Appendices

For Online Publication

A Mathematical Proofs

A.1 Mechanisms

The DA mechanism works as follows:

Round 1: Every student applies to her first choice. Each school temporarily holds top applicants based on the normal priority no more than its total capacity, and rejects other applicants.

In general:

Round $k > 1$ Each rejected student i from Round $k-1$ applies to the next choice on her list. Each school pools new applicants and those who are held from Round $k-1$ together, then it temporarily holds top applicants based on the normal priority no more than its total capacity, and rejects other applicants.

The process terminates when no rejections are issued. Each school admits students who are currently held.

In the main context, we have discussed the cadet-optimal mechanism with BRADSO program (Greenberg, Pathak, and Sonmez 2021), which is a variation of the cadet-optimal stable mechanism. We have used the term "COSM" to refer to the cadet-optimal mechanism with BRADSO program, although it deviates slightly from the standard notation. Now, let us describe the matching algorithm of the cadet-optimal stable mechanism defined in Sönmez and Switzer (2013) as follows:

Round 1: Each student applies to her first choice. Each school j tentatively holds the top q_j^a students whose first choices are (j, c_0) based on the normal priority in its normal pool. Among the remaining applicants, the school tentatively holds the top q_j^z students whose first choices are (j, c_1) or (j, c_0) based on the ZX priority in its ZX pool. Other applicants are rejected.

Round $k > 1$ Each rejected student applies to her next choice. Each school j considers the new applicants whose choices are (j, c_0) along with those who are held in the normal pool from the previous round; then each j tentatively holds the top q_j^a (with their contracts) applicants in the normal pool based on the normal priority. Among the remaining applicants, j considers the new applicants whose choice is (j, c_1) or (j, c_0) along with those who are held in its ZX pool with their holding contracts from the previous round; it then holds the top q_j^z applicants based on the ZX priority. The other applicants are rejected.

This algorithm terminates when each student is tentatively held by a school, at which point the tentative assignments become final. A student i who is assigned a seat in j pays tuition c_0 if her assignment is (j, c_0) or pays c_1 if the assignment is (j, c_1).

A.2 Example to indicate that the CPPS mechanism is not strategy-proof

For the CPPS mechanism with permanency-execution period $(e_1, e_2, ...)$ with $e_1 \geq 1$, there are three students i_1, i_2, i_3 and three high schools j_1, j_2, j_3. Each school has one ZX seat and no normal seat. Students are ordered as $i_1 \succ i_2 \succ i_3$ by schools under the normal priority. Suppose student i_3's true preference over schools and tuitions is as follows:

$$(j_1, c_0)\pi_{i_3}(j_2, c_0)\pi_{i_3}(j_1, c_1)\pi_{i_3}(j_2, c_1)\pi_{i_3}(j_3, c_0).$$

So student i_3's true preference over schools is $j_1\widetilde{\pi}_{i_3}j_2\widetilde{\pi}_{i_3}j_3$.

We need to show that no truthful strategy weakly dominates all other strategies.

Case 1: i_3 chooses the ZX option for j_1 and $i_3 \in A_j$.

Given i_2 and i_3 choose the same strategy as $\{(j_1, 0), (j_3, 0), (j_2, 0)\}$, where 1 represents choosing the ZX option for the school and 0 otherwise.

If i_3 chooses the strategy as $\{(j_1, 1), (j_2, 0), (j_3, 0)\}$, then i_3 will receive the assignment (j_1, c_1). If i_3 switches to the strategy $\{(j_2, 0), (j_1, 1), (j_3, 0)\}$, she gets better off by receiving the assignment (j_2, c_0).

Case 2: i_3 does not choose the ZX option for j_1.

Given i_1's strategy as $\{(j_1, 0), (j_2, 0), (j_3, 0)\}$, and i_2's strategy as $\{(j_2, 0), (j_1, 0), (j_3, 0)\}$.

Subcase 2.1: $e_1 > 1$.

i_3 cannot receive an assignment better than (j_2, c_1) if she put j_1 as the first choice and does not choose the ZX option for it, because her normal priority is lower than i_1 and i_2. In this situation, if i_3 switches to the strategy $\{(j_2, 0), (j_1, 1), (j_3, 0)\}$, she gets better off by receiving the allocation (j_1, c_1).

Subcase 2.2: $e_1 = 1$.

In this mechanism, i_3 will be assigned to j_3 if she put j_1 as the first choice. If she switches to the strategy $\{(j_2, 1), (j_1, 0), (j_3, 0)\}$, then she gets better off by receiving the allocation (j_2, c_1).

Therefore, revealing true preferences over schools might not always be the best strategy for student i_3.

Proposition A.1. (i) *Nash equilibrium outcomes under the CPPS mechanism with* $e_1 > 1$ *can be unstable and may also be Pareto inferior to outcomes under the COSM.*

(ii) *The set of Nash equilibrium outcomes under the BMPS is equal to the set of stable matchings.*

(iii) *Nash equilibrium outcomes of the BMPS are Pareto dominated by the outcome of the COSM.*

Proof of Proposition A.1. Part 1: There are four students i_1, i_2, i_3, i_4 and four schools j_1, j_2, j_3, j_4 with one ZX seat each and no normal seat. Schools order the students in the same way as $i_1 \succ i_2 \succ i_3 \succ i_4$. Students' preferences are as follows:

π_{i_1}: $(j_1, c_0)\pi_{i_1}(j_2, c_0)\pi_{i_1}(j_1, c_1)\pi_{i_1}(j_3, c_0)\cdots$

π_{i_2}: $(j_1, c_0)\pi_{i_2}(j_1, c_1)\pi_{i_2}(j_2, c_0)\pi_{i_2}(j_2, c_1)\pi_{i_2}(j_4, c_0)\pi_{i_2}(j_4, c_1)\pi_{i_2}(j_3, c_0)\pi_{i_2}(j_3, c_1)$.

π_{i_3}: $(j_1, c_0)\pi_{i_3}(j_3, c_0)\pi_{i_3}(j_1, c_1)\pi_{i_3}(j_2, c_0)\pi_{i_3}(j_3, c_1)\pi_{i_3}(j_2, c_1)\pi_{i_3}(j_4, c_0)\pi_{i_3}(j_4, c_1)$.

π_{i_4}: $(j_4, c_0)\pi_{i_4}(j_2, c_0)\pi_{i_4}(j_4, c_1)\pi_{i_4}(j_2, c_1)\cdots$

Consider the following strategy profile under the CPPS mechanism:

$a_{i_1} = \{(j_1, 1), (j_2, 0), (j_3, 0), (j_4, 0)\}$,

$a_{i_2} = \{(j_1, 0), (j_2, 0), (j_4, 1), (j_3, 0)\}$,

$a_{i_3} = \{(j_1, 1), (j_3, 0), (j_2, 0), (j_4, 0)\}$,

$a_{i_4} = \{(j_4, 0), (j_2, 1), (j_1, 0), (j_3, 0)\}$.

Then the matching outcome is

$\{(i_1, j_1, c_1), (i_2, j_2, c_0), (i_3, j_3, c_0), (i_4, j_4, c_0)\}$.

