{U_{CC}}(C, N)\right)} \frac{1}{N}.$$
(A.25)

For the felicity function,

$$U(C,N) = \frac{\left(C^{\eta}(1-N)^{1-\eta}\right)^{1-\sigma}}{1-\sigma} = \frac{C^{\eta(1-\sigma)}(1-N)^{(1-\eta)(1-\sigma)}}{1-\sigma},$$
(A.26)

we get

$$U_N = -(1-\eta) \left(C^{\eta} \right)^{1-\sigma} (1-N)^{(1-\eta)(1-\sigma)-1} = -(1-\eta)(1-\sigma) \frac{U(C,N)}{(1-N)}, \quad (A.27)$$

$$U_{NN} = (1 - \eta)(1 - \sigma) \left((1 - \eta)(1 - \sigma) - 1 \right) \frac{U(C, N)}{(1 - N)^2},$$
(A.28)

$$U_C = \eta C^{\eta(1-\sigma)-1} (1-N)^{(1-\eta)(1-\sigma)} = \eta (1-\sigma) \frac{U(C,N)}{C},$$
(A.29)

$$U_{CC} = \eta \big(\eta (1 - \sigma) - 1 \big) (1 - \sigma) \frac{U(C, N)}{C^2},$$
(A.30)

$$U_{CN} = -\eta (1 - \eta)(1 - \sigma)C^{\eta(1 - \sigma) - 1}(1 - N)^{(1 - \eta)(1 - \sigma) - 1}$$

= $-\eta (1 - \eta)(1 - \sigma)(1 - \sigma)\frac{U(C, N)}{C(1 - N)}.$ (A.31)

After a lot of tedious algebra, we get that

$$\eta^{\lambda} = \frac{U_N(C,N)}{\left(U_{NN}(C,N) - \frac{U_{CN}^2(C,N)}{U_{CC}}(C,N)\right)} \frac{1}{N} = \frac{1 - \eta(1-\sigma)}{1 - (1-\sigma)} \frac{1-N}{N}.$$
 (A.32)

Supplementary Material

A.3 Steady state

The stochastic discount factor, equation (A.14), in steady state evaluates to

$$M = \beta, \tag{A.33}$$

while the first-order condition for investment, equation (A.9), gives Tobin's marginal q as

$$q = 1. \tag{A.34}$$

Plugging this into (A.8) yields

$$R_K = \frac{1}{\beta} - (1 - \delta) \tag{A.35}$$

and the pricing FOC (A.7) in steady state implies that

$$\Xi_{t,p} = \frac{\theta_p}{\theta_p - 1}.\tag{A.36}$$

The wage setting FOC (A.12) implies

$$V_N = \frac{V_C}{1 + \tau^c} \bigg[(\theta_w - 1) \big(1 - \tau_t^n \big) \frac{W}{P} N \bigg].$$
(A.37)

Using the definition of marginal utility, (A.11),

$$V_C = \eta \frac{\left(C^{\eta} (1-N)^{1-\eta}\right)^{1-\sigma}}{C}$$
(A.38)

and the definition of V_N , (A.13),

$$V_N = -(1-\eta) \frac{\left(C^{\eta}(1-N)^{1-\eta}\right)^{1-\sigma}}{1-N},$$
(A.39)

equation (A.37) reduces to

$$\frac{1-\eta}{1-N}\theta_w = \frac{\eta}{1+\tau^c} \frac{1}{C} \bigg[(\theta_w - 1) \big(1-\tau^n\big) \frac{W}{P} \bigg].$$
(A.40)

With net output normalized to 1 by appropriately setting Y^{norm} , which is determined later, and the labor and capital share given by \aleph and $1 - \aleph$, respectively, we have

$$\aleph = \frac{\frac{W}{P}N}{Y} = \frac{\frac{W}{P}N}{1} \quad \Rightarrow \quad W/P = \frac{\aleph}{N} \tag{A.41}$$

and similarly

$$K = \frac{1 - \aleph}{R^K}.\tag{A.42}$$

Equation (A.42) can be used with (A.35) to directly compute *K* and via the law of motion for capital, equation (A.17), also investment

$$I = \delta K. \tag{A.43}$$

Next, substituting for the real wage in (A.40) from (A.41), one obtains

$$\frac{1-\eta}{\eta}\frac{C}{1-N} = \frac{\theta_w - 1}{\theta_w}\frac{1-\tau^n}{1+\tau^c}\frac{\aleph}{N}.$$
(A.44)

Solving this equation for consumption yields

$$C = \frac{\theta_w - 1}{\theta_w} \frac{1 - \tau^n}{1 + \tau^c} \otimes \frac{1 - N}{N} \frac{\eta}{1 - \eta}.$$
(A.45)

Consolidating the household and government budget constraints, equations (A.19) and (A.20), and using equation (A.43) and the definition of firm dividends, equation (A.4), yields

$$C + \delta K = Y = 1. \tag{A.46}$$

Plugging in from (A.45) for consumption yields

$$\frac{\theta_w - 1}{\theta_w} \frac{1 - \tau^n}{1 + \tau^c} \otimes \frac{1 - N}{N} \frac{1 - \eta}{\eta} + \delta K = 1, \tag{A.47}$$

where K is already known from (A.42).

The Frisch elasticity η^{λ} is calibrated to 1. From (A.32) then follows that

$$\eta = \frac{\theta}{1 - \sigma} \left[1 - \eta^{\lambda} \left(1 - \frac{1 - \sigma}{\theta} \right) \frac{N}{1 - N} \right].$$
(A.48)

Plugging (A.48) into (A.47), one obtains a nonlinear equation for N:

$$0 = \frac{\theta_w - 1}{\theta_w} \frac{1 - \tau^n}{1 + \tau^c} \approx \frac{1 - N}{N} \frac{1 - \frac{1}{1 - \sigma} \left(1 - (1 - 1 - \sigma) \frac{N}{1 - N} \right)}{\frac{1}{1 - \sigma} \left(1 - (1 - 1 - \sigma) \frac{N}{1 - N} \right)} + \delta K - 1.$$
(A.49)

This equation is solved numerically for hours worked *N*. Consumption immediately follows from (A.45), η from (A.48), the real wage from (A.41), and dividends from (A.4).

Up to this point, we have assumed that net output is normalized to 1. We are now in a position to compute the variables and parameters of the production side of our model, including the normalizing technology factor Y^{norm} that allowed working with Y = 1.

Fixed costs Φ are set equal to steady-state profits, which are the difference between output and factor payments:

$$\Phi = Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^o \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}} - K R^K - W N.$$
(A.50)

With technology being in steady state, that is, Z = 1, the firm FOCs, equations (A.2)–(A.6), imply

$$R^{K} = \Xi Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}-1} \alpha K^{\frac{\psi-1}{\psi}-1}, \tag{A.51}$$

$$\frac{W}{P} = \Xi Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^o \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}-1} (1-\alpha) \left(N - N^o \right)^{\frac{\psi-1}{\psi}-1}$$
(A.52)

so that (A.50) with $N^o = \phi_o N$ becomes

$$\begin{split} \Phi &= Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}} \\ &- \Xi Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1} - 1} \alpha K^{\frac{\psi-1}{\psi}} \\ &- \Xi Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1} - 1} (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \frac{N}{N - N^{o}} \\ &= Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{1}{\psi-1}} \\ &\times \left(1 - \Xi \frac{\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \frac{1}{(1-\phi_{o})}}{\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N_{t} - N^{o} \right)^{\frac{\psi-1}{\psi}}} \right). \end{split}$$
(A.53)

In the absence of overhead labor, this reduces to

$$\Phi = (1 - \Xi) Y^{\text{norm}} \left(\alpha K^{\frac{\psi - 1}{\psi}} + (1 - \alpha) N^{\frac{\psi - 1}{\psi}} \right)^{\frac{1}{\psi - 1}}.$$
(A.54)

Net output *Y* is given by production minus fixed costs:

$$Y = Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}} - \Phi$$

$$\stackrel{(A.53)}{=} Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}}$$

$$\times \Xi \frac{\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \frac{1}{(1-\phi_{o})}}{\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}}}, \qquad (A.55)$$

which in the absence of overhead labor reduces to

$$Y = Y^{\text{norm}} \left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^o \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}}.$$

Equation (A.55) implies that the normalizing technology factor Y^{norm} is given by

$$Y^{\text{norm}} = \left[\left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)^{\frac{\psi}{\psi-1}} \times \Xi \frac{\left(\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \frac{1}{(1-\phi_{o})} \right)}{\left(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(N - N^{o} \right)^{\frac{\psi-1}{\psi}} \right)} \right]^{-1}.$$
 (A.56)

All the previous equations require knowledge of the labor share parameter α , which is not a true structural parameter in the sense that it depends on the units of the model variables (see Cantore and Levine (2012), for details). It can be computed from the actual labor share \aleph using

$$1 - \aleph$$

$$= \frac{KR^{K}}{Y}$$

$$= \frac{K\overline{Z}Y^{\text{norm}}(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha)(N-N^{o})^{\frac{\psi-1}{\psi}})^{\frac{\psi-1}{\psi-1}-1}\alpha K^{\frac{\psi-1}{\psi}-1}}{Y^{\text{norm}}(\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha)(N-N^{o})^{\frac{\psi-1}{\psi}})^{\frac{\psi}{\psi-1}}\overline{\Xi}\frac{\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha)(N-N^{o})^{\frac{\psi-1}{\psi}}\frac{1}{1-\phi_{o}}}{\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha)(N-N^{o})^{\frac{\psi-1}{\psi}}}$$

$$= \frac{\alpha K^{\frac{\psi-1}{\psi}}}{\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha)(N-\bar{N}^{o})^{\frac{\psi-1}{\psi}}\frac{1}{1-\phi_{o}}}.$$
(A.57)

Solving for α yields

$$\alpha = \frac{\aleph (N - N^o)^{\frac{\psi - 1}{\psi}} \frac{1}{1 - \phi_o}}{(1 - \aleph)K^{\frac{\psi - 1}{\psi}} + \aleph (N - N^o)^{\frac{\psi - 1}{\psi}} \frac{1}{1 - \phi_o}},$$
(A.58)

allowing us to compute the normalizing technology factor Y^{norm} from (A.56) and the fixed costs Φ from (A.53).

