

# Identification of income–leisure preferences and evaluation of income tax policy

CHARLES F. MANSKI

Department of Economics and Institute for Policy Research, Northwestern University

The merits of alternative income tax policies depend on the population distribution of preferences for income and leisure. Standard theory, which supposes that persons want more income and more leisure, does not predict how they resolve the tension between these desires. Empirical studies of labor supply have imposed strong preference assumptions that lack foundation. This paper examines anew the problem of inference on income–leisure preferences and considers the implications for evaluation of tax policy. I first perform a basic revealed-preference analysis assuming only that persons prefer more income and leisure. This shows that observation of a person's time allocation under a status quo tax policy may bound his allocation under a proposed policy or may have no implications, depending on the tax schedules and the person's status quo time allocation. I next explore the identifying power of two classes of assumptions that restrict the distribution of income–leisure preferences. One assumes that groups of persons who face different choice sets have the same preference distribution. The second restricts the shape of this distribution. The generic finding is partial identification of preferences. This implies partial prediction of tax revenue under proposed policies and partial knowledge of the welfare function for utilitarian policy evaluation.

**KEYWORDS.** Revealed preference analysis, partial identification, labor supply.

**JEL CLASSIFICATION.** C14, C25, H21, H24, J22.

## 1. INTRODUCTION

Economists have long recognized that the relative merits of alternative income tax policies depend on the preferences of individuals for income and leisure. The profession has also recognized the difficulty of inference on these preferences. In the conclusion to his seminal study of optimal income taxation, James Mirrlees (1971, p. 207) wrote, “The examples discussed confirm, as one would expect, that the shape of the optimum

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Charles F. Manski: [cfmanski@northwestern.edu](mailto:cfmanski@northwestern.edu)

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earned-income tax schedule is rather sensitive to the distribution of skills within the population, and to the income–leisure preferences postulated. Neither is easy to estimate for real economies.”

Income–leisure preferences play both positive and normative roles in analysis of tax policy, and it is important to distinguish them. The positive role is that preferences yield labor supply and other decisions that determine tax revenue. The normative role is that social welfare aggregates individual preferences in utilitarian policy evaluation. Thus, comparison of tax policies requires knowledge of preferences both to predict tax revenues and to compute the welfare achieved by alternative policies.

As I see it, we lack the knowledge of preferences necessary to credibly evaluate income tax policies. Analysis of the identification problem in Sections 2 and 3 explains how I have reached this sober conclusion. As prelude, I first offer my perspective on research that studies the response of labor supply to taxation, a subject that has received enormous attention. I then outline the paper.

### *Taxation and labor supply*

To begin, recall that standard economic theory does not predict the response of labor supply to income taxation. To the contrary, it shows that a worker may rationally respond in disparate ways. As tax rates increase, a person may rationally decide to work less, work more, or not change his labor supply at all.

The silence of theory on labor supply has long been appreciated; see [Robbins \(1930\)](#). Modern labor economics envisions labor supply as a complex sequence of schooling, occupation, and work effort decisions made under uncertainty over the life course. However, we need only consider a simple static scenario to see that a person may respond to income taxes in disparate ways.

Suppose that a person who has a predetermined wage and no unearned income allocates each day between paid work and the various nonpaid activities that economists have traditionally called leisure. Let a proportional income tax reduce his wage by the prevailing tax rate, yielding his net wage. Assume that the person allocates time to maximize utility, which is an increasing function of net income and leisure.

Different utility functions imply different relationships between the tax rate and labor supply. The labor supply implied by utility functions in the constant-elasticity-of-substitution (CES) family increases or decreases with the tax rate depending on the elasticity of substitution. Other utility functions imply that labor supply is *backward-bending*. That is, hours worked may initially increase as net wage rises from zero, but above some threshold, decrease as net wage rises further. Still other utility functions yield more complex nonmonotone relationships between net wage and labor supply. The review article of [Stern \(1986\)](#) describes a broad spectrum of possibilities.

Given that theory does not predict how income taxation affects labor supply, prediction requires empirical analysis. [Robbins \(1930, p. 129\)](#) emphasized this, concluding his article as follows: “we are left with the conclusion. . . that any attempt to predict the effect of a change in the terms on which income is earned must proceed by inductive investigation of elasticities.”

Economists have performed numerous empirical studies of labor supply. Their methodologies, data, and findings have been summarized and critiqued in multiple lengthy review articles including [Pencavel \(1986\)](#), [Killingsworth and Heckman \(1986\)](#), [Blundell and MaCurdy \(1999\)](#), [Meghir and Phillips \(2010\)](#), [Keane \(2011\)](#), and [Saez, Slemrod, and Giertz \(2012\)](#). Attempting to distill the huge literature, [Meghir and Phillips \(2010, p. 204\)](#) wrote, “Our conclusion is that hours of work do not respond particularly strongly to the financial incentives created by tax changes for men, but they are a little more responsive for married women and lone mothers. On the other hand, the decision whether or not to take paid work at all is quite sensitive to taxation and benefits for women and mothers in particular.” Focusing on compensated (Hicksian) elasticities of labor supply, [Saez, Slemrod, and Giertz \(2012, p. 1\)](#) wrote, “With some exceptions, the profession has settled on a value for this elasticity close to zero for prime-age males, although for married women the responsiveness of labor force participation appears to be significant. Overall, though, the compensated elasticity of labor appears to be fairly small.” [Keane \(2011, p. 1071\)](#) expressed a different perspective about compensated elasticities, writing, “My review suggests that labor supply of men may be more elastic than conventional wisdom suggests.”

Reading the recent literature concerned with uncompensated (Marshallian) elasticities of labor supply, I have been struck to find that while authors may differ on the magnitude of elasticities, they largely agree on the sign. The consensus is that increasing tax rates usually reduces work effort.<sup>1</sup> [Keane \(2011, p. 963\)](#) stated the directionality of the effect without reservation, writing, “the use of labor income taxation to raise revenue causes people to work less.” Considering the effect of a rise in a proportional tax, [Meghir and Phillips \(2010, p. 207\)](#) wrote, “in most cases this will lead to less work, but when the income effect dominates the substitution effect at high hours of work it may increase effort.” Here and elsewhere, researchers may recognize the theoretical possibility that effort may increase with tax rates, but view this as an empirical rarity rather than a regularity. This view has been accepted in official government forecasts of the response of labor supply to income taxation; see [Congressional Budget Office \(2007\)](#).

Examining the models of labor supply used in empirical research, I have become concerned that the prevailing consensus on the sign of uncompensated elasticities may be an artifact of model specification rather than an expression of reality.<sup>2</sup> Models differ across studies, but they generally share two key assumptions. First, they suppose that labor supply varies monotonically with net wages. Thus, model specifications do not generally permit backward-bending labor supply functions or other nonmonotone relationships. Second, they suppose that the response of labor supply to net wage is homoge-

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<sup>1</sup>Curiously, the opposite consensus prevailed early in the twentieth century. [Gilbert and Pfouts \(1958\)](#) cited assertions by [Pigou and Knight](#) in the 1920’s that increasing tax rates increases work effort.

<sup>2</sup>The literature contains some precedent for my concern that empirical findings on labor supply may be artifacts of model specification. Concluding his detailed comparison of alternative labor supply functions, [Stern \(1986, p. 173\)](#) wrote, “Our general conclusion must be in favour of diversity of functions and great caution in drawing policy conclusions on results based on a particular form.” [Stern](#) and other writers such as [Blundell and MaCurdy \(1999\)](#) have called attention to the potential detrimental consequences of restrictive functional-form assumptions.

neous within broad demographic groups. With occasional exceptions, researchers specify hours-of-work equations that permit hours to vary additively across group members, but that assume constant treatment response. That is, they assume that all group members would adjust hours worked in the same way in response to a conjectural change in net wage.<sup>3</sup>

The reality may be that persons have heterogeneous income–leisure preferences and, consequently, heterogeneous labor-supply functions. Some may increase work effort with net wage, others may decrease effort, and still others may exhibit a nonmonotone wage–effort relationship. If so, estimates of models that assume monotonicity and homogeneity of labor supply can at most characterize the behavior of an artificial “representative” person. The estimates may not have even this limited interpretation.

### *This paper*

In light of the above discussion, this paper examines anew the problem of identification of income–leisure preferences and draws implications for evaluation of tax policy. I study inference when data on time allocation under status quo tax policies are interpreted through the lens of standard theory. To illuminate elemental issues, I find it productive to study the classical static model in which persons with separable preferences for private and public goods must allocate one unit of time to work and leisure.

The classical static model is obviously simple, but it serves well the purpose of showing the difficulty of inference on income–leisure preferences. Inference becomes even more difficult when one considers models of dynamic labor supply (e.g., [MaCurdy \(1985\)](#)), models that consider how income taxation may affect decisions other than time allocation (e.g., [Feldstein \(1995\)](#)), models that conjecture socially interdependent preferences (e.g., [Lindbeck, Nyberg, and Weibull \(1999\)](#)), models that suppose persons may

<sup>3</sup>The review article by Keane (2011, p. 972) uses this hours-of-work equation to illustrate common practices in the literature:

$$\ln h_{it} = \beta + e \cdot \ln[w_{it}(1 - \tau_t)] + \beta_I \cdot N_{it} + \varepsilon_{it}.$$

Here  $i$  denotes a person,  $t$  is a time period,  $h$  is hours worked,  $w$  is wage,  $\tau$  is a proportional tax rate,  $N$  is unearned income, and  $(\beta, e, \beta_I)$  are parameters. Keane wrote, “Crucial is the addition of the stochastic term  $\varepsilon_{it}$ , which enables the model to explain heterogeneity in behavior.” However, the model permits only a very restricted form of heterogeneity. Hours of work may vary additively across persons through  $\varepsilon_{it}$ , but the parameters  $(\beta, e, \beta_I)$  do not vary across persons. In particular, the elasticity parameter  $e$  expresses the assumption of constant treatment response.

The occasional exceptions to such strong homogeneity assumptions begin with the important early research of [Burtless and Hausman \(1978\)](#), who emphasized the potential importance of heterogeneity, but were more limited in their ability to operationalize it. The authors ultimately assumed homogeneous response of hours worked to changes in net wage. The recent contributions of [Blundell and Shephard \(2012\)](#) and [Blundell, Bozio, and Laroque \(2011\)](#) also emphasized heterogeneity and are able to do more to operationalize it within a parametric random utility framework.

An exception to parametric modeling is [Blomquist and Newey \(2002\)](#). They initially proposed general nonparametric mean regression of hours worked on wage and on tax schedule variables that characterize a person’s (income, leisure) choice set. Concerned with the dimensionality of the variables needed to characterize the choice set, they subsequently studied estimation of a model that takes desired hours of work to be a nonparametric but increasing function of a scalar measure of preference heterogeneity.

be boundedly rational (e.g., [Chetty \(2012\)](#)), or models that do not assume separability of preferences for private and public goods (e.g., [Wildasin \(1979\)](#)).<sup>4</sup> These developments all generalize the classical static model. Hence, they logically pose more difficult inferential problems.