This strategy profile is a Nash equilibrium but not stable. It is because i_1 prefers (j_2, c_0) to her assignment (j_1, c_1), and j_2 prefers i_1 to i_2 under the normal priority. Furthermore, this outcome is Pareto dominated by the outcome of the COSM:

$\{(i_1, j_2, c_0), (i_2, j_1, c_1), (i_3, j_3, c_0), (i_4, j_4, c_0)\}$.

Part 2: For any Nash equilibrium strategy profile $(a_1, ..., a_n)$ and matching outcome τ of the BMPS mechanism, suppose τ is not stable under the true preference. Then there is an contract (i, j, c) such that student i prefers assignment (j, c) to her assignment in τ and either school j has an empty seat for tuition c or i has higher priority at school j than another student who receives a seat with tuition c. In the first case, the unstable matching implies i does not put j as the first choice if $c = c_0$, then i can move school j to the first choice and receives the assignment (j, c). In the second case, if $c = c_1$, the unstable matching implies either i does not choose j as the first choice and choose the ZX option for it. Then i can put j as the first choice and choose the ZX option for it, and i can receive the assignment (j, c). In either case, student i has the incentive to deviate, so the matching result is not an equilibrium.

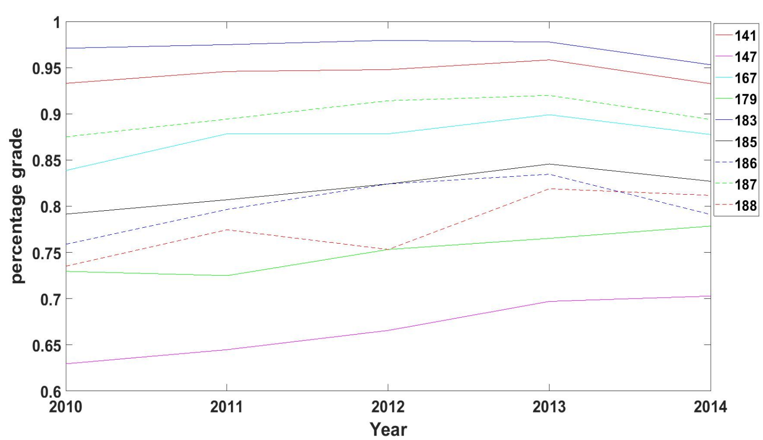
For a stable matching outcome τ, student i's assignment is (j, c). Then consider a strategy profile A as follow; if $c = c_0$, then student i put j as the first choice, if $c = c_1$, then student i put j as the first choice and choose the ZX option for it. Under this profile, every student receives the assignment in the first round and receives the same assignment as in τ. For student i, if she prefers an assignment (j', c') to the current assignment (j, c), since τ is stable, it implies the seats with tuition c' in school j' have assigned to other students who have higher priority to receive the seats. When $c' = c_1$, since the students who receive assignment (j', c') must put j' as the first choices and choose the ZX options for j', therefore even if student i put j as the first choice and choose the ZX option for j', she still cannot receive the assignment (j', c'). When $c' = c_0$, similarly, putting j' as the first choice cannot help i to receive (j', c_0). Therefore, student i has no way to get a better assignment by deviating to other strategies, and the strategy profile A is a Nash equilibrium.

Part 3 is straightforward because it is already proven in Sönmez and Switzer (2013) that students prefer the outcome under the COSM over any stable outcomes.

□

B More Results of Summary Statistics

Figure B.1: Fluctuation of Admission Cutoffs



Notes: This figure indicates the fluctuation of admission cutoffs of popular schools as measured by percentage grade. The y-axis represents the percentage grade, and the x-axis represents the year.

Table B.1: Survey Length

	Freq.	Percent
2 schools	175	12.09%
3 schools	130	8.98%
4 schools	242	16.72%
5 schools	900	62.20%
Total	1447	100%

Notes: This table indicates how many schools surveyed students listed.

Table B.2: School Frequency of Occurrence in the Survey

School	Total	above 90th	90th-80th	80th-70th	below 70th
141	416	166	105	74	71
142*	122	1	5	32	84
147	294	5	36	91	162
165†	58	20	30	6	2
166†	51	6	25	18	2
167	737	158	191	206	182
169†	95	31	45	15	4
173	94	3	10	30	51
177†	91	19	47	19	6
179	690	35	106	233	316
180†	55	2	29	12	12
181*	147	2	9	32	104
183	297	119	63	51	64
184*	231	8	11	72	140
185	960	117	190	306	347
186	748	64	144	259	281
187	586	152	157	147	130
188	289	25	51	90	123
200†	34	9	20	5	0

Notes: This table displays the frequency of occurrence of each school in the survey. * indicates the leftover schools, while † marks the special classes. The second column presents the total frequency of occurrence, and the third to sixth columns show the frequency of occurrence across different student scoring groups.

Table B.3: School Quota

School	2012	2013	2014
141	137	120	175
142	500	500	500
147	240	400	400
167	130	121	123
173	400	320	320
179	168	160	139
181	500	500	300
183	189	182	210
184	600	600	600
185	132	125	139
186	167	140	139
187	129	133	123
188	42	45	54
28†	40	40	NaN
165†	40	40	40
166†	NaN	NaN	40
169†	NaN	40	80
177†	40	40	40
180†	NaN	40	80
200†	NaN	NaN	40

Notes: This table indicates the normal quota of each school. † indicates the special classes.

C Identification of Parameters $\boldsymbol{\alpha}$

In this section, we demonstrate the identification of parameters denoted $\boldsymbol{\alpha} \equiv \{\alpha_k\}$. Student i's action $a_i = ((j_i^1, v_i^1), (j_i^2, v_i^2), (j_i^3, v_i^3), r_i)$. Considering that there is no opportunity for admission into the third choice as a ZX student, we can omit v_i^3. For simplicity, and without causing confusion, we can also omit the subscript i. Thus, a student's action can be abbreviated to $((j^1, v^1), (j^2, v^2), j^3, r)$. Given any school set (j^1, j^2, j^3) and and a random assignment choice r in i's ROL, her decision regarding the ZX option can be viewed as selecting from four possibilities, $(v^1, v^2) \in \{(1,1), (1,0), (0,1), (0,0)\}$. Let $U^{(v^1,v^2)}$ represent i's expected utility for the choice (v^1, v^2), it can be written as

$$U^{(v^1,v^2)} = (\bar{P}_1^{(v^1,v^2)} + v^1\hat{P}_1^{(v^1,v^2)})\hat{u}_1 + (\bar{P}_2^{(v^1,v^2)} + v^2\hat{P}_2^{(v^1,v^2)})\hat{u}_2 + \bar{P}_3^{(v^1,v^2)}\hat{u}_3 + \tilde{P}^{(v^1,v^2)}\tilde{u}$$
$$- [v^1\hat{P}_1^{(v^1,v^2)}(\sum_k (c_1 - c_0)x^k\alpha^k) + v^2\hat{P}_2^{(v^1,v^2)}(\sum_k (c_2 - c_0)x^k\alpha^k)]$$
$$+ (\bar{P}_1^{(v^1,v^2)} + v^1\hat{P}_1^{(v^1,v^2)})\varepsilon_1 + (\bar{P}_2^{(v^1,v^2)} + v^2\hat{P}_2^{(v^1,v^2)})\varepsilon_2 + \bar{P}_3^{(v^1,v^2)}\varepsilon_3.$$

Here, $\bar{P}_j^{(v^1,v^2)}$ is the probability that i is admitted by school j as the normal student; $\hat{P}_j^{(v^1,v^2)}$ is the probability that i is admitted by her jth choice as the ZX student; $\tilde{P}^{(v^1,v^2)}$ is the utility that i is randomly assigned to a school; $\hat{u}_j$ is the deterministic part of the utility when i attends school j; $\tilde{u}$ is the utility that i is randomly assigned to a school; c_j is the ZX tuition i pays for school j. ε_j is the error term when i attends school j.