We also need to compute the steady states of our auxiliary variables in the model. In steady state, the wage markup between marginal rate of substitution is

$$MRS = \frac{1 - \eta}{\eta} \frac{C}{1 - N},\tag{A.59}$$

while the real wage is given by

$$\Xi^w = \frac{\theta_w}{\theta_w - 1}.\tag{A.60}$$

A.4 Particle filter details and smoothed volatilities

We employ a Sequential Importance Resampling (SIR) filter (Gordon, Salmond, and Smith (1993)) with 20,000 particles to construct the likelihood of the stochastic volatility processes. Draws from the posterior are generated using the Metropolis–Hastings algorithm. We generate a Monte Carlo Markov Chain with 205,000 draws of which 5000 are used as a burn-in. As the proposal density, we use a multivariate normal distribution with the identity matrix as the covariance matrix, scaled to achieve an acceptance rate of about 25%. Smoothed objects are constructed using the backward-smoothing routine of Godsill, Doucet, and West (2004) with 20,000 particles for the smoother. More details can be found in Appendix B of Born and Pfeifer (2014).



FIGURE A.1. Median smoothed volatilities from the particle smoother, based on 20,000 particles for the forward pass and 20,000 particles for the backward smoothing routine. Shaded areas denote 90% highest posterior density intervals.

A.5 Convergence diagnostics

TABLE A.1.	Geweke (1992)	convergence tests,	based on means	of draws	5001 to	45001 vs	3. 105001
to 205000. p	-values are for ;	χ^2 -test for equality	of means.				

Parameter	Mean	Std	No Taper	4% Taper	8% Taper	15% Taper
ρ_{σ^z}	0.5177	0.1242	0.0591	0.8851	0.8846	0.8869
ρ_z	0.7730	0.0498	0.0000	0.2467	0.2427	0.2611
η_{σ^z}	0.0023	0.0003	0.2213	0.8546	0.8504	0.8503
$\overline{\sigma}^{z}$	0.0070	0.0006	0.1086	0.5670	0.5420	0.5090

TABLE A.2. Geweke (1992) convergence tests, based on means of draws 5001 to 45001 vs. 105001 to 205000. *p*-values are for χ^2 -test for equality of means.

Parameter	Mean	Std	No Taper	4% Taper	8% Taper	15% Taper
ρ_{σ^g}	0.5041	0.1226	0.0000	0.1551	0.1687	0.1578
ρ_g	0.9380	0.0343	0.0000	0.4864	0.5655	0.6271
η_{σ^g}	0.0030	0.0004	0.1448	0.8583	0.8514	0.8298
$\overline{\sigma}^{g}$	0.0076	0.0007	0.1993	0.8189	0.8289	0.8423
ϕ_{gy}	0.0222	0.0343	0.0001	0.4009	0.4399	0.5008

A.6 Additional model IRFs



FIGURE A.2. Model IRFs with sticky prices and flexible wages. *Notes:* IRFs to a two-standard deviation shock measured in percentage deviations from the stochastic steady state.



FIGURE A.3. Model IRFs with sticky wages and flexible prices. *Notes:* IRFs to a two-standard deviation shock measured in percentage deviations from the stochastic steady state.



FIGURE A.4. Model IRFs with flexible prices and wages. *Notes*: IRFs to a two-standard deviation shock measured in percentage deviations from the stochastic steady state.



FIGURE A.5. Model IRFs to level shocks with sticky prices and wages. *Notes:* IRFs to a one-standard deviation shock, measured in percentage deviations or percentage point deviations (annualized inflation and interest rates) from the stochastic steady state.



FIGURE A.6. Model IRFs to a two-standard deviation technology uncertainty shock using our estimated TFP process (left panel) and the TFP process estimated in Leduc and Liu (2016) (right panel), with the model solved at order 4 instead of 3 as in the baseline. The left panel displays the output response for the baseline calibration (solid line), the baseline calibration with higher firm risk aversion (dashed line), and the latter calibration with lower real and higher nominal rigidities as in Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez (2015) (dotted line). The right panel combines the last calibration with the Leduc and Liu (2016) TFP process. See the main text for details. *Notes:* IRFs measured in percentage deviations from the stochastic steady state.



FIGURE A.7. Output responses under different four-standard deviation technology uncertainty shock scenarios using the TFP process estimated in Leduc and Liu (2016) with higher firm risk aversion as well as lower real and higher nominal rigidities as in Fernández-Villaverde et al. (2015), with the model solved at order 4. The solid line shows two times the response to a two-s-tandard deviation technology uncertainty shock, i.e. what would happen if the model scaled linearly in the shock size. The dashed line displays the response to a true four-standard deviation shock, while the yellow dotted line shows the response to a cascading four-standard deviation uncertainty shock, that is, two consecutive two-standard deviation shocks happening at t = 1 and t = 2. Notes: IRFs measured in percentage deviations from the stochastic steady state.

APPENDIX B: MARGINAL PRODUCT OF LABOR FOR MARKUP COMPUTATION Given our production function, the marginal product of labor is equal to

$$MPL_{t} = Y^{\text{norm}} \Big[\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \Big(Z_{t} \big(N_{t} - N^{o} \big) \Big)^{\frac{\psi-1}{\psi}} \Big]^{\frac{\psi}{\psi-1}-1} \\ \times \frac{(1-\alpha) \big(e^{Z_{t}} \big(N_{t} - N^{o} \big) \big)^{\frac{\psi-1}{\psi}}}{N_{t} - N^{o}}.$$
(B.1)

This is equal to

$$MPL_{t} = \left(Y^{\text{norm}} \left[\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(e^{Z_{t}} \left(N_{t} - N^{o}\right)\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}\right)^{\frac{1}{\psi}} \\ \times \left(Y^{\text{norm}}\right)^{\frac{\psi-1}{\psi}} \frac{(1-\alpha) \left(e^{Z_{t}} \left(N_{t} - N^{o}\right)\right)^{\frac{\psi-1}{\psi}}}{N_{t} - N^{o}}.$$
(B.2)

Using (A.1), we have that

$$Y_t + \Phi = Y^{\text{norm}} \Big[\alpha K_t^{\frac{\psi - 1}{\psi}} + (1 - \alpha) \Big(e^{Z_t} \big(N_t - N^o \big) \Big)^{\frac{\psi - 1}{\psi}} \Big]^{\frac{\psi}{\psi - 1}}$$
(B.3)

so that

$$MPL_t = (1 - \alpha) \left(Y^{\text{norm}} \right)^{\frac{\psi - 1}{\psi}} \left(e^{Z_t} \right)^{\frac{\psi - 1}{\psi}} \left(\frac{Y_t + \Phi}{N_t - N^o} \right)^{\frac{1}{\psi}}.$$
 (B.4)

In case of no fixed costs and no overhead labor, this reduces to the familiar

$$MPL_t = (1 - \alpha) \left(Y^{\text{norm}} \right)^{\frac{\psi - 1}{\psi}} \left(e^{Z_t} \right)^{\frac{\psi - 1}{\psi}} \left(\frac{Y_t}{N_t} \right)^{\frac{1}{\psi}}.$$
 (B.5)

In logs, we have from (B.4)

$$\log(MPL_t) = \log\left((1-\alpha)\left(Y^{\text{norm}}\right)^{\frac{\psi-1}{\psi}}\right) + \frac{\psi-1}{\psi}\log\left(e^{Z_t}\right) + \frac{1}{\psi}\log\left(\frac{Y_t + \Phi}{N_t - N^o}\right), \tag{B.6}$$

where the first term is a constant that depends on the units of measurement. For the second term, we need a measure of labor-augmenting technology Z_t . Thus, the price markup can be computed as

$$\xi_t^p = \log\left((1-\alpha)\left(Y^{\text{norm}}\right)^{\frac{\psi-1}{\psi}}\right) + \frac{\psi-1}{\psi}Z_t + \frac{1}{\psi}\log\left(\frac{Y_t + \Phi}{N_t - N^o}\right) - \log\left(\frac{W_t}{P_t}\right). \tag{B.7}$$

Technology movements are approximated using the Fernald (2012) utilizationadjusted TFP measure. This TFP measure, based on growth accounting, originally assumes a unit elasticity of output with respect to technology, which would correspond to Hicks-neutral technology growth. Starting from a general production function

$$Y = Y(K, L, TFP), \tag{B.8}$$

the contribution of TFP to output growth is effectively computed via the total differential as the part of output growth not accounted for by utilization adjusted factor growth:

$$\frac{dTFP_t}{TFP_t} = \frac{dY_t}{Y_t} - \varepsilon_{K,t} \frac{dK_t}{K_t} - \varepsilon_{N,t} \frac{dN_t}{N_t},\tag{B.9}$$

where ε denotes the respective output elasticities and where by construction $\varepsilon_{TFP,t} = 1$. Thus, we need to transform this TFP measure to correspond to our measure of laboraugmenting (Kaldor-neutral) technology $A_t = e^{Z_t}$ as

$$\frac{dTFP_t}{TFP_t} = \varepsilon_{A,t} \frac{dA_t}{A_t} \quad \Rightarrow \quad \log A_t = \frac{1}{\varepsilon_{A,t}} \log TFP_t, \tag{B.10}$$

where the integration constant has been set to 0. Thus, when knowing the elasticity of TFP with respect to labor-augmenting technology, $\varepsilon_{A,t}$, the Fernald (2012) measure can be transformed into our required technology measure.² As $\varepsilon_{A,t}$ is invariant to multiplicative transformations of output, we first normalize output by steady state/balanced growth path output Y to get gross deviations from steady state.³

$$\hat{Y} = \frac{Y_t}{Y} = \frac{\left[\alpha K_t^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(Ae^{\hat{Z}_t} \left(N_t - N^o\right)\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}} - \Phi}{\left[\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(A \left(N - N^o\right)\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}} - \Phi},$$
(B.11)

where *A* is a constant capturing the unknown level of labor-augmenting technology and all other normalizations, for example, the one introduced by using an index for output.

