Supplementary Material

# Supplement to "A scale-free transportation network explains the city-size distribution"

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Keywords. Zipf's law, city-size distribution, scale-free network.
JEL classification. L14, R12, R40.

## Graphic files

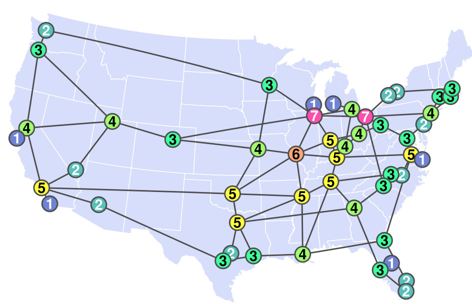

Figure 6. The Interstate route map (abridged).

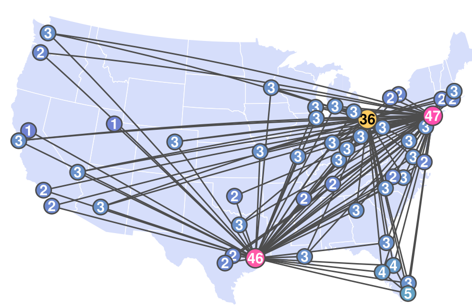

Figure 7. A typical airline's route configuration (premerger Continental Airlines).

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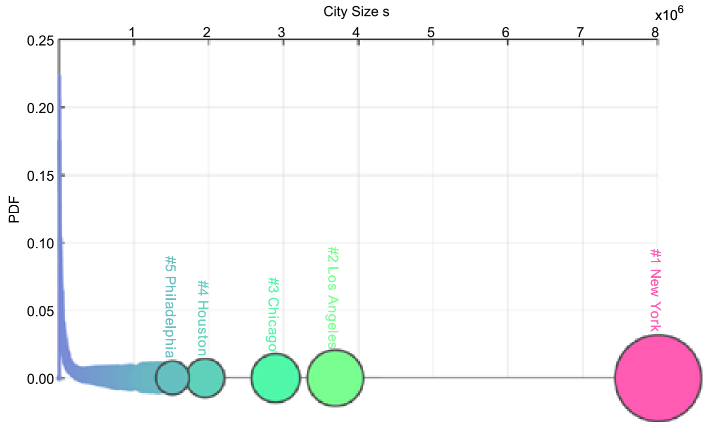
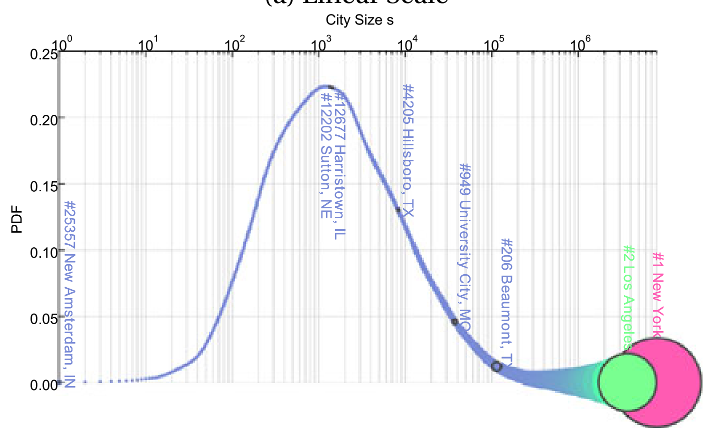
Available at http://qeconomics.org. https://doi.org/10.3982/QE619




(a) Linear Scale




(b) Log Scale

FIGURE 8. Frequency plot of the city-size distribution. Dots are size proportionate. See Table 1 for explanation of the cities selected in the figure. *Data source*: US Census 2000.