If i chooses $(v^1, v^2) = (1, 1)$, then the probability that we observe this choice is

$$Pr(U^{(1,1)} > U^{(1,0)}, U^{(1,1)} > U^{(0,1)}, U^{(1,1)} > U^{(0,0)}) =$$
$$Pr\{\Delta\bar{\mathbf{P}}_{12} \cdot \hat{\mathbf{u}}' - (\hat{P}_1^{(1,1)} - \hat{P}_1^{(1,0)})(\sum_k (c_1 - c_0)x^k\alpha^k) - \hat{P}_2^{(1,1)}(\sum_k (c_2 - c_0)x^k\alpha^k) > \tilde{\varepsilon}_{12};$$
$$\Delta\bar{\mathbf{P}}_{13} \cdot \hat{\mathbf{u}}' - \hat{P}_1^{(1,1)}(\sum_k (c_1 - c_0)x^k\alpha^k) - (\hat{P}_2^{(1,1)} - \hat{P}_2^{(0,1)})(\sum_k (c_2 - c_0)x^k\alpha^k) > \tilde{\varepsilon}_{13};$$
$$\Delta\bar{\mathbf{P}}_{14} \cdot \hat{\mathbf{u}}' - \hat{P}_1^{(1,1)}(\sum_k (c_1 - c_0)x^k\alpha^k) - \hat{P}_2^{(1,1)}(\sum_k (c_2 - c_0)x^k\alpha^k) > \tilde{\varepsilon}_{14}\}$$

Here $\Delta\bar{\mathbf{P}}_{12} = (\bar{P}_1^{(1,1)} + \hat{P}_1^{(1,1)} - \bar{P}_1^{(1,0)} - \hat{P}_1^{(1,0)}, \bar{P}_2^{(1,1)} + \hat{P}_2^{(1,1)} - \bar{P}_2^{(1,0)}, \bar{P}_3^{(1,1)} - \bar{P}_3^{(1,0)}, \tilde{P}^{(1,1)} - \tilde{P}^{(1,0)})$; $\Delta\bar{\mathbf{P}}_{13} = (\bar{P}_1^{(1,1)} + \hat{P}_1^{(1,1)} - \bar{P}_1^{(0,1)}, \bar{P}_2^{(1,1)} + \hat{P}_2^{(1,1)} - \bar{P}_2^{(0,1)} - \hat{P}_2^{(0,1)}, \bar{P}_3^{(1,1)} - \bar{P}_3^{(0,1)}, \tilde{P}^{(1,1)} - \tilde{P}^{(0,1)})$ $\Delta\bar{\mathbf{P}}_{14} = (\bar{P}_1^{(1,1)} + \hat{P}_1^{(1,1)} - \bar{P}_1^{(0,0)}, \bar{P}_2^{(1,1)} + \hat{P}_2^{(1,1)} - \bar{P}_2^{(0,0)}, \bar{P}_3^{(1,1)} - \bar{P}_3^{(0,0)}, \tilde{P}^{(1,1)} - \tilde{P}^{(0,0)})$; $\Delta\hat{\mathbf{u}} = (\hat{u}_1, \hat{u}_2, \hat{u}_3, \tilde{u})$; $\tilde{\varepsilon}_{12} = \Delta\bar{\mathbf{P}}_{12} \cdot \hat{\varepsilon}'$; $\tilde{\varepsilon}_{13} = \Delta\bar{\mathbf{P}}_{13} \cdot \hat{\varepsilon}'$; $\hat{\varepsilon} = -(\varepsilon_1, \varepsilon_2, \varepsilon_3)$

Now we let $F_{11} \equiv Pr(U^{(1,1)} > U^{(1,0)}, U^{(1,1)} > U^{(0,1)}, U^{(1,1)} > U^{(0,0)})$, and F_{11} can be considered as the cdf function of the joint distribution of $(\tilde{\varepsilon}_{12}, \tilde{\varepsilon}_{13}, \tilde{\varepsilon}_{14})$. Similarly, we have $F_{10} \equiv Pr(U^{(1,0)} > U^{(1,1)}, U^{(1,0)} > U^{(0,1)}, U^{(1,0)} > U^{(0,0)})$, which represents probability of i chooses $(v^1, v^2) = (1, 0)$; $F_{01} \equiv Pr(U^{(0,1)} > U^{(1,1)}, U^{(0,1)} > U^{(1,0)}, U^{(0,1)} > U^{(0,0)})$, which represents probability of i chooses $(v^1, v^2) = (1, 0)$; $F_{00} \equiv Pr(U^{(0,0)} > U^{(1,1)}, U^{(0,0)} > U^{(1,0)}, U^{(0,0)} > U^{(0,1)})$, which represents probability of i chooses $(v^1, v^2) = (0, 0)$.

Therefore, for student i, the log-likelihood of an observation $((j^1, v^1), (j^2, v^2), j^3, r)$ is

$$\begin{aligned} L(\alpha) &= \ln(F_{11}^{v^1 v^2} F_{10}^{v^1(1-v^2)} F_{01}^{(1-v^1)v^2} F_{00}^{(1-v^1)(1-v^2)}) \\ &= v^1 v^2 \ln F_{11} + v^1(1-v^2) \ln F_{10} + (1-v^1)v^2 \ln F_{01} + (1-v^1)(1-v^2) \ln F_{00} \end{aligned} \tag{13}$$

For the first term of this equation, we have $\frac{\partial \ln F_{11}}{\partial \boldsymbol{\alpha}} = \frac{1}{F_{11}} \frac{\partial F_{11}}{\partial \boldsymbol{\alpha}}$. By the regularity condition, to prove the concavity of $\ln F_{11}$, we just need to show that $\frac{\partial F_{11}}{\partial \boldsymbol{\alpha}} \frac{\partial F_{11}}{\partial \boldsymbol{\alpha'}}$ is positive definite.

$$\begin{aligned} \frac{\partial F_{11}}{\partial \boldsymbol{\alpha}} \frac{\partial F_{11}}{\partial \boldsymbol{\alpha'}} &= f_{11}^2 \begin{pmatrix} x^1 \\ x^2 \\ \vdots \\ x^k \end{pmatrix} \begin{pmatrix} A_1 & A_2 & A_3 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \begin{pmatrix} x^1 & x^2 & \cdots & x^k \end{pmatrix} \\ &= f_{11}^2 (\sum_r A_r^2) \begin{pmatrix} x^1 x^1 & x^1 x^2 & \cdots & x^1 x^k \\ \cdots & \cdots & \cdots & \cdots \\ x^k x^1 & x^k x^2 & \cdots & x^k x^k \end{pmatrix} \end{aligned} \tag{14}$$

Here, f_{11} is the pdf of F_{11}; $A_1 = (\hat{P}_1^{(1,1)} - \hat{P}_1^{(1,0)})(c_1 - c_0) + \hat{P}_2^{(1,1)}(c_2 - c_0)$; $A_2 = \hat{P}_1^{(1,1)}(c_1 - c_0) + (\hat{P}_2^{(1,1)} - \hat{P}_2^{(0,1)})(c_2 - c_0)$; $A_3 = \hat{P}_1^{(1,1)}(c_1 - c_0) + \hat{P}_2^{(1,1)}(c_2 - c_0)$.