Noting that in steady state

$$Y = \frac{1}{(1+\phi_{\text{fix}})} \Big[\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \Big(A \Big(N - N^o \Big) \Big)^{\frac{\psi-1}{\psi}} \Big]^{\frac{\psi}{\psi-1}}, \tag{B.12}$$

$$\Phi = \frac{\phi_{\text{fix}}}{(1+\phi_{\text{fix}})} \Big[\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \Big(A \Big(N - N^o \Big) \Big)^{\frac{\psi-1}{\psi}} \Big]^{\frac{\psi}{\psi-1}}$$
(B.13)

equation (B.11) can be rewritten as

$$\hat{Y} = \frac{(1+\phi_{\text{fix}}) \left[\alpha K_t^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(A e^{\hat{Z}_t} \left(N_t - N^o \right) \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{1}{\psi}}}{\left[\alpha K^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(A \left(N - N^o \right) \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}}} - \phi_{\text{fix}}.$$
(B.14)

Using the corresponding firm first-order conditions,

$$\frac{W_t}{P_t} = \Xi \left[\frac{(1-\alpha) \left(A e^{\hat{Z}_t} \left(N_t - N^o \right) \right)^{\frac{\psi-1}{\psi}}}{\alpha K_t^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(A e^{\hat{Z}_t} \left(N_t - N^o \right) \right)^{\frac{\psi-1}{\psi}}} \right] \frac{Y_t + \Phi}{N_t - N^o}$$
(B.15)

²In the Cobb–Douglas case, we have $Y_t = K_t^{\alpha} (A_t L_t)^{1-\alpha} = A_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$ so that a 1% change in laboraugmenting technology A_t moves measured TFP by $\varepsilon_{A,t} = 1 - \alpha$ percent (up to first order).

³We suppress the assumed deterministic loglinear trend in A for notational convenience.

and

$$R_{t}^{K} = \Xi \left[\frac{\alpha K_{t}^{\frac{\psi-1}{\psi}}}{\alpha K_{t}^{\frac{\psi-1}{\psi}} + (1-\alpha) \left(Ae^{\hat{Z}_{t}} \left(N_{t} - N^{o} \right) \right)^{\frac{\psi-1}{\psi}} \right] \frac{Y_{t} + \Phi}{K_{t}}, \tag{B.16}$$

equation (B.14) becomes

$$\begin{split} \hat{Y} &= (1 + \phi_{\text{fix}}) \\ \times \left[\frac{\alpha K_t^{\frac{\psi-1}{\psi}}}{\alpha K^{\frac{\psi-1}{\psi}} + (1 - \alpha) \left(A(N - N^o) \right)^{\frac{\psi-1}{\psi}}} + \frac{(1 - \alpha) \left(Ae^{\hat{Z}_t} (N_t - N^o) \right)^{\frac{\psi-1}{\psi}}}{\alpha K^{\frac{\psi-1}{\psi}} + (1 - \alpha) \left(A(N - N^o) \right)^{\frac{\psi-1}{\psi}}} \right]^{\frac{\psi}{\psi-1}} \\ &- \phi_{\text{fix}} \\ &= (1 + \phi_{\text{fix}}) \left[\frac{1}{\Xi} \frac{R^K K}{(Y + \Phi)} \left(\frac{K_t}{K} \right)^{\frac{\psi-1}{\psi}} + \frac{1}{\Xi} \frac{W}{P} \frac{(N - N^o)}{(Y + \Phi)} \left(\frac{Ae^{\hat{Z}_t} (N_t - N^o)}{A(N - N^o)} \right)^{\frac{\psi-1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \\ &- \phi_{\text{fix}}. \end{split}$$
(B.17)

Defining the share of nonoverhead labor compensation in output as

$$\aleph^{o} \equiv \frac{\frac{W}{P}(N-N^{o})}{Y} = \frac{\frac{W}{P}N}{Y}\frac{N-N^{o}}{N} = \aleph(1-\phi^{o})$$
(B.18)

and noting that the prefactors in front of capital and labor sum up to 1, equation (B.14) can be rewritten as

$$\hat{Y}_{t} = (1 + \phi_{\text{fix}}) \left[\left(1 - \frac{\aleph^{o}}{\Xi(1 + \phi_{\text{fix}})} \right) \hat{K}_{t}^{\frac{\psi - 1}{\psi}} + \frac{\aleph^{o}}{\Xi(1 + \phi_{\text{fix}})} \left(e^{\hat{Z}_{t}} \hat{N}_{t} \right)^{\frac{\psi - 1}{\psi}} \right]^{\frac{\psi}{\psi - 1}} - \phi_{\text{fix}}.$$
(B.19)

The elasticity of output with respect to technology A_t can then be computed by differentiating net output deviations from steady state with respect to \hat{Z}_t ,

$$\begin{split} \varepsilon_{A,t} &= \frac{\partial (\hat{Y}_{t} - 1)}{\partial \hat{Z}_{t}} \\ &= (1 + \phi_{\text{fix}}) \bigg[\bigg(1 - \frac{\aleph^{o}}{\Xi (1 + \phi_{\text{fix}})} \bigg) \hat{K}_{t}^{\frac{\psi - 1}{\psi}} + \frac{\aleph^{o}}{\Xi (1 + \phi_{\text{fix}})} \big(e^{\hat{Z}_{t}} \hat{N}_{t} \big)^{\frac{\psi - 1}{\psi}} \bigg]^{\frac{\psi}{\psi - 1} - 1} \\ &\times \frac{1}{\Xi (1 + \phi_{\text{fix}})} \aleph^{o} \big(e^{\hat{Z}_{t}} \hat{N}_{t} \big)^{\frac{\psi - 1}{\psi}} \\ & \stackrel{(\text{B.19})}{=} \bigg[\frac{\hat{Y}_{t} + \phi_{\text{fix}}}{1 + \phi_{\text{fix}}} \bigg]^{\frac{1}{\psi}} \frac{1}{\Xi} \aleph^{o} \big(e^{\hat{Z}_{t}} \hat{N}_{t} \big)^{\frac{\psi - 1}{\psi}}. \end{split}$$
(B.20)

In the Cobb–Douglas case in steady state, this simplifies to the well-known

$$\varepsilon_{A,t} = \frac{1}{\Xi} \aleph. \tag{B.21}$$

To operationalize the aforementioned, we first need to detrend output with the rate of labor-augmenting technology growth.

Appendix C: Data

C.1 Macro data

The data for the VARs is taken from FRED-MD (McCracken and Ng (2016)), except for (i) our constructed markup measure, (ii) the respective uncertainty measure, (iii) the shadow federal funds rate, which is taken from Wu and Xia (2016), and (iv) real new orders, which are taken from Conference Board as the sum of "Orders: consumer goods" (A1M008) and "Orders: capital goods" (A1M027) and are deflated using the "PCE Implicit Price Deflator" (PCEPI) from FRED-MD.

For the particle filtering, we use *Government Consumption Expenditures and Gross Investment* (FRED: GCE) as our measure of government spending and *Real Gross Domestic Product* (FRED: GDPC1) as our output measure. Both are transformed to per capita values via division by *Civilian noninstitutional population* (FRED: CNP16OV), smoothed with an HP-filter with $\lambda = 10,000$ to solve the best levels problem (Edge and Gurkaynak (2010)). The resulting per capita series are then logged and detrended using a one-sided HP-filter with $\lambda = 1600$.

For TFP, we cumulate the utilization adjusted TFP growth rates of Fernald (2012) (dtfp_util, transformed from annualized to quarterly growth rates), and detrend using a one-sided HP-filter with $\lambda = 1600$. The vintage of TFP data used already incorporates recent methodological changes in the computation of utilization (see Kurmann and Sims (forthcoming)).

C.2 Uncertainty measures

- The Jurado, Ludvigson, and Ng (2015) macro uncertainty measure and the Ludvigson, Ma, and Ng (forthcoming) financial uncertainty measure are available at Sydney Ludvigson's homepage at https://www.sydneyludvigson.com/data-and-appendixes/. We use the h = 1 measures.
- The Baker, Bloom, and Davis (2016) economic policy uncertainty measure is taken from FRED (USEPUINDXM)
- The VIX index is taken from FRED (VIXCLS) and averaged across months. Before the VIX becomes available in 1990, we use the realized stock return volatility. For that purpose, we compute the monthly standard deviation of the daily S&P 500 stock price index returns. The stock price index values are taken from Datastream (S&PCOMP(PI)). The resulting index of realized volatilities is normalized to have the same mean and variance as the VIX index when they overlap from 1990 onwards. The correlation between the two during that period is 0.8776.