I consider the use of revealed preference analysis to predict labor supply and tax revenue under a proposed policy that would alter persons' status quo tax schedules. The policies that I have in mind use tax revenue to produce public goods and/or to redistribute income from persons who pay positive income tax to those who pay negative tax. I do not consider policies that extract positive income tax from persons and then compensate them through provision of lump-sum transfers that yield pre-tax utility levels. Thus, this paper concerns uncompensated rather than compensated responses of labor supply to income taxation.<sup>5</sup>

My objective is to shed light on how maintained assumptions affect the conclusions that one may draw about counterfactual labor supply and tax revenue. As in my past research, I find it illuminating to begin with weak assumptions and then to characterize the identifying power of stronger assumptions. I have explicated this approach to empirical research in [Manski \(2007a\)](#) and elsewhere.

Section 2 assumes only that persons prefer to have more income and leisure. Basic revealed-preference analysis of the type pioneered by [Samuelson \(1938\)](#) then shows that observation of a person's time allocation under a status quo tax policy may bound his allocation under a proposed policy or may have no implications, depending on the tax schedules and the person's status quo time allocation. Basic analysis that assumes only that more-is-better generically does not predict the sign of labor-supply response to change in the tax schedule.

Section 3 explores the identifying power of assumptions that restrict the distribution of preferences across persons. I suppose that one observes the time allocation of each person in a population whose members may have heterogeneous preferences and wages, and face various status quo tax schedules. I find it analytically helpful to suppose that persons choose among a finite set of feasible (income, leisure) values rather than the continuum often assumed in the literature. I then use the discrete-choice framework of [Manski \(2007b\)](#) to characterize preferences and to predict aggregate labor supply and tax revenue when various assumptions restrict the distribution of preferences.

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<sup>4</sup>[Wildasin \(1979, pp. 63–64\)](#) notably called attention to the last difficulty, observing, "The proper way of taking the effects of distortionary taxes into account in evaluating public expenditure depends sensitively on complement-substitute relations between public and private goods." He went on (p. 64), "Most bothersome of all is the fact that we have very little empirical information on the interaction between public good provision and private demand." This observation remains accurate today.

<sup>5</sup>Much research in public economics has used the idea of utility-preserving lump-sum transfers as a device to motivate interest in compensated elasticities of labor supply. I find this motivation puzzlingly remote from actual tax policy, which generally does not contemplate return of tax revenues to the population as utility-preserving lump-sum transfers. [Keane \(2011, p. 965\)](#) called attention to the fact that the elasticity of interest depends on how the government uses tax revenue, writing, "In a static model where the revenue is returned to the population via lump-sum transfers, it is again the Hicks elasticity concept that is most relevant. But if the revenue is used to finance public goods that do not influence labor supply, it is the Marshallian elasticity concept that is relevant."

I show how to determine the identifying power of two classes of assumptions. The first assumes exogenous variation in choice sets, in the formal sense that groups of persons who face different choice sets are assumed to have the same distribution of preferences. For example, one may assume that groups of persons who have different wages or who face different tax schedules have the same preference distribution. The second assumption restricts the shape of this distribution. For example, one may assume that all persons have preferences in the CES family, with possibly heterogeneous parameters. The generic finding is partial identification of the preference distribution. This implies partial ability to predict tax revenue under proposed policies. I use a computational experiment to illustrate.

The analysis in Sections 2 and 3 reaches highly cautionary findings about the present knowledge of income–leisure preferences. Section 4 discusses implications for evaluation of tax policy. A familiar exercise in normative public economics poses a utilitarian social welfare function and ranks tax policies by the welfare they achieve. This requires knowledge of income–leisure preferences to predict tax revenues and compute welfare. I observe that partial knowledge of preferences implies two difficulties for policy evaluation. One can only partially predict tax revenue and one can only partially evaluate the utilitarian welfare of policies.

The only route I see open to improve our capacity to evaluate tax policy is to collect richer data that shed more light on income–leisure preferences. The concluding Section 5 discusses two ideas. One possibility, difficult to achieve, would be for governments to promote exogenous variation in choice sets by decentralizing tax policy or by performing experiments that randomize persons into alternative tax schedules. Another possibility, easier to accomplish, is to enhance the data collected on individual behavior. The analysis in Sections 2 and 3 supposes that a researcher observes one status quo time allocation per person. The identifying power of revealed preference analysis grows if a researcher can observe individual behavior in multiple choice settings. The source of multiple observations may be longitudinal data on individual time allocation under varying wages or tax schedules, or it may be responses to questions that ask persons to predict the choices they would make in various scenarios. Implementing these ideas for data enrichment may lessen but not eliminate the difficulties for policy evaluation discussed in Section 4.

## 2. BASIC ANALYSIS OF REVEALED PREFERENCE

To begin, Section 2.1 formalizes the conventional description of income taxation and labor supply that I use. Section 2.2 applies basic revealed-preference analysis to derive partial predictions of individual labor supply under counterfactual alternatives to a status quo tax policy. Section 2.3 studies simple cases where the net-income functions under the status quo and proposed tax schedules cross once. I call this section a “basic” analysis of revealed preference because I maintain no assumptions about preferences except that individual utility is an increasing function of net income and leisure. In short, more is better.

In the absence of assumptions that restrict the population distribution of preferences, predicting population labor supply under a proposed tax policy simply requires aggregation of individual predictions. Hence, the analysis below focuses on one person.

### 2.1 Tax policy and labor supply

Suppose that a person labeled  $j$  is endowed with wage  $w_j$ , unearned income  $z_j$ , and one unit of time. The person must allocate his time endowment between leisure and work. If he allocates a fraction  $L \in [0, 1]$  to leisure and  $1 - L$  to work, he receives gross income  $w_j(1 - L) + z_j$ .

The status quo tax policy, denoted  $S$ , subtracts the work-dependent tax revenue  $R_{jS}(L)$  from gross income, leaving  $j$  with net income

$$Y_{jS}(L) \equiv w_j(1 - L) + z_j - R_{jS}(L). \quad (1)$$

Taxes may take positive or negative values. The  $R_{jS}(\cdot)$  notation allows the status quo tax schedule to be specific to person  $j$ . The schedule that  $j$  faces depends on the current tax policy of the jurisdiction where he resides. Within a tax jurisdiction, the tax levied may depend on unearned income and on personal attributes that determine eligibility for exemptions and deductions.

Person  $j$  chooses a value of  $L$  from a set  $\Lambda_j \subset [0, 1]$  of feasible leisure alternatives. Most analysis of labor supply supposes that  $\Lambda_j = [0, 1]$ . However, it may be more realistic to suppose that only a few allocations are feasible. For example,  $\Lambda_j = \{0, 1/2, 1\}$  means that the feasible options are full-time work ( $L = 0$ ), half-time work ( $L = 1/2$ ), and no work ( $L = 1$ ).

Preferences are expressed in person  $j$ 's utility function  $U_j(\cdot, \cdot)$ , whose arguments are (net income, leisure). Utility is strictly increasing in both arguments. Let  $L_{jS} \in \Lambda_j$  denote the amount of leisure that  $j$  chooses under tax schedule  $R_{jS}(\cdot)$ . Utility maximization implies the inequalities

$$U_j[Y_{jS}(L_{jS}), L_{jS}] \geq U_j[Y_{jS}(L), L], \quad \text{all } L \in \Lambda_j. \quad (2)$$

Using  $U_j(\cdot, \cdot)$  to express preferences suppresses the possible dependence of preferences on the public goods produced with tax revenue under policy  $S$ . This is innocuous if preferences are separable in private and public goods.

### 2.2 Predicting labor supply under a proposed tax schedule

Suppose that one observes the wage, unearned income, and other tax-relevant attributes of person  $j$ . One also observes the leisure  $L_{jS}$  chosen by  $j$  under tax schedule  $R_{jS}(\cdot)$ . Empirical microeconomic research on labor supply typically assumes observability of most of these quantities, the data source being surveys such as the Panel Study of Income Dynamics or the National Longitudinal Survey of Youth. A prominent exception is the usual absence of wage data for persons who do not work at all. I abstract from this well known difficulty of empirical research.

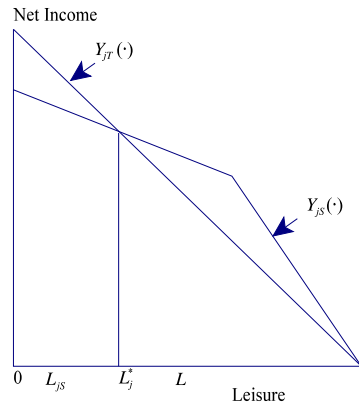


FIGURE 1. Net income with progressive and proportional tax schedules.

Let  $R_{jT}(\cdot)$  denote the tax schedule if  $j$  were to face a proposed tax policy  $T$ . What can one predict about time allocation under  $R_{jT}(\cdot)$ ? The answer depends on the value of  $L_{jS}$  and on the budget sets  $\{[Y_{jS}(L), L], L \in \Lambda_j\}$  and  $\{[Y_{jT}(L), L], L \in \Lambda_j\}$  that  $j$  faces under the status quo and proposed tax schedules.

*Illustration* I begin with an illustration, which modestly extends the original revealed-preference argument of Samuelson (1938). Consider the tax schedules in Figure 1. Policy  $S$  has a two-rate progressive schedule and policy  $T$  has a proportional one, the latter crossing the former from above when leisure equals  $L_j^*$ . Person  $j$  has no unearned income.

Suppose that  $L_{jS} \in [0, L_j^*]$  and consider any feasible  $L > L_j^*$ . The (income, leisure) pair  $[Y_{jT}(L_{jS}), L_{jS}]$  is feasible under policy  $T$ . Since more income is better than less, this pair is preferred to the pair  $[Y_{jS}(L_{jS}), L_{jS}]$  chosen under  $S$ . Pair  $[Y_{jS}(L), L]$  is feasible under policy  $S$ . Since more income is preferred to less, this pair is preferred to  $[Y_{jT}(L), L]$ . The observation that  $j$  chose  $[Y_{jS}(L_{jS}), L_{jS}]$  reveals that  $j$  prefers this pair to  $[Y_{jS}(L), L]$ . Combining these preference inequalities implies that he prefers  $[Y_{jT}(L_{jS}), L_{jS}]$  to  $[Y_{jT}(L), L]$ . Thus, if person  $j$  were to face the proposed tax schedule, he would not choose any  $L > L_j^*$ . It is shown below that this approach exhausts the predictive power of basic revealed-preference analysis.<sup>6</sup>

*General analysis* I now formally perform basic revealed-preference analysis without restricting the tax schedules generated by the status quo and proposed policies. The analysis requires a few definitions.

Given two bivariate real vectors  $(a, b)$  and  $(a', b')$ , define  $(a, b) < (a', b')$  and say that  $(a, b)$  is strictly smaller than  $(a', b')$  if  $(a \leq a'$  and  $b \leq b')$  and  $(a < a'$  or  $b < b')$ . Define

<sup>6</sup>I have written that this derivation modestly extends Samuelson (1938). Samuelson considered use of data on the commodity bundle chosen under a status quo linear budget set to predict the bundle that would be chosen under a counterfactual linear budget set. Figure 1 differs only in that the budget set produced by policy  $S$  is not linear. Linearity of either budget set is immaterial. Samuelson's argument holds whenever the status quo and counterfactual budget sets cross one another.



$(a, b) \leq (a', b')$  and say that  $(a, b)$  is weakly smaller than  $(a', b')$  if  $(a \leq a'$  and  $b \leq b')$ . Define strictly and weakly larger analogously.