It is easy to show Matrix (14) is positive definite. Similarly, we can prove the same result for F_{10}, F_{01} and F_{00}. Therefore, the information matrix of L is positive definite and $\boldsymbol{\alpha}$ is locally identified from the observed decisions.

D Maximum Simulated Likelihood Estimate

This appendix describes the algorithm used in the maximum simulated likelihood estimation to estimate the ZX-related parameters with a logit-smoothed accept-reject simulator. The procedure is implemented in the following steps, similar to the steps in Chapter 5 of Train (2009).

Step 1. Draw a J dimensional vector of errors, ϵ_i from type I extreme value distribution. Label the draw ϵ_i^r with r, and denote the elements of the draw as $\epsilon_{i1}^r, ..., \epsilon_{iJ}^r$.

Step 2. Calculate the utility for each alternative. That is, $u_{i,j,c}^r = \tilde{u}_{i,j,c} + \epsilon_{ij}^r$, where $\tilde{u}_{i,j,c}$ is the deterministic part of the utility when student i enters school j and pays tuition c, and $u_{i,o}^r = \tilde{F}_o + \epsilon_{ij}^r$ that is denoted the utility when student i attends a non-public high school.

Step 3. Given the beliefs and thus the admission probabilities, calculate the expected utility, $EU_i^r(a)$, of submitting a ROL $a = \{(j^1, v^1), (j^2, v^2), j^3\}$

In this step, the utility that student i attends one of her chosen school is $u_{i,j,c}^r$ obtained from step 2. The utility of being randomly assigned into a leftover school is $(\sum_{k=1,\dots,n_e}^{n_e} \frac{n_{j_{lo}^k}}{n_{to}} u_{i,j_{lo}^k,c_0}^r)$ where n_{to} is the total number of available seats in all leftover schools, $n_{j_{lo}^k}$ is the available seats in leftover school j_{lo}^k in year e, u_{i,j_{lo}^k,c_0}^r is the utility of i attending the leftover school j_{lo}^k by paying the basic tuition c_0.

Given student i's ROL $a = \{(j^1, v^1), (j^2, v^2), j^3\}$ and exam score s_i, the probability of i being admitted by school j^k as a normal student or by a non-public school can be calculated as follows:

$$P_{i,j_i^k,c_0} \text{ or } P_{i,o} = \max\{0, P_i^{k-1} - \Phi((\bar{S}_{j_i^k}^k - s_i)/\eta)\}.$$

Here Φ is the cdf of the standard normal; $P_i^{k-1} = 1$ if $k = 1$, which represents the normal admission for the first choice; $P_i^{k-1} = \Phi((\bar{S}_{j_i^{k-1}}^{k-1} - s_i)/\eta)$ if $v^{k-1} = 0$, which represents the probability that i is rejected by the $k-1$-th choice and gets into the k-th choice when she does not choose the ZX option for her $k-1$-th choice; and $P_i^{k-1} = \Phi((\hat{S}_{j_i^{k-1}}^{k-1} - s_i)/\eta)$ if $v^{k-1} = 1$, which represents the probability that i is rejected by her $k-1$-th choice with the

ZX option and gets into her kth choice. Let $\bar{S}_j^k = \bar{S}^2$ and $\hat{S}_j^k = \hat{S}^2$ when $k = 1, 2$

The probability of being admitted by school j_i^k as a ZX student with tuition c is

$$P_{i,j_i^k,c} = \sum_{t=1}^{4} I(c_t = c)[\max\{0, \Phi((\bar{S}_{j_i^k}^k - 10(t-1) - s_i)/\eta) - \max\{\Phi((\bar{S}_{j_i^k}^k - 10t - s_i)/\eta), \Phi((\hat{S}_{j_i^k}^k - s_i)/\eta)\}\}].$$

This formula represents the situation that i is rejected by her kth choice as a normal student but gets into it as a ZX student with tuition c.

Finally, the probability of being randomly assigned to a leftover school can be calculated as one minus the probability of being rejected by all three choices.

Step 4. For any student i in group 1, put these expected utilities into the logit formula, i.e.,

$$S_i^r = \frac{exp(Eu_i^r(a_i)/\lambda)}{\sum_{i'} exp(Eu_i^r(a_{i'})/\lambda)}, \tag{15}$$

where a_i is student i's observed choice, $a_{i'}$ is her alternatives including a_i, and $\lambda > 0$ is a scale factor ($\lambda = 0.01$ in the reported results).

For any student i in group 2, calculate $S_i^{r,2+}$ and $S_i^{r,2-}$ by using $a_i^{2+} = \{(j_i^1, v^1), (j_i^2, 1), j^3\}$ and $a_i^{2-} = \{(j_i^1, v^1), (j_i^2, 0), j^3\}$ to replace a_i in equation (15) respectively. Similarly, for any student i in group 3, calculate $S_i^{r,3+}$ and $S_i^{r,3-}$ by using $a_i^{3+} = \{(j_i^1, 1), (j_i^2, v^2), j^3\}$ and $a_i^{3-} = \{(j_i^1, 0), (j_i^2, v^2), j^3\}$ to replace a_i in equation (15) respectively.

Step 5. Repeat step 1-4 for R times, so that r takes the value from 1 to R.

Step 6. The simulated probability of student i in group 1 choosing the observed ROL a_i is the average of the values of the logit formula: $\hat{P}(a_i \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R} S_i^r$. For students in group 2, the simulated probability of observing a_i^2 is $\hat{P}(a_i^{2+} \in A_i^*) + \hat{P}(a_i^{2-} \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R}(S_i^{r,2+} + S_i^{r,2-})$. Similarly, for students in group 3, the the simulated probability of observing a_i^3 is $\hat{P}(a_i^{3+} \in A_i^*) + \hat{P}(a_i^{3-} \in A_i^*) = \frac{1}{R}\sum_{r=1}^{R}(S_i^{r,3+} + S_i^{r,3-})$.

Finally, the log-likelihood function can be calculated using the following equation.

$$
\begin{aligned}
LogL_2 &= \sum_{i \in G_1} log(P(a_i \in A_i^*)) \\
&+ \sum_{i \in G_2} log[P(a_i^{2+} \in A_i^*) + P(a_i^{2-} \in A_i^*)] + \sum_{i \in G_3} log[P(a_i^{3+} \in A_i^*) + P(a_i^{3-} \in A_i^*)].
\end{aligned}
$$

E Simulations in Counterfactual Analysis

The section describes the simulation procedure used for welfare comparison analysis. We use students' profiles from 2014. To simplify the calculation, special classes and non-public schools are excluded because they do not admit any ZX students and contribute a small proportion to the total capacity. To calculate the equilibrium outcomes under different mechanisms, the procedure is described as follows:

Part 1 Generate utility functions:

For each student i, draw a value of J-dimensional vector of errors, ϵ_i from a type I extreme value distribution. Label the draw as ϵ_i^r with r, and label the elements of the draw as $\epsilon_{i1}^r, ..., \epsilon_{iJ}^r$. Then, we calculate the utility function as $u_{i,j,c}^r = \tilde{u}_{i,j,c} + \epsilon_{ij}^r$, where $\tilde{u}_{i,j,c}$ is the deterministic part of the utility (the parameters come from Column 3 of Table 5).