C.3 Wage markup

For the wage markup, that is, the wedge between the marginal rate of substitution and the real wage, we focus on an encompassing measure of hours. Recall the equation for computing the wage markup in the baseline case of a Cobb–Douglas utility function

$$\xi_t^w = \log\left(\frac{1-\tau_t^n}{1+\tau_t^c}\right) + \log\left(\frac{W_t N_t}{P_t Y_t}\right) + \log\left(\frac{Y_t}{C_t}\right) - \log\left(\frac{1-\eta}{\eta}\right) + \log\left(\frac{1-N_t}{N_t}\right).$$
(C.1)

Demeaning yields:

$$\begin{aligned} \xi_t^w - \xi^w &= \left[\log\left(\frac{W_t N_t}{P_t Y_t}\right) - \log\left(\frac{WN}{PY}\right) \right] + \left[\log\left(\frac{Y_t}{C_t}\right) - \log\left(\frac{Y}{C}\right) \right] \\ &+ \left[\log\left(\frac{1 - N_t}{N_t}\right) - \log\left(\frac{1 - N}{N}\right) \right] \\ &+ \left[\log\left(\frac{1 - \tau_t^n}{1 + \tau_t^c}\right) - \log\left(\frac{1 - \tau^n}{1 + \tau^c}\right) \right], \end{aligned}$$
(C.2)

where the first term on the right-hand side is the labor share. Expanding the fractions to get the wedge in terms of the labor share and the consumption to output ratio has the advantage of avoiding problems with different trends that may be contained in different data sources.⁴

In case of isoelastic preferences with external habits:

$$U(C_t, N_t) = \frac{(C_t - \phi_c C_{t-1})^{1-\sigma} - 1}{1 - \sigma} - \psi \frac{N_t^{1+\frac{1}{\kappa}}}{1 + \frac{1}{\kappa}},$$
(C.3)

where κ is the inverse Frisch elasticity, σ the risk aversion, ϕ_c the habit persistence parameter, and ψ the weight of labor in the utility function, we get

$$\xi_t^w = -\log(\phi) + \log(Y_t) - \sigma \log(C_t - \phi_c C_{t-1}) - (1+\kappa) \log(N_t) + \log\left(\frac{1-\tau_t^n}{1+\tau_t^c}\right) + \log\left(\frac{W_t N}{P_t Y_t}\right),$$
(C.4)

which, with $\sigma = 1$ and $\phi_c = 0$, simplifies to

$$\xi_t^w = \log\left(\frac{1-\tau_t^n}{1+\tau_t^c}\right) + \log\left(\frac{W_t N_t}{P_t Y_t}\right) + \log\left(\frac{Y_t}{C_t}\right) + \log(\psi) + (1+\kappa)\log(N_t).$$
(C.5)

In case of Cobb-Douglas preferences with external habits of the form

$$U(C_t, N_t) = \frac{\left((C_t - \phi_c C_{t-1})^{\eta} (1-N)^{1-\eta}\right)^{1-\sigma} - 1}{1-\sigma}$$
(C.6)

⁴For example, the trend in NIPA GDP and *Average hourly earnings of production and nonsupervisory workers in the private sector* differs, although theory says they should be the same.

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we obtain

$$\xi_t^w = \log\left(\frac{1-\tau_t^n}{1+\tau_t^c}\right) + \log\left(\frac{W_t N_t}{P_t Y_t}\right) + \log(Y_t) - \sigma \log(C_t - \phi_c C_{t-1}) - \log\left(\frac{1-\eta}{\eta}\right) + \log\left(\frac{1-N_t}{N_t}\right),$$
(C.7)

which nests our baseline specification (4.1) with $\sigma = 1$ and $\phi_c = 0$.

For GHH preferences with

$$U(C_t, N_t) = \frac{\left(C_t - \psi N_t^{1+\kappa}\right)^{1-\sigma} - 1}{1-\sigma},$$
(C.8)

where $\sigma \ge 0$ determines the intertemporal elasticity of substitution ($\sigma = 1$ corresponds to log utility), $\psi > 0$ determines weight of the disutility of labor, and κ is the inverse of the Frisch elasticity, we get

$$\xi_t^w = -\log(\psi(1+\kappa)) - (1+\kappa)\log(N_t) + \log\left(\frac{1-\tau_t^n}{1+\tau_t^c}\right) + \log\left(\frac{W_tN}{P_tY_t}\right) + \log(Y_t).$$
(C.9)

It should be noted that GHH preferences and isoelastic preferences with $\sigma \neq 1$ are not consistent with a balanced growth path unless additional stationarizing devices are used (as in, e.g., Mertens and Ravn (2011)). Including a log-linear trend in our empirical VAR allows us to deal with such remaining trends.

In order to compute the wage markup, the right-hand-side variables are mapped to the data in the following way:

• $\frac{W_i N_t}{P_l Y_t}$: to compute the labor share, we take the share of employees' compensation *Compensation of Employees, Paid* (FRED: COE) in net national income (NNI), where net national income is compute as *National Income* (FRED: NICUR) minus net indirect taxes, computed as the difference between taxes on production and imports (FRED: GDITAXES) and subsidies (FRED: GDISUBS). To this, we add part of the ambiguous proprietor's income (FRED: PROPINC). The share of proprietor's income assigned to labor is computed as the share of unambiguous labor income in total unambiguous income resulting in

$$\frac{WN}{PY} = \frac{COE}{NNI - PROPINC}.$$

- *P_t*: Gross Domestic Product: Implicit Price Deflator (FRED: GDPDEF).
- *Y_t*: *Gross Domestic Product* (FRED: GDP), deflated by the GDP deflator and divided by population *Pop_t* (defined below).
- C_t : real private consumption is computed as the sum of *Personal Consumption Expenditures: Nondurable Goods* (FRED: PCND) and *Personal Consumption Expenditures: Services* (FRED: PCESV), each deflated by the GDP deflator and divided by population Pop_t .⁵

⁵Due to chain-weighting, this separate deflating is required to preserve additivity.

- N_t : We use a quarterly total hours measure following Cociuba, Prescott, and Ueber-feldt (2018), divided by population Pop_t . For this purpose, we extend their measure to include more recent periods.
 - 1. Compute the civilian noninstitutional population between 16 and 65 years by subtracting the *(Unadj) Population Level—65 yrs. & over* (BLS: LNU00000097) from *Civilian Noninstitutional Population* (BLS: LNU00000000), both averaged over the respective quarter.
 - 2. To compute the number of military personell, we first download the most recent vintage from Simona Cociuba's website at https://sites.google.com/site/simonacociuba/research and then update *Military Personnel—Total Worldwide* using data from https://www.dmdc.osd.mil/appj/dwp/dwp_reports.jsp: Military Personnel -> Active Duty Military Personnel by Service by Rank/Grade (Updated Monthly); for the current year, we use the monthly PDFs. There, we use *GRAND TOTAL—Total services*. Again we average monthly values to get a quarterly series.
 - 3. Civilian employment and weekly hours worked before 1976, which are based on Census and BLS data in printed books, are taken from the most recent vintage from Simona Cociuba's website.
 - 4. Civilian employment after 1976 is taken from *Number Employed*, *At Work* (BLS: LNU02005053), while their weekly hours worked are from *Average Hours*, *Total At Work*, *All Industries* (BLS: LNU02005054).

The series in 2 to 4 are first averaged over the quarter. When doing so for the civilian series in 3 and 4, we follow Cociuba, Prescott, and Ueberfeldt (2018) and check for outliers on the low side, i.e. we check whether $d_t \equiv \text{mean}(m_i)/\text{min}(m_i) < 0.95$, where m_i denotes the months belonging to a quarter. If $d_t < 0.95$, we use $(3 \times \text{mean}(m_i) - \text{min}(m_i))/2$ and $\text{mean}(m_i)$ otherwise. The civilian quarterly series are then seasonally adjusted using the X13 routine of Eviews 8. Total quarterly hours are computed as the sum of civilian and military hours, both computed as the product of employment times weekly hours worked in the respective category. For military weekly hours, we assume a workweek of 40 hours. To get from weekly to quarterly hours, we assume 4 quarters with 13 weeks.

- *Pop*_{*t*}: we use the sum of civilian noninstitutional population between 16 and 65 and military personell, based on our update of Cociuba, Prescott, and Ueberfeldt (2018).
- Leisure $1 N_t$: Following Karabarbounis (2014), who in turn is motivated by Aguiar, Hurst, and Karabarbounis (2013), we normalize the discretionary time endowment available to 92 hours per week per person and compute leisure as the difference between this endowment and N_t . Again, the measure is transformed to per capita values by dividing by Pop_t .