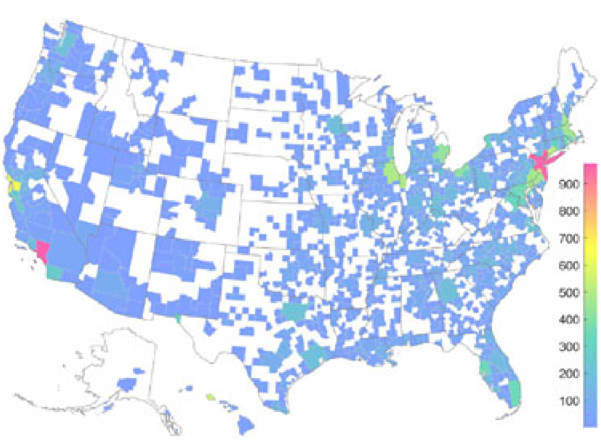
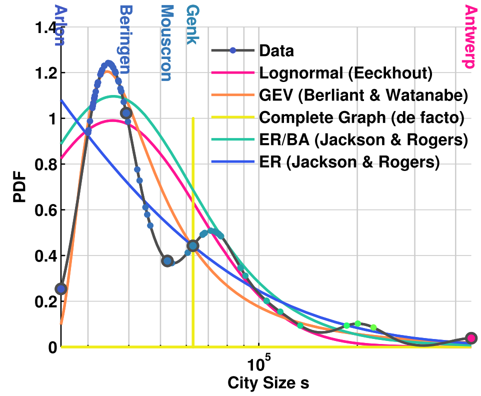



FIGURE 9. Population density by CBSA (persons/km$^2$). *Data source*: Census 2000.

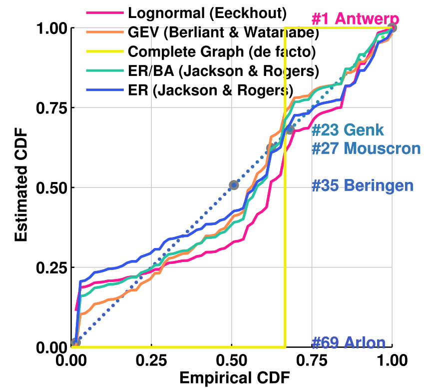
Table 2. Model comparison.