Part 2 Simulate the matching process:

Case 1: The DA mechanism and COSM :

These two mechanisms are strategy-proof. For the DA mechanism, we treat students' true preferences across all schools as their reported ROLs. Then we run the serial dictatorship algorithm (based on their exam scores) to match students to schools. For the COSM, as there are three tuition levels for each school, we treat students' true preferences across school-tuition pairs as their ROLs. Then we follow the algorithm of the COSM defined in Section 3 to match students to schools.

Case 2, The CPPS mechanism:

The CPPS mechanism is not strategy-proof. The calculation of the equilibrium outcomes is described as follows:

Step 1: Use the admission cutoffs generated by the DA mechanism as the first prior belief for all students.

Step 2: Use the prior belief to calculate each student's optimal choice. When the optimal choice is not unique, we randomly select one of them. Then each student reports the selected optimal choice as her ROL.

Step 3: Given the submitted ROLs, run the matching algorithm based on the CPPS mechanism's definition to match students to schools. Then rank all students by their exam scores. The matching outcome from this step generates new admission cutoffs of schools. Then use these cutoffs as students' new belief.

Start from the first student and let $k = 1$.

Step 4: Fix all other students' strategy, calculate the k-th student's best response to the belief from Step 3. If there exists at least one new choice for this student to strictly increase her expected payoff, then jump to Step 5. If there does not exist any new choice for this student to get strictly better off, then repeat Step 4 for the $k+1$-th student and set $k = k+1$ when $k < N$; when $k = N$, the algorithm moves to Step 6.

Step 5: Choose the optimal choice of the student who is from Step 4 as the new ROL in the submitted ROL. When the optimal choice is not unique, then randomly choose one of them. Thereafter, repeat Step 3.

Step 6: The current ROLs are the equilibrium strategies of the students.

After calculating one equilibrium outcome for each mechanism, repeat Part 1 and 2 R times ($R = 5000$ in the reported results).

F More results for the estimation and welfare comparison

Table F.1: Admission Patterns (%)

	Within Sample						Out of Sample					
	2012			2013			2012			2014		
	True	Predicted	Diff	True	Predicted	Diff	True	Predicted	Diff	True	Predicted	Diff
1st choice admitted	29.4	32.8	-3.4	30.7	35	-4.3	29.4	31	-1.6	26.5	22.7	3.8
Normal	15.5	20.9	-5.4	15.6	21.6	-6	15.5	17	-1.5	26.5	22.7	3.8
ZX	13.8	12	1.8	15.0	13.5	1.5	13.8	14	-0.2			
2nd choice admitted	36.7	27.4	9.3	38.9	30.6	8.3	36.7	33.4	3.3	39.0	46.6	-7.6
Normal	31.0	23.6	7.4	32.0	26.4	5.6	31.0	27.4	3.6	39.0	46.6	-7.6
ZX	5.7	3.8	1.9	6.6	4.2	2.4	5.7	6	-0.3			

Notes: This table indicates the within- and out-of-sample test of the matching patterns for the 1st and 2nd choices in the ROLs.

Table F.2: Winners and Losers

	DA-COSM				DA-CPPS			
	10%		30%		10%		30%	
	W	L	W	L	W	L	W	L
HHP+HS %	2.6	5.3	7	14.4	3.4	10.7	6.2	22.3
HHP+MS %	6.7	11.5	10.7	14.2	14	19.4	10.8	33.4
HHP+LS %	4.1	1.4	6.9	1.2	7.1	3.6	6.7	3.1
MHP+HS %	3	4.5	5.8	16.3	3.4	9	5	23.5
MHP+MS %	5.3	13.2	8.8	17.4	14.4	16.8	12.1	29.7
MHP+LS %	2.7	1.3	3.6	1.1	5.3	2.3	4.1	2
LHP+HS %	0.1	8.6	0.1	36.2	0	9.5	0	40.2
LHP+MS %	9	12	15.2	16	15.9	16.3	15	25.1
LHP+LS %	0.3	0.4	0.6	0.4	2.1	0.8	1.6	0.5
HHP+HS ¥	1073	-1260	1079	-1304	1458	-1534	1218	-1304
HHP+MS ¥	819	-927	1017	-972	1254	-914	1116	-1016
HHP+LS ¥	534	-809	543	-808	996	-1069	835	-496
MHP+HS ¥	1639	-1526	1684	-1440	1985	-1428	1881	-1444
MHP+MS ¥	1049	-1002	1192	-1029	1500	-761	1267	-978
MHP+LS ¥	431	-741	418	-746	1072	-892	970	-538
LHP+HS ¥	591	-1223	604	-1587	1369	-1019	718	-1628
LHP+MS ¥	1045	-953	1045	-994	1940	-637	1520	-849
LHP+LS ¥	322	-562	335	-557	884	-620	812	-343

Notes: The first panel of this table indicates the percentage change in the number of students whose utilities increase (winners) or decrease (losers) when the DA mechanism is replaced by the COSM and CPPS mechanisms. The second panel indicates the welfare change measured by yuan. "W" represents winners, and "L" represents losers. For each mechanism change, utility changes are measured in three scenarios in which the ZX quotas are 10% and 30% of the total quotas. "HHP" represents students from high housing price communities, 'MHP" denotes students from moderate housing price communities, and "LHP" denotes students from low housing price communities.

Table F.3: Students' Strategies

		1st choice	ZX ratio	2nd choice	ZX ratio
HHP+HS	CP	1.12		3.06	
	CPPS (10%)	1.01	0.47	3.00	0.80
	CPPS (30%)	1.02	0.50	3.04	0.80
HHP+MS	CP	1.71		4.14	
	CPPS (10%)	1.50	0.26	4.00	0.48
	CPPS (30%)	1.52	0.25	3.96	0.47
HHP+LS	CP	1.98		4.26	
	CPPS (10%)	2.00	0.15	4.32	0.07
	CPPS (30%)	1.99	0.15	4.31	0.07
MHP+HS	CP	1.15		3.11	
	CPPS (10%)	1.02	0.34	2.92	0.81
	CPPS (30%)	1.03	0.37	2.98	0.82
MHP+MS	CP	1.68		4.15	
	CPPS (10%)	1.58	0.22	4.14	0.32
	CPPS (30%)	1.62	0.21	4.13	0.31
MHP+LS	CP	2.19		4.46	
	CPPS (10%)	2.26	0.08	4.52	0.03
	CPPS (30%)	2.26	0.08	4.51	0.03
LHP+HS	CP	1.28		3.61	
	CPPS (10%)	1.05	0.32	3.07	0.83
	CPPS (30%)	1.08	0.33	3.19	0.81
LHP+MS	CP	1.62		4.06	
	CPPS (10%)	1.64	0.21	4.15	0.20
	CPPS (30%)	1.75	0.20	4.2	0.18
LHP+LS	CP	1.96		3.97	
	CPPS (10%)	2.03	0.18	4.04	0.03
	CPPS (30%)	2.05	0.17	4.03	0.03

Notes: This table displays students' strategic behaviors in their ROLs. The third and fifth columns show the average positions of students' first and second choices according to their true preferences. For instance, under the CP mechanism, students' first choice is, on average, their 1.12th choice in their true preferences. The fourth and sixth columns indicate the percentages of students who choose the ZX options for their first and second choices in the ROLs.