Supplementary Material

• Labor tax rate τ_t^n : The average labor income tax rate is computed as the sum of taxes on labor income, τ^{LI} , plus the "tax rate" on social insurance contributions, τ^{SI} ,

$$\tau^n = \tau^{\mathrm{LI}} + \tau^{\mathrm{SI}}.$$

We closely follow Mendoza, Razin, and Tesar (1994), Jones (2002), and Leeper, Plante, and Traum (2010) and compute the tax rate from the national accounts by dividing the tax revenue by the respective tax base. For labor income tax rates, we need to compute the portion of personal income tax revenue that can be assigned to labor income. We first compute the average personal income tax rate

$$\tau^p = \frac{IT}{W + PRI/2 + CI}$$

where *IT* is personal current tax revenues computed as the sum of *Federal government current tax receipts: Personal current taxes* and *State and local government current tax receipts: Personal current taxes* (Table 3.1 line 3, FRED: A074RC1Q027SBEA) + W071RC1Q027SBEA), *W* is *Compensation of Employees: Wages and Salary Accruals* (Table 1.12 line 3, FRED: WASCUR), *PRI* is *Proprietors' Income with Inventory Valuation Adjustment(IVA) and Capital Consumption Adjustment (CCAdj)* (Table 1.12 line 9, FRED: PROPINC), and *CI* is capital income. It is computed as

$$CI \equiv PRI/2 + RI + CP + NI,$$

where *RI* is *Rental Income of Persons with Capital Consumption Adjustment (CCAdj)* (Table 1.12 line 12, FRED: RENTIN), *CP* is *Corporate Profits with Inventory Valuation Adjustment (IVA) and Capital Consumption Adjustment (CCAdj)* (Table 1.12 line 13, FRED: CPROFIT), and *NI* denotes *Net interest and miscellaneous payments on assets* (Table 1.12 line 18, FRED: W255RC1Q027SBEA). In doing so, the ambiguous proprietor's income is assigned in equal parts to capital and labor income. The labor income tax can then be computed as

$$\tau^{\rm LI} = \frac{\tau^p (W + PRI/2)}{EC + PRI/2},$$

where *EC* is *National Income: Compensation of Employees, Paid* (Table 1.12 line 2, FRED: COE), which, in addition to wages, includes contributions to social insurance and untaxed benefits. The social insurance "tax rate" is given by

$$\tau^{\rm SI} = \frac{CSI}{EC + PRI/2},$$

where *CSI* denotes *Government current receipts: Contributions for government social insurance* (Table 3.1 line 7, FRED: W782RC1Q027SBEA).

• Consumption tax rate τ_t^c : The tax revenue from consumption taxes, *CT*, requires apportioning the indirect tax revenue to investment and consumption.⁶ We do this

⁶We opt to not attribute sales tax revenues to government purchases due to the different tax-exemption status of local, state, and federal purchases in different states. For example, government entities are sales tax-exempt in New York, but are tax-liable in California.

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as:

$$CT = \frac{PC}{PC + I}INDT,$$

where *PC* is *Personal Consumption Expenditures* (FRED: PCE), *I* is *Gross Private Domestic Investment* (FRED: GPDI), and *INDT* is net indirect taxes, computed as the difference between *Gross Domestic Income: Taxes on Production and Imports* (FRED: GDITAXES) and *Gross Domestic Income: Subsidies* (FRED: GDISUBS).⁷ The consumption tax rate is then computed as

$$\tau^c = \frac{CT}{PC - CT}$$

C.4 Price markup

For the price markup, that is, the wedge between the real wage and the marginal product of labor, we focus on the private business sector. Recall the equation for computing the price markup:

$$\xi_t^p = \log\left((1-\alpha)\left(Y^{\text{norm}}\right)^{\frac{\psi-1}{\psi}}\right) + \frac{\psi-1}{\psi}Z_t + \frac{1}{\psi}\log\left(\frac{Y_t + \Phi}{N_t - N^o}\right) - \log\left(\frac{W_t}{P_t}\right). \tag{4.1}$$

Demeaning this expression yields

$$\xi_t^p - \xi^p = \frac{\psi - 1}{\psi} \log(e^{Z_t}) + \frac{1}{\psi} \left[\log\left(\frac{Y_t + \Phi}{N_t - N^o}\right) - \log\left(\frac{Y + \Phi}{N - N^o}\right) \right] - \left[\log\left(\frac{W_t}{P_t}\right) - \log\left(\frac{W}{P}\right) \right], \tag{C.10}$$

where

$$e^{Z_t} = \frac{1}{\varepsilon_{A,t}} \log TFP_t, \tag{C.11}$$

$$\varepsilon_{A,t} = \left[\frac{\hat{Y}_t + \phi_{\text{fix}}}{1 + \phi_{\text{fix}}}\right]^{\frac{1}{\psi}} \frac{1}{\Xi} \aleph^o \left(e^{\hat{Z}_t} \hat{N}_t\right)^{\frac{\psi - 1}{\psi}}.$$
(B.20)

We can then compute the price markup by using the following sources:

- *W_t*: following the approach in Nekarda and Ramey (2013), we use the *Average hourly earnings of production and nonsupervisory workers in the private sector* (BLS: CES050000008).⁸
- *P_t*: *Gross Domestic Product: Implicit Price Deflator* (FRED: GDPDEF).

⁷The use of net indirect taxes follows Karabarbounis (2014) and differs from e.g. Mendoza, Razin, and Tesar (1994) who use gross indirect taxes.

⁸This implicitly assumes that all nonproduction and supervisory workers are overhead labor, which probably is an upper bound (see Ramey (1991)).

- $N_t N^o$: Average weekly hours of production and nonsupervisory employees, private business (BLS: CES050000006) multiplied by Production and nonsupervisory employees, private business (CES: CES050000006), divided by civilian noninstitutional population.
- Y_t : *Current dollar output, private business* (BLS: PRS84006053), deflated using the GDP deflator and divided by *civilian noninstitutional population*, detrended by an exponential trend.
- Φ: Consistent with our model, we assume additional fixed costs of 2.96% of steadystate output per capita, which we approximate using the average detrended log output per capita.
- Population: *Civilian noninstitutional population* (FRED: CNP16OV), smoothed with an HP-filter with $\lambda = 10,000$ to solve the best levels problem (Edge and Gurkay-nak (2010)).
- *TFP*_{*t*}: cumulated sum of the utilization adjusted or nonutilization adjusted TFP growth rates of Fernald (2012) (dtfp_util or dtfp, starting value initialized to 1, transformed from annualized to quarterly growth rates), detrended by an exponential trend.
- \aleph^o : The labor share not accounting for overhead labor, \aleph is computed as 1 minus *Capital's share of income* from Fernald (2012),⁹ which is "[B]ased primarily on NIPA data for the corporate sector." To derive the share of non-overhead labor \aleph^o , we use equation

$$\aleph^{o} \equiv \frac{\frac{W}{P}(N-N^{o})}{Y} = \frac{\frac{W}{P}N}{Y}\frac{N-N^{o}}{N} = \aleph(1-\phi^{o})$$
(B.18)

with $\phi^o = 0.11$ as discussed in the calibration section.

In the Cobb–Douglas case, the price markup simplifies to

$$\xi_t^p = \log\left(\frac{Y_t + \Phi}{N_t - N^o}\right) - \log\left(\frac{W_t}{P_t}\right),\tag{C.12}$$

which, in the absence of fixed costs, reduces to the inverse labor share. In the robustness checks, we use three different measures:

• The labor share based on total compensation in the nonfinancial business sector is computed as *Net value added of nonfinancial corporate business: Compensation of employees* (FRED: A460RC1Q027SBEA), divided by *Gross value added of nonfinancial corporate business* (FRED: A455RC1Q027SBEA) minus *Net value added of non-financial corporate business: Taxes on production and imports less subsidies* (FRED: W325RC1Q027SBEA).

⁹This series substitutes for *Business Sector: Labor Share* (FRED: PRS84006173), which is unfortunately only available in index form.

- The labor share in the private business sector is based on *Business Sector: Labor Share* (FRED: PRS84006173).
- The labor share based on total compensation in the private business sector is computed as the product of *Production and Nonsupervisory Employees: Total Private* (FRED: CES050000006), *Average Weekly Hours of Production and Nonsupervisory Employees: Total private* (FRED: AWHNONAG) and *Average Hourly Earnings of Production and Nonsupervisory Employees: Total Private* (FRED: AHETPI) divided by *Business Sector: Current Dollar Output* (FRED: PRS84006053).

C.5 Industry-level markups

The majority of our data needed to construct industry-level price markups comes from the NBER-CES manufacturing industry database, which covers the SIC2 industries 20 to 39 at a four-digit granularity for the years 1958–2011.

We compute industry-level price markups using equations (B.20), (C.10), and (C.11). As we have no information on fixed costs, we assume the absence of fixed costs such that

$$\xi_{i,t}^{p} = \frac{\psi - 1}{\psi} \log(e^{Z_{i,t}}) + \frac{1}{\psi} \log\left(\frac{Y_{t}}{N_{i,t} - N_{i}^{o}}\right) - \log\left(\frac{W_{i,t}}{P_{i,t}}\right), \tag{C.10'}$$

where

$$e^{Z_{i,t}} = \frac{1}{\varepsilon_{A,i}} \log TFP_{i,t}.$$
(C.13)

Here, we use the steady-state elasticity $\varepsilon_{A,i}$ given by

$$\varepsilon_{A,i} = \Xi_i^{-1} \aleph_i^o, \tag{B.20'}$$

where \aleph_i^o is the labor share and Ξ_i^{-1} is the gross markup.

