

# Monetary policy and long-term interest rates\*

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## Abstract

We study the relationship between monetary policy and long-term rates in a structural, general equilibrium model estimated on both macro- and yield-data from the United States. Regime shifts in the conditional variance of productivity shocks, or “uncertainty shocks”, are a crucial driver of bond risk premia. We highlight three main results. First, our term premia on 10-year bonds are highly correlated with estimates from the affine literature, even if less markedly volatile. Second, uncertainty shocks also induce an increase in equity premia and exert downward pressure on consumption and inflation. An increase in equity premia will therefore be accompanied by a cut in policy interest rates, even if the policy rule does not directly react to equity prices. This model mechanism is consistent with the empirical evidence on the “Fed put”. Third, model-implied long-term inflation expectations are less dogmatically anchored than survey-based measures over the 2000s.

### JEL classification:

**Keywords:** monetary policy, Fed put, risk premia, term structure of interest rates, regime switches, Bayesian estimation.

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# 1 Introduction

Following Smets and Wouters (2007), estimated general equilibrium models with sticky prices have become popular tools for monetary policy analysis in central banks. Partly as a result of the 2008 financial crisis and the ensuing Great recession, these models have become much richer in a number of directions, notably including financial frictions (e.g. Christiano, Motto and Rostagno, 2014, or Del Negro et al., 2017). Due to linearization, however, they are not suited to describe financial asset prices and risk premia and they in fact almost always abstract from such prices in calibration or estimation. This is especially surprising for bond yields, because nominal bonds play an important role in the models – after all the key monetary policy instrument is the return on a short-term nominal bond. Moreover, long-term rates and risk premia are often an object of interest in monetary policy analysis – see for example Goodfriend’s (1993) discussion of an “inflation scare” in bond yields in the early 1990s, or the famous Greenspan (2005) speech on the “bond markets conundrum”.<sup>1</sup>

In this paper we solve and estimate the nonlinear version of a new-Keynesian model to study its joint implications for the macroeconomy and bond risk premia in the United States over the past 50 years. The model has two notable ingredients. First, as in many existing analyses of asset prices in production economies with endogenous labor supply, we also rely on non-expected utility preferences.<sup>2</sup> Second, consistently with a large empirical literature, we allow for regime switching in selected model parameters to capture dimensions of structural change: the parameters include the central bank’s inflation target and the conditional variance of structural shocks.<sup>3</sup> We demonstrate that, taken together, these two model ingredients allow for non-negligible time-variation in bond risk premia in response to regime switches. More specifically, time-varying risk premia can be observed already when the model is solved to second order. Since second-order solutions can be

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<sup>1</sup>Goodfriend (1993) defines an inflation scare as a significant increase in long term nominal interest rates in the absence of an increase in policy rates.

<sup>2</sup>See e.g. Tallarini (2000), Rudebusch and Swanson (2012), van Binsbergen et al. (2012), Andreasen (2012a), Colacito and Croce (2013), Swanson (2018).

<sup>3</sup>See McDonnell and Perez-Quiros (2000), Sims and Zha (2006), Primiceri (2005), Justiniano and Primiceri (2008), Liu, Waggoner and Zha (2011). See also Gourio (2012, 2013), Andreasen (2012b) for the case of time variations in disaster risk. Schorfheide (2005) documents time variation in the Federal Reserve’s inflation target.

computed fast, the reduced-form is amenable to estimation using Bayesian, maximum likelihood methods. We can therefore analyse the time series properties of macroeconomic and financial variables in an internally consistent, micro-founded framework.

We show that the estimated time series of bond risk premia and term premia produced by our model are significantly different from zero from a statistical perspective. The crucial driver of variation in bond risk premia are switches in the conditional standard deviation of technology shocks, which we also refer to as "uncertainty shocks". Since these switches are highly correlated with business cycles, bond premia are countercyclical: they are large during recessions, when uncertainty over the future is high, and lower during expansions, when uncertainty falls to normal levels. Switches in the standard deviations of other shocks, including monetary policy shocks, also produce time-variation in risk premia, but their quantitative impact is much smaller.

We investigate how our model-based estimates of bond risk premia compare to results obtained using a pure-finance approach. As a benchmark for comparison, we use the estimates in Adrian et al. (2013) that are based on a flexible implementation of the affine, term structure approach (see Piazzesi, 2010, for a survey). In spite of the fact that our model is much more tightly parameterized, cyclical fluctuations in the two term premia measures are largely synchronised and highly statistically correlated with each other. However, our estimated term premia are smaller and less highly volatile. One interpretation of this difference is that our model misses relevant sources of bond premia volatility, for example those induced by swings in market sentiment. At the same time, reduced-form estimates are often based on highly parameterised frameworks, which may lead to overfitting for risk premia.

Even if we do not use equity prices in estimation, we can explore the implications of our model for equity premia. We show that uncertainty shocks also have an impact on equity premia and produce an important link between such premia and the macroeconomy. The increase in uncertainty during recessions leads to an increase in households' precautionary saving and, by implication, a fall in their demand for consumption and disinflationary pressure. If monetary policy aims to stabilize inflation, policy rates must be cut in the face of such developments. As a result, a sharp increase in equity premia is endogenously accompanied by a reduction in monetary policy rates even if the policy rule does not directly react to the stock price. This mechanism can account for the evidence on the

“Fed put”, i.e. the Fed’s tendency to ease monetary policy sharply in the wake of large stock market declines (Cieslak and Vissing-Jorgensen, 2020).

Finally, a by-product of our model are model-based, long-term inflation expectations. Given that we use long-term bonds when estimating the model, filtered long-term inflation expectations are tightly disciplined. We show that their evolution is broadly comparable to that of survey measures available from the Federal Reserve Bank of Philadelphia’s quarterly Survey of Professional Forecasters at the end of the XX century. In both cases, we observe a progressive parallel fall in expectations over the 1980s and 1990s. However, model-based expectations are less dogmatically anchored over the 2000s.

Our paper is related to different strands of the literature. First, it contributes to the integration of asset pricing and macroeconomics—see e.g. Piazzesi and Schneider (2006), Rudebusch and Swanson (2012), Swanson (2012, 2018, 2019). Compared to these papers, that study calibrated versions of microfounded models, our main contribution is to push the model further and study its likelihood when confronted with U.S. data. From this empirical perspective, our paper is also related to De Graeve, Emiris and Wouters (2007), which estimates the loglinearized version of a medium-scale DSGE model using both macroeconomic and term structure data and must therefore introduce additional parameters to allow for (constant) risk-premia. Christoffel, Jaccard and Kilponen (2011) also estimate the linearized version of a new Keynesian model, and then draw bond pricing implications using a higher order approximation. Bekaert, Cho and Moreno (2010) and Campbell, Pflueger and Viceira (2013) follow an intermediate route and study asset prices in a linearized New Keynesian model assuming a stochastic discount factor that is related to the new Keynesian model’s equations in a reduced-form manner. The papers most similar to ours are Doh (2011, 2012), van Binsbergen *et al.* (2012) and Andreasen (2012), which estimate nonlinear models with macroeconomic and term structure data. In contrast to all these papers, we allow for regime switches in the variance of key shocks and in the inflation target. We argue that these are important model features to fit bonds and macro data. All these papers are also related to the huge consumption-based asset pricing literature and build on the results of either Campbell and Cochrane (1999) or Bansal and Yaron (2004). In a recent contribution to this literature, Schorfheide, Song and Yaron (2018) highlight the importance of allowing for measurement error in consumption in a long-run risk model. We also allow for measurement errors in our estimation.

Our paper is also related to the literature documenting time variation in macroeconomic volatility in a reduced form setting, including e.g. McDonnell and Perez-Quiros (2000), Sims and Zha (2006), Primiceri (2005). Justiniano and Primiceri (2008) allows for shifts in the volatility of structural shocks in a linearized, medium-scale DSGE model applied to the U.S. economy. Compared to Justiniano and Primiceri's, we rely on a smaller, but non-linear model, which allows us to explore the effects of uncertainty shocks on households' demand for precautionary saving and on bond risk premia. Our modelling of second moments is however more parsimonious and less flexible in uncovering trend shifts in volatility. A strand of this literature, spawned from Bloom (2009), has analysed the implications of shocks to conditional variances, or uncertainty shocks. While in Bloom (2009) an increase in uncertainty induces firms to temporarily reduce investment and hiring, in our model it induces households to increase their precautionary saving and reduce consumption. Uncertainty shocks therefore act like demand shocks. This is consistent with the results in Basu and Bundick (2012), which relies on a more comprehensive, calibrated model of the U.S. economy. Bianchi, Ilut and Schneider (2014) put forward a model with ambiguity averse investors, where regime shifts generate large low frequency movements in asset prices.