Table F.4: Tuition Collection vs Student Quality (%)

School		DA-COSM		DA-CPPS	
		10%	30%	10%	30%
183	Δ Tuition	11.48	34.3	11.29	42.43
	Δ Qual.	-0.05	-0.45	-0.09	-0.59
141	Δ Tuition	12.19	27.73	13.74	43.61
	Δ Qual.	-0.16	-0.5	-0.98	-1.04
187	Δ Tuition	0.32	-28	7.04	14.01
	Δ Qual.	-0.13	0.23	-0.54	-0.03
167	Δ Tuition	11.31	27.48	12.26	53.49
	Δ Qual.	0.23	-0.18	0.75	0.64
185	Δ Tuition	12.16	15.36	11.19	30.01
	Δ Qual.	-0.66	-1.36	-1.24	-2.05
186	Δ Tuition	10.9	-13.89	6.74	13.43
	Δ Qual.	0.02	0.33	1.03	1.21
179	Δ Tuition	12.69	-0.28	6.39	10.48
	Δ Qual.	-0.08	-0.3	0.18	1.15
184	Δ Tuition	-7.96	-15.81	-5.24	-13.32
	Δ Qual.	-0.79	-1.47	-1.03	-1.07
147	Δ Tuition	-1.53	-6.15	-6.34	-8.29
	Δ Qual.	0.03	0.02	0.45	0.58
181	Δ Tuition	-2.15	-23.14	0.26	1.78
	Δ Qual.	-0.04	-0.22	0.04	0.19
173	Δ Tuition	-1.77	-5.63	4.18	2.29
	Δ Qual.	-0.02	-0.18	1.06	0.82
142	Δ Tuition	-3.63	-8.7	-8.06	-18.05
	Δ Qual.	-0.2	-0.39	0.11	0.07

Notes: This table shows the percentage change in tuition collection and student quality when the DA mechanism is replaced by the COSM and CPPS mechanisms. For each mechanism change, utility changes are measured in three cases where the ZX quotas are 10%, and 30% of the total quotas.

Table F.5: Standard Deviation of Student Quality

School ID	DA	COSM		CPPS	
		10%	30%	10%	30%
183	0.015	0.016	0.023	0.016	0.024
141	0.026	0.028	0.041	0.039	0.049
187	0.035	0.038	0.033	0.046	0.038
167	0.045	0.045	0.054	0.053	0.063
185	0.044	0.044	0.049	0.053	0.062
186	0.05	0.053	0.053	0.065	0.059
179	0.042	0.042	0.046	0.05	0.053
184	0.035	0.032	0.029	0.032	0.032
147	0.039	0.039	0.039	0.043	0.044
181	0.032	0.032	0.032	0.034	0.035
173	0.03	0.03	0.03	0.038	0.037
142	0.028	0.027	0.027	0.028	0.029

Notes: This table shows the standard deviation (s.d.) of the admitted students' quality under different mechanisms. Except for the DA mechanism, the standard deviations are measured in two scenarios where the ZX quotas are 10% and 30% of the total quotas.

G Estimate without Survey in 2014

In this section, we use the admission records from 2014 to re-estimate the non-ZX related coefficients in the first step. Student i's indirectly utility function is denoted as

$$u_{i,j,c} = \sum_l \beta^l y_j^l + \sum_w \beta^w x_i^w y_j^w + \beta^D d_{ij} + \varepsilon_{ij} \tag{16}$$

and that the utility from being assigned to nonpublic high school o is

$$u_{i,o} = F_o + \varepsilon_{io}. \tag{17}$$

Now we consider students' decision problem in the first step is

$$\max_{a_i \in A_i} EU(a_i, s_i) \tag{18}$$

where $a_i = (j_1, j_2, j_3)$ and A_i is the student i's choice set. Here we assume that students all accept the random assignment if they are rejected by all three choices in the ROLS. A student i

We use the backward induction approach at Section 5.2 to estimate non-ZX related coefficients. The identification strategy is the same as we present at Section 5.2. The student's decision problem becomes

$$V^k(s_i) = \max_{j_i^k} \{ P_{i,j,c_0}^k \cdot u_{i,j,c_0} + (1 - P_{i,j,c_0}^k) \cdot V^{k+1}(s_i) \},$$

where V^k is the value function of i in round k, P_{i,j,c_0}^k is the probability of student i is admitted by school j in round k with tuition c_0. More precisely, $P_{i,j_1,c_0}^1 = Pr(\bar{S}_{j_1}^2 \leq s_i)$, $P_{i,j_2,c_0}^2 = Pr(\bar{S}_{j_2}^2 \leq s_i | s_i < \bar{S}_{j_1}^2)$, and $P_{i,j_3,c_0}^3 = Pr(\bar{S}_{j_3}^3 \leq s_i | s_i < \bar{S}_{j_2}^2)$.

The log-likelihood function for the entire sampel can be denoted as

$$\log L_1(\beta) = \sum_i log(Pr(a_i \in A_i^*)) \tag{19}$$

where A_i^* is student i's optimal choice set.

The estimated results are reported in Table G.1, in which the ZX-related coefficient are estimated by the same approach in Section 5.2. We also conduct the out-of-sample test in Table G.3 and Table G.2 to show the predicted schools cutoffs and admission patterns in 2012 and 2013.

Table G.1: Preference Parameters

	Estimated Result Using Admission Records
Reputation × HS	0.543
	(0.172)
Reputation × MS	0.348
	(0.044)
Reputation × LS	0.293
	(0.061)
Capacity × HS	-0.918
	(0.538)
Capacity × MS	-2.811
	(0.554)
Capacity × LS	-0.898
	(0.135)
Special class × H	-8.038
	(2.208)
Special class × MS	3.168
	(2.071)
Special class × LS	4.778
	(2.105)
Distance	-1
Distance × Male	0.971
	(0.863)
Same district	-2.113
	(0.310)
Same district × Male	2.933
	(0.439)
Dorm	4.861
	(0.893)
Dorm × Male	-0.757
	(0.950)
Cost × HS	-1.472
	(0.019)
Cost × MS	-1.661
	(0.037)
Cost × LS	-1.821
	(0.021)
Cost × HHP	-1.002
	(0.071)
Cost × MHP	-1.274
	(0.092)
Cost × LHP	-1.889
	(0.094)
Non-public high school	1.052
	(0.540)
School Fixed Effect	Y

Notes: Standard errors are reported in parentheses. Distance is measured by kilometer. The coefficient of female's attitude to home-school distance is normalized to -1. Capacity is measured by 100 seats. The unit of cost(Tuition) is 1000 Yuan. "HS", "MS", and "LS" represent high-, medium- and low-scoring students respectively. "HHP", "MHP", and "LHP" represent students from high-, moderate- and low-housing price communities respectively.