| Data | Distribution | ⟨log LH⟩ ▲ | KS ▼ | ⟨log step⟩ ▲ | geo/arith ▲ | $\lvert\theta\rvert$ | BIC ▼ | AIC ▼ | $\hat{r}$ | $\hat{\delta}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Belgium | Lognormal (Eeckhout) | -11.69 | 0.1986 | -5.266 | 0.005166/0.01449 | 2 | 1621 | 1617 | | |
| Belgium | GEV (Berliant & Watanabe) | **-11.40** | **0.1122** | **-4.981** | **0.006870**/0.01449 | 5 | **1594** | **1583** | | |
| Belgium | Complete Graph (de facto) | $-\infty$ | 0.6812 | $-\infty$ | 0/0.01449 | 1 | $\infty$ | $\infty$ | | |
| Belgium | ER/BA (Jackson & Rogers) | **-11.47** | **0.1348** | **-5.072** | **0.006268**/0.01449 | 5 | 1604 | **1593** | 0.002745 | 2.536 |
| Belgium | ER (Jackson & Rogers) | -11.49 | 0.1766 | -5.086 | 0.006185/0.01449 | 4 | **1603** | 1594 | $\infty$ | |
| MA | Lognormal (Eeckhout) | -14.28 | 0.1036 | -6.232 | 0.001996/0.003623 | 2 | 7891 | 7884 | | |
| MA | GEV (Berliant & Watanabe) | **-14.13** | **0.04334** | **-6.089** | **0.002267**/0.003623 | 5 | **7828** | **7810** | | |
| MA | Complete Graph (de facto) | $-\infty$ | 0.7935 | $-\infty$ | 0/0.003623 | 1 | $\infty$ | $\infty$ | | |
| MA | ER/BA (Jackson & Rogers) | **-14.17** | **0.06102** | **-6.134** | **0.002168**/0.003623 | 5 | **7852** | **7834** | 0.001154 | 1.275 |
| MA | ER (Jackson & Rogers) | -14.21 | 0.1057 | -6.173 | 0.002084/0.003623 | 4 | 7860 | 7851 | $\infty$ | |
| CBSA | Lognormal (Eeckhout) | -13.05 | 0.09402 | -7.548 | 0.0005270/0.001085 | 2 | 2.407e+04 | 2.406e+04 | | |
| CBSA | GEV (Berliant & Watanabe) | **-12.91** | **0.02606** | **-7.409** | **0.0006056**/0.001085 | 5 | **2.384e+04** | **2.382e+04** | | |
| CBSA | Complete Graph (de facto) | $-\infty$ | 0.8362 | $-\infty$ | 0/0.001085 | 1 | $\infty$ | $\infty$ | | |
| CBSA | ER/BA (Jackson & Rogers) | **-12.95** | **0.05922** | **-7.449** | **0.0005819**/0.001085 | 5 | **2.391e+04** | **2.389e+04** | 0.0004526 | 1.278 |
| CBSA | ER (Jackson & Rogers) | -13.29 | 0.1762 | -7.794 | 0.0004121/0.001085 | 4 | 2.450e+04 | 2.450e+04 | $\infty$ | |
| Places | Lognormal (Eeckhout) | **-9.258** | **0.01895** | **-8.840** | 0/3.944e-05 | 2 | **4.696e+05** | **4.696e+05** | | |
| Places | GEV (Berliant & Watanabe) | **-9.254** | **0.008847** | **-8.836** | 0/3.944e-05 | 5 | **4.694e+05** | **4.693e+05** | | |
| Places | Complete Graph (de facto) | $-\infty$ | 0.8342 | $-\infty$ | 0/3.944e-05 | 1 | $\infty$ | $\infty$ | | |
| Places | ER/BA (Jackson & Rogers) | -9.268 | 0.02198 | -8.849 | 0/3.944e-05 | 5 | 4.701e+05 | 4.700e+05 | 0.0003171 | 0.9911 |
| Places | ER (Jackson & Rogers) | -9.392 | 0.1134 | -8.974 | 0/3.944e-05 | 4 | 4.764e+05 | 4.763e+05 | $\infty$ | |
| Places | Lognormal as Degree Dist. | -9.258 | 0.01896 | -8.840 | 0/3.944e-05 | 4 | 4.696e+05 | 4.696e+05 | | |
| Places | GEV as Degree Dist. | -9.255 | 0.01159 | -8.836 | 0/3.944e-05 | 5 | 4.694e+05 | 4.694e+05 | | |

*Note*: Row color corresponds to the line colors in Figures 10 to 13. ▲ denotes a statistic the higher value of which indicates a better fit and ▼, the other way around. In the first row, ⟨log LH⟩ denotes the average of the log of likelihood values, KS denotes the Kolomogorov–Smirnov statistic, ⟨log step⟩ measures the geometric mean of the step $F(s_i;\theta) - F(s_{i-1};\theta)$ in the logarithmic scale. Geo/arith measures the ratio between geometric mean and arithmetic mean of the step. The closer the geometric mean is to the arithmetic mean, the better the fit is. It is zero for Places due to multiple cities having the same size. $\lvert\theta\rvert$ counts the number of parameters. BIC and AIC stand for Bayesian and Akaike Information Criteria for detecting overfitting. **Boldface with white foreground** marks the winner and **boldface with black foreground** denotes the runner-up among the five distributions tested. See the last paragraph in Section 4.2 for an explanation of the last two rows.

