Supplement to "Family job search and wealth: The added worker effect revisited"

(*Quantitative Economics*, Vol. 11, No. 4, November 2020, 1431–1459)

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Appendix B: Numerical solution of the model

Continuous and discrete variables

In the numerical solution of the model, wealth is a continuous variable, only discretized to support the computation of any value on its domain, while wages are discretized. Table B1 gives further details of this discretization.

Table B1. Discretization of variables.

	Wealth	Wages
Original variable	A	w
Discretized variable	$A[i]$	$w[j]$
Gridpoints	$i = 1, \dots, N_A$	$j = 1, \dots, N_w$
Gridpoint location	Left	Middle
Number of gridpoints	$N_A = 101$	$N_w = 101$
Number of intervals	$N_A - 1$	N_w
Lower bound	$\underline{A} = -s\frac{(1+r)(b_1+b_2)}{r}$	$\underline{w} = 700$
Upper bound	$\overline{A} = 500{,}000$	$\overline{w} = 10{,}000$
Gridsize	$\Delta_A = \frac{\overline{A}-\underline{A}}{N_A-1}$	$\Delta_w = \frac{\ln\overline{w}-\ln\overline{w}}{N_w}$

The lower bound on wealth is set at a fraction of the natural borrowing limit, so that a household can borrow up to some fraction of the present discounted value of their lowest possible income. We also define $w[0] = b_1$ and $w[0] = b_2$.

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 https://doi.org/10.3982/QE1092

Wage offer distribution

For each discretized wage, $j = 1, N_w$, and for each agent, $l = 1, 2$, we compute discrete probabilities integrating the wage interval defined by the grid:

$$g(j,l) = \frac{\Phi\left(\frac{\ln w_j + \Delta_w/2 - \mu_l}{\sigma_l}\right) - \Phi\left(\frac{\ln w_j - \Delta_w/2 - \mu_l}{\sigma_l}\right)}{\Phi\left(\frac{\ln \overline{w} - \mu_l}{\sigma_l}\right) - \Phi\left(\frac{\ln \underline{w} - \mu_l}{\sigma_l}\right)}.$$

Value function, policy rules, and expected value function

These are approximated by:

$$V(A_t, w_1, w_2) = V[i, j, k],$$
$$A_{t+1}(A_t, w_1, w_2) = A[i, j, k],$$
$$EV(A_{t+1}, w_1, w_2) = EV\left[i', j, k\right].$$

Solution to the dynamic problem

The following steps are done for each i, j, and k:

1. Initialization. We initialize the value function at the deterministic value of consuming all wealth and income forever with the instantaneous value of leisure, which admits an explicit expression:

$$V[i, j, k] = c_1 \frac{\left(A[i] + \left(1 + \frac{1}{r}\right)\left(w[j] + w[k]\right)\right)^{1-\gamma}}{1-\gamma} - \frac{1}{1-\gamma}\frac{1}{1-\beta} + \vartheta[j,k],$$

where $c_1 = (1 - \beta^{\frac{1}{\gamma}}(1+r)^{\frac{1-\gamma}{\gamma}})^{-\gamma}$, and $\vartheta[j,k] = \vartheta_1 I(j=0) + \vartheta_2 I(k=0) + \vartheta_3 I(j=0)I(k=0)$.

2. Integration. For each combination i', j, k integrate over all admissible values of j and k. For instance, for $V[i', 0, 0]$, we calculate the following three summations:

$$EV_{11}\left[i', 0, 0\right] = \sum_{j=1}^{N_w}\sum_{k=1}^{N_w} \max\left[V\left[i', j, k\right], V\left[i', j, 0\right], V\left[i', 0, k\right], V\left[i', 0, 0\right]\right] \times g(j,1) g(k,2),$$

$$EV_{10}\left[i', 0, 0\right] = \sum_{j=1}^{N_w} \max\left[V\left[i', j, 0\right], V\left[i', 0, 0\right]\right] g(j,1),$$

$$EV_{01}\left[i', 0, 0\right] = \sum_{k=1}^{N_w} \max\left[V\left[i', 0, k\right], V\left[i', 0, 0\right]\right] g(k,2).$$

With them, we build the integral

$$EV[i',0,0] = \lambda_1\lambda_2 EV_{11}[i',0,0] + \lambda_1(1-\lambda_2)EV_{10}[i',0,0] \\ + (1-\lambda_1)\lambda_2 EV_{01}[i',0,0] + (1-\lambda_1)(1-\lambda_2)V[i',0,0].$$

We repeat this process for the expected value functions of the other three joint employment statuses.