Table G.2: Admission Cutoffs

	Within Sample					
		2012			2013	
	(1)	(2)	(3)	(4)	(5)	(6)
School ID	True	Predicted	Diff	True	Predicted	Diff.
141	607	604.7	2.3	604	599.7	4.3
142	535	535	0	530	530	0
147	555.5	535	20.5	552.5	548.1	4.4
167	592.5	591.8	0.7	590	586.4	3.6
173	535	535	0	530	530	0
177	597	588.4	8.6	590.5	583.6	6.9
179	571.5	553.1	18.4	565	560.3	4.7
181	535	535	0	530	530	0
183	617	617	0	611	611.5	-0.5
184	535	535	0	530	530	0
185	583	577.1	5.9	580	574.2	5.8
186	583	555.5	27.5	578	565.8	12.2
187	599.5	594.7	4.8	594.5	588.1	6.4
188	571.5	590.3	-18.8	575	579.7	-4.7
28†	594.5	591.6	2.9	589	583.3	5.7
165†	608.5	603.6	4.9	605.5	597.2	8.3
166†						
169†				599	596.7	2.3
180†				576.5	574	2.5
200†						

Notes: This table indicates the out-of-sample tests for the schools' cutoffs, using the estimated coefficients from the 2014 admission records. The full mark is 665. The threshold is 535 in 2012, and 530 in 2013. † indicates the special class. The number of special classes varies with years

Table G.3: Admission Patterns (%)

	With in Sample					
	2012			2013		
	True	Predicted	Diff	True	Predicted	Diff
1st choice admitted	29.4	9.5	19.9	30.7	9.7	21
Normal	15.5	5.4	10.1	15.6	5.4	10.2
ZX	13.8	4.2	9.6	15	4.2	10.8
2nd choice admitted	36.7	34.8	1.9	38.9	41.4	-2.5
Normal	31	25.4	5.6	32	31	1
ZX	5.7	9.4	-3.7	6.6	10.4	-3.8

Notes: This table indicates the out-of-sample test of the matching patterns for the 1st and 2nd choices in the ROLs.

H Student Survey in 2014

Survey Overview

We cooperated with the local education bureau to conduct the student survey in mid-May 2014. 27 out of 42 of these schools agreed to cooperate with our research and let us survey their 9th grade students.

It takes about 10 minutes for one to finish answering all the questions on the survey at most. Two weeks before running the survey, we ran a pilot study of the survey to 60 students one week prior to our fieldwork.

The team of surveyors was led by a retired professor in educational psychology. The members of the team consisted of 20 college students. They were instructed in detail the survey process and their accountability to supervise the survey.

The survey asked the 9th grade students about:

- What aspects of a high school do they think as important when selecting schools.
- Students' true preferences over high schools based their study ability.
- For how many years' cutoff lines do the students look at before submitting their rank order lists?

Survey Process

Each day, starting from 7:00am, the survey team started to travel together to the targeted schools. They arrived at the first school at about 7:45am, then started the survey immediately after their arrival. Each member of our surveyor team supervised the survey for one classroom. The responsibility of our surveyor team members were distributing the paper form surveys and watching the students to make sure they are answering the questions and also to prevent them from looking at others' answers or communicating with each other.

After finishing collecting the answered surveys, the surveyor team would start traveling to the next middle school. In each survey day, the surveyor team surveyed 5 to 10 middle schools, depending on the distance between one school and another. During the survey dates, the surveys were all conducted before morning classes started, during class breaks, at noon before afternoon classes started, and after afternoon classes ended. The starting times were about 7:45am, 9:45am, 1:15pm, 2:45pm and 4:15pm. Each member of the surveyor team were paid by 300 yuan (approximately 45 USD) per survey day.

Outreach

At the beginning of the survey, we stated clearly that this survey was not related to students' high school admission and was for research only. Also, it would be kept completely confidential. Every time before starting the survey, the surveyors announced these points to the students and requested them answer according to the truth.

Questionnaire

Dear students: We are researchers of Educational Science Research Department. Please take a few minutes to complete this questionnaire. This questionnaire is only for research – it has no relationship with the results of high school entrance exam, neither does it have any relationship with high school admission. Any personal information in this questionnaire will be treated as highly confidential. Please answer the questions carefully. Thank you!

School: Class: Name: Gender: Student ID:

Q1. Are you Arts or Sports Specialty Student? A. Yes B. No

Q2. Are you directly upgrading student? A. Yes B. No.

Q3. Are you quota student? A. Yes B. No.

Q4. Are you a student who graduated in previous years? A. Yes B. No.

Q5. Please choose the level of importance of the following factors that you consider when you choose general high schools or vocational schools:

Q5A. (1) The academic quality (e.g. college entrance exam scores):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

(2) The employment condition after graduation and the professional training (Please answer this question if you are possible to choose vocational schools; do not answer if you do not consider vocational schools):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5B. The facility condition of schools (e.g. equipment, computers, sports fields):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5C. Whether the school provides scholarship or tuition waive:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5D. The distance from school to home:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5E. Low pressure at school:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5F. Good study atmosphere of the school:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5G. The school's especially good performance at arts or sports:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5H. The strict management in students' study and life:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5I. School's environment (e.g. beautiful and clean campus, good safety condition around the campus):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5J. School's living condition for students (e.g. the quality of food, school bus condition, accommodation condition):

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5K. The outside-class life condition

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5L. Good classmates:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q5M. Whether the school has special classes:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Please list other factors not listed above that you think important:

Q6. When you are considering the choice of high schools, how important are your opinion and other people's opinion:

Q6A. Your own opinion:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q6B. Parents' opinion:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q6C. Your teacher's suggestion:

5. Very important 4. Relatively important 3. Normal 2. Not so important 1. Not important at all

Q7. With your current study ability and scores, please list 7 schools - ordinary high schools or vocational schools – you may consider as your choice (do not consider order):

Q8. Please pick up 5 schools that you want to get in most from the above 7, and list them in the order of intensity of your willingness to get in:

Q9. When filling your rank order list, how many previous years' admission lines for the schools will you refer to:

A. Do not refer to any previous year admission lines

B. Refer to the admission lines for only last year

C. Refer to the admission lines for the past two years

D. Refer to the admission liners for the past three years

E. Refer to the admission lines for more than past three years

I ZX policies in other Chinese cities

In this section, we describe the implementation of ZX policy in three directly-controlled municipalities of China, i.e., Beijing, Shanghai, and Tianjin.

Beijing integrated the ZX policy into its centralized high school admission system in 2005. After the Ministry of Education announced the cancellation of the ZX policy in 2012, the percentage of ZX students of each school decreased from 18% to 15% and further to 10% in 2013. The ZX policy was fully terminated in 2014. The basic tuition of public high schools was 1,600 Yuan/year for a normal student in 2011, while for a ZX student, it could not exceed 10,000 Yuan/year.

The admission mechanism of the ZX policy applied in Beijing was an adjusted constrained DA mechanism with purchasing seat options. In this process, no more than eight schools could be selected in the ROL. Each student could select no more than two options from each specific school choice. The options of a school include normal, ZX, special class, and dorm. This mechanism is a special case of CPPS mechanism, wherein the matching algorithm follows the CPPS mechanism with permanency-execution period (8, 0, 0,...).