Finally, our paper contributes to the econometrics literature on Markov switching models—see Kim (1994), Kim and Nelson (1999), Schorfheide (2005) and Liu, Waggoner and Zha (2011). So far this literature has focused on linear state space model. Given that our model has a quadratic, reduced-form representation, the likelihood is non-standard. We approximate it based on a combination of the extended Kalman filter and the Kim (1994) filter. As a result, the likelihood can be computed fast and we can study and compare different model variants. Our preferred specification allows for regime-switching in the inflation target and in the conditional variances of monetary policy and technology shocks. We provide indications that the filter we use performs reasonably well in a DSGE environment such as ours, in which nonlinearities unrelated to Markov switching are not very pronounced.

The rest of the paper is organized as follows. Section 2 describes the model, focusing on its distinguishing features: the distribution of the shocks and the utility function, which is of the class proposed by Epstein and Zin (1989) and Weil (1990), but extended to allow for habit persistence in consumption. The methods that we adopt to solve and estimate

the model are described next, in section 3. Such methods are non-standard, because we need to solve the model to a second order approximation in order to capture precautionary savings effects. We demonstrate that the reduced form of the model is quadratic in the state variables with continuous support and includes regime-switching intercepts, as well as variances. We then estimate the non-linear reduced form using Bayesian methods. Section 4 described the estimation results, analyses different versions of the model through marginal likelihood comparisons, and illustrates the robustness of parameter estimates to an extension of the estimation sample to the post-Great Recession period. The role of uncertainty shocks and our estimates of bond risk premia are presented in Section 5, which also compares our estimates to those obtained in the standard, affine literature. Section 5 also illustrates the implications of the model for long-term inflation expectations. Finally, Section 6 analyses equity premia and their implications for the Fed put. Section 7 offers some concluding remarks.

## **2 The model**

We start from a simple version of the new-Keynesian model sufficient to account for the dynamics of key nominal and real macroeconomic variables – see also Woodford (2003). We thus assume nominal price rigidities, external habit persistence, inflation indexation, and a monetary policy rule with “interest rate smoothing”. Since our interest is in the model’s implications for long-term interest rates, we abstract from capital accumulation and real wage rigidities. Our results suggest that even with these simplifications our model can go a long way in explaining the data of interest to us.

We incorporate in the standard model two non-standard features in macroeconomic applications: regime switching in selected model parameters and non-expected utility preferences. In the rest of this section we describe these two features in more detail, before sketching the other model ingredients, which are standard.

### **2.1 Regime-switching**

In macroeconomic applications, exogenous shocks are almost always assumed to be log-normal. However, Hamilton (2008) argues that a correct modelling of conditional variances is always necessary, for example because inference on conditional means can be inappro-

priately influenced by outliers and high-variance episodes. In macroeconomics there is by now a long tradition of papers documenting time variation in macroeconomic volatility, including e.g. McDonnell and Perez-Quiros (2000), Sims and Zha (2006), Primiceri (2005).

In this paper, we therefore assume that conditional variances of structural shocks are subject to regime switches. We have investigated model versions in which various shocks are assumed to be heteroskedastic, but in our preferred specification regime-switching will only affect the second moments of productivity and monetary policy shocks. More specifically, we will assume that the technology shock  $z_t$  and the monetary policy shocks  $\eta_t$  have standard deviations that can independently switch between a high and a low regime. Denoting the low variance regime by 1 and the high variance regime by 0, we write

$$\begin{aligned}\sigma_{z,s_{z,t}} &= \sigma_{z,H}s_{z,t} + \sigma_{z,L}(1 - s_{z,t}) \\ \sigma_{\eta,s_{\eta,t}} &= \sigma_{\eta,H}s_{\eta,t} + \sigma_{\eta,L}(1 - s_{\eta,t})\end{aligned}$$

where the variables  $s_{z,t}$  and  $s_{\eta,t}$  can assume the discrete values  $H$  (mnemonic for high) and  $L$  (mnemonic for low). For each variable  $s_{j,t}$  ( $j = z, \eta$ ), the probabilities of remaining in states  $H$  and  $L$  are constant and equal to  $p_{j,H}$  and  $p_{j,L}$ , while the probabilities of switching to the other state will be  $1 - p_{j,H}$  and  $1 - p_{j,L}$ , respectively.

Beside the conditional variances of structural shocks, in our preferred specification the Fed's inflation target  $\Pi_{s_{\pi,t}}^*$  is also stochastic from the public's perspective.<sup>4</sup> This assumption follows Schorfheide (2005) and Liu, Waggoner and Zha (2011).<sup>5</sup> As in these papers, we assume that the log-target can independently switch between a high and a low regime

$$\pi_{s_{\pi,t}}^* = \pi_{s_{\pi,H}}^* s_{\pi,t} + \pi_{s_{\pi,L}}^* (1 - s_{\pi,t})$$

where the probabilities to remain in the high and low regimes are constant and equal to  $p_{\pi,H}$  and  $p_{\pi,L}$ , respectively.

## 2.2 Households

We extend the non-expected utility specification proposed by Epstein and Zin (1989) and Weil (1990) to account for habit persistence in consumption and labour-leisure choice.

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<sup>4</sup>In the paper we denote gross inflation and gross interest rates by capital letters.

<sup>5</sup>See also Favero and Rovelli (2003); Erceg and Levin (2003), Gürkaynak, Sack and Swanson (2005).

More specifically, we assume that households provide differentiated labor services to firms. Following Erceg, Henderson and Levin (2000), we also assume that households are monopolistic suppliers of each type of labour. Each household  $i$  will therefore maximize its inter-temporal utility with respect to consumption and its wage rate, subject to firms' demand for its labour  $N_t(i) = L_t \left( \frac{w_t(i)}{w_t} \right)^{-\theta_{w,t}}$ , for a time varying elasticity of substitution  $\theta_{w,t}$ , and the budget constraint

$$P_t C_t(i) + E_t Q_{t,t+1} W_{t+1}(i) \leq W_t(i) + w_t(i) N_t(i) + \int_0^1 \Psi_t(j) dj \quad (1)$$

In the budget constraint,  $C_t$  is a consumption index satisfying  $C_t = \left( \int_0^1 C_t(z)^{\frac{\theta-1}{\theta}} dz \right)^{\frac{\theta}{\theta-1}}$ ,  $W_t$  denotes the beginning-of-period value of a complete portfolio of state contingent assets,  $Q_{t,t+1}$  is their price and  $\Psi_t(j)$  are the profits received from investment in firm  $j$ . The price level  $P_t$  is defined as the minimal cost of buying one unit of  $C_t$ , hence equal to

$$P_t = \left( \int_0^1 p(z)^{1-\theta} dz \right)^{\frac{1}{1-\theta}}. \quad (2)$$

Households' preferences are described by the Kreps and Porteus (1978) specification proposed by Epstein and Zin (1989) and Weil (1990). We generalize the utility function proposed in those papers by allowing for habit formation and a labour-leisure choice, as in standard, general equilibrium macro-models. The generalization to allow for the labour-leisure choice has already been used, for example, in Rudebusch and Swanson (2012). We additionally allow for habit formation because it has been shown to be important to match the dynamic behavior of aggregate consumption – see e.g. Fuhrer (2000).

As a result, the utility of household  $i$  can be defined recursively through an aggregator of current and future utility

$$V_t = \left\{ (1 - \beta) u_t^{1-\psi} + \beta \left( E_t V_{t+1}^{1-\gamma} \right)^{\frac{1-\psi}{1-\gamma}} \right\}^{\frac{1}{1-\psi}}, \quad \psi, \gamma \neq 1, \quad (3)$$

where current utility is a function of consumption and leisure

$$u_t = u \{ C_t(i) - h \Xi_t C_{t-1}, 1 - N_t(i) \}$$

where leisure is written as  $1 - N_t$  because total hours are normalized to 1, the  $h$  parameter represents the force of external habits and  $\Xi_t$  is the rate of growth of technology.<sup>6</sup>

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<sup>6</sup>Guariglia and Rossi (2002) also use expected utility preferences combined with habit formation to study precautionary savings in UK consumption. Koskiewicz (1999) studies an inter-temporal consumption-leisure model with non-expected utility.



With our more general preferences specification,  $\gamma$  is no-longer related one-to-one to risk aversion. Swanson (2012) discusses the appropriate measures of risk aversion in a dynamic setting with consumption and leisure entering the utility function. However, the Appendix demonstrates that  $1/\psi$  continues to measure the long-run elasticity of intertemporal substitution of consumption.