Let  $\Lambda_{j<}$  denote the feasible leisure values such that the (income, leisure) pairs in the  $R_{jT}(\cdot)$  budget set are strictly smaller than some pair in the  $R_{jS}(\cdot)$  budget set; that is,

$$\Lambda_{j<} \equiv \{L_{<} \in \Lambda_j : [Y_{jT}(L_{<}), L_{<}] < [Y_{jS}(L), L] \text{ for some } L \in \Lambda_j\}. \quad (3)$$

Let  $\Lambda_{j>}$  denote the leisure values such that the (income, leisure) pairs in the  $R_{jT}(\cdot)$  budget set are strictly larger than  $[Y_{jS}(L_{jS}), L_{jS}]$ , the pair that  $j$  chooses under the status quo policy; that is,

$$\Lambda_{j>} \equiv \{L_{>} \in \Lambda_j : [Y_{jT}(L_{>}), L_{>}] > [Y_{jS}(L_{jS}), L_{jS}]\}. \quad (4)$$

Define  $\Lambda_{j\leq}$  and  $\Lambda_{j\geq}$  analogously, with weak inequalities replacing the strict ones. The sets  $(\Lambda_{j<}, \Lambda_{j>}, \Lambda_{j\leq}, \Lambda_{j\geq})$  are functions of the tax schedules  $R_{jS}(\cdot)$  and  $R_{jT}(\cdot)$ , but I keep this implicit to simplify the notation.

Revealed-preference analysis combines three facts. First, the choice of  $L_{jS}$  yields the inequalities (2). Second, the assumption that more is better implies that for each  $L_{<} \in \Lambda_{j<}$ , there exists an  $L \in \Lambda_j$  such that

$$U_j[Y_{jS}(L), L] > U_j[Y_{jT}(L_{<}), L_{<}]. \quad (5)$$

Similarly, for each  $L_{\leq} \in \Lambda_{j\leq}$ , there exists an  $L \in \Lambda_j$  such that

$$U_j[Y_{jS}(L), L] \geq U_j[Y_{jT}(L_{\leq}), L_{\leq}]. \quad (5')$$

Third, more-is-better implies that for each  $L_{>} \in \Lambda_{j>}$ ,

$$U_j[Y_{jT}(L_{>}), L_{>}] > U_j[Y_{jS}(L_{jS}), L_{jS}]. \quad (6)$$

Similarly, for each  $L_{\geq} \in \Lambda_{j\geq}$ ,

$$U_j[Y_{jT}(L_{\geq}), L_{\geq}] \geq U_j[Y_{jS}(L_{jS}), L_{jS}]. \quad (6')$$

Combining (2), (5), and (6') yields

$$U_j[Y_{jT}(L_{\geq}), L_{\geq}] \geq U_j[Y_{jS}(L_{jS}), L_{jS}] > U_j[Y_{jT}(L_{<}), L_{<}], \quad (7)$$

all  $L_{<} \in \Lambda_{j<}$  and  $L_{\geq} \in \Lambda_{j\geq}$ .

Similarly, combining (2), (5'), and (6) yields

$$U_j[Y_{jT}(L_{>}), L_{>}] > U_j[Y_{jS}(L_{jS}), L_{jS}] \geq U_j[Y_{jT}(L_{\leq}), L_{\leq}], \quad (7')$$

all  $L_{\leq} \in \Lambda_{j\leq}$  and  $L_{>} \in \Lambda_{j>}$ .

**Proposition 1** summarizes the implications for prediction of time allocation.

**PROPOSITION 1.** *Let  $\Lambda_{j<}$  and  $\Lambda_{j\geq}$  be nonempty. Then if person  $j$  were to face tax schedule  $R_{jT}(\cdot)$ , he would not choose any leisure value in  $\Lambda_{j<}$ . Let  $\Lambda_{j\leq}$  and  $\Lambda_{j>}$  be nonempty. Then if  $j$  were to face  $R_{jT}(\cdot)$ , he would not choose any leisure value in  $\Lambda_{j\leq}$ .*

These conclusions exhaust the predictive power of basic revealed-preference analysis for time allocation under the proposed tax schedule.

*Welfare implications* We may also draw partial conclusions about the welfare implications of replacing the status quo tax schedule with the proposed one. If  $\Lambda_{j<} = \Lambda_j$ , replacement of the status quo tax schedule with the proposal strictly decreases the welfare of person  $j$ . This follows purely from the assumption that more is better and does not require observation of  $j$ 's time allocation under the status quo. If  $\Lambda_{j>}$  is nonempty, replacement of the status quo with the proposal strictly increases welfare. This holds because nonemptiness of  $\Lambda_{j>}$  means that the budget set under the proposed schedule contains an (income, leisure) pair larger than the one  $j$  obtained under the status quo. Basic analysis yields no welfare conclusions if  $\Lambda_{j<}$  is a proper subset of  $\Lambda_j$  and  $\Lambda_{j>}$  is empty.

### 2.3 Prediction when downward-sloping net-income functions cross once

The above analysis was entirely general. I henceforth suppose that the net-income functions  $Y_{jS}(\cdot)$  and  $Y_{jT}(\cdot)$  implied by the status quo and proposed tax schedules are downward sloping. This condition is substantively innocuous, it being violated only if the marginal tax rate is above 100 percent at some level of work effort.

Given that the schedules are downward sloping, basic revealed-preference analysis can have predictive power for labor supply only if the two net-income functions cross at least once. Formally,  $Y_{jT}(\cdot)$  crosses  $Y_{jS}(\cdot)$  from above at some  $L_j^* \in (0, 1)$  if  $Y_{jT}(L_j^*) = Y_{jS}(L_j^*)$ ,  $Y_{jT}(L) > Y_{jS}(L)$  for  $L < L_j^*$ , and  $Y_{jT}(L) < Y_{jS}(L)$  for  $L > L_j^*$ . Similarly,  $Y_{jT}(\cdot)$  crosses  $Y_{jS}(\cdot)$  from below at  $L_j^*$  if the inequalities are reversed.

To see that a crossing is necessary for predictive power, consider a policy  $T$  such that  $Y_{jT}(L) \geq Y_{jS}(L)$  for all  $L \in [0, 1]$ . Thus, at all  $L$ , a person pays weakly less tax under  $T$  than under  $S$ . Then set  $\Lambda_{j<}$  is empty because  $Y_{jT}(L_{<}) < Y_{jS}(L) \Rightarrow L_{<} > L$ . Similarly, consider  $T$  such that  $Y_{jT}(L) \leq Y_{jS}(L)$  for all  $L \in [0, 1]$ . Thus, at all  $L$ , a person pays weakly more tax under  $T$  than under  $S$ . Then  $\Lambda_{j\geq}$  is empty because  $Y_{jT}(L_{<}) > Y_{jS}(L_{jS}) \Rightarrow L_{<} < L_{jS}$ .

Policy comparison is simple when the two tax schedules cross exactly once, as in Figure 1. When, as in the figure,  $Y_{jT}(\cdot)$  crosses  $Y_{jS}(\cdot)$  from above,  $\Lambda_{j<} = [L \in \Lambda_j : L > L_j^*]$  and  $\Lambda_{j\leq} = [L \in \Lambda_j : L \geq L_j^*]$ .<sup>7</sup> The set  $\Lambda_{j>}$  is nonempty if and only if  $L_{jS} < L_j^*$ , and  $\Lambda_{j\geq}$  is nonempty if and only if  $L_{jS} \leq L_j^*$ .<sup>8</sup> Hence, observation that  $L_{jS} < L_j^*$  implies that if

<sup>7</sup>To see this, first observe that  $L > L_j^* \Rightarrow [Y_{jT}(L), L] < [Y_{jS}(L), L]$  and  $L = L_j^* \Rightarrow [Y_{jT}(L), L] = [Y_{jS}(L), L]$ . Hence,  $[L \in \Lambda_j : L > L_j^*] \subset \Lambda_{j<}$  and  $[L \in \Lambda_j : L \geq L_j^*] \subset \Lambda_{j\leq}$ . Now consider  $L < L_j^*$ . Then  $Y_{jT}(L) > Y_{jS}(L)$ . Given that  $Y_{jS}(\cdot)$  is downward sloping,  $Y_{jT}(L) < Y_{jS}(L') \Rightarrow L' < L$ . Hence, there exists no  $L'$  such that  $[Y_{jT}(L), L] < [Y_{jS}(L'), L']$ . Hence,  $L < L_j^* \Rightarrow L \notin \Lambda_{j<}$ .

<sup>8</sup>To see this, first observe that  $L_{jS} < L_j^* \Rightarrow [Y_{jT}(L_{jS}), L_{jS}] > [Y_{jS}(L_{jS}), L_{jS}]$  and  $L_{jS} = L_j^* \Rightarrow [Y_{jT}(L_{jS}), L_{jS}] = [Y_{jS}(L_{jS}), L_{jS}]$ . Hence,  $L_{jS} < L_j^*$  implies that  $\Lambda_{j>}$  is nonempty and  $L_{jS} = L_j^*$  implies that

person  $j$  were to face tax schedule  $R_{jT}(\cdot)$ , he would not choose any  $L \geq L_j^*$ ; observation that  $L_{jS} \leq L_j^*$  implies that he would not choose any  $L > L_j^*$ ; observation that  $L_{jS} > L_j^*$  has no predictive power.<sup>9</sup>

The above illustration shows that the sign of the response of labor supply to the change in tax schedule is predictable only if status quo leisure is at a corner or equals  $L_j^*$ . If status quo leisure equals 0 or 1, then labor supply under the proposed schedule obviously cannot increase or decrease, respectively. If status quo leisure is  $L_{jS} = L_j^*$ , then basic analysis of revealed preference predicts that labor supply will not decrease under the proposed schedule. Thus, basic analysis predicts the sign of the labor-supply response only in the latter very special case.

*Numerical illustration* To illustrate, suppose that the progressive schedule taxes income at rate 15 percent up to \$50,000 per year and at rate 25 percent above \$50,000. Let the proportional schedule tax all income at rate 20 percent. Thus, the tax schedules cross when gross income equals \$100,000, when both take \$20,000 tax and yield net income \$80,000. As in Figure 1, suppose that person  $j$  has no unearned income. The conclusions that may be drawn about time allocation and welfare under the proposed tax schedule depend on person  $j$ 's wage, his feasible set of time allocations, and the allocation he chooses under the status quo schedule. In what follows, I measure wage as the annual gross labor income that  $j$  would receive for full-time work and I suppose that all time allocations are feasible.

Suppose first that  $w < \$100,000$ . Then net income under the proposed schedule is lower than net income under the status quo schedule for all  $L \in [0, 1]$ . Hence, the proposed schedule definitely lowers welfare. Revealed-preference analysis yields no prediction of time allocation.

Next let  $w = \$150,000$ . Then  $L^* = 1/3$ , which is roughly the case in Figure 1. Basic revealed-preference analysis has predictive power if  $L_{jS} \leq 1/3$ ; that is, if  $j$  works at least  $2/3$  of the time in the status quo. Then we can conclude that leisure under the proposed schedule would lie in the interval  $[0, 1/3]$  and that welfare would increase. However, no prediction of time allocation or welfare is possible if  $L_{jS} > 1/3$ .