3. Differentiation. Compute the derivative of this object over wealth using a cubic interpolation:

$$EV_A[i',j,k] = \frac{-EV[i'+2,j,k] + 4EV[i'+1,j,k] - 3EV[i',j,k]}{A[i'+2] - A[i']}, \quad \text{if } i' = 1;$$
$$= \frac{EV[i'+1,j,k] - EV[i'-1,j,k]}{A[i'+1] - A[i'-1]}, \quad \text{if } N_A > i' > 1;$$
$$= \frac{3EV[i',j,k] - 4EV[i'-1,j,k] + EV[i'-2,j,k]}{A[i'] - A[i'-2]}, \quad \text{if } i' = N_A.$$

4. Policy rule inversion. We use the endogenous gridpoints method as in Carroll (2006). For each i', j, and k, optimal consumption $C[i', j, k]$ is found:

$$C[i',j,k] = \big(\beta(1+r)EV_A[i',j,k]\big)^{-\frac{1}{\gamma}}.$$

5. Smoothing. Conditional on j, k, regress $C[i', j, k]$ on $A(i')$. Whenever there are nonmonotonicities in $C[i', j, k]$ over $A(i')$, use predicted consumption instead of actual consumption:

$$\widehat{C}[i',j,k] = \widehat{b}_0 + \widehat{b}_1 A[i'] + \widehat{b}_2[A[i']]^2.$$

6. Inverse solution. Find wealth at time t as a function of i' and j, k, denoted by $\widetilde{A}$, for each j, k:

$$\widetilde{A}[i',j,k] = \widehat{C}[i',j,k] - w[j] - w[k] - \frac{A[i']}{1+r}.$$

7. Conditional solution. Reposition current liquid wealth $\widetilde{A}$ to find the solution.
   Interior solution. For each i locate $i\prime$ such that $\widetilde{A}[i', j, k] < A[i] < \widetilde{A}[i' + 1, j, k]$, then compute the linear interpolations

$$A'[i,j,k] = aA[(i'] + (1-a)A[i'+1],$$
$$EV^* = aEV[i',j,k] + (1-a)EV[i'+1,j,k],$$

where $a = \frac{A(i)-\widetilde{A}(i',j,k)}{\widetilde{A}(i'+,j,k)-\widetilde{A}(i',j,k)}$.
   Corner solutions. If $A(i) < \widetilde{A}(1, j, k)$, then let $i^* = 1$; if $A(i) > \widetilde{A}(N_A, i, k)$, then $i^* = N_A$:

$$A'[i,j,k] = A[i^*],$$
$$EV^* = EV[i^*,j,k].$$

8. Then compute the value function using
$$C^*[i,j,k] = A[i] + w[j] + w[k] - \frac{A'[i,j,k]}{1+r},$$
$$V[i,j,k] = U\big(C^*[i,j,k]\big) - \vartheta[j,k] + \beta EV^* + \vartheta[j,k].$$

9. Evaluate convergence. If $\|V' - V\| < \varepsilon$, stop; otherwise go back to step 2, and repeat the process.

Appendix C: Simulation procedure

We start the construction of the simulated dataset, when household wealth is first observed, for each couple and each of the four types, 11, 12, 21, and 22.

1. At period $t = 1$, we have $A = A^{\text{obs}}$, $w_1 = w_1^{\text{obs}}$, and $w_2 = w_2^{\text{obs}}$. As explained, we denote unemployment by $w = 0$. The household enters the next period with $A_2 = A'(A, w_1, w_2)$.

2. At period t, conditional on household's wealth, joint employment status, and individual wages, (A, w_1, w_2) job offers or job separations are realized.

   If a household member is nonemployed, he or she may receive a job offer, which we determine by taking random draws from a Bernoulli distribution with parameter λ.

   If the household member is employed, we take two similar draws from Bernoulli distributions with parameters π, and θ, which respectively determine whether the employed household member received an offer or if he or she was fired.

   If there is a job offer, we determine the specific offered wage by taking draws from the lognormal distribution F.

3. Once job offers are realized, the household decides whether to accept or reject them, using the reservation wage rules $w_1^*(A, w_2)$, $w_2^*(A, w_1)$, $w_1^{**}(A)$, $w_2^{**}(A)$ so that the maximum value of $V(A, w_1, w_2)$ over each available set of arguments is attained.

4. At the end of period t, there is a new joint employment status and wages w_1 and w_2, and the household enters the next period with $A_{t+1} = A'(A, w_{1t}, w_{2t})$.

5. Go back to step 2. This process is repeated until reaching the last observed period T for this household.

This process is repeated 100 times for each household of the actual dataset, so that a larger simulated dataset is built. We then compute for this simulated dataset the same moments that we calculated for the actual dataset. From these type-specific moments, using the four proportion of types of couples, p_{11}, p_{12}, p_{21}, p_{22}, we compute weighted simulated moments for the whole sample. Finally, we measure the distance between simulated moments and actual moments in the way that we described in Section 4.

Appendix D: Age distribution at first observation

Our setup is an infinite horizon model in which age or marriage duration are not state variables. However, the role of search frictions vs. unobserved heterogeneity in explaining the dispersion in accepted wages is different for households with heads of, say, age 35 who has been married for 1 year compared to households with heads of age 49 who have been married for 19 years.[1] The longer a couple has been making joint search decisions together, the larger a role the interdependence due to family search will have in the observed wage dispersion of such households. Unfortunately, the wave of the SIPP that we are using, 1996, does not contain information on the duration of marriage. This information was only incorporated to the data in 2014. We can, however, report the composition of households according to their age of head at the first period in which wealth is observed.