The first order conditions for consumption-leisure choice are a function of the mark-up  $\mu_{w,t} \equiv (\theta_{w,t} - 1) / \theta_{w,t}$ , which is assumed to follow an exogenous autoregressive process

$$\mu_{w,t+1} = \mu_w^{1-\rho_\mu} (\mu_{w,t})^{\rho_\mu} e^{\varepsilon_{t+1}^\mu}, \quad \varepsilon_{t+1}^\mu \sim N(0, \sigma_\mu).$$

### 2.3 Closing the model

The rest of the model is standard. We assume a continuum of monopolistically competitive firms (indexed on the unit interval by  $j$ ), each of which produces a differentiated good. Demand arises from households' consumption and from the exogenous component  $G_t$ , which is an aggregate of differentiated goods of the same form as households' consumption. Hence the aggregate demand index  $Y_t^D$  satisfies  $Y_t^D = C_t + G_t$ .

Firms' production function is

$$Y_t(j) = A_t L_t^\alpha(j)$$

where  $L_t$  is the aggregator of differentiated labour inputs  $L_t = \left[ \int_0^1 N_t(i)^{\frac{\theta_{w,t}-1}{\theta_{w,t}}} di \right]^{\frac{\theta_{w,t}}{\theta_{w,t}-1}}$ , and  $A_t$  is a mixture of two shocks  $A_t = Z_t B_t$  such that, in logs,

$$\begin{aligned} b_t &= b_{t-1} + \xi_t \\ \xi_t &= \xi + \varepsilon_t^\xi, \quad \varepsilon_{t+1}^\xi \sim N(0, \sigma_\xi) \\ z_t &= \rho_z z_{t-1} + \varepsilon_t^z, \quad \varepsilon_{t+1}^z \sim N(0, \sigma_{z,s_{z,t}}) \end{aligned}$$

where  $\xi_t = \log(\Xi_t)$ ,  $\xi$  is the long run productivity growth rate. This specification allows for both a standard, stationary technology shock and for a stochastic trend, represented by  $B_t$ .

As in Rotemberg (1982), we assume the firms face quadratic costs in adjusting their prices – see also Schmitt-Grohé and Uribe (2004b). We assume partial indexation to past inflation with coefficient  $\iota$ . More specifically, quadratic price adjustment costs are specified as  $\zeta/2 \left( P_t^j / P_{t-1}^j - \left( \Pi_{s_{\pi,t}}^* \right)^{1-\iota} \Pi_{t-1}^\iota \right)^2 Y_t$ .

Finally, monetary policy follows the simple Taylor-type policy rule

$$I_t = \left( \frac{\Pi_{s\pi,t}^* \Xi_t^\psi}{\beta} \right)^{1-\rho_I} \left( \frac{\Pi_t}{\Pi_{s\pi,t}^*} \right)^{\psi_\Pi} \left( \frac{\tilde{Y}_t}{\tilde{Y}} \right)^{\psi_Y} I_{t-1}^{\rho_I} e^{\eta_t} \quad (4)$$

where  $\tilde{Y}_t \equiv Y_t/B_t$  is detrended aggregate output,  $\tilde{Y}$  its steady state level and  $\eta_t$  is a policy shock such that

$$\eta_{t+1} = e^{\varepsilon_{t+1}^\eta}, \quad \varepsilon_{t+1}^\eta \sim N(0, \sigma_{\eta, s\eta, t}).$$

Market clearing in the labour market requires  $L_t = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{\alpha}}$  and in the goods market

$$Y_t = C_t + G_t + \frac{\zeta}{2} \left( \Pi_t - \left( \Pi_{s\pi,t}^* \right)^{1-\iota} \Pi_{t-1}^\iota \right)^2 Y_t$$

where  $G_t$  is an exogenous stochastic process which captures additional non-interest-rate-sensitive components of output and which we specify in deviation from the stochastic growth trend  $B_t$ , so that

$$\frac{G_t}{B_t} = \left( \frac{gY}{B} \right)^{1-\rho_g} \left( \frac{G_{t-1}}{B_{t-1}} \right)^{\rho_g} e^{\varepsilon_t^g} \quad \varepsilon_{t+1}^G \sim N(0, \sigma_G)$$

where the long run level  $g$  is specified in percent of output, so that  $g \equiv G/Y$ .

The variable  $G_t$  is a common way to introduce demand shocks in the model (see e.g. Rudebusch and Swanson, 2012). It also breaks the theoretical equivalence between GDP (net of price adjustment costs) and consumption. As a result, we will use both these variables in estimation.

### 3 Solution and estimation methods

#### 3.1 Functional forms

In our empirical analysis we need to choose a functional form for the utility aggregator  $u\{C_t - h\Xi_t C_{t-1}, 1 - N_t\}$ . As shown by King, Plosser and Rebelo (1988), consistency with long run growth requires a functional form of the following type

$$u = (C_t - h\Xi_t C_{t-1}) v(N_t)$$

where  $v(N_t)$  is a decreasing function. Various options are available for  $v(N_t)$ . We rely on the particular specification proposed by Trabandt and Uhlig (2011), which implies a constant Frisch elasticity of labour supply in the absence of habits and with standard, expected-utility preferences. The specification implies  $v(N_t) = \left( 1 - \eta(1 - \psi) N_t^{1 + \frac{1}{\phi}} \right)^{\frac{\psi}{1 - \psi}}$ .

### 3.2 Solution

To solve the model, we first detrend all real variables by the technological level  $B_t$ . In the solution, we expand variables around their natural logarithms, which are denoted by lower-case letters. We then collect all (detrended) predetermined variables (including both lagged endogenous predetermined variables and exogenous states with continuous support) in a vector  $\mathbf{x}_t$  and all the non-predetermined variables in a vector  $y_t$ .

The reduced form of the model can thus be written in compact form as

$$y_t = g(\mathbf{x}_t, \tilde{\sigma}, \mathbf{s}_t) \quad (5)$$

$$\mathbf{x}_{t+1} = h(\mathbf{x}_t, \tilde{\sigma}, \mathbf{s}_t) + \tilde{\sigma} \Sigma(\mathbf{s}_t) \mathbf{u}_{t+1} \quad (6)$$

for matrix functions  $g(\cdot)$ ,  $h(\cdot)$ , and  $\Sigma(\cdot)$  and a vector of i.i.d. innovations  $\mathbf{u}_t$ . The vector  $\mathbf{s}_t$  includes the state variables that index the discrete regimes.  $\tilde{\sigma}$  is a perturbation parameter.

Following Hamilton (1994), we can write the law of motion of the discrete processes  $\mathbf{s}_t$  as

$$\mathbf{s}_{t+1} = \kappa_0 + \kappa_1 \mathbf{s}_t + \nu_{t+1} \quad (7)$$

for a vector  $\kappa_0$  and a matrix  $\kappa_1$ . The law of motion of state  $s_{z,t}$ , for example, is written as  $s_{z,t+1} = (1 - p_{z,L}) + (-1 + p_{z,L} + p_{z,H}) s_{z,t} + \nu_{z,t+1}$ , where  $\nu_{z,t+1}$  is an innovation with mean zero and heteroskedastic variance.

For the model variants in which we assume a constant inflation target, we follow the solution approach described in Amisano and Tristani (2011). The approach exploits the property that regime switching do not affect the non-stochastic steady state. We can therefore apply standard perturbation methods (as in, for example, Schmitt-Grohé and Uribe, 2004a, or Gomme and Klein, 2011) and approximate the solution as a function of the state vector  $\mathbf{x}_t$  and perturbation parameter  $\tilde{\sigma}$  around the point where  $\mathbf{x}_t = \bar{\mathbf{x}}$  and  $\tilde{\sigma} = 0$ . Amisano and Tristani (2011) demonstrate that the second order approximation can be written as

$$g(\mathbf{x}_t, \tilde{\sigma}, \mathbf{s}_t) = F \hat{\mathbf{x}}_t + \frac{1}{2} (I_{n_y} \otimes \hat{\mathbf{x}}_t') E \hat{\mathbf{x}}_t + k_{y,s_t} \tilde{\sigma}^2 \quad (\text{Sol1})$$

and

$$h(\mathbf{x}_t, \tilde{\sigma}, \mathbf{s}_t) = P \hat{\mathbf{x}}_t + \frac{1}{2} (I_{n_x} \otimes \hat{\mathbf{x}}_t') G \hat{\mathbf{x}}_t + k_{x,s_t} \tilde{\sigma}^2 \quad (\text{Sol2})$$

where  $F$ ,  $E$ ,  $P$  and  $G$  are constant vectors and matrices and only the vectors  $k_{y,s_t}$  and  $k_{x,s_t}$  are regime dependent. As a result, regime-switching affect the conditional means of vectors  $\mathbf{x}_t$  and  $y_t$ .