Finally let  $w = \$500,000$ . Then  $L^* = 4/5$ . If  $L_{jS} \leq 4/5$ , we can conclude that leisure under the proposed schedule would lie in the interval  $[0, 4/5]$  and that welfare would increase. No prediction is possible if  $L_{jS} > 4/5$ .

*Inequalities on aggregate labor supply* Although basic analysis rarely predicts the sign of individual labor-supply response, it generically yields some simple inequalities that relate population labor supply under the status quo and proposed tax policies. Let the

$A_{j\geq}$  is nonempty. Now consider  $L_{jS} > L_j^*$ . Then  $Y_{jT}(L_{jS}) < Y_{jS}(L_{jS})$ . Given that  $Y_{jT}(\cdot)$  is downward sloping,  $Y_{jT}(L') \geq Y_{jS}(L_{jS}) \Rightarrow L' < L_{jS}$ . Hence, there exists no  $L'$  such that  $[Y_{jT}(L'), L'] \geq [Y_{jS}(L_{jS}), L_{jS}]$ . Hence,  $L_{jS} > L_j^*$  implies that  $A_{j\geq}$  is empty.

<sup>9</sup>Analogous findings hold when  $Y_{jT}(\cdot)$  crosses  $Y_{jS}(\cdot)$  from below. Then  $A_{j<} = [L_{<} \in A_j : L_{<} < L_j^*]$  and  $A_{j\leq} = [L_{\leq} \in A_j : L_{\leq} \leq L_j^*]$ . The set  $A_{j>} = [L_{>} \in A_j : L_{>} > L_j^*]$  is nonempty if  $L_{jS} > L_j^*$  and  $A_{j\geq}$  is nonempty if  $L_{jS} \geq L_j^*$ . Hence, observation that  $L_{jS} > L_j^*$  implies that if  $j$  were to face schedule  $R_{jT}(\cdot)$ , he would not choose any  $L \leq L_j^*$ ; observation that  $L_{jS} \geq L_j^*$  implies that he would not choose any  $L < L_j^*$ ; observation that  $L_{jS} < L_j^*$  has no predictive power.

proposed tax schedule cross the status quo from above. Then, among persons whose tax schedules cross at the same point  $l$ , the fraction who choose leisure less than  $l$  is higher under the proposed schedule than under the status quo schedule.

To see this, let  $J$  denote a population of persons formalized as a probability space  $(J, \Omega, P)$ . Let  $L_{jT}$  denote the leisure that person  $j$  would choose under the proposed schedule  $R_{jT}(\cdot)$ . We have found that for each  $j \in J$ ,  $L_{jS} < L_j^* \Rightarrow L_{jT} < L_j^*$  and  $L_{jS} \leq L_j^* \Rightarrow L_{jT} \leq L_j^*$ . It follows that for all  $l \in [0, 1]$ ,<sup>10</sup>

$$P(L_S < L^* | L^* = l) \leq P(L_T < L^* | L^* = l), \quad (8a)$$

$$P(L_S \leq L^* | L^* = l) \leq P(L_T \leq L^* | L^* = l). \quad (8b)$$

Inequalities (8a) and (8b) formalize a rather limited sense in which decreasing the progressivity of a tax schedule must weakly increase aggregate labor supply. If schedule  $T$  crosses  $S$  from above at point  $l$ , then  $T$  is less progressive than  $S$  in the specific sense that, among people with crossing point  $l$ ,  $T$  yields higher net income than  $S$  when leisure is less than  $l$  and yields lower net income than  $S$  when leisure is larger than  $l$ .

I say that the inequalities formalize a “rather limited” relationship between progressivity and labor supply because they hold only at the crossing point of the two tax schedules. Consider (8a). It is consistent with rational choice and the more-is-better assumption to have the reverse inequality hold at any  $L$  other than  $l$ ; that is, it can happen that  $P(L_S < L | L^* = l) > P(L_T < L | L^* = l)$  for any  $L \neq l$ .

### 3. RESTRICTIONS ON THE PREFERENCE DISTRIBUTION

A huge distance separates basic revealed-preference analysis from the practice of empirical analysis of labor supply. As noted in the [Introduction](#), the models used in empirical studies usually suppose that labor supply responds monotonically to variation in net wage. Moreover, it is common to assume that time allocation differs across persons only via a person-specific additive constant.

To explore the vast middle ground between basic analysis and current practice, I find it productive to use the discrete-choice framework developed in [Manski \(2007b\)](#). Section 3.1 lays out basic concepts and notation. Section 3.2 shows how to determine the identifying power of alternative assumptions that place restrictions on the population distribution of preferences. Section 3.3 discusses the mathematics and the substance of the identification problem. Section 3.4 presents a computational experiment using the framework to predict tax revenue under a proposed tax policy.

<sup>10</sup>Hoderlein and Stoye (2012, Lemma 1) reported a version of this inequality in the context of the classical problem of consumer demand for two goods and cited earlier, more abstract, research that implies the result. In the classical demand setting, one observes demand under status quo product prices and wants to predict demand under counterfactual prices. Among persons with the same income, the status quo and counterfactual budget lines necessarily cross once, at the same point.

### 3.1 Preferences on a finite universe of alternatives

Consider again a population  $J$  of persons formalized as a probability space  $(J, \Omega, P)$ . Suppose that the status quo and proposed tax policies together generate a finite universe of potential values for (net income, leisure); that is, the universe

$$A \equiv \{[Y_{jS}(L), L], [Y_{jT}(L), L], L \in \Lambda_j, j \in J\} \quad (9)$$

of distinct  $(Y, L)$  pairs that members of the population can realize under either policy has finite cardinality. Current practice usually treats net income and leisure as continuous variables, but some authors have viewed time allocation as a choice among a discrete set of leisure options; see, for example, Blundell and Shephard (2012), Hoynes (1996), and Keane and Moffitt (1998).

Viewing the universe of feasible  $(Y, L)$  pairs as finite rather than continuous enables one to explore flexibly the identifying power of a spectrum of assumptions on the preferences of population members. Analysis is most straightforward if almost all persons have strict preference orderings.<sup>11</sup> There are  $|A|!$  potential strict orderings on  $A$ . The number of feasible orderings is smaller than  $|A|!$  because the assumption that more-is-better excludes some orderings, the number excluded depending on the composition of  $A$ .

Let  $A_k, k \in K$ , denote the orderings of set  $A$  that are consistent with the assumption that more is better. Then the population distribution of preferences is multinomial with at most  $|K|$  mass points, where  $|K| \leq |A|!$ . All assumptions about population preferences take the form of restrictions on this multinomial distribution.<sup>12</sup>

*Numerical illustration* Consider again the illustration of Section 2.3. A status quo progressive schedule taxes income at rate 15 percent up to \$50,000 per year and at rate 25 percent above \$50,000. A proposed proportional schedule taxes all income at rate 20 percent.

Consider a population of high-skilled, prime-age persons. Let each person have two time-allocation alternatives, full-time or half-time work. Thus,  $\Lambda_j = \{0, 1/2\}$  for all  $j \in J$ .

<sup>11</sup>The assumption that almost all persons have strict preference orderings can be weakened. The number of possible ordering is still finite, but larger, if we allow weak orderings that may exhibit indifference among some  $(Y, L)$  pairs. However, when considering prediction, it is essential that at least some known positive fraction of orderings be strict rather than weak. The concept of rationality makes no prediction about choice among equally ranked alternatives. Revealed-preference analysis is vacuous if one permits an unknown fraction of the population to be indifferent among all elements of  $A$ .

<sup>12</sup>In Manski (2007b), I observed that prediction of counterfactual choice behavior does not require that one distinguish all feasible preference orderings from one another. Multiple preference orderings may yield identical choices when facing all status quo and proposed choice sets. Thus, it is only necessary to distinguish groups of preference orderings that have different *choice functions*; that is, groups yielding different choices in some relevant choice setting. This permits one to reduce the set  $K$  of distinct preference orderings to a smaller set of distinct choice functions, which can be computationally advantageous in the analysis below.

I will, however, not reduce  $K$  to the set of distinct choice functions. The reason is that prediction of counterfactual choice behavior is not the sole concern of this paper. A further concern, discussed in Section 4, is to compare policies via a utilitarian social welfare function. Such policy comparison requires one to distinguish persons by their utility functions, not just by their choice functions.

Let the population divide into two groups with different annual wages for full-time work,  $w = \$150,000$  or  $w = \$500,000$ . Then the  $(Y, L)$  choice sets are as follows:

	Status Quo Schedule		Proposed Schedule	
Group 1 ( $w = \$150,000$ )				
Full-time work	a	[117,500, 0]	e	[120,000, 0]
Half-time work	b	[61,250, 1/2]	f	[60,000, 1/2]
Group 2 ( $w = \$500,000$ )				
Full-time work	c	[380,000, 0]	g	[400,000, 0]
Half-time work	d	[192,500, 1/2]	h	[200,000, 1/2]

The universe  $A$  of potential values of  $(Y, L)$  contains eight elements, denoted a–h. Thus, the total number of strict preference orderings is  $8! = 40,320$ . However, the assumption that more is better eliminates many orderings. In particular, it implies that feasible orderings satisfy the conditions  $g > c > e > a$ ,  $h > d > b > f$ , and  $d > e$ . Eliminating the orderings that do not satisfy these conditions leaves 53 feasible orderings.

### 3.2 Identification analysis

*Assumptions* The analysis in this section shows how to determine the identifying power of two classes of assumptions on preferences. The first assumes that groups of persons who face different status quo choice sets have the same distribution of preferences. Versions of this assumption have been common in empirical research, often described as *exogenous* variation in choice sets. Studies regularly assume that within a tax jurisdiction, groups of persons with different wages have the same preference distribution. Studies often assume that groups of persons facing different tax schedules have the same preference distribution. Variation in tax schedule may occur geographically across tax jurisdictions or it may occur temporally as a given jurisdiction changes its tax policy. It may also occur through performance of experiments that randomize tax schedules across persons, such as the American negative income tax experiments of the 1970's.<sup>13</sup>

The second assumption adds restrictions on the shape of the preference distribution. For example, one may assume that all persons have preferences in the CES family but place no restrictions on the parameters of each person's CES utility function or one may go further and constrain the variation of parameters across persons. Empirical studies have typically made strong assumptions of preference homogeneity.

<sup>13</sup>It is important to distinguish two bodies of research that use observed variation in labor supply across persons facing different tax schedules to draw conclusions about policy impacts. Structural econometric research, which includes the present paper, uses the data to learn about the population distribution of income–leisure preferences and then uses knowledge of preferences to predict the impacts of counterfactual tax policies. Atheoretic analyses of treatment response study policy impacts directly, viewing tax policy as a treatment and labor supply as a response. For example, researchers may perform before-and-after studies, comparing labor supply in a given tax jurisdiction before and after a change in tax policy, or they may compare the labor supply of persons living in different tax jurisdictions. Such studies do not enable prediction of the impacts of counterfactual policies; they only enable comparison of policies that have actually been enacted in some tax jurisdiction.

*Analysis* As in Section 2, I suppose that one observes the wage, unearned income, and other tax-relevant attributes of the members of the population. One also observes the leisure chosen by each person under his status quo tax schedule.