The NBER-CES database only contains information on wages paid. But what matters for the labor margin is the total compensation of employees. For that reason, we follow the approach of Chang and Hong (2006) and Nekarda and Ramey (2011) and multiply the wage bill in the CES database by the ratio of the total compensation (NIPA Table 6.2, Compensation of Employees by Industry) to wages (NIPA Table 6.3 Wages and Salaries by Industry) at the two-digit industry level. The respective mapping is displayed in Tables C.3 and C.4. When the SIC classifications in the NIPA tables change, we splice the respective adjustment factor series by giving precedence to the 1987 SIC series (NIPA Table B) when there is overlap and multiplying the earlier/later series by the ratio of the two series in the first/last period of overlap to ensure smooth pasting. Similarly, the database only contains hours of production workers (NBER-CES code: *prodh*). To compute total hours (*toth*), we compute the number of production workers as the difference between total employment (*emp*) and production workers (*prode*). We then assume that non-production workers are salaried and work 1960 hours per year as in Nekarda and Ramey (2011):

$$toth = prodh + (emp - prode) \times 1960.$$
(C.14)

The database contains information about real shipments which is not equal to output due to inventories. To compute real output accounting for inventories, we follow Nekarda and Ramey (2011). A problem is that only the total value of inventories $I_{i,t}^{nom}$ (*invent*) is reported, which also includes inventories of materials that need to be subtracted. The first step is to compute the change in nominal finished-goods and work-inprocess inventories $\Delta I_{i,t}^{f,nom}$, which is equal to nominal value added $V_{i,t}^{nom}$ (*vadd*) minus the value of shipments $S_{i,t}^{nom}$ (*vship*) plus nominal material costs $M_{i,t}^{nom}$ (*matcost*):

$$\Delta I_{i,t}^{f,\text{nom}} = V_{i,t}^{\text{nom}} - S_{i,t}^{\text{nom}} + M_{i,t}^{\text{nom}}.$$
 (C.15)

The change in materials inventories $\Delta I_{i,t}^{m,\text{nom}}$ can then be computed as the difference between total inventory changes and changes in nominal finished-goods and work-in-process inventories:

$$\Delta I_{i,t}^{m,\text{nom}} = \Delta I_{i,t}^{\text{nom}} - \Delta I_{i,t}^{f,\text{nom}}.$$
(C.16)

Real output $Y_{i,t}$ can then be computed as¹⁰

$$Y_{i,t} \approx \frac{S_{i,t}^{\text{nom}}}{P_{i,t}} + \left[\frac{I_{i,t}^{\text{nom}}}{P_{i,t}} - \frac{I_{i,t-1}^{\text{nom}}}{P_{i,t-1}}\right] - \frac{\Delta I_{i,t}^{m,\text{nom}}}{P_{i,t}}.$$
(C.17)

To implement the above formulas, we need a sectoral TFP estimate and the elasticity of labor productivity with respect to labor-augmenting technology $\varepsilon_{A,i}$.

Elasticity of labor productivity with respect to labor-augmenting technology To compute the elasticity, we need to know both the average markup and the labor share. In the absence of fixed costs, the average markup can be directly computed from the average profit share, as one minus the profit share is then equal to the inverse steady-state gross industry markup. The profit share in industry *i*, $\Pi_{i,t}^{ps}$ is computed as

$$\Pi_{i,t}^{\text{ps}} = \left(Y_{i,t}^{\text{nom}} - W_{i,t}^{\text{comp,nom}} - (0.05 + \bar{\delta})K_{i,t}P_{i,t}^{\text{inv}} - M_{i,t}^{\text{nom}}\right) / Y_{i,t}^{\text{nom}},$$
(C.18)

where $Y_{i,t}^{\text{nom}}$ is nominal output defined as real output $Y_{i,t}$ times the shipment deflator ("pship"), $W_{i,t}^{\text{comp,nom}}$ it total compensation of employees, $M_{i,t}^{\text{nom}}$ is nominal materials costs (*matcost*), and (0.05 + $\bar{\delta}$) $K_{i,t}P_{i,t}^{\text{inv}}$ is the imputed nominal cost of capital, where we assume an interest rate of 5% per year. We compute the average depreciation rate from

$$\delta_{i,t} = 1 - (K_{i,t} - I_{i,t}) / K_{i,t-1}, \tag{C.19}$$

where real investment is obtained by dividing nominal investment ("invest") by the investment deflator $P_{i,t}^{\text{inv}}$ ("piinv") and $K_{i,t}$ is the real capital stock ("cap"). When computing the average depreciation rate $\bar{\delta}$ over the sample, we discard observations that show negative depreciation rates and depreciation rates larger than 50%.

¹⁰See the Technical Appendix (A.5) of Nekarda and Ramey (2011) and their discussion of the approximation error involved.

	Iddatus .c.c. TTTT	þ					7	
60200B Ann		Code	Line	60200C Ann	Code	Line	60200D Ann	Code
Lumber and wood products		J4115C0	15	Lumber and wood products	B4115C0	15	Wood products	N4115C0
Furniture and fixt	ures	J4116C0	16	Furniture and fixtures	B4116C0	24	Furniture and related products	N4124C0
Stone, clay, and gl nroducts	ass	J4117C0	17	Stone, clay, and glass products	B4117C0	16	Nonmetallic mineral moducts	N4116C0
Primary metal inc	lustries	J4118C0	18	Primary metal industries	B4118C0	17	Primary metals	N4117C0
Fabricated metal	products	J4119C0	19	Fabricated metal products	B4119C0	18	Fabricated metal products	N4118C0
Machinery, excep electrical	t	J4120C0	20	Industrial machinery and equipment	B4120C0	19	Machinery	N4119C0
Electric and elect	ronic	J4121C0	21	Electronic and other	B4121C0	21	Electrical equipment,	N4121C0
equipment				electric equipment			appliances, and components	
Motor vehicles ar	pr	J4122C0	22	Motor vehicles and	B4122C0	22	Motor vehicles, bodies	N4122C0
equipment				equipment			and trailers, and parts	
Other transporta	tion	J4123C0	23	Other transportation equinment	B4123C0	23	Other transportation equinment	N4123C0
Instruments and	related	J4124C0	24	Instruments and related	B4124C0	19	Machinery	N4119C0
products				products				
Miscellaneous		J4125C0	25	Miscellaneous	B4125C0	25	Miscellaneous	N4125C0
manufacturing ir	idustries			manufacturing industries			manufacturing	
Food and kindred	_	J4127C0	27	Food and kindred	B4127C0	27	Food and beverage and	N4127C0
products			0	products		l	tobacco products	
Tobacco manuta	ctures	Q4128BC0	28	Tobacco products	Q4128C0	27	Food and beverage and tobacco products	N4127C0
Textile mill produ	acts	J4129C0	29	Textile mill products	B4129C0	28	Textile mills and textile product mills	N4129C0
Apparel and othe products	r textile	J4130C0	30	Apparel and other textile products	B4130C0	29	Apparel and leather and allied products	N4130C0

				TA	BLE C.3. Continued.				
SIC Code	Line	60200B Ann	Code	Line	60200C Ann	Code	Line	60200D Ann	Code
26	31	Paper and allied products	J4131C0	31	Paper and allied products	Q4131C0	30	Paper products	N4132C0
27	32	Printing and publishing	Q4132BC0	32	Printing and publishing	Q4132C0	31	Printing and related	N4133C0
								support activities	
28	33	Chemicals and allied	J4133C0	33	Chemicals and allied	B4133C0	33	Chemical products	N4135C0
		products			products				
29	34	Petroleum and coal	J4134C0	34	Petroleum and coal	B4134C0	32	Petroleum and coal	N4134C0
		products			products			products	
30	35	Rubber and miscellaneous	J4135C0	35	Rubber and miscellaneous	B4135C0	34	Plastics and rubber	N4136C0
		plastics products			plastics products			products	
31	36	Leather and leather	J4136C0	36	Leather and leather	B4136C0	29	Apparel and leather and	N4130C0
		products			products			allied products	
<i>Note</i> : In machinery a	Table "6 nd equip	0200D Ann." we do not assign NIP ment" and later became a separate	A line 20 "Comp category, introdi	outer an acing a s	d electronic products" (N4020C0) i tructural break.	to any two-dig	git indus	try, because in SIC 1987 it was pa	art "Industrial

ć ¢ C

					D			D	
SIC Code	Line	60300B Ann	Code	Line	60300C Ann	Code	Line	60300D Ann	Code
24	15	Lumber and wood	J4115C0	15	Lumber and wood	B4115C0	15	Wood products	N4115C0
25	16	products Furniture and fixtures	J4116C0	16	products Furniture and fixtures	B4116C0	24	Furniture and related	N4124C0
33	17	Stone clay and glace	1411700	17	Stone claw and clace	B4117C0	16	products Nonmetallic mineral	NALLECO
10	-	products	00 ITTE	-	products	OC ITTER	01	products	CONTIENT
33	18	Primary metal industries	J4118C0	18	Primary metal industries	B4118C0	17	Primary metals	N4117C0
34	19	Fabricated metal products	J4119C0	19	Fabricated metal products	B4119C0	18	Fabricated metal products	N4118C0
35	20	Machinery, except	J4120C0	20	Industrial machinery and	B4120C0	19	Machinery	N4119C0
		electrical			equipment				
36	21	Electric and electronic	J4121C0	21	Electronic and other	B4121C0	21	Electrical equipment,	N4121C0
		equipment			electric equipment			appliances, and	
								components	
371	22	Motor vehicles and	J4122C0	22	Motor vehicles and	B4122C0	22	Motor vehicles, bodies	N4122C0
		equipment			equipment			and trailers, and parts	
37	23	Other transportation	J4123C0	23	Other transportation	B4123C0	23	Other transportation	N4123C0
		equipment			equipment			equipment	
38	24	Instruments and related	J4124C0	24	Instruments and related	B4124C0	19	Machinery	N4119C0
		products			products				
39	25	Miscellaneous	J4125C0	25	Miscellaneous	B4125C0	25	Miscellaneous	N4125C0
		manufacturing industries			manufacturing industries			manufacturing	
20	27	Food and kindred	J4127C0	27	Food and kindred	B4127C0	27	Food and beverage and	N4127C0
		products			products			tobacco products	
21	28	Tobacco manufactures	Q4128BC0	28	Tobacco products	Q4128C0	27	Food and beverage and	N4127C0
								tobacco products	
22	29	Textile mill products	J4129C0	29	Textile mill products	B4129C0	28	Textile mills and textile	N4129C0
								product mills	
23	30	Apparel and other textile products	J4130C0	30	Apparel and other textile products	B4130C0	29	Apparel and leather and allied products	N4130C0
		ΓΓ			T T				

TABLE C.4. Mapping between SIC two digit codes and NIPA Table 6 lines: Wages.