TABLE D1. Age distribution of head of household for first observation, by sample.

	Education:	High School		College	
Age	Children:	0–1	2+	0–1	2+
26–30		10.81	9.83	14.82	3.16
31–35		14.05	25.28	21.91	14.33
36–40		16.82	32.44	15.94	31.83
41–45		23.11	25.37	15.78	31.23
46–50		35.21	7.08	31.55	19.45

In Table D1, we report the age distribution of the head of household when wealth is first observed for the four samples used in this paper. We can see that agents' ages are pretty dispersed, which suggests that this issue may be important, even after analyzing samples defined by education and number of children. We leave this matter for future research.

APPENDIX E: TABLES NOTED IN THE ARTICLE

TABLE E1. Nonemployment rate and average wage by spouse's wage segment. High school. 0 or 1 child.

	Nonemployment Rate (%)				Average Wage ($)			
	Husband		Wife		Husband		Wife	
Spouse is	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
Nonemployed	4.08	4.53	15.85	19.50	1739	1718	1106	1060
[300, 1000)	5.64	5.42	24.05	24.95	1436	1484	847	952
[1000, 1500)	6.33	5.30	20.10	21.09	1600	1563	989	990
[1500, 2500)	5.47	4.85	17.45	18.94	1901	1843	1139	1178
[2500, 3500)	5.85	6.87	25.02	22.11	2038	1665	1271	1158
[3500, 10,000)	7.21	4.41	38.59	44.25	2090	1693	1270	1086

[1]We thank an anonymous referee for this suggestion.

Table E2. Household employment transitions. (Rows add to 100.) High school. 0 or 1 child.

	t							
	Actual				Predicted			
$t-1$	uu	ue	eu	ee	uu	ue	eu	ee
uu	76.51	3.56	17.44	2.49	81.97	3.38	14.15	0.50
ue	0.46	85.13	0.33	14.08	0.79	84.49	0.29	14.44
eu	0.59	0.10	95.17	4.13	0.72	0.05	95.66	3.57
ee	0.07	0.90	1.19	97.84	0.01	0.77	1.17	98.06

Table E3. Variations of the nonemployment rate under an economic downturn and increasing nonemployment transfers, by type of couple. High school. 0 or 1 child.

		Economic Downturn		Non-employment Transfers		
Types	Spouse	Husband $+\theta_1$	Wife $+\theta_2$	Husband $+b_1$	Wife $+b_2$	Both $+b_1, +b_2$
$p_{11}=40\%$	Husband	4.46	0.03	0.30	0.01	0.13
	Wife	−0.52	6.93	−1.25	0.80	−0.14
$p_{22}=1\%$	Husband	4.65	0.03	0.71	0.01	0.33
	Wife	−0.13	6.95	−0.09	0.67	0.21
$p_{21}=59\%$	Husband	4.65	0.07	0.68	0.03	0.34
	Wife	−0.36	6.90	−0.11	0.82	0.38
All	Husband	4.58	0.05	0.53	0.02	0.26
	Wife	−0.42	6.91	−0.56	0.81	0.17

Table E4. Employment, wages, and wealth by household employment status. Actual.

		High School		College			
	Children:	2 or More		0 or 1		2 or More	
Sample	Variable Spouse:	Non-E	Emp	Non-E	Emp	Non-E	Emp
Joint employment status							
Husband nonemployed		1.52	3.28	0.61	2.66	0.55	1.93
Husband employed		31.70	63.50	17.02	79.71	31.54	65.98
Nonemployment rate							
Husband		4.56	4.91	3.48	3.22	1.71	2.84
Wife		31.58	33.30	18.76	17.60	22.19	32.34
Wages							
Husband		1749	1535	3554	2470	3723	2881
		(1839)	(1055)	(3834)	(2190)	(3487)	(2848)
Wife		1012	916	1645	1651	1909	1451
		(695)	(907)	(1180)	(1331)	(1579)	(1481)
Wealth if husband							
Nonemployed		34,079	27,542	96,251	53,861	55,251	114,711
		(92,946)	(58,402)	(86,826)	(73,112)	(81,465)	(218,981)
Employed		38,001	42,304	159,466	103,391	160,141	120,416
		(68,945)	(70,329)	(382,556)	(189,525)	(277,900)	(210,548)

Table E5. Wage segment comparison between partners, by sample.

Education:	High School				College			
Children:	0–1		2+		0–1		2+	
Comparison	Actual	Predicted	Actual	Predicted	Actual	Predicted	Actual	Predicted
Same for both	34.68	32.04	31.37	36.00	26.68	24.39	21.59	20.19
Husband is higher	52.16	53.33	56.68	49.49	51.53	53.91	61.56	66.57
Wife is higher	13.15	14.63	11.93	14.51	21.39	21.63	15.67	13.10

References

Carroll, C. D. (2006), "The method of endogenous gridpoints for solving dynamic stochastic optimization problems." *Economics Letters*, 91 (3), 312–320. [3]

Co-editor Christopher Taber handled this manuscript.

Manuscript received 13 March, 2018; final version accepted 6 April, 2020; available online 5 May, 2020.