To formalize the assumptions placed on the distribution of preferences, divide the population into a collection  $(J_m, m \in M)$  of mutually exclusive and exhaustive groups according to the  $(Y, L)$  choice sets they face under the status quo and proposed tax schedules. Thus, all members of group  $m$  face a common choice set  $C_m \subset A$  of  $(Y, L)$  pairs under the status quo schedule and a common set  $D_m \subset A$  under the proposed schedule. Groups  $m$  and  $m'$  are distinct if  $C_m \neq C_{m'}$  or  $D_m \neq D_{m'}$ . Substantively, groups are composed of persons who have the same tax-relevant attributes. Persons who have the same wage, unearned income, exemptions/deductions, and live in the same tax jurisdiction face the same income–leisure options.

Next, divide the population into a finite collection  $X$  of groups of persons with common observed covariates. These covariates need not be tax relevant. For example, covariate groups may be defined by gender, race, and/or education. Let  $J_{mx}$  denote the subpopulation of persons who are members of tax group  $m$  and covariate group  $x$ .

Let  $\pi_{mx} \equiv (\pi_{mxk}, k \in K)$  denote the multinomial distribution of preferences in (tax, covariate) group  $(m, x)$ . Thus, for each  $k \in K$ ,  $\pi_{mxk}$  is the fraction of group  $(m, x)$  who have preference ordering  $A_k$ . With no assumptions on the population distribution of preferences,  $(\pi_{mx}, m \in M, x \in X)$  is an unrestricted set of multinomial distributions, one for each (tax, covariate) group. The analysis of this section first assumes that a specified set of groups have the same preference distribution and then places restrictions on the shape of this distribution. Formally, consider the following assumptions.

**ASSUMPTION 1.** *For a specified class of (tax, covariate) groups  $N \subset M \times X$ , there exists a single multinomial distribution  $\pi_N \equiv (\pi_{Nk}, k \in K)$  such that  $\pi_{mx} = \pi_N$ , all  $(m, x) \in N$ .*

**ASSUMPTION 2.** *The distribution  $\pi_N$  lies in a specified set  $\Pi_N$  of multinomial distributions with at most  $|K|$  mass points.*

Assumption 1 poses a substantive requirement whose credibility depends on the specification of  $N$ . One may think it credible to assume that some  $(m, x)$  groups have the same preference distribution but not others. Assumption 1 does not restrict the shape of the distribution  $\pi_N$  of (income, leisure) preferences. Assumption 2 generically adds such restrictions. Particularly simple are *linear* restrictions that take  $\Pi_N$  to be a set of multinomial distributions that satisfy specified linear equalities or inequalities. These include assumptions that restrict the preference orderings that appear in the population. Suppose one believes that some orderings, say  $K_{N0} \subset K$ , do not occur in class  $N$ . Then  $\pi_{Nk} = 0, k \in K_{N0}$ .

The identifying power of Assumptions 1 and 2 is easy to characterize, but we first need some new notation. For each nonempty set  $B \subset A$ , let  $c_k(B)$  denote the  $(Y, L)$  pair that a person with preference ordering  $k$  would choose if he were to face choice set  $B$ . Let  $c_j(B)$  denote the pair that person  $j$  would choose and let the random variable  $c(B)$  denote the choice of a person randomly drawn from  $J$ . For each  $(m, x) \in N$  and

$(Y, L) \in C_m$ , let  $P[c(C_m) = (Y, L)|J_{mx}]$  give the observed fraction of subpopulation  $J_{mx}$  who make choice  $(Y, L)$  when facing their status quo choice set  $C_m$ . Similarly, for each  $(m, x) \in N$  and  $(Y, L) \in D_m$ , let  $P[c(D_m) = (Y, L)|J_{mx}]$  give the unobserved fraction of persons in  $J_{mx}$  who would choose  $(Y, L)$  if they were to face choice set  $D_m$ .

Assumption 1 implies that the observed choice probabilities  $P[c(C_m) = (Y, L)|J_{mx}]$ ,  $(Y, L) \in C_m$ ,  $(m, x) \in N$ , are related to the preference distribution  $\pi_N$  through the linear equations

$$P[c(C_m) = (Y, L)|J_{mx}] = \sum_{k \in K} 1[c_k(C_m) = (Y, L)] \cdot \pi_{Nk},$$

$$(Y, L) \in C_m, (m, x) \in N. \quad (10)$$

For each value of  $(m, x)$  and  $(Y, L)$ , the right-hand side adds up the fractions of  $J_{mx}$  whose preference orderings make  $(Y, L)$  their most preferred alternative in  $C_m$ . Distribution  $\pi_N$  also satisfies the adding-up and nonnegativity conditions

$$\sum_{k \in K} \pi_{Nk} = 1 \quad (11a)$$

and

$$\pi_{Nk} \geq 0, \quad k \in K. \quad (11b)$$

Finally, Assumption 2 states that

$$\pi_N \in \Pi_N. \quad (12)$$

Proposition 2 states the implications for identification of the distribution of preferences.

**PROPOSITION 2.** *Given Assumptions 1 and 2, the identification region for  $\pi_N$  is the set  $H(\pi_N)$  of multinomial distributions that solve (10), (11a), (11b), and (12).*

The set  $H(\pi_N)$  has a simple structure when  $\Pi_N$  places linear restrictions on  $\pi_N$ . Then  $H(\pi_N)$  is the convex set of vectors in  $R^{|K|}$  that solve a set of linear equalities and inequalities. The set  $H(\pi_N)$  is nonempty if the preference assumptions are correct; that is, if almost all persons have strict preferences that satisfy more-is-better and if Assumptions 1 and 2 hold. If  $H(\pi_N)$  is empty, then some assumption is incorrect.

When  $H(\pi_N)$  is nonempty, it is straightforward to predict the choice behavior of class  $N$  under the proposed tax schedule. The unobserved choice probabilities  $P[c(D_m) = (Y, L)|J_{mx}]$ ,  $d \in D_m$ ,  $(m, x) \in N$ , are related to  $\pi_N$  through the linear equations

$$P[c(D_m) = (Y, L)|J_{mx}] = \sum_{k \in K} 1[c_k(D_m) = (Y, L)] \cdot \pi_{Nk},$$

$$(Y, L) \in D_m, (m, x) \in N. \quad (13)$$



The feasible preference distributions are the elements of  $H(\pi_N)$ . Hence, the identification region for the unobserved choice probabilities is

$$\begin{aligned} & H\{P[c(D_m) = (Y, L)|J_{mx}], (Y, L) \in D_m, (m, x) \in N\} \\ &= \left\{ \left\{ \sum_{k \in K} 1[c_k(D_m) = (Y, L)] \cdot p_k, (Y, L) \in D_m, (m, x) \in N \right\}, \right. \\ & \quad \left. \text{all } (p_k, k \in K) \in H(\pi_N) \right\}. \end{aligned} \quad (14)$$

A question of policy interest is to predict tax revenue under the proposed tax schedule. Recall that  $L_{jT}$  denotes the leisure that person  $j$  would choose under the proposed schedule  $R_{jT}(\cdot)$ . Let the random variable  $R_T(L_T)$  denote the tax paid by a randomly drawn member of  $N$ . Then the per capita tax paid by the persons in  $N$  under the proposed schedule is  $E[R_T(L_T)|J_N]$ . This is related to the preference distribution as

$$\begin{aligned} & E[R_T(L_T)|J_N] \\ &= \sum_{(m,x) \in N} P(m, x) \sum_{(Y,L) \in D_m} R_{mT}(L) \sum_{k \in K} 1[c_k(D_m) = (Y, L)] \cdot \pi_{Nk}, \end{aligned} \quad (15)$$

where  $R_{mT}(\cdot)$  is the common tax schedule faced by members of tax group  $m$  and where  $P(m, x)$  is the fraction of the population who belong to group  $(m, x)$ . Hence, the identification region for per capita tax revenue in class  $N$  is the set

$$\begin{aligned} & H\{E[R_T(L_T)|J_N]\} \\ &= \left\{ \sum_{(m,x) \in N} P(m, x) \sum_{(Y,L) \in D_m} R_{mT}(L) \sum_{k \in K} 1[c_k(D_m) = (Y, L)] \cdot p_k, \right. \\ & \quad \left. \text{all } (p_k, k \in K) \in H(\pi_N) \right\}. \end{aligned} \quad (16)$$

When  $\Pi_N$  places linear restrictions on  $\pi_N$ , convexity of  $H(\pi_N)$  and the linearity of equation (14) in  $\pi_N$  imply that  $H\{E[R_T(L_T)]\}$  is an interval. The lower and upper bounds of this interval solve two linear programming problems. The lower bound solves

$$\begin{aligned} & \min_{(p_k, k \in K) \in H(\pi_N)} \sum_{(m,x) \in N} P(m, x) \\ & \quad \times \sum_{(Y,L) \in D_m} R_{mT}(L) \sum_{k \in K} 1[c_k(D_m) = (Y, L)] \cdot p_k. \end{aligned} \quad (17)$$

The upper bound solves the analogous problem, with max replacing min.

When the restrictions imposed by  $\Pi_N$  are not linear, minimization problem (16) still gives the sharp lower bound and the corresponding maximization problem gives the sharp upper bound. However, the identification region may or may not be the entire interval that connects these bounds, depending on the structure of  $\Pi_N$ .

### 3.3 Discussion

The identification analysis of Section 3.2 is mathematically simple. When  $\Pi_N$  places linear restrictions on  $\pi_N$ , computation is tractable as well. However, two matters are less straightforward: one is analytical and the other is substantive.

Analytically, one would like to be able to characterize succinctly the identifying power of alternative assumptions. When  $\Pi_N$  places linear restrictions on  $\pi_N$ , one can crudely count the number of linear equations and inequalities that (10), (11a), (11b), and (12) place on the  $|K|$ -dimensional vector  $\pi_N$  of multinomial probabilities. Unfortunately, doing so does not reveal much about the size and shape of  $H(\pi_N)$  or about the width of bounds on derived quantities such as tax revenue under proposed tax schedules. The structure of  $H(\pi_N)$  depends delicately on the choice sets that the various tax groups in class  $N$  face under the status quo tax policy. The implied bounds on outcomes under a proposed tax schedule depend as well on the choice sets that arise under the proposed schedule.

To learn the identifying power of alternative assumptions, it appears necessary to impose them and compute the resulting identification regions. There does not seem to be an effective shortcut to characterize  $H(\pi_N)$  or derived quantities. This should not be surprising in light of the common difficulty of deriving succinct descriptions of identification for structural econometric models. At one pole of research, a body of nonparametric analysis of discrete choice that uses Assumption 1 but places no restrictions on preference distributions has found that even simple configurations of observed binary choice probabilities imply subtle restrictions on preference distributions; see Marschak (1960), Fishburn (1992), and McFadden (2005). At another pole, researchers performing parametric structural analysis of labor supply have often been unable to give succinct necessary and sufficient conditions for point identification of their models. The present analysis, which combines Assumptions 1 and 2 with the assumption that more is better, studies models in the middle ground between these poles.

The substantive issue is that we often lack a credible basis for placing informative restrictions on preference distributions. Standard economic theory only suggests that persons have preference orderings that satisfy more-is-better. This is a compelling starting point for analysis, but it is not clear how to go beyond it. The strong assumptions regarding the homogeneity and structure of preferences used in empirical research on labor supply have been motivated by a desire to obtain tractable point estimates, not by empirical evidence. Researchers regularly assert highly restrictive versions of Assumptions 1 and 2 without offering a rationale beyond analytical convenience.