When we also allow for a time-varying inflation target, it is no longer the case that the non-stochastic steady state is independent of regime switching. In this case we follow Liu, Waggoner and Zha (2011) and Schorfheide (2005) and perturb the inflation target around its ergodic mean.<sup>7</sup> As a result, the matrices  $F$ ,  $E$ ,  $P$  and  $G$  will remain constant.

### 3.3 Estimation

Given the form of the solution, the reduced form system of equations (5) and (6) can be re-written as

$$y_t = k_{y,j} + F\hat{x}_{t+1} + \frac{1}{2} \left( I_{n_y} \otimes \hat{x}'_{t+1} \right) E\hat{x}_{t+1} + Dv_{t+1} \quad (8)$$

$$x_{t+1} = k_{x,i} + P\hat{x}_t + \frac{1}{2} \left( I_{n_x} \otimes \hat{x}'_t \right) G\hat{x}_t + \tilde{\sigma} \times \Sigma_i \times u_{t+1} \quad (9)$$

$$s_t \sim \text{Markov switching with } (8 \times 8) \text{ transition probability } T \quad (10)$$

where

$$k_{y,j} = k_{y,s_{t+1}=j}$$

$$k_{x,i} = k_{x,s_t=i}$$

$$\Sigma_i = \Sigma(s_t = i).$$

The vector  $y_t$  includes all observable variables, and  $v_{t+1}$  and  $u_{t+1}$  are measurement and structural shocks, respectively. In this representation, the regime switching variables affect the system by changing the intercepts  $k_{y,j}$ ,  $k_{x,i}$  and the loadings of the structural innovations  $\Sigma_i$  (we indicate here with  $i$  the value of the discrete state variables at  $t$  and with  $j$  the value of the discrete state variables at  $t + 1$  – not to be confused with the  $j$  used above to denote monopolistically competitive firms).

If a linear approximation were used, we would have a linear state space model with Markov switching (see Kim, 1994, and Kim and Nelson, 1999).

In the quadratic case, the likelihood cannot be obtained in closed form. One possible approach to compute the likelihood is to rely on Sequential Monte Carlo techniques<sup>8</sup>.

<sup>7</sup>Foerster et al. (2016) develops a more accurate, but highly computationally expensive method for constructing first- and second-order approximate solutions of Markov-switching DSGE models.

<sup>8</sup>See, for example, Herbst and Schorfheide (2015), Part III.

The convergence of these methods, however, can be very slow in a case, such as the one of our model, in which both nonlinearities and non-Gaussianity of the shocks characterise the economy.<sup>9</sup>

Based on the observation that quadratic terms  $1/2 \left( I_{n_y} \otimes \hat{x}'_{t+1} \right) E \hat{x}_{t+1}$  and  $1/2 \left( I_{n_x} \otimes \hat{x}'_t \right) G \hat{x}_t$  in equations (8) and (9) tend to be small, we therefore proceed as follows.

At any point in time, we first linearise the two quadratic terms around the conditional mean of the continuous state variables. In a homoskedastic setting, this would correspond to applying the extended Kalman filter. In our model with regime switching, the linearisation must be conditional on each regime. As a result, at any point in time we can rewrite equations (8) and (9) as

$$\begin{aligned} y_t &= \tilde{k}_{y,t+1}^{(i,j)} + \tilde{F}_{t+1}^{(i,j)} \hat{x}_{t+1} + Dv_{t+1} \\ \hat{x}_{t+1} &= \tilde{k}_{x,t}^{(i)} + \tilde{P}_t^{(i)} \hat{x}_t + \Sigma_i u_{t+1} \end{aligned} \tag{11}$$

for suitably defined coefficients  $\tilde{k}_{y,t+1}^{(i,j)}$ ,  $\tilde{F}_{t+1}^{(i,j)}$ ,  $\tilde{k}_{x,t}^{(i)}$  and  $\tilde{P}_t^{(i)}$ . Note that, in contrast to the original system (5)-(6), in the above equations both the intercepts  $\tilde{k}_{y,t+1}^{(i,j)}$ ,  $\tilde{k}_{x,t}^{(i)}$  and the slope coefficients  $\tilde{F}_{t+1}^{(i,j)}$ ,  $\tilde{P}_t^{(i)}$  become regime-dependent. Nevertheless, we are still in the world of linear state space models with Markov switching. To compute the likelihood, we can therefore apply Kim's (1994) approximate filter—see section B.3 in the Appendix for a description of the algorithm that we use to compute the likelihood. We then combine the likelihood with a prior and sample from the posterior using a tuned Metropolis-Hastings algorithm. This approach based on the extended Kalman Filter linearisation is computationally much faster than using sequential Monte Carlo methods<sup>10</sup>.

## 4 Empirical results

This section presents the estimation results. It starts with a description of the data. It then analyses different versions of the model through a comparison of marginal like-

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<sup>9</sup>Another reason why we decided not use Sequential Monte Carlo methods is that these techniques are not easily applied in situations where the measurement error standard deviations are treated as free parameters to be estimated, as in this paper, and not calibrated to some arbitrary value. See in particular the discussion in Herbst and Schorfheide (2015), Sections 8.1.3 and 8.4.2.

<sup>10</sup>In section B.3.1 of the Appendix we empirically characterise the properties of the likelihood approximation used in this paper.

likelihoods. Finally, it describes key parameter estimates and shows their robustness to a longer estimation sample including the recent period of binding zero lower bound.

## 4.1 Data description

Our benchmark estimates are based on quarterly US data over the sample period from 1966Q1 to 2009Q1. We start in 1966, because this is the date when a Taylor rule begins providing a reasonable characterization of Federal Reserve policy.<sup>11</sup> We end in 2009Q1 when the zero bound constraint, which we do not explicitly include in our model, becomes binding. In section 4.5 we extend the estimation sample to 2019 by treating the short-term (policy) rate as unobservable.

Concerning the macro data, we use per capita total real personal consumption, per capita GDP and inflation. We need to use both GDP and consumption because the former enters the Taylor rule and the latter the Euler equation. Inflation is measured as the logarithmic first-difference in the consumption deflator (all macro variables are from the FRED database of the St. Louis Fed). We use the Effective Federal Funds Rate as shortest maturity (3-month), taken from FRED, and continuously compounded yields on 3-year and 10-year zero-coupon bonds from Gürkaynak, Sack and Wright (2007).<sup>12</sup>

Prior to the analysis, we take logarithmic first differences for consumption and GDP, which in the model are assumed to follow a stochastic trend. No other data transformations are applied. All variables are expressed as quarterly rates, so that 0.25 represents an annualized interest rate, inflation rate, or growth rate equal to 1 percent.

## 4.2 Prior distributions

Prior distributions for our model are presented in Table (1).

Concerning regime switching processes, we assume beta priors for transition probabilities. We expect the states to be relatively persistent, so we centre all distributions around a value of 0.9, which implies a persistence of 2.5 years for each state. For the regime-

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<sup>11</sup>According to Fuhrer (1996), "since 1966, understanding the behaviour of the short rate has been equivalent to understanding the behaviour of the Fed, which has since that time essentially set the federal Funds rate at a target level, in response to movements in inflation and real activity". Goodfriend (1991) argues that even under the period of official reserves targeting, the Federal Reserve had in mind an implicit target for the Funds rate.

<sup>12</sup>See section B.1 for a detailed description of the data used for estimation and their mnemonics

switching inflation target we use priors centred around 0.97, that imply a persistence of 8 years. This corresponds to the modal term of office of the Chair of the Federal Reserve over our sample period (Chairs Burns, Volcker and Bernanke all served for two 4-year terms).

Since it is well known that in mixture models the likelihood is invariant to label permutations of the discrete states, for each of the two discrete volatility states (transitory component of technology and monetary policy shocks) and for the inflation target, we achieve identification through non-overlapping prior distributions. We call state 1 the low variance, or low target, state. Prior and posterior draws not complying with the inequality constraint are therefore suitably permuted (see Geweke 2007). Table (1) reports the resulting empirical distribution for the prior of regime-switching coefficients once the inequality constraint is imposed.

We use inverse gamma priors for the standard deviations of the shocks and beta priors for their persistence.

For the policy rule, we centre priors around parameter values estimated from quarterly data over a pre-sample period running from 1953 to 1965, namely  $\rho_I = 0.85$ ,  $\psi_{\Pi} = 0.2$  and  $\psi_Y = 0.02$ .

The priors for all the remaining parameters are specified broadly in line with the rest of the literature. The details are presented in Section B.2 of the online Appendix.

### 4.3 Comparison across different model specifications

We performed a number of specification tests to identify the most likely version of the model.

First, in preliminary analyses we considered versions of the model where regime switching affects only the conditional variances of structural shocks. In the cases of demand, mark-up and technology growth shocks, the regime-switching hypothesis received little support from the data. We therefore focused on regime-switching variances only for (level) technology shocks and monetary policy shocks.