Shanghai is one of the cities that discontinued the ZX policy immediately after the announcement from the Ministry of Education in 2012. The total percentage of ZX students was restricted within 15% for each school in 2011, which is the percentage for ZX policy in the previous year. the ZX tuition was charged according to the type of school. In district-level key high schools, the basic tuition for students was 2,400 Yuan/year, whereas the ZX tuition was 6,000 Yuan/year before 2011 and 4,266 Yuan/year in 2011. For the city-level key high schools, the basic tuition was 3,000 Yuan/year, whereas the ZX tuition was 10,000 Yuan/year before 2011 and 7,000 Yuan/year in 2011. For the boarding schools, the basic tuition was 4,000 Yuan/year, whereas the ZX tuition was 13,333 Yuan/year before 2011 and 9,333 Yuan/year in 2011. The admission mechanism adopted in Shanghai was the constrained COSM where no more than 15 schools could be selected from the ROL.

Tianjin cancelled its ZX policy in 2015. Before 2015, the ZX tuition was standardized across all general high schools at 8,000 Yuan/year, which was a fourfold increase in the basic tuition (2,000 Yuan/year). The matching algorithm used in Tianjin was a constrained CPPS mechanism with permanency-execution period (2, 8). The students could select two key high schools in the first round and eight ordinary high schools in the second round.

J Random Assignment

In Section 5.2, we use the backward induction method to solve the student problem, and reduce the dimension used in the estimate. We assume that a student accepts the randomly assigned school if she is rejected by all her listed schools. In this section, we relax this assumption and allow students to choose whether to accept the random assignment. In this way, we may estimate the outside option of students. In the background of the local high school admission, we introduce that all type of schools, such as public high schools, private high schools, and vocational schools, must attend this centralized admission mechanism. In other words, a student has no "outside options" if she wants to continue her education in this city. However, from the ex ante perspective, if a student does not want to attend a randomly assigned school, she can leave the city to find other education opportunities or join the labor market directly. Therefore, technically the "outside option" still exists for students, although only 150 students were observed not accepting the random assignment across three years in our sample. It is necessary to estimate the outside option to justify that the assumption that students all accept the random assignment is reasonable.

It is worth noting that we do not observe the choice of $r_i = 0$ or 1 for students who are accepted by one of the schools in their ROLs. The admission record only reveals r_i for those who are rejected by all three choices. Therefore we have 1559 observations and 150 of them chose $r_i = 0$. By the backward induction approach, student i faces the choice of r_i only when she is rejected by all three choices. Therefore, her decision problem is

$$V^4(s_i) = \max_{r_i \in \{0,1\}} \{I_{r_i=1} \cdot (\sum_j \widetilde{P}_j \cdot u_{i,j,c_0}) + (1 - I_{r_i=1}) \cdot \widetilde{u}_i\} \tag{20}$$

which is Equation 9 in Section 5.2. Here $\widetilde{P}_j = \frac{1}{N}$, where N is the number of leftover schools. u_{i,j,c_0} is i's utility of attending school j with tuition c_0, and it has been estimated by the survey data in Section 5.1. $\widetilde{u}_i$ is the utility of the outside option. We simply define it as $\widetilde{u} = \varphi + \nu_i$ where φ is the average payoff outside option and ν_i is the error term.

We may observe $r_i = 1$ when $\sum_j \widetilde{P}_j \cdot u_{i,j,c_0} > \widetilde{u}_i$, and 0 otherwise. To estimate φ, we use the maximum simulated likelihood estimation as that in Section 5.2 and the technical detail is the same as that described in Section D. The estimated result is reported as follow: The coefficient of "outside option" is 12.63 with standard error 45.71, the p-value is 0.78.

This result indicates that the outside option is insignificant. We also try to estimate the heterogeneity of φ for different scoring student and years, however none of these results are signification or stable due to the very low number of students who choose $r_i = 0$. Therefore, our assumption that all students choose $r_i = 1$ is a reasonable simplification in the main content.

K Persistence of Preferences

In the estimation, we use the survey from 2014 to estimate the non-ZX related parameters, and adopt the results from the first stage to estimate the ZX-related parameters. This procedure has an assumption that the distribution of students' preferences does not change from 2012 to 2014. It is difficult to test this assumption directly, but we provide some empirical evidences to support this assumption in this section.

The first panel in Table 2 shows that the distributions of exam scores are stable across years. It implies the overall structure of the student ability and the middle school education quality is stable from 2012 to 2014. To further investigate the information from the student side, we focus on the middle schools that have at least 100 graduates every year in our data set.[67] Table K.1 shows that 28.8% of students in these schools receive scores higher than 90th percentile in 2012, and this number is still 31.5% in 2013 and slightly increased to 31.8% in 2014. Similarly the distribution of medium- and low- scoring students are also consistent across years.

We also investigate another issue on the student side, which is the students' living areas. Since the middle school admission is based on the school zone in the city. Table K.1-K.2 imply that the student body in each middle school and education quality are stable across years, and no significant demographic change is detected. On the supply side, except the observable information of high schools we have reported in Table 1, no school has changed the location. Other dramatic changes, such as government funding or security incidents, have not been found in any of high schools from 2012 to 2014 in the city. Therefore, we think the assumption that students' preferences over schools are stable in this short period is reasonable. Another evidence of this assumption is the sample test results in Section 5.4 (Model fitness and robustness check section). Table 6 of the main context and Table F.1 of

[67] The number of graduates from each of other middle schools fluctuates significantly, it is difficult to tell the reason. To avoid the measurement error or other unobservable reason that causing these variations, we focus on the middle schools with a large and stable number of graduates.

the Appendix show that our estimated preference has reasonable predictive power to fit the data. If the students' preferences had systematically changed over years, then the sample test results could be worse and more biased.

Table K.1: Score distribution in Large Middle Schools

	2012	2013	20.14
High-scoring	28.8%	31.5%	31.8%
Medium-scoring	45.4%	46.4%	49.0%
Low-scoring	25.8%	22.1%	19.2%
Total number of students	1706	1747	1719

Notes: This table indicates score distributions by percentage in middles schools with at least 100 graduates. "High-scoring" means the student score is high than 90th percentile, "Mediam-scoring" means the student score is between 70th percentile and 90th percentile, and "Low-scoring" means the student score is below 70th percentile.

Table K.2: Home Distribution in Large Middle Schools

	2012	2013	20.14
HHP	52.6%	53.9%	58.8%
MHP	43.9%	43.1%	36.8%
LHP	2.6%	2.5%	2.6%
Total number of students	1706	1747	1719

Notes: This table indicates the percentage distribution of students in different housing price area in the largest 11 middle schools. "HHP" means the high housing price communities, "MHP" means the moderate housing price communities, and "LHP" means the low housing price communities.

L Survey Sample vs. Whole Population

In this section, we compare the survey sample with the entire sample to determine whether our survey accurately represents the full sample.

Table L.1: Survey vs. Population

Whole Population			Surveyed Students		
Mean	s.d.	Median	Mean	s.d	Median
575.0	24.9	572.5	570.0	23.0	567

Notes: This table compares descriptive statistics between the survey sample and the whole population

Table L.2: Survey vs. Population: Score distribution

	Whole Population	Surveyed Students
High-Scoring	18.4%	16.6%
Medium-Scoring	49.3%	49.6%
Low-Scoring	32.3%	33.8%

Notes: This table indicates the score distribution of the survey sample and the whole population

Table L.3: Survey vs. Population: Score distribution: Home allocation

	Whole Population	Surveyed Students
HHP	36.3%	34.3%
MHP	48.9%	47.9%
LHP	14.8%	17.8%

Notes: This table indicates the score distribution of the survey sample and the whole population