Researchers sometimes try to motivate preference assumptions by viewing time allocation as a neoclassical problem of budget allocation, with income and leisure being goods akin to apples and oranges. They may thus introspect on whether leisure is a normal good, whether the income effect of a change in net wage outweighs the substitution effect, and so on. Such introspection may perhaps be a helpful way to conjecture plausible forms for income–leisure preferences. However, it is important to understand that the analogy of time allocation and neoclassical budget allocation is inexact.

The neoclassical problem presumes that a person allocates a specified money budget between two goods with linear pricing determining the set of feasible allocations.

In contrast, the time-allocation problem presumes that a person begins with two asymmetric endowments, one of time and the other of unearned income. The asymmetry is that the person can transform time into income through paid work, but he cannot transform unearned income into time. Moreover, nonlinear pricing determines the set of feasible transformations of time into income except in the special case of a proportional tax.<sup>14</sup> Further distinctions arise if the set of feasible time allocations is finite rather than a continuum as assumed in neoclassical theory. All of these factors make me wary of using neoclassical consumer theory to introspect about time allocation.

### 3.4 *A computational experiment*

The classical static model is too simplistic to warrant application in substantive study of labor supply. Nevertheless, it is revealing to perform computational experiments that show the identifying power of alternative assumptions on the preference distribution. I describe a representative such experiment here.

Given data on labor supply under a status quo progressive tax schedule, the problem is to predict the tax revenue per capita that would materialize under a proposed proportional schedule. I show the predictive power of a sequence of increasingly strong assumptions: (i) more is better; (ii) additionally, persons in specified wage groups have the same distribution of preferences; (iii) additionally, preferences have the CES form; and (iv) additionally, all CES utility functions in the specified wage groups have the same elasticity of substitution.

*Tax policies, wages, choice sets, and preferences* The progressive status quo policy taxes income at rate 20 percent up to \$100,000 per year and at rate 30 percent above \$100,000. The proposed proportional policy taxes all income at rate 25 percent. Thus, the tax schedules cross when gross income equals \$200,000, when both take \$50,000 tax and yield net income \$150,000.

The population is composed of persons who have no unearned income and who have one of eight annual wages for full-time work: {50, 100, 150, 200, 250, 300, 350, 400} thousand dollars per year. The assumed population wage distribution is

$$\begin{aligned} P(w = 50) &= 0.70, & P(w = 100) &= 0.20, & P(w = 150) &= 0.05, \\ P(w = 200) &= 0.02, & P(w = 250) &= 0.01, & P(w = 300) &= 0.0075, \\ P(w = 350) &= 0.0075, & P(w = 400) &= 0.005. \end{aligned}$$

This distribution roughly approximates that of the 2011 population of adult males in the United States. The tax schedule under the status quo policy is a simplified idealization of the prevailing American tax schedule.

<sup>14</sup>Beginning with Hall (1973), it has been common to model nonproportional tax schedules as if they are proportional. The idea is to view an individual facing a nonproportional status-quo schedule as if he were facing a proportional tax equal to the marginal tax rate at his chosen time allocation. His inframarginal net income is treated as a “virtual” form of unearned income. This modeling strategy has become so prevalent that researchers often neglect to cite its maintained assumptions. These include that income–leisure preferences have convex indifference curves and that all time allocations in the  $[0, 1]$  interval be feasible.

I suppose that each person can work full time, three-quarter time, half time, or not at all. Thus,  $A_j = \{0, 1/4, 1/2, 1\}$  for all  $j \in J$ . [Appendix Table A.1](#) gives the resulting universe of potential values of  $(Y, L)$ .

When using the assumption that persons have CES preferences, I suppose that person  $j$  has a utility function of the form

$$U_j(Y, L) = [\alpha_j(Y/400,000)^{\rho_j} + (1 - \alpha_j)L^{\rho_j}]^{1/\rho_j}. \quad (18)$$

Dividing  $Y$  by 400,000, which is the maximum annual wage for full-time work, rescales income to the interval  $[0, 1]$ , to make income and leisure have a comparable range. The identification analysis presumes that the parameter  $\alpha$  may take one of the 101 values  $\{0, 0.01, 0.02, \dots, 0.99, 1\}$  and that  $\rho$  may take one of the 211 values  $\{-100, -90, \dots, -20, -10, -1, -0.99, -0.98, \dots, 0.98, 0.99, 1\}$ . Hence, there are  $101 \times 211 = 21,311$  possible CES preferences. Income and leisure are essentially perfect complements when  $\rho = -100$ . They are perfect substitutes when  $\rho = 1$ . The value  $\rho = 0$  gives Cobb–Douglas preferences.

The final step in specification of the experiment is to choose the actual distribution of preferences in the population. I suppose that the population actually contains persons who have 20 distinct CES preferences, with  $(\alpha, \rho) \in \{0.25, 0.5, 0.65, 0.75\} \times \{-100, -0.5, 0, 0.5, 1\}$ . Moreover, I suppose that the population distribution of  $(\alpha, \rho)$  is uniform conditional on wage. Thus, the population subgroups with different wages share the same preference distribution, placing probability  $1/20$  on each feasible  $(\alpha, \rho)$  pair. This distribution of  $(\alpha, \rho)$  values was selected so the population would contain a modest fraction of persons who choose different time allocations under policies  $S$  and  $T$ .<sup>15</sup>

*Findings* Given this specification, actual per capita tax revenues under the status quo and proposed tax policies are \$7652 and \$9211, respectively. [Appendix Table A.2](#) details the labor supply and taxes paid under the two policies by persons with different wages.

If one were to know the wage distribution but not know the status quo labor supply, one would only be able to predict that tax revenue per capita under the proposed policy must lie in the interval  $[0, \$18,969]$ . The upper and lower bounds are the taxes paid if everyone were to work full time or not at all, respectively.

Given data on status quo labor supply, one's ability to predict tax revenue under the proposed policy depends on the assumptions imposed on preferences. The basic assumption that more is better yields the revenue bound  $[\$593, \$18,969]$ . This result follows from the analysis of [Section 2.3](#). The upper bound is the same as the upper bound with no labor supply data because working full time under the proposed policy is always consistent with the assumption that more is better. The lower bound is larger than zero because any person  $j$  for whom  $L_{jS} \leq L_j^*$  would not choose  $L_{jT} > L_j^*$ . In the context of the computational experiment, this condition only raises the lower bound on per capita tax revenue from 0 to \$593.

<sup>15</sup>Persons with the 6  $(\alpha, \rho)$  pairs  $\{(0.5, -0.5), (0.65, -0.5), (0.65, 0), (0.65, 0.5), (0.65, 1), (0.75, 0.5)\}$  change their choices if they have certain values of  $w$ . Those with the other 14  $(\alpha, \rho)$  pairs never change regardless of  $w$ .

I next combine the assumption that more is better with the assumption that persons within each of two broad wage groups have the same distribution of preferences. I assume that persons with  $w \leq \$200,000$  have the same preference distribution and that those with  $w > \$200,000$  have the same preference distribution. I do not assume that persons with wage below and above  $\$200,000$  have the same preference distribution. The resulting bound on per capita tax revenue is  $[\$3744, \$14,149]$ . This result follows from application of equation (16) to each of the two wage groups and aggregation of the findings across the two groups.<sup>16</sup>

The above result applies a version of Assumption 1, but does not use any version of Assumption 2. I next suppose that all persons have CES utility functions. Then application of equation (16) to each wage group and aggregation across the two groups yields the bound  $[\$6883, \$10,444]$  on per capita tax revenue.<sup>17</sup>

The assumption that everyone has CES preferences is strong. However, it is still much weaker than the assumptions of homogeneous preferences that have been traditionally maintained in parametric empirical analyses of labor supply. My final step is to assume that the persons in each broad wage group have the same elasticity of substitution. Thus, I assume that all persons with  $w \leq 200,000$  have the same value of  $\rho$  in their CES utility functions and, similarly, all persons with  $w > 200,000$  have the same  $\rho$ . I place no homogeneity assumption on personal values of  $\alpha$ . It turns out that assuming homogeneity of  $\rho$  yields an empty identification region for the preference distribution within the second broad wage group. Thus, the assumption of homogeneous  $\rho$  within each wage group is rejected.

*Computation of the bounds* Computation of the above bounds ranges from easy to challenging. The bound making only the basic assumption that more is better is simple to compute. It uses only the analytical result of Section 2.3 and does not require the identification analysis of Section 3.2.

The bound assuming that preferences have the CES form uses the analysis of Section 3.2, but it is not hard to compute because the number of distinct choice functions is fairly small. Although persons may have 21,311 distinct values of  $(\alpha, \rho)$ , this set partitions into a much smaller number of equivalency classes, within which distinct  $(\alpha, \rho)$  values yield the same choices under both tax schedules. It turns out that 108 and 99 equivalency classes suffice to describe the behavior of persons with  $w \leq \$200,000$  and  $w > \$200,000$ , respectively. Thus, the relevant multinomial distributions  $\pi$  place probability on 108 and 99 mass points, respectively. Solution of the resulting linear programming problem (17) is tractable.

<sup>16</sup>Within the groups with  $w \leq \$200,000$  and  $w > \$200,000$ , the bounds on per capita tax revenue are  $[\$2941, \$12,455]$  and  $[\$29,701, \$68,906]$  respectively. The fractions of the population with wages in these two groups are 0.97 and 0.03 respectively. Averaging the bounds with these fractional weights yields the aggregate result  $[\$3744, \$14,149]$ .

<sup>17</sup>Within the groups with  $w \leq \$200,000$  and  $w > \$200,000$ , the bounds on per capita tax revenue are  $[\$5947, \$9143]$  and  $[\$37,148, \$52,500]$  respectively. The fractions of the population with wages in these two groups are 0.97 and 0.03 respectively. Averaging the bounds with these fractional weights yields the aggregate result  $[\$6883, \$10,444]$ .

The most challenging task is to compute the intermediate bound assuming that persons within each wage group have the same distribution of preferences, but not assuming that preferences have the CES form. Inspection of Table A.1 shows that the members of each wage group face 24 distinct  $(Y, L)$  pairs under the status quo and proposed tax schedules. Thus, the total number of strict preference orderings in each group is  $24! \cong 6.2 \times 10^{23}$ . Laborious application of an algorithm that determines and excludes orderings inconsistent with the more-is-better assumption drastically reduces the number of feasible orderings to 7654 and 4000 in the groups with  $w \leq \$200,000$  and  $w > \$200,000$ , respectively. Hence, the relevant multinomial distributions  $\pi$  place probability on 7654 and 4000 mass points, respectively. Solution of the resulting linear programming problem (17) is still tractable.<sup>18</sup>

#### 4. IMPLICATIONS FOR UTILITARIAN POLICY EVALUATION

A familiar exercise in normative public economics poses a utilitarian social welfare function and ranks tax policies by the welfare they achieve. Performing this ranking requires knowledge of income–leisure preferences both to predict tax revenues and to compute the welfare achieved by alternative policies. Thus, Mirrlees (1971, p. 176) wrote in his introductory section, “The State is supposed to have perfect information about the individuals in the economy, their utilities and, consequently, their actions.” This assumption has been standard in the subsequent literature on optimal income taxation.