Second, we consider model specifications in which the inflation target is not subject to stochastic regime switches.

Third, following widespread practice in estimating nonlinear DSGE models (see, for example, Herbst and Schorfheide, 2015), we consider a version of the model where mea-

surement errors are not estimated, but calibrated to 20% of the unconditional variance of the data. This approach is usually adopted to facilitate the use of SMC techniques.

Finally, we analyse a version of the model without uncertainty shocks, i.e. without regime-switching in the variance of technology shocks. This case is interesting in view of the important role of uncertainty shocks in our results.

A comparison of these various specifications is presented in Table (2), both for the 1966-2009 estimation sample and for the longer sample including the zero lower bound period. For both periods, the version of the model with regime-switching inflation target attains a higher marginal likelihood value. We therefore focus on this specification in the rest of the paper.

The specification with uncertainty shocks is also preferred in terms of marginal likelihood.

#### 4.4 Posterior distributions

The posterior distributions of structural parameters for our preferred model specification is shown in Table (3).

We observe that the quarterly standard deviations of monetary policy shocks in the low and high regimes are equal to 0.12 percent and 0.37 percent, respectively. The standard deviation of technology shocks change between 1.02 percent in the low volatility regime and 2.03 percent in the high volatility regime. In annualised terms the estimated targets are 3.4% to 7.4%, respectively. These values are roughly consistent with the estimates in Schorfheide (2005).

The posterior mode of the transition probabilities suggests that the low-volatility states are more persistent for monetary policy and technology shocks. Based on these results, we refer to low-volatility regimes as “normal regimes”. By contrast the high and low regimes for the inflation target are equally persistent.

The estimates of the other structural parameters are roughly consistent with the existing literature. We underline that the  $\gamma$  parameter is equal to 7.14 and the habit parameter  $h = 0.82$ . Together, these two parameters are suggestive of a high level of risk aversion, which is in line with the results in Piazzesi and Schneider (2006), or in Rudebusch and Swanson (2012). Both setting  $\gamma = \psi$  and setting  $h = 0$  would result in a sharp fall of the



level and volatility of estimated bond risk premia.<sup>13</sup>

#### 4.5 Extending the sample to 2019

In this section we check the robustness of our parameter estimates to an extension of the sample until 2019 – see Table (4).

Our solution method does not explicitly take the zero-lower bound into account. In a standard new-Keynesian model solved under the Taylor rule, it is well-known that the zero-lower bound would induce a strong nonlinearity in the reduced-form – see e.g. Nakov (2008). In reality, however, the deployment of credit policy has been an alternative means of monetary accommodation at the zero lower bound. Debortoli et al. (2020) argue that the response of long-term interest rates to shocks during the zero-lower-bound period is very similar to its counterpart in previous years. Motivated by their results, we treat the policy rate as unobservable after 2008 and estimate the model over the full available sample (that is, until 2019). Our implicit assumption is that unconventional policy measures adopted by the Federal Reserve after the Great Recession were such as to replicate the degree of monetary accommodation which would have been produced by a negative Federal Funds rate, had the zero-lower bound constraints not existed.

As shown in Tables (3) and (4), posterior estimates are remarkably similar over the two estimation samples. The largest changes in economic terms can be observed in the value of the inflation target in the two target regimes. The low-regime target becomes about 1.3 pp lower (in annual terms) in the longer sample, consistently with the persistently low inflation observed after the Great Recession. By contrast, the high-regime target tends to increase somewhat. In the longer sample there is also a noticeable increase in  $\gamma$  from 7 to 13. From a statistical viewpoint, these differences are all irrelevant, since the point estimates in the longer samples all fall within the 95% credibility bounds generated by using the shorter sample.

As a by-product, this extension delivers a model-consistent measure of the monetary policy interest rate which would had been observed in the absence of the zero-lower bound. Our estimates suggest that it would have been only mildly negative over the zero-lower bound years.

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<sup>13</sup>See Section C in the online appendix.

## 5 Bond risk premia and inflation expectations

This section explores the implications of our model for bond risk premia. It then compares our estimates of 10-year term premia to those obtained from the affine literature in finance. Finally, it compares model-implied 10-year inflation expectations to survey-based measures.

### 5.1 Bond risk premia and uncertainty shocks

Nominal bonds reflect risk premia associated with both consumption risk and with inflation risk. It is well known that models with homoskedastic shocks solved to a second order approximation can only generate constant risk premia – see for example Hördahl, Tristani and Vestin (2008). Our model can produce changes in risk premia when there is a change in the standard deviation of the structural shocks. In other words, time variation in risk premia is associated with switches in the variance regimes.

To illustrate this point we consider a typically used measure of risk premia, i.e. the expected excess holding period return on a bond of maturity  $n$ . This corresponds to the expected return that can be earned by holding an  $n$ -maturity bond for one quarter in excess of the 1-quarter interest rate.<sup>14</sup>

The (filtered) expected excess holding period return generated by our model for 10-year bonds is displayed in the upper panel of Figure (1). We note that these estimates are characterised by filtering and parameter uncertainty: 95 percent confidence sets are around 2 percentage points. Focusing on mean estimates, after remaining below 1 percentage point (in annualised terms) over the first decade of our sample, excess holding period returns increase over the second half of the 1970s and peak to 2.6 percentage points ahead of the 1981 recession. They fall again in the mid-1980s and climb sharply again ahead of all subsequent recessions.

The key source of quantitatively sizable time-variation in risk premia are switches in the variance of technology shocks, which we also refer to as uncertainty shocks. Uncertainty tends to increase during recessions and to fall back to normal levels during expansions. Recursive utility implies that households fear downward revisions in consumption

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<sup>14</sup>Excess holding period returns are defined as  $XHPR_{n,t} = HPR_{n,t}/I_t$ , where  $HPR_{n,t}$  is the return on holding a bond of maturity  $n$  for one period given by  $HPR_{n,t} = E_t B_{n-1,t+1}/B_{n,t}$ .

expectations (see for example Piazzesi and Schneider, 2006).<sup>15</sup> Since increases in future technological uncertainty also fuel the volatility of future consumption, risk premia also become larger after uncertainty shocks. Consequently, and consistently with the finding in the finance literature (see also Fama and French, 1989), risk premia are countercyclical.

Changes in the conditional variance of monetary policy shocks also affect risk premia, but their impact is estimated to be quantitatively small.

A different notion of bond risk premia that is frequently used is that of term premia. For a bond of a given maturity, term premia denote the component of the yield curve slope which is due to risk premia, as opposed to the path of expected future interest rates.<sup>16</sup> Figure (1) also shows our term premia estimates and compares them to a frequently used estimate from the affine literature (lower panel) – see Adrian et al. (2013).

The two measures are quite correlated with each other – the correlation coefficient is 0.74. The Adrian et al. (2013) estimate, however, is also characterised by higher-frequency volatility, while we find variations in term premia to be more infrequent. Depending on the time period, the point estimates of our term premia can be up to about 4 times smaller than estimates from the affine model.

One interpretation of this difference is that our model misses relevant sources of bond premia volatility, for example those induced by swings in market sentiment. While such swings are ruled out by our model, they could be captured by more flexible models which do not connect financial price movements to macroeconomic dynamics. At the same time, reduced-form estimates are often based on highly parameterised frameworks, which may lead to overfitting for risk premia.

## 5.2 Model-based vs. survey inflation expectations

A by-product of our model are estimates of long-term inflation expectations derived from bond yields. We show in Figure (2) model-implied, average inflation expectations over

<sup>15</sup>In our model with endogenous labour supply, revisions in the expected stream of overall future utility influences risk premia. See the Appendix, sections A.3 and A.4 .

<sup>16</sup>More specifically we define term premia as in Rudebusch and Swanson (2012). While the actual bond price for an  $n$ -maturity bond is defined as  $B_{t,n} = E_t [Q_{t,t+1} B_{t+1,n-1}]$ , where  $Q_{t,t+1}$  is the household's stochastic discount factor, the price of a bond net of term premia is discounted at the nominal interest rate  $I_t$ , i.e.  $B_{t,n}^{EH} = \frac{E_t [B_{t+1,n-1}^{EH}]}{I_t}$ . The (gross) term premium is defined as the ratio between the yields on  $B_{t,n}$  and on  $B_{t,n}^{EH}$ .

the next 10 years based on the longer estimation sample.<sup>17</sup> As a benchmark for comparison, we use expectations by the Federal Reserve Bank of Philadelphia’s quarterly Survey of Professional Forecasters combined with the Blue Chip Economic Indicators, which is available since 1979:Q4.<sup>18</sup>

Model-implied and survey expectations share a common downward trend over the 1980s and 1990s. Starting from peaks of 6 percent or higher, they both come down by several percentage points. However, the descent of model-implied expectations is more gradual and less monotonic. Model-implied expectations increase again sharply in 1993, consistently with the idea of an “inflation scare”, which was put forward by some commentators in this period.<sup>19</sup> They continue hovering around 3 percent over the 2000s, while survey expectations remain completely constant at 2.5 percent. Model-implied expectations fall more sharply during the Great recession.