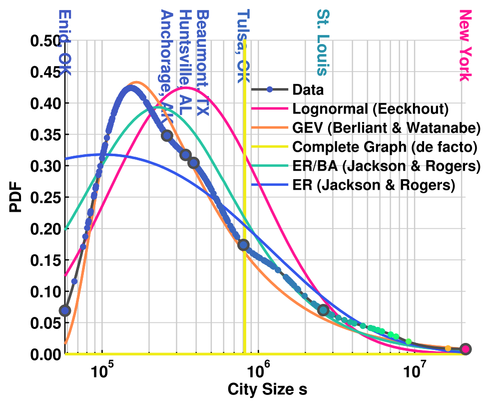



(a) PDF

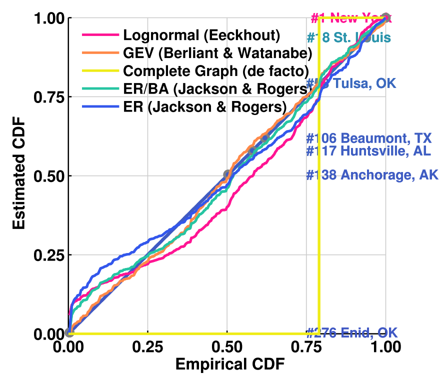



(b) PP Plot

FIGURE 10. Model comparison (Belgium).

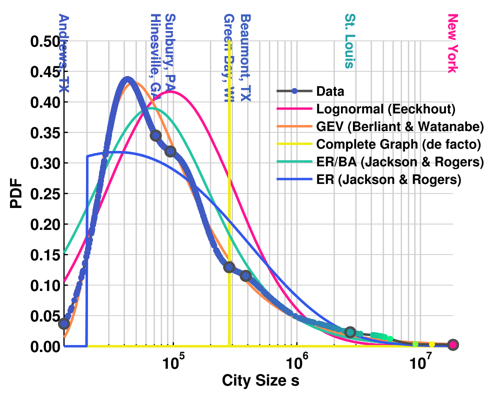



(a) PDF

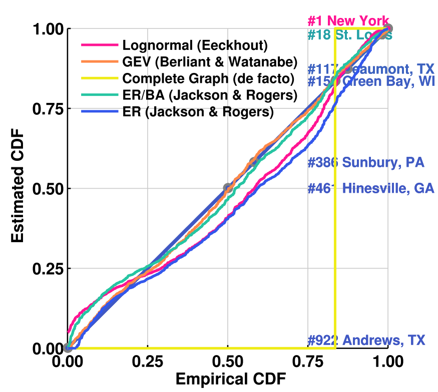



(b) PP Plot

FIGURE 11. Model comparison (MA).

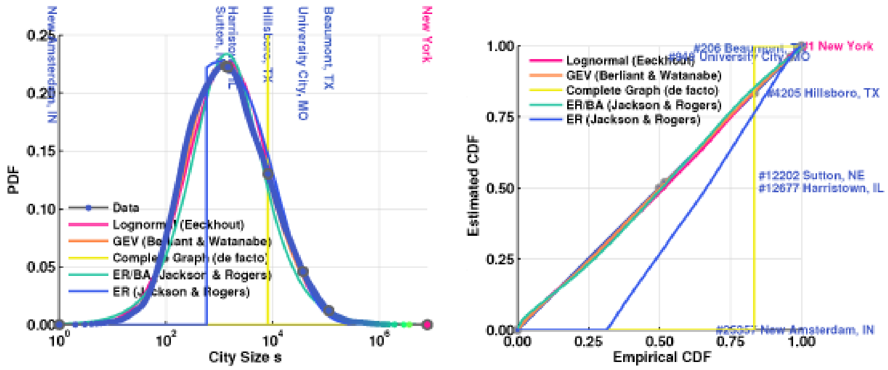



(a) PDF




(b) PP Plot

FIGURE 12. Model comparison (CBSA).




(a) PDF (b) PP Plot

Figure 13. Model comparison (Places).

Co-editor Karl Schmedders handled this manuscript.

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