Consider a status quo policy  $S$  with tax schedules  $[R_{jS}(\cdot), j \in J]$  and a proposed policy  $T$  with corresponding  $[R_{jT}(\cdot), j \in J]$ . Given cardinal representations for persons’ ordinal utility functions, these policies yield utilitarian welfare  $E\{U[(Y_S(L_S), L_S)]\}$  and  $E\{U[Y_T(L_T), L_T]\}$ , respectively. Policy  $T$  is strictly preferred to  $S$  if

$$E\{U[(Y_T(L_T), L_T)]\} - E\{U[Y_S(L_S), L_S]\} > 0. \quad (19)$$

Based on the analysis in Sections 2 and 3, I conclude that we lack the knowledge of preferences necessary to credibly rank policies. Considering the classical static model, Section 2 showed that basic revealed-preference analysis has little power to predict labor supply under proposed policies. Importantly, it does not predict whether increasing tax rates reduces or increases work effort. Section 3 showed that Assumptions 1 and 2 add predictive power, but credible versions of these assumptions generally yield bounds rather than point predictions. The computational experiment of Section 3.4 is only an illustration, but I think it typifies the situations that empirical researchers using the classical static model face in practice. Studies using more general models of time allocation face more difficult inferential problems.

Partial identification of income–leisure preferences creates two distinct difficulties for policy evaluation. First, partial knowledge of the tax revenue yielded by a proposed policy may make it impossible to ensure that the policy yields a balanced budget. Whatever the intent ex ante, a budget surplus or deficit may materialize ex post. Then it is

<sup>18</sup>I am grateful to Jörg Stoye for making available the MATLAB code used to compute the intermediate bounds. This code was developed for his analysis of related testing and prediction problems that arise in applications of neoclassical consumer theory. See Kitamura and Stoye (2011).

infeasible to evaluate policies within a deterministic static framework of the type used by Mirrlees and others. One requires a suitable dynamic perspective on private time allocation and social planning under uncertainty.

Second, partial identification of preferences implies partial knowledge of the utilitarian welfare achieved by a policy. Thus, choice of tax policy becomes a problem of planning under ambiguity. Basic decision theory prescribes that a planner should first eliminate dominated policies and then use some reasonable criterion to choose among the undominated policies. Studying other planning problems, I have found it instructive to compare the policy choices that result from maximization of subjective expected welfare and from application of the maximin and minimax regret criteria (Manski (2011)). I have recently begun to explore application of these criteria to choice of taxation–spending policy in settings where a planner can ensure budget balance by choosing policy components sequentially rather than simultaneously (Manski (forthcoming)).

A challenge for future research is to develop tractable approaches to the study of choice of tax policy under ambiguity. One issue, already mentioned above, is the potential need for a dynamic perspective on planning. Another is that the knowledge of preferences required to compute welfare is more extensive than the knowledge needed to predict labor supply. The transformation from preferences to time allocation under status quo and proposed tax policies is many-to-one; that is, distinct preferences may imply the same choices and, hence, be equivalent from the perspective of prediction of labor supply.<sup>19</sup> However, such distinct preferences may have different utilitarian welfare implications.

Yet another complication is that specification of a welfare function requires that the planner select cardinal representations of ordinal individual utility functions. These representations are already consequential for policy comparison in the Mirrlees setting where all members of the population have the same known preferences. They are even more consequential when preferences may be heterogeneous and the planner has only partial knowledge of the preference distribution.

## 5. ENRICHING THE DATA FOR IDENTIFICATION OF INCOME–LEISURE PREFERENCES

This paper has reached highly cautionary findings about the present knowledge of income–leisure preferences and has concluded that we lack the knowledge of preferences necessary to credibly rank tax policies. Yet I will end by expressing modest hope for progress.

In principle, identification problems may be mitigated by development of new theory that yields credible assumptions or by collection of richer data. I do not expect that new theory will sharpen our knowledge of preferences. To the contrary, efforts to enhance the realism of the theory of labor supply have steadily generalized the classical static model and have thus made more evident the difficulty of inference.

<sup>19</sup>The computational experiment in Section 3.4 gave an example. Recall that the 21,311 distinct values of the CES preference parameters  $(\alpha, \rho)$  partitioned into a much smaller number of 108 or 99 equivalency classes, within which distinct  $(\alpha, \rho)$  values yielded the same choices under both tax schedules.

The only route I see open to improve our capacity to evaluate tax policy is to collect richer data that shed more light on income–leisure preferences. I will discuss two ideas for data collection. Implementation of these ideas will not eliminate the difficulties in policy evaluation discussed in Section 4, but it may lessen their severity.

### 5.1 *Promoting exogenous variation in choice sets*

Economists have long appreciated the identifying power of exogenous variation in choice sets. In this paper, exogenous variation is expressed through Assumption 1. It has identifying power by placing linear restrictions of form (10) on the preference distribution assumed to be invariant across groups facing different status quo choice sets.

When studying income–leisure preferences, economists have usually been able to exploit only the natural variation in choice sets that arises from wage heterogeneity and from the diversity of tax schedules across jurisdictions. Governments may be able to enhance variation by decentralizing tax policy and by performing experiments that randomize persons into alternative tax schedules.

In the United States, tax policy is already somewhat decentralized as federalism gives states the power to enact state-specific income taxes and, in turn, some states permit cities to enact local taxes. As a consequence, residents of different American tax jurisdictions face different combined federal–state–city tax schedules. Proponents of federalism have long appreciated that policy decentralization can help to learn policy impacts. Following Roosevelt (1912) and a remark made by Justice Brandeis in his dissent to the 1932 Supreme Court case *New York State Ice Co. v. Liebmann* (285 U.S. 311), it has become common to refer to the states as the *laboratories of democracy*. A caveat is that it may not be credible to assume that the observed variation in tax schedules across jurisdictions is exogenous. State and city tax policies may vary systematically with the preferences of residents.

Experimental randomization of tax schedules may seem farfetched, but something of the sort was done in the United States in the negative income tax experiments of the 1970's. Economists have used the resulting data to infer income–leisure preferences. See Hausman and Wise (1985) and Munnell (1986) for assessment of the lessons learned. The structural econometric studies summarized in these volumes notably differ from traditional atheoretic analysis of treatment response in randomized experiments. Analysis of the latter type only enables assessment of observable outcomes of treatments administered in the experiment. It does not provide the knowledge of preferences needed to predict the outcomes of counterfactual policies and to compute utilitarian welfare.

### 5.2 *Observation of individual behavior in multiple choice settings*

It may be naive to expect government action to promote exogenous variation in tax schedules. A more realistic hope may be to enhance the data collected on individual behavior.

The analysis of Sections 2 and 3 supposed that a researcher observes one status quo time allocation per person. The identifying power of revealed-preference analysis grows



if it is feasible to observe individual behavior in multiple choice settings. The source of multiple observations may be longitudinal data on individual time allocation under varying wages or tax schedules. Alternatively, it may be responses to stated-choice questions asking persons to predict their behavior in various scenarios. I first explain how observation of multiple choices helps identification. I then discuss the possible two data sources.

*Identification* The stream of basic revealed-preference analysis running from Samuelson (1938, 1948) through Afriat (1967), Varian (1982), and others recognizes that observation of neoclassical consumer demand has increasing power to predict counterfactual behavior as choices from more budget sets are observed. The findings, taking the form of bounds on counterfactual demands, stem jointly from the transitivity of preferences and the assumption that more is better. Research of this type continues. For example, Blundell, Browning, and Crawford (2008) have reported tighter bounds using the additional preference assumption that all goods are normal.

The analyses in the literature cited above do not apply directly to revealed-preference analysis of time allocation. I pointed out in Section 3.3 that the choice sets faced in time allocation with income taxes and in neoclassical budget allocation differ in several respects. Nevertheless, I expect that basic revealed-preference analysis of time allocation in settings with multiple choice observations would yield similar qualitative results. That is, transitivity and the assumption that more is better should yield tighter bounds on counterfactual labor supply than those reported in Section 2, where only one status quo time allocation was observed. Extension of Section 2 in this direction is beyond the scope of this paper, but I think it an apt subject for future research.

Extension of the analysis of Section 3 to settings with multiple choice observations per person requires only a straightforward generalization of equation (10) and Proposition 2. The basic idea is that observation of multiple choices enlarges the set of linear restrictions that Assumption 1 places on the preference distribution. Hence, it tightens the identification region.

To formalize this, let the time allocation of each member of the population be observed in  $I$  choice settings,  $I$  being a positive integer. Extend the definition of group membership so that all members of group  $m$  face  $I$  common status quo choice sets  $C_{im}$ ,  $I = 1, \dots, I$ , and a common proposed choice set  $D_m$ . Now consider the joint probability that a member of group  $m$  makes a given sequence of choices  $[(Y_i, L_i), i = 1, \dots, I]$ , namely  $P[c(C_{im}) = (Y_i, L_i), i = 1, \dots, I | J_{mx}]$ .

Assumption 1 implies that observed joint choice probabilities are related to the preference distribution  $\pi_N$  through the linear equations

$$\begin{aligned} & P[c(C_{im}) = (Y_i, L_i), i = 1, \dots, I | J_{mx}] \\ &= \sum_{k \in K} 1[c_k(C_{im}) = (Y_i, L_i), i = 1, \dots, I] \cdot \pi_{Nk}, \\ & (Y_i, L_i) \in C_{im}, i = 1, \dots, I, (m, x) \in N. \end{aligned} \tag{10'}$$

This equation, generalizing (10), states for each value of  $(m, x)$  and vector  $[(Y_i, L_i) \in C_{im}, i = 1, \dots, I]$  of possible choice sequences, the right-hand side adds up the frac-

tions of  $J_{mx}$  whose preference orderings make this vector their most preferred choice sequence. Applying (10') gives this immediate generalization of Proposition 2.

**PROPOSITION 2'.** *Given Assumptions 1 and 2, the identification region for  $\pi_N$  is the set  $H(\pi_N)$  of multinomial distributions that solve (10'), (11a), (11b), and (12).*

This proposition provides the mathematical foundation for computation of identification regions as illustrated in the computational experiment of Section 3.4. Hence, it is possible to explore numerically how identification regions shrink as more choices are observed. However, as I observed earlier, there does not seem to be an effective shortcut to characterize identification regions or derived quantities.

*Data* There are two potential sources of data on individual time allocation in multiple choice settings. Both are imperfect, but, I think, worthy of analysis with due caution.

One data source, familiar to labor economists, is longitudinal observation of time allocation under varying wages or tax schedules. Personal wages change over time for multiple reasons. Tax schedules may change as jurisdictions change their tax policies or as persons migrate across jurisdictions.

A potentially serious analytical issue is that interpretation of longitudinal data in the manner of Sections 2 and 3 rests on acceptance of the classical static model. Suppose, to the contrary, that one views labor supply in dynamic terms, as in MaCurdy (1985) and elsewhere. Then repeated observations of time allocation provide data on a single sequential choice path rather than data on choice from multiple independent choice sets. Interpretation of the data from a dynamic perspective requires assumptions about the information that persons possess when making choices, the way they form expectations for relevant future events, and the criteria they use to make decisions under uncertainty.

Another data source is performance of stated-choice analysis in which a researcher poses multiple hypothetical choice settings to a person and asks the person to predict the choice he would make in each setting. Inference on preferences from data on stated choices has a long history in econometric analysis of discrete choice. See, for example, Beggs, Cardell, and Hausman (1981), Ben-Akiva and Morikawa (1990), and Blass, Lach, and Manski (2010). Stated choice data are also collected and analyzed regularly by experimental economists.