To summarise, our model-implied estimate of long-term inflation expectations implicit in bond prices complements information available from survey data. It suggests that long-term inflation expectations are somewhat less rigidly anchored than one would conclude, based on survey data.

Our model also provides us with measures of long-term inflation expectations over the late 1960s and 1970s, a period for which survey measures are not available. Levin and Taylor (2013) compute far forward inflation expectations over this period based on a simple methodology and suggest that they started drifting up steadily as of 1965 reaching an estimated peak of about 4.5 percent in 1970 and then remained between 3.5 and 4.5 percent over the next several years. Our estimates are broadly consistent with these results. Average 10-year inflation expectations increase to 4 percent in 1971 and then hover between 3 and 3.5 percent in the early 1970s. In the second half of the 1970s they increase again to levels around 4.5 percent and remain at levels above 4 percent until 1980.

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<sup>17</sup>Estimates based on the sample until 2009 are comparable over the common sample.

<sup>18</sup>Both surveys report forecasts for the average rate of CPI inflation over the next 10 years. The Blue Chip survey reports long-term inflation forecasts taken twice a year (March and October). Prior to 1983, and in 1983:4, the variable was the GNP deflator rather than the CPI. As of 1991:Q4, we rely on the Philadelphia Survey of Professional Forecasters.

<sup>19</sup>For example, Goodfriend (2002) states: “Starting from a level of 5.9 percent [in October 1993], the 30-year bond rate rose through 1994 to peak at 8.2 percent just before election day in November. The nearly 2 1/2 percentage point increase in the bond rate indicated that the Fed’s credibility for low inflation was far from secure in 1994.”

These estimates are also consistent with the results in Ang, Bekaert and Wei (2008), which are based on a no-arbitrage factor model of the term structure.

## 6 Uncertainty shocks, macro dynamics and the Fed put

A joint model of macroeconomic variables and bond yields allows us to explore the effects on the macroeconomy of shocks which generate time variation in risk premia. We report in Figure (3) the impulse responses to an increase in the variance of technology from the low to the high regime.<sup>20</sup>

The regime switch has cyclical implications.<sup>21</sup> Increases in the variance of technology shocks generate an increase in the demand for precautionary saving. As a result, the demand for consumption goods falls. Given that prices are sticky and output is demand determined, lower demand for consumption goods generates a fall in output and inflation. Whether this translates into an actual recession with disinflationary shocks depends on the monetary policy response.<sup>22</sup> Figure (3) also reports impulse responses to the natural rate of interest – i.e. the real interest rate which, if "tracked" by the central bank, would ensure that inflation is stable at all times. The natural rate falls markedly and persistently. The policy rate also falls but, due to the high smoothing coefficient in the Taylor rule, very slowly over time. After one to two years, consumption, output and inflation return closer to their initial level, but the policy rate remains low for much longer, because the persistent nature of the uncertainty shock keeps the demand for precautionary saving high and it drives down real rates. As a result, 3-year and 10-year yields net of risk premia also

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<sup>20</sup>In Section B.6 of the Appendix we provide details on how impulse responses are computed in our model. The reported impulse responses take into account that new switches in the technology variance regime are possible in the future, and averages across all possible values of all other shocks, discrete and continuous.

<sup>21</sup>Uncertainty shocks in technology look like demand shocks, in the sense of being associated with a fall in output, consumption and prices at the same time. Our results corroborate, in the context of an estimated model, Basu and Bundick's (2012) finding that a persistent fall in nominal interest rates is an important part of the macroeconomic adjustment mechanism, following an uncertainty shock. If the fall in the nominal interest rate were prevented by the zero lower bound, the macro-economic effects of the shock would be even larger.

<sup>22</sup>Reifschneider, Tetlow and Williams (1999) also underlines the key role of monetary policy in reaction to an exogenous increase in risk premia in the context of the FRB/US model – see Brayton, Laubach, and Reifschneider (2014).

fall. By contrast, actual yields fall more mutedly, because term premia increase.

Even if we have not used equity prices in estimation, the stochastic discount factor generated by our model can be used to compute equity prices and the equity premium.<sup>23</sup>

Figure (3) also shows impulse responses to equity prices and equity premia. An uncertainty shock leads to an increase in equity premia and an equity market crash. From a reduced-form perspective, the same shock which causes the equity market crash will lead to a persistent reduction of monetary policy interest rates. This combination of equity market developments and monetary policy interest rate is reminiscent of the so-called “Fed put”. Cieslak and Vissing-Jorgensen (2020) argues that, since the mid-1990s, the Fed has tended to ease policy in the wake of large stock market declines. The pattern emerges in the late 1990s and holds through the 2007/2009 financial crisis and beyond. Through an analysis of Fed minutes, Cieslak and Vissing-Jorgensen conclude that the Fed likely believes the stock market crash to have adverse implications on the economy through a wealth effect. Hence the Fed would appear to react to the stock market crash because it predicts a future downturn with deflationary effects.

Our model reproduces this type of mechanism, given that the source of the stock market crash and of the increase in equity premia is the same uncertainty shock which is also associated with a reduction in the demand for consumption. From the perspective of an econometrician, therefore, an increase in equity premium and a drop in equity prices will be associated with a progressive reduction in interest rates. This is, however, not the result of a direct policy response to the stock market, since, by construction, our policy rule does not react to movements in the equity premium.

Overall, our model incorporate a mechanism which can account for part of the evidence in Cieslak and Vissing-Jorgensen (2020). However, our model, at least to a second order approximation, is unable to account for the observed asymmetry in the Fed put. In our model, a sudden stock market boom would warrant a symmetrically sharp monetary policy tightening.

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<sup>23</sup>See Section A.10 in the Appendix

## 7 Conclusions

This paper presents the results of the estimation of a nonlinear macro-yield curve model with Epstein-Zin-Weil preferences, in which the variance of structural shocks is subject to changes of regime.

From the finance viewpoint, the model produces term premia estimates that are highly correlated with those from the affine literature. It also incorporates a mechanism of shocks to the equity premium which can account for the empirical evidence on the "Fed put". From the macroeconomic perspective, the model provides a framework to exploit information from bond prices which is useful for monetary policy. It delivers model-based estimates of long-term inflation expectations that are largely consistent with survey-based measures, but less dogmatically anchored than those measures since the 2000s.

In our application we have relied on a parsimonious new-Keynesian model, but our solution and estimation procedure is sufficiently fast to be easily extended to larger frameworks. For example, it would be feasible to apply our approach to models with financial frictions, such as Gertler and Karadi (2011), that are often linearized. Our approach would allow one to analyse the interaction of financial frictions and pure risk premia, for example at times of crisis when financial constraints are especially tight.

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	prior mean	prior std	prior 2.5 %	prior 97.5 %
$p_{\pi_{11}^*}$	0.97	0.02	0.93	0.99
$p_{\pi_{00}^*}$	0.97	0.02	0.93	0.99
$p_{\eta,11}$	0.90	0.07	0.74	0.99
$p_{\eta,00}$	0.90	0.06	0.74	0.99
$p_{z,11}$	0.90	0.06	0.75	0.99
$p_{z,00}$	0.90	0.07	0.74	0.99
$\pi_1^*$	0.50	0.18	0.21	0.90
$\pi_0^*$	2.00	0.22	1.59	2.45
$\sigma_{\eta,1}$	0.22	0.08	0.12	0.42
$\sigma_{\eta,0}$	0.39	0.29	0.17	1.04
$\sigma_{z,1}$	0.21	0.08	0.12	0.42
$\sigma_{z,0}$	0.39	0.28	0.17	1.03
$\sigma_{\mu}$	0.30	0.23	0.12	0.83
$\sigma_{\xi}$	0.12	0.05	0.06	0.24
$\sigma_G$	3.03	2.11	1.26	8.09
$\rho_{\mu}$	0.86	0.09	0.64	0.98
$\rho_z$	0.86	0.09	0.64	0.98
$\rho_G$	0.86	0.09	0.64	0.98
$\psi_{\pi}$	0.20	0.02	0.16	0.24
$\psi_y$	0.02	0.01	0.00	0.04
$\rho_I$	0.85	0.02	0.82	0.88
$\xi$	0.50	0.10	0.32	0.71
$\iota$	0.50	0.19	0.15	0.86
$\phi$	1.00	0.50	0.01	1.98
$\gamma$	11.03	7.12	2.23	28.91
$\psi$	1.20	0.29	1.00	2.04
$\zeta$	15.06	7.02	1.25	28.99
$h$	0.50	0.19	0.15	0.86
$100 \times (1/\beta - 1)$	0.14	0.14	0.00	0.53
$\sigma_{me,\pi}$	0.01	0.01	0.01	0.04
$\sigma_{me,\Delta c}$	0.01	0.01	0.01	0.04
$\sigma_{me,\Delta y}$	4.97	2.49	2.40	11.18
$\sigma_{me,i}$	0.01	0.01	0.01	0.04
$\sigma_{me,i12}$	13.93	11.73	5.64	38.09
$\sigma_{me,i40}$	13.83	9.87	5.67	37.45