				TA	BLE C.4. <i>Continued.</i>				
SIC Code	Line	60300B Ann	Code	Line	60300C Ann	Code	Line	60300D Ann	Code
26	31	Paper and allied products	J4131C0	31	Paper and allied products	Q4131C0	30	Paper products	N4132C0
27	32	Printing and publishing	Q4132BC0	32	Printing and publishing	Q4132C0	31	Printing and related	N4133C0
								support activities	
28	33	Chemicals and allied	J4133C0	33	Chemicals and allied	B4133C0	33	Chemical products	N4135C0
		products			products				
29	34	Petroleum and coal	J4134C0	34	Petroleum and coal	B4134C0	32	Petroleum and coal	N4134C0
		products			products			products	
30	35	Rubber and miscellaneous	J4135C0	35	Rubber and miscellaneous	B4135C0	34	Plastics and rubber	N4136C0
		plastics products			plastics products			products	
31	36	Leather and leather	J4136C0	36	Leather and leather	B4136C0	29	Apparel and leather and	N4130C0
		products			products			allied products	
<i>Note</i> : In machinery a	Table "6 md equif	0300D Ann." we do not assign NIP oment" and later became a separate	A line 20 "Comp category, introdi	uter an acing a s	d electronic products" (N4020C0) i tructural break.	to any two-dig	git indust	ry, because in SIC 1987 it was pa	ırt "Industrial

ć C The elasticity of labor productivity with respect to labor-augmenting technology is then given by the mean labor share, $1/T \sum_{t=1}^{T} W_{i,t}^{\text{comp,nom}} / Y_{i,t}^{\text{nom}}$, times the inverse markup.¹¹

Industry-level TFP To get a measure of productivity, we follow Nekarda and Ramey (2013) and run a Galí (1999)-type VAR with labor productivity and hours in first differences. We compute labor productivity by dividing real output $Y_{i,t}$ by either total hours (*toth*) or hours of production workers (*prodh*). Technology shocks are identified as the only shocks that moves productivity in the long-run. An estimated TFP series is then computed by cumulating the productivity growth rates resulting from simulating the long-run VAR with only the identified technology innovations.¹²

Appendix D: Mixed-frequency VARs

D.1 Prior, estimation, and convergence diagnostics

We use a shrinking prior of the Independent Normal–Wishart type (Kadiyala and Karlsson (1997)), where the mean and precision are derived from from a Minnesota-type prior (Litterman (1986), Doan, Litterman, and Sims (1984)). Denote the vector of stacked coefficients with $\beta = \text{vec}([\mu \alpha A_1, \dots, A_p]')$. It is assumed to follow a normal prior

$$\beta \sim N(\underline{\beta}, \underline{V}).$$
 (D.1)

For the prior mean $\underline{\beta}$, we assume the variables to follow a univariate AR(1)-model with mean of 0.9 for levels and mean 0 for growth rates, while all other coefficients are 0. The prior precision \underline{V} is assumed to be a diagonal matrix with the highest precision for the first lag and exponential decay for the other lags. The weighting of cross-terms is conducted according to the relative size of the error terms in the respective equations, while a rather diffuse prior is used for deterministic terms. The diagonal element corresponding to the *j*th variable in equation *i*, $\underline{V}_{i,jj}$ is

$$\underline{V}_{i,jj} = \begin{cases} \frac{\underline{a}_1}{r^2}, & \text{for coefficients on own lag } r \in \{1, \dots, p\}, \\ \frac{\underline{a}_2 s_i^2}{r^2 s_j^2}, & \text{for coefficients on lag } r \in \{1, \dots, p\} \text{ of variable } j \neq i, \end{cases}$$
(D.2)
$$\underline{\underline{a}_3 s_i^2}, & \text{for coefficients on exogenous variables,} \end{cases}$$

where s_i^2 is the OLS estimate of the error variance of an AR(*p*) model with constant and trend estimated for the *i*th variable (see Litterman (1986)).¹³ We follow Koop and Korobilis (2010) and set $\underline{a}_1 = 0.2$, $\underline{a}_2 = 0.5$ and $\underline{a}_3 = 10^4$. The prior error covariance is

¹¹The labor share is computed by dividing an appropriate measure of worker compensation by a output measure. Depending on the concept used, the worker compensation is either the one for production or production and supervisory workers. As the output measure, we use either total value added or total value added minus material costs. The latter provides a labor share after abstracting from materials.

¹²Note this assumes the equality between labor productivity movements caused by techn. shocks and TFP.

 $^{^{13}}$ In case of the quarterly variable, we estimate the AR(p) model on linearly interpolated data.

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assumed to follow

$$\underline{\Sigma} \sim \mathrm{IW}(\underline{S}, \underline{\nu}) \tag{D.3}$$

with $\underline{\nu} = 60$ "pseudo-observations", corresponding to $\approx 10\%$ of the observations, and <u>S</u> being the OLS covariance matrix.

As a practical matter, we use z-scored the data (including the trend) to avoid numerical problems arising from under/overflow during the posterior computations that involve sum of squares. We also impose a stability condition on our VAR by drawing from the conditional distribution for β until all eigenvalues of the companion form matrix are smaller than 1.

In the Gibbs sampler, we use 50,000 draws, of which we discard the first 5000 draws as a burn-in. The Raftery and Lewis (1992) convergence diagnostics with quantile q = 0.025, precision r = 0.01, and probability of attaining this precision s = 0.95 suggests that this is sufficient for convergence.

D.2 11+1 variable VAR

The Jurado, Ludvigson, and Ng (2015) 11+1-variable VAR is given by (FRED-MD Acronyms in brackets, see Appendix C for details on other variables)

log(real IP (INDPRO))	
log(employment (PAYEMS))	
log(real consumption (DPCERA3M086SBEA))	
log(PCE Deflator (PCEPI))	
log(real new orders)	
log(real wage (CES300000008))	(D.4)
hours (AWHMAN)	. (D.4)
shadow federal funds rate	
log(S&P 500 Index (S&P 500))	
growth rate of M2 (M2SL)	
uncertainty proxy	
log(markup)	

D.3 8+1 variable VAR

The Bloom (2009) 8+1 variable VAR is given by

log(S&P 500 Index (S&P 500))uncertainty proxyshadow federal funds ratelog(real wage (CES300000008))log(CPI (CPIAUCSL))hours (AWHMAN)log(manufacturing employment (MANEMP))log(real manufacturing production (IPMANSICS))log(markup)

(D.5)



FIGURE D.8. IRFs to JLN-based two-standard deviation uncertainty shock in the 11+1 variable mixed-frequency VAR. *Notes*: Bands are pointwise 90% HPDIs. The macroeconomic uncertainty index is measured in arbitrary units and has a mean of 0.65.



FIGURE D.9. IRFs to JLN-based two-standard deviation uncertainty shock in the 11+1 variable mixed-frequency VAR including the total markup. *Notes:* Bands are pointwise 90% HPDIs. The total markup is computed as the sum of the price and wage markup. The macroeconomic uncertainty index is measured in arbitrary units and has a mean of 0.65.

Y	Emp.	С	Р	Orders	W/P	Ν	R	S&P	$\Delta M2$	Uncert.	Markup
					Price M	larkup VA	AR				
12.97	12.85	11.31	6.55	15.02	7.57	11.20	6.95	10.16	4.79	23.53	7.73
					Wage N	farkup V/	AR				
12.86	12.71	11.56	6.48	13.82	8.46	11.28	6.80	11.89	4.64	23.41	14.88
					Total M	larkup VA	AR				
12.39	11.85	11.15	6.06	13.39	7.86	11.44	6.72	11.14	4.58	22.93	10.64

TABLE D.5. Unconditional forecast error variance explained by uncertainty shock.

Note: Mean posterior forecast error variance share explained by the uncertainty shock in the 11+1 variable mixed-frequency VAR with the Jurado, Ludvigson, and Ng (2015) uncertainty measure ordered second-to-last. Based on 1000 posterior draws. First row: VAR with price markup measure; second row: VAR with wage markup measure; third row: VAR with total markup measure.



FIGURE D.10. IRFs to VIX-based two-standard deviation uncertainty shock in the 8+1 variable mixed-frequency VAR where uncertainty is ordered second. *Notes:* Bands are pointwise 90% HPDIs. The VIX is measured as the annualized volatility in percentage points.