A practical advantage of this data source is that the choice settings considered are not limited by what nature offers up. A researcher can elicit predictions of behavior in a wide spectrum of hypothetical choice settings. A potentially serious analytical issue is that interpretation of stated-choice data requires assumptions about the way that persons construe the scenarios posed and the cognitive processes they use when responding to questions. See Fischhoff, Welch, and Frederick (1999) and Manski (1999) for perspectives on this subject.

Thus, collecting longitudinal and stated-choice data will not “solve” the problem of identifying income–leisure preferences. Yet I think that careful efforts to collect and study such data can add significantly to the little we now know.

## APPENDIX

TABLE A.1. Universe of  $(Y, L)$  values in the computational experiment.

Wage	(Net Income, Leisure)		Wage	(Net Income, Leisure)	
	Status Quo	Proposed		Status Quo	Proposed
50,000	[40,000, 0]	[37,500, 0]	250,000	[185,000, 0]	[187,500, 0]
	[30,000, 1/4]	[28,125, 1/4]		[141,250, 1/4]	[140,625, 1/4]
	[20,000, 1/2]	[18,750, 1/2]		[97,500, 1/2]	[93,750, 1/2]
	[0, 1]	[0, 1]		[0, 1]	[0, 1]
100,000	[80,000, 0]	[75,000, 0]	300,000	[220,000, 0]	[225,000, 0]
	[60,000, 1/4]	[56,250, 1/4]		[167,500, 1/4]	[168,750, 1/4]
	[40,000, 1/2]	[37,500, 1/2]		[115,000, 1/2]	[112,500, 1/2]
	[0, 1]	[0, 1]		[0, 1]	[0, 1]
150,000	[115,000, 0]	[112,500, 0]	350,000	[255,000, 0]	[262,500, 0]
	[88,750, 1/4]	[84,375, 1/4]		[193,750, 1/4]	[196,875, 1/4]
	[60,000, 1/2]	[56,250, 1/2]		[132,500, 1/2]	[131,250, 1/2]
	[0, 1]	[0, 1]		[0, 1]	[0, 1]
200,000	[150,000, 0]	[150,000, 0]	400,000	[290,000, 0]	[300,000, 0]
	[115,000, 1/4]	[112,500, 1/4]		[220,000, 1/4]	[225,000, 1/4]
	[80,000, 1/2]	[75,000, 1/2]		[150,000, 1/2]	[150,000, 1/2]
	[0, 1]	[0, 1]		[0, 1]	[0, 1]

TABLE A.2. Actual labor supply and tax revenues in the computational experiment.

Wage	Status Quo Policy					Proposed Policy				
	Leisure Choice				Tax per Capita	Leisure Choice				Tax per Capita
	$L = 0$	$L = 1/4$	$L = 1/2$	$L = 1$		$L = 0$	$L = 1/4$	$L = 1/2$	$L = 1$	
50,000	0	0.45	0.25	0.30	4625	0	0.45	0.25	0.30	5781
100,000	0	0.45	0.30	0.25	9750	0	0.45	0.30	0.25	12,188
150,000	0	0.45	0.30	0.25	15,188	0	0.45	0.30	0.25	18,281
200,000	0.05	0.40	0.35	0.20	23,500	0.05	0.45	0.30	0.20	26,875
250,000	0.05	0.40	0.35	0.20	31,375	0.05	0.45	0.30	0.20	33,594
300,000	0.05	0.25	0.55	0.15	37,625	0.10	0.30	0.45	0.15	41,250
350,000	0.10	0.25	0.50	0.15	47,938	0.10	0.30	0.45	0.15	48,125
400,000	0.10	0.25	0.50	0.15	56,000	0.10	0.25	0.50	0.15	53,750

## REFERENCES

Afriat, S. (1967), “The construction of utility functions from expenditure data.” *International Economic Review*, 8, 67–77. [169]

Beggs, S., S. Cardell, and J. Hausman (1981), “Assessing the potential demand for electric cars.” *Journal of Econometrics*, 17, 1–19. [170]

Ben-Akiva, M. and T. Morikawa (1990), “Estimation of switching models from revealed preferences and stated intentions.” *Transportation Research: Part A*, 24, 485–495. [170]

Blass, A., S. Lach, and C. Manski (2010), "Using elicited choice probabilities to estimate random utility models: Preferences for electricity reliability." *International Economic Review*, 51, 421–440. [170]

Blomquist, S. and W. Newey (2002), "Nonparametric estimation with nonlinear budget sets." *Econometrica*, 70, 2455–2480. [148]

Blundell, R., A. Bozio, and G. Laroque (2011), "Extensive and intensive margins of labour supply: Working hours in the US, UK and France." Working Paper 01/11, Institute for Fiscal Studies. [148]

Blundell, R., M. Browning, and I. Crawford (2008), "Best nonparametric bounds on demand responses." *Econometrica*, 76, 1227–1262. [169]

Blundell, R. and T. MaCurdy (1999), "Labor supply: A review of alternative approaches." In *Handbook of Labor Economics*, Vol. 3 (O. Ashenfelter and D. Card, eds.), 1559–1695, North-Holland, Amsterdam. [147]

Blundell, R. and A. Shephard (2012), "Employment, hours of work and the optimal taxation of low-income families." *Review of Economic Studies*, 79, 481–510. [148, 157]

Burtless, G. and J. Hausman (1978), "The effect of taxes on labor supply." *Journal of Political Economy*, 86, 1103–1130. [148]

Chetty, R. (2012), "Bounds on elasticities with optimization frictions: A synthesis of micro and macro evidence on labor supply." *Econometrica*, 80, 969–1018. [149]

Congressional Budget Office (2007), "The effect of tax changes on labor supply in CBO's microsimulation tax model." Background paper, Congress of the United States, Congressional Budget Office. [147]

Feldstein, M. (1995), "The effect of marginal tax rates on taxable income: A panel study of the 1986 Tax Reform Act." *Journal of Political Economy*, 103, 551–572. [148]

Fischhoff, B., N. Welch, and S. Frederick (1999), "Construal processes in preference assessment." *Journal of Risk and Uncertainty*, 19, 139–164. [170]

Fishburn, P. (1992), "Induced binary probabilities and the linear ordering polytope: A status report." *Mathematical Social Sciences*, 23, 67–80. [162]

Gilbert, F. and R. Pfouts (1958), "A theory of the responsiveness of hours of work to changes in wage rates." *Review of Economics and Statistics*, 40, 116–121. [147]

Hall, R. (1973), "Wages, income and hours of work in the US labor force." In *Income Maintenance and Labor Supply* (G. Cain and H. Watts, eds.), University of Chicago Press, Chicago. [163]

Hausman, J. and D. Wise (1985), *Social Experimentation*. University of Chicago Press, Chicago. [168]

Hoderlein, S. and J. Stoye (2012), "Testing stochastic rationality and predicting stochastic demand: The case of two goods." Working paper, Department of Economics, Cornell University. [156]

- Hoynes, H. (1996), “Welfare transfers in two-parent families: Labor supply and welfare participation under AFDC-UP.” *Econometrica*, 64, 295–332. [157]
- Keane, M. (2011), “Labor supply and taxes: A survey.” *Journal of Economic Literature*, 49, 961–1075. [147, 148, 149]
- Keane, M. and R. Moffitt (1998), “A structural model of multiple welfare program participation and labor supply.” *International Economic Review*, 39, 553–589. [157]
- Killingsworth, M. and J. Heckman (1986), “Female labor supply: A survey.” In *Handbook of Labor Economics*, Vol. 1 (O. Ashenfelter and R. Layard, eds.), 103–204, North-Holland, Amsterdam. [147]
- Kitamura, Y. and J. Stoye (2011), “Nonparametric analysis of random utility models.” Working paper, Department of Economics, Cornell University. [166]
- Lindbeck, A., S. Nyberg, and J. Weibull (1999), “Social norms and economic incentives in the welfare state.” *Quarterly Journal of Economics*, 114, 1–35. [148]
- MaCurdy, T. (1985), “Interpreting empirical models of labor supply in an intertemporal framework with uncertainty.” In *Longitudinal Analysis of Labor Market Data* (J. Heckman and B. Singer, eds.), Cambridge University Press, Cambridge. [148, 170]
- Manski, C. (1999), “Analysis of choice expectations in incomplete scenarios.” *Journal of Risk and Uncertainty*, 19, 49–66. [170]
- Manski, C. (2007a), *Identification for Prediction and Decision*. Harvard University Press, Cambridge, Massachusetts. [149]
- Manski, C. (2007b), “Partial identification of counterfactual choice probabilities.” *International Economic Review*, 48, 1393–1410. [149, 156, 157]
- Manski, C. (2011), “Choosing treatment policies under ambiguity.” *Annual Review of Economics*, 3, 25–49. [167]
- Manski, C. (forthcoming), “Choosing size of government under ambiguity: Infrastructure spending and income taxation.” *The Economic Journal*. [167]
- Marschak, J. (1960), “Binary choice constraints on random utility indicators.” In *Stanford Symposium on Mathematical Methods in the Social Sciences* (K. Arrow, ed.), Stanford University Press, Stanford, California. [162]
- McFadden, D. (2005), “Revealed stochastic preference: A synthesis.” *Economic Theory*, 26, 245–264. [162]
- Meghir, C. and D. Phillips (2010), “Labour supply and taxes.” In *Dimensions of Tax Design: The Mirrlees Review* (T. Besley, R. Blundell, M. Gammie, and J. Poterba, eds.), 202–274, Oxford University Press, Oxford. [147]
- Mirrlees, J. (1971), “An exploration in the theory of optimal income taxation.” *Review of Economic Studies*, 38, 175–208. [145, 166]

Munnell, A., ed. (1986), *Lessons From the Income Maintenance Experiments*. Federal Reserve Bank of Boston, Boston. [168]

Pencavel, J. (1986), "Labor supply of men: A survey." In *Handbook of Labor Economics*, Vol. 1 (O. Ashenfelter and R. Layard, eds.), 3–102, North-Holland, Amsterdam. [147]

Robbins, L. (1930), "On the elasticity of demand for income in terms of effort." *Economica*, 29, 123–129. [146]

Roosevelt, T. (1912), "Introduction to C. McCarthy." In *The Wisconsin Idea*, McMillan, New York. [168]

Saez, E., J. Slemrod, and S. Giertz (2012), "The elasticity of taxable income with respect to marginal tax rates: A critical review." *Journal of Economic Literature*, 50, 3–50. [147]

Samuelson, P. (1938), "A note on the pure theory of consumer behavior." *Economica*, 5, 61–71. [149, 152, 169]

Samuelson, P. (1948), "Consumption theory in terms of revealed preferences." *Economica*, 15, 243–253. [169]

Stern, N. (1986), "On the specification of labour supply functions." In *Unemployment, Search and Labour Supply* (R. Blundell and I. Walker, eds.), 143–189, Cambridge University Press, Cambridge. [146, 147]

Varian, H. (1982), "The nonparametric approach to demand analysis." *Econometrica*, 50, 945–973. [169]

Wildasin, D. (1979), "Public good provision with optimal and non-optimal commodity taxation." *Economic Letters*, 4, 59–64. [149]

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