Table 1: Prior specification 1966-2009

Legend: standard deviations of structural shocks ( $\sigma_i$ ), inflation targets ( $\pi_{s_t}^*$ ) and the growth rate of technology ( $\xi$ ) are reported in percent per quarter, while standard deviations of measurement errors ( $\sigma_{me,i}$ ) in basis points per quarter. "std" denotes the standard deviation; "2.5 %" and "97.5 %" denote the corresponding percentiles of the distribution. Prior moments reported are based on 20,000 draws. Note that the standard errors of shocks that have switching volatility are drawn from priors that are symmetric across regimes and assigned to the high or low state depending on their values.

	1966-2009	1966-2019
baseline	4622.1	5628.2
cal. meas. errors	4512.6	5498.1
no MS in $\pi^*$	4619.3	5591.3
no MS in $z$	4571.6	5537.9

Table 2: Marginal likelihood comparisons

Legend: The table reports log marginal likelihoods for the models of interest. The models of interest are the "baseline", the model with "calibrated measurement error standard deviations", the model with a constant inflation target ("no MS in  $\pi^*$ "), the model

with a constant variance on the transitory technology shock ("no MS in  $z$ ").

	post. mean	post. std	post. 2.5	post. 97.5
$p_{\pi_{11}^*}$	0.96	0.02	0.92	0.99
$p_{\pi_{00}^*}$	0.97	0.01	0.94	0.99
$p_{\eta,11}$	0.95	0.02	0.89	0.99
$p_{\eta,00}$	0.90	0.05	0.78	0.98
$p_{z,11}$	0.97	0.01	0.95	0.99
$p_{z,00}$	0.94	0.02	0.90	0.97
$\pi_1^*$	0.86	0.26	0.32	1.24
$\pi_0^*$	1.85	0.19	1.51	2.27
$\sigma_{\eta,1}$	0.12	0.01	0.10	0.15
$\sigma_{\eta,0}$	0.37	0.05	0.29	0.50
$\sigma_{z,1}$	1.02	0.21	0.64	1.50
$\sigma_{z,0}$	2.03	0.40	1.34	2.96
$\sigma_{\mu}$	12.25	1.87	8.99	16.30
$\sigma_{\xi}$	0.50	0.04	0.42	0.59
$\sigma_G$	1.89	0.24	1.46	2.38
$\rho_{\mu}$	0.72	0.04	0.63	0.81
$\rho_z$	0.98	0.00	0.98	0.99
$\rho_G$	0.90	0.02	0.86	0.93
$\psi_{\pi}$	0.24	0.02	0.21	0.28
$\psi_y$	0.03	0.01	0.01	0.05
$\rho_I$	0.85	0.02	0.82	0.88
$\xi$	0.45	0.04	0.38	0.53
$\iota$	0.27	0.10	0.11	0.49
$\phi$	0.78	0.11	0.58	1.01
$\gamma$	7.14	2.50	3.45	13.11
$\psi$	1.49	0.16	1.23	1.84
$\zeta$	37.26	5.88	25.61	48.68
$h$	0.82	0.04	0.74	0.89
$100 \times (1/\beta - 1)$	0.14	0.09	0.01	0.33
$\sigma_{me,\pi}$	0.01	0.01	0.01	0.04
$\sigma_{me,\Delta c}$	36.79	4.33	28.35	45.14
$\sigma_{me,\Delta y}$	43.80	6.10	30.92	55.43
$\sigma_{me,i}$	0.01	0.01	0.01	0.04
$\sigma_{me,i12}$	4.20	0.49	3.34	5.25
$\sigma_{me,i40}$	3.49	0.35	2.86	4.24

Table 3: Posterior estimates 1966-2009

Legend: standard deviations of structural shocks ( $\sigma_i$ ), inflation targets ( $\pi_{s_t}^*$ ) and the growth rate of technology ( $\xi$ ) are reported in percent per quarter, while standard deviations of measurement errors ( $\sigma_{me,i}$ ) in basis points per quarter. "std" denotes the standard deviation; "2.5 %" and "97.5 %" denote the corresponding percentiles of the distribution. Prior moments reported are based on 20,000 draws.

	post. mean	post. std	post. 2.5	post. 97.5
$p_{\pi_{11}^*}$	0.94	0.02	0.90	0.97
$p_{\pi_{00}^*}$	0.96	0.02	0.92	0.98
$p_{\eta,11}$	0.96	0.02	0.91	0.99
$p_{\eta,00}$	0.89	0.06	0.76	0.97
$p_{z,11}$	0.98	0.01	0.96	0.99
$p_{z,00}$	0.93	0.02	0.90	0.95
$\pi_1^*$	0.53	0.20	0.18	0.94
$\pi_0^*$	2.13	0.21	1.76	2.56
$\sigma_{\eta,1}$	0.15	0.01	0.13	0.18
$\sigma_{\eta,0}$	0.38	0.06	0.29	0.52
$\sigma_{z,1}$	0.95	0.19	0.62	1.35
$\sigma_{z,0}$	1.98	0.37	1.32	2.75
$\sigma_{\mu}$	13.88	1.68	10.86	17.38
$\sigma_{\xi}$	0.60	0.03	0.55	0.66
$\sigma_G$	1.95	0.22	1.54	2.39
$\rho_{\mu}$	0.70	0.04	0.62	0.77
$\rho_z$	0.98	0.00	0.98	0.99
$\rho_G$	0.91	0.01	0.89	0.94
$\psi_{\pi}$	0.21	0.01	0.19	0.23
$\psi_y$	0.03	0.01	0.01	0.05
$\rho_I$	0.84	0.01	0.82	0.86
$\xi$	0.43	0.04	0.36	0.50
$\iota$	0.28	0.09	0.13	0.49
$\phi$	0.85	0.11	0.64	1.08
$\gamma$	12.65	3.84	6.81	21.67
$\psi$	1.28	0.07	1.16	1.42
$\zeta$	42.10	5.59	30.69	52.70
$h$	0.87	0.03	0.81	0.91
$100 \times (1/\beta - 1)$	0.10	0.06	0.01	0.23
$\sigma_{me,\pi}$	0.01	0.01	0.01	0.04
$\sigma_{me,\Delta c}$	0.01	0.01	0.01	0.04
$\sigma_{me,\Delta y}$	45.75	5.46	34.41	55.62
$\sigma_{me,i}$	0.01	0.01	0.01	0.03
$\sigma_{me,i12}$	3.92	0.43	3.15	4.84
$\sigma_{me,i40}$	3.50	0.33	2.90	4.22

Table 4: Posterior estimates 1966-2019

Legend: standard deviations of structural shocks ( $\sigma_i$ ), inflation targets ( $\pi_{s,t}^*$ ) and the growth rate of technology ( $\xi$ ) are reported in percent per quarter, while standard deviations of measurement errors ( $\sigma_{me,i}$ ) in basis points per quarter. "std" denotes the standard deviation; "2.5 %" and "97.5 %" denote the corresponding percentiles of the distribution. Prior moments reported are based on 20,000 draws.