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FIGURE D.11. IRFs to JLN-based two-standard deviation uncertainty shock in the 8+1 variable mixed-frequency VAR where uncertainty is ordered second. *Notes:* Bands are pointwise 90% HPDIs. The macroeconomic uncertainty index is measured in arbitrary units and has a mean of 0.65.

D.4 MF-VAR: Other uncertainty measures

Recently, Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek (2016) and Ludvigson, Ma, and Ng (forthcoming) have argued that it is important to distinguish between macroeconomic and financial uncertainty, with the latter driving the former. In Figure D.12, we therefore display the VAR-IRFs in response to the Ludvigson, Ma, and Ng (forthcoming) financial uncertainty measure. The results are similar to the ones of the JLN-macro uncertainty measure.

Carriero, Clark, and Marcellino (2018) also provide measures of macroeconomic and financial uncertainty. Figures D.13 to D.16 display the results. Again, the wage markup increases while the price markup tends to fall. The only slight exception is their financial uncertainty proxy for which we see an initial, insignificant increase in the price markup before it drops again.

Baker, Bloom, and Davis (2016) have constructed an index of economic policy uncertainty. It is more narrow than the JLN-uncertainty measure in that it only captures the political dimension of uncertainty, but is at the same time broader in that it not only captures risk, but also Knightian uncertainty. Despite these differences, the responses of the respective markups, displayed in Figure D.17, show a familiar pattern: the wage markup increases while the price markup falls.¹⁴

¹⁴In this case, due to non-availability of the EPU measure, the sample only starts in 1985, potentially explaining the non-significant drop in industrial production.



FIGURE D.12. IRFs to Ludvigson, Ma, and Ng (forthcoming) two-standard deviation financial uncertainty shock in the 11+1 variable mixed-frequency VAR. *Notes:* Bands are pointwise 90% HPDIs. The financial uncertainty index is measured in arbitrary units and has a mean of 0.91.



FIGURE D.13. IRFs to Carriero, Clark, and Marcellino (2018) two-standard deviation macro uncertainty shock in the 11+1 variable mixed-frequency VAR. *Notes*: Bands are pointwise 90% HPDIs. The macro uncertainty series is measured in arbitrary units and has a mean of 1.0.

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FIGURE D.14. IRFs to Carriero, Clark, and Marcellino (2018) two-standard deviation macro uncertainty shock in the 8+1 variable mixed-frequency VAR. *Notes:* Bands are pointwise 90% HPDIs. The macro uncertainty series is measured in arbitrary units and has a mean of 1.0.



FIGURE D.15. IRFs to Carriero, Clark, and Marcellino (2018) two-standard deviation financial uncertainty shock in the 11+1 variable mixed-frequency VAR. *Notes:* Bands are pointwise 90% HPDIs. The financial uncertainty series is measured in arbitrary units and has a mean of 1.06.

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FIGURE D.16. IRFs to Carriero, Clark, and Marcellino (2018) two-standard deviation financial uncertainty shock in the 8+1 variable mixed-frequency VAR. *Notes:* Bands are pointwise 90% HPDIs. The financial uncertainty series is measured in arbitrary units and has a mean of 1.06.



FIGURE D.17. IRFs to EPU-based two-standard deviation uncertainty shock in the 11+1 variable mixed-frequency VAR. *Notes:* Bands are pointwise 90% HPDIs. The EPU is measured in arbitrary units and has a mean of 100.

Appendix E: Proof of precautionary pricing in stylized example

Denote marginal costs with γ and the optimal relative price chosen by the firm with p. Due to uncertainty about the aggregate price level, this relative price is due to a mean preserving spread. The spread is parameterized by $0 \le \varepsilon < 1$. The demand elasticity is given by $\theta > 0$

The firm faces the problem

$$\max_{p} \mathbb{E}\Pi = \max[(1+\varepsilon)p - \gamma][(1+\varepsilon)p]^{-\theta} + [(1-\varepsilon)p - \gamma][(1-\varepsilon)p]^{-\theta}.$$

The FOC is given by:

$$\frac{\partial \mathbb{E}\Pi}{\partial p} = (1-\theta)p^{-\theta}(1+\varepsilon)^{1-\theta} + \theta\gamma(1+\varepsilon)^{-\theta}p^{-\theta-1} + (1-\theta)p^{-\theta}(1-\varepsilon)^{1-\theta} + \theta\gamma(1-\varepsilon)^{-\theta}p^{-\theta-1} \stackrel{!}{=} 0$$

which simplifies to

$$(1-\theta)p^*(1+\varepsilon)^{1-\theta} + (1-\theta)p^*(1-\varepsilon)^{1-\theta} = -\theta\gamma \big[(1+\varepsilon)^{-\theta} + (1-\varepsilon)^{-\theta} \big]$$

and thus

$$p^* = \frac{-\theta\gamma \left[(1+\varepsilon)^{-\theta} + (1-\varepsilon)^{-\theta} \right]}{(1-\theta) \left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta} \right]}.$$

Now check whether the optimal price increases in the spread ε

$$\begin{split} \frac{\partial p*}{\partial \varepsilon} &= \frac{-\theta\gamma}{(1-\theta)} \bigg\{ \frac{\left[-\theta(1+\varepsilon)^{-\theta-1} - \theta(1-\varepsilon)^{-\theta-1}(-1)\right] \left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^2} \\ &- \frac{\left[(1+\varepsilon)^{-\theta} + (1-\varepsilon)^{-\theta}\right] \left[(1-\theta)(1+\varepsilon)^{-\theta} + (1-\theta)(1-\varepsilon)^{-\theta}(-1)\right]}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^2} \bigg\} \end{split}$$

simplify

$$= \frac{-\theta\gamma}{(1-\theta)} \bigg\{ \frac{\left[-\theta(1+\varepsilon)^{-\theta-1} + \theta(1-\varepsilon)^{-\theta-1}\right] \left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^2} \\ - (1-\theta) \frac{\left[(1+\varepsilon)^{-\theta} + (1-\varepsilon)^{-\theta}\right] \left[(1+\varepsilon)^{-\theta} - (1-\varepsilon)^{-\theta}\right]}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^2} \bigg\}.$$

Now split in two terms, factor out $(-\theta)$ in the first term and use the binomial formula on the second term

$$= \frac{-\theta\gamma}{(1-\theta)} \\ \times \left\{ \frac{-\theta(1+\varepsilon)^{-2\theta} - \theta(1+\varepsilon)^{-\theta-1}(1-\varepsilon)^{1-\theta} + \theta(1-\varepsilon)^{-\theta-1}(1+\varepsilon)^{1-\theta} + \theta(1-\varepsilon)^{-2\theta}}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^2} \right\}$$

$$- (1-\theta) \frac{\left[\left((1+\varepsilon)^{-\theta}\right)^{2} - \left((1-\varepsilon)^{-\theta}\right)^{2}\right]}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^{2}} \right\}$$

$$= \frac{-\theta\gamma}{(1-\theta)} \left\{ (-\theta) \frac{(1+\varepsilon)^{-2\theta} + (1+\varepsilon)^{-\theta-1}(1-\varepsilon)^{1-\theta} - (1-\varepsilon)^{-\theta-1}(1+\varepsilon)^{1-\theta} - (1-\varepsilon)^{-2\theta}}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^{2}} - (1-\theta) \frac{\left[\left((1+\varepsilon)^{-\theta}\right)^{2} - \left((1-\varepsilon)^{-\theta}\right)^{2}\right]}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^{2}} \right\}.$$

Now cancel the $-\theta(1+\varepsilon)^{-2\theta}$ and $-\theta(1-\varepsilon)^{-2\theta}$ terms present in both terms of the curly brackets to get

$$= \frac{-\theta\gamma}{(1-\theta)} \bigg\{ (-\theta) \frac{(1+\varepsilon)^{-\theta-1}(1-\varepsilon)^{1-\theta} - (1-\varepsilon)^{-\theta-1}(1+\varepsilon)^{1-\theta}}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^2} \\ - \frac{\left[\left((1+\varepsilon)^{-\theta}\right)^2 - \left((1-\varepsilon)^{-\theta}\right)^2\right]}{\left[(1+\varepsilon)^{1-\theta} + (1-\varepsilon)^{1-\theta}\right]^2} \bigg\}.$$

Finally, factor out $(1 + \varepsilon)^{-\theta}(1 - \varepsilon)^{-\theta}$ in the first term after the big curly bracket:

$$=\underbrace{\frac{-\theta\gamma}{(1-\theta)}}_{>0}\left\{\underbrace{\underbrace{(-\theta)}_{<0}\underbrace{\frac{(1+\varepsilon)^{-\theta}(1-\varepsilon)^{-\theta}\left(\frac{1-\varepsilon}{1+\varepsilon}-\frac{1+\varepsilon}{1-\varepsilon}\right)}{\left[(1+\varepsilon)^{1-\theta}+(1-\varepsilon)^{1-\theta}\right]^{2}}}_{<0}-\underbrace{\frac{\left[\left((1+\varepsilon)^{-\theta}\right)^{2}-\left((1-\varepsilon)^{-\theta}\right)^{2}\right]}{\left[(1+\varepsilon)^{1-\theta}+(1-\varepsilon)^{1-\theta}\right]^{2}}}_{<0}\right\}$$

Thus, both parts are positive, establishing that the optimal price increases in response to a mean preserving spread. As marginal costs were constant, the markup increases.

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Co-editor Kjetil Storesletten handled this manuscript.

Manuscript received 22 February, 2019; final version accepted 8 September, 2020; available online 19 October, 2020.