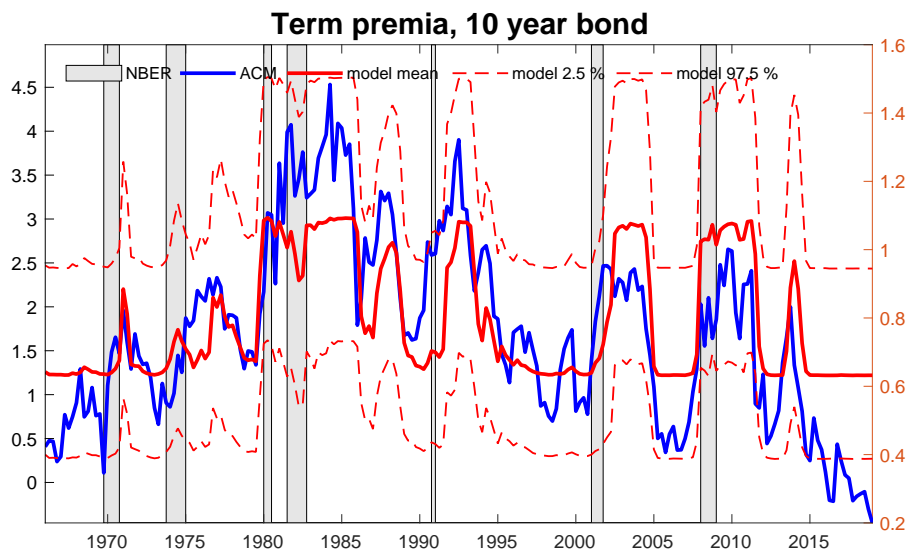
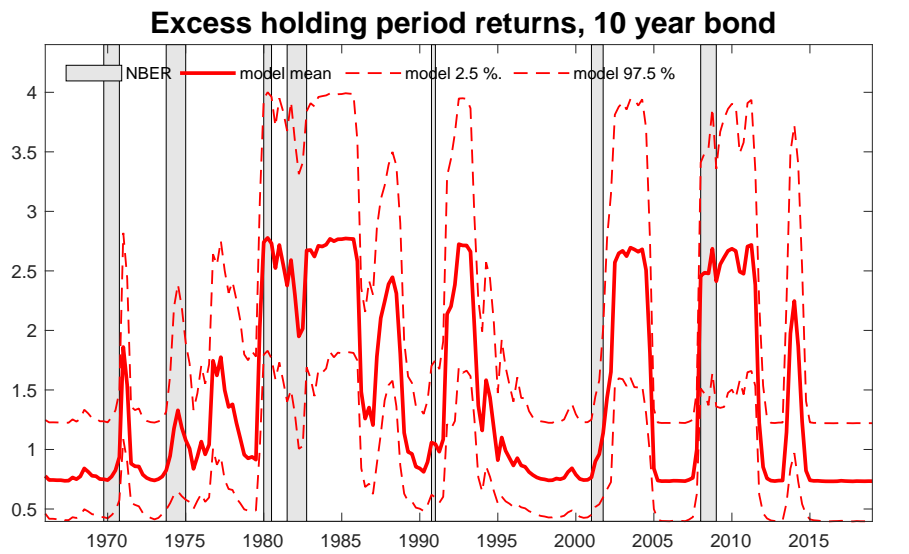


Figure 1: Upper panel: model-based expected excess holding period returns. Lower panel: comparisons of model-based term premia with term premia estimates from Adrian et al. (2013), denoted by "ACM". Shaded areas denote NBER recessions.

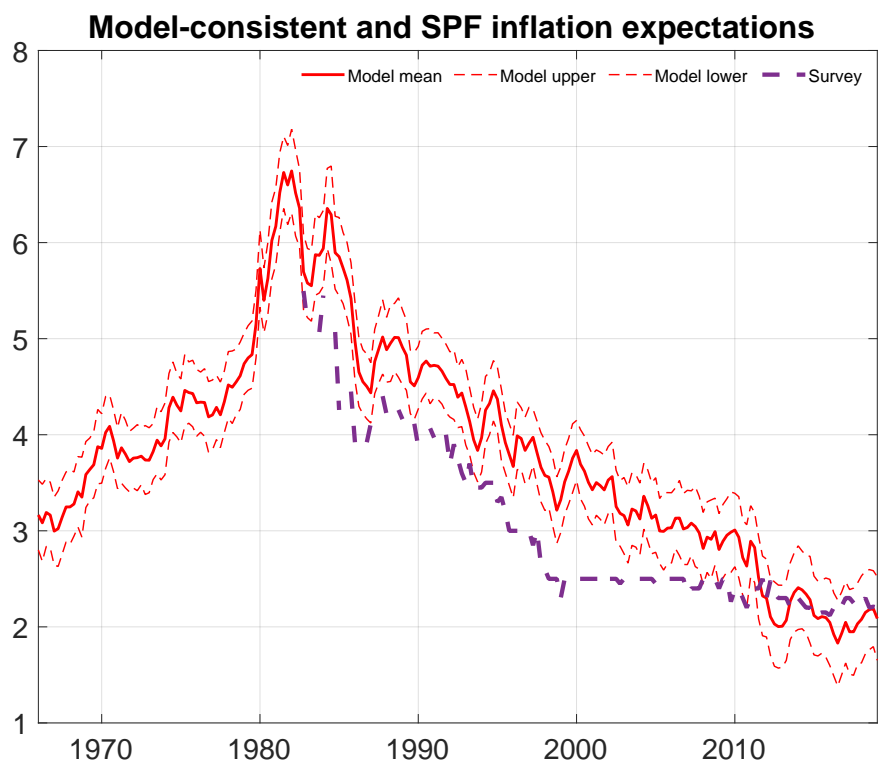


Figure 2: Average inflation expectations over the next 10 years: model vs. survey of professional forecasters (SPF)

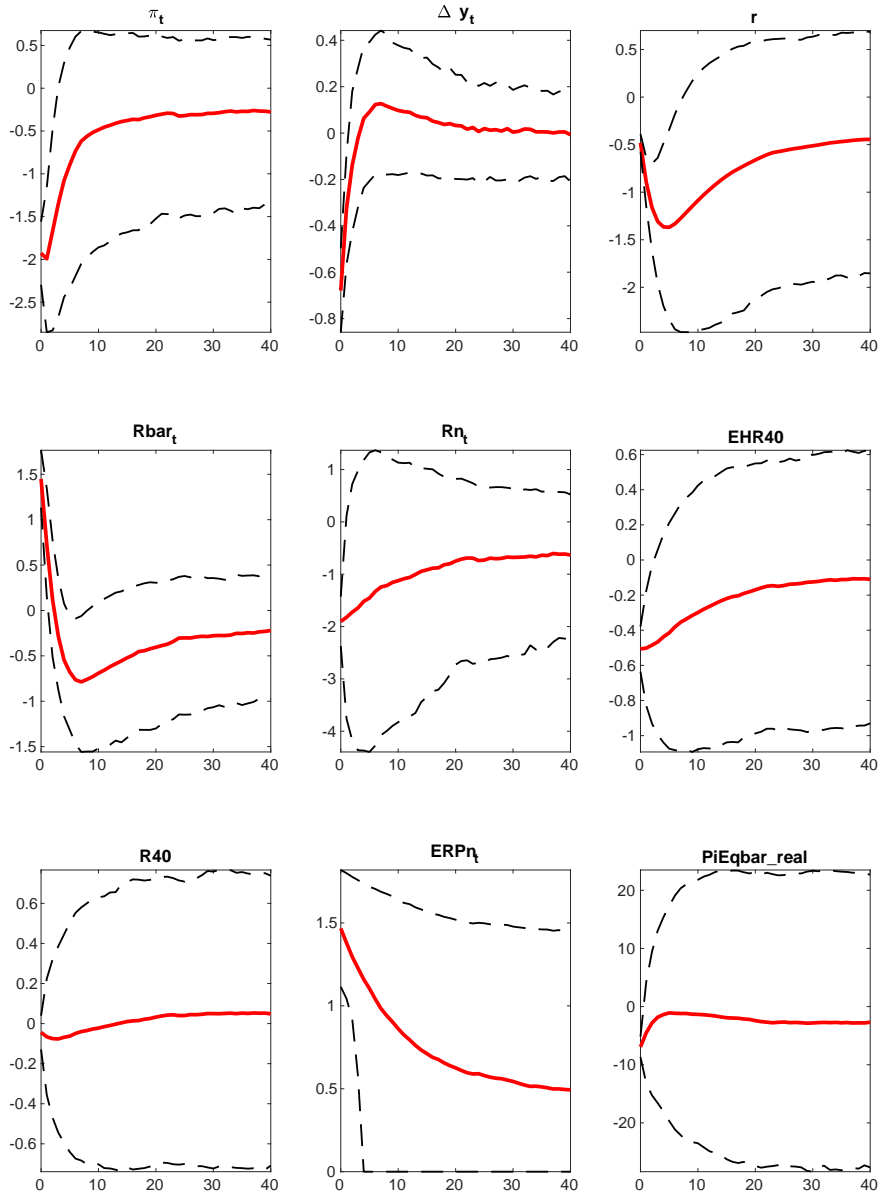


Figure 3: Impulse responses to a jump in the variance of the technology shock. Legend:  $\pi$ ,  $\Delta y$ ,  $i$ ,  $Rbar$ ,  $R_n$ ,  $EHR40$ ,  $R40$ ,  $ERP$ ,  $PiEq$  denote, respectively, inflation, output growth, the short term interest rate, the short real rate, the natural rate of interest, the long-run (40-period) rate compatible with the expectation hypothesis, the actual long-run rate, the equity premium, and the deflated and detrended equity price.