B Online Appendix

B.1 Sticky Prices [Matějka, 2016]

Additional Figures. Both the GAP-SQP and BA algorithms replicate the results in Matějka [2016] very closely. Figure 9 shows the marginal distributions over prices for the GAP-SQP algorithm (panel (a)) and BA algorithm (panel(b)), together with the numerical solutions from AMPL provided by Filip Matějka. Solutions are so close that we had to offset the histograms for visibility. We find that increasing grid precision for actions does not meaningfully alter the solution.

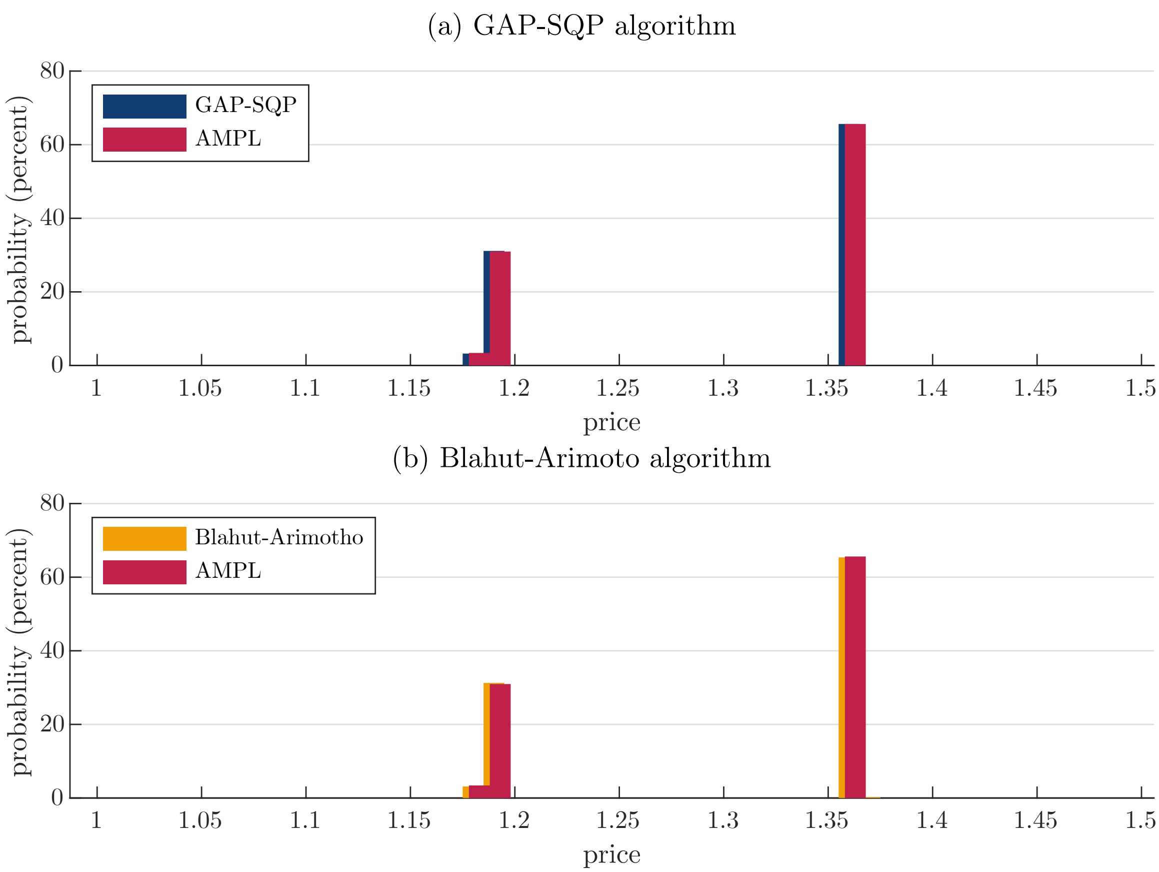

Figure 9: Replication of Matějka [2016]

Figure 10 reports the differences in the objective function value, at the computed

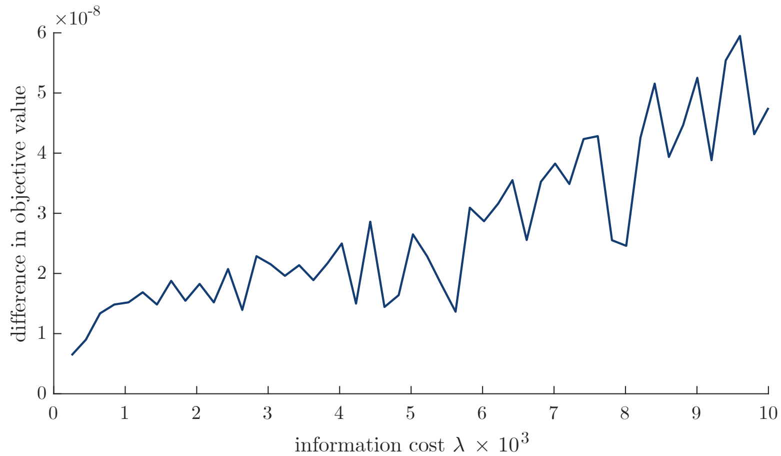

Figure 10: Objective function: GAP-SQP minus Blahut-Arimoto algorithm.

maximum, between the GAP-SQP and BA algorithms, for the benchmark case. The difference is positive for all the information values, indicating that the GAP-SQL algorithm achieves greater precision despite running on a fraction of the time of the BA algorithm. The difference, though, is very small by our choice of stopping values.

B.2 Portfolio Choice [Jung et al., 2019]

Derivation of the optimal Gaussian solution. Thanks to the properties of the CARA utility function, it is possible to rewrite Equation (6) as

$$U(\boldsymbol{\theta}, \boldsymbol{Y}) = -\exp\left(-\alpha\left(1.03 + \sum_{j=1}^{2}(0.01 + Y_j)\theta_j\right) + \frac{\alpha^2}{2}(\theta_1^2 + \theta_2^2)\sigma_z^2\right),$$

or in matrix notation with $\boldsymbol{x} = [\theta_1, \theta_2, Y_1, Y_2]^\top$ as

$$U(\boldsymbol{x}) = -\exp\left(-1.03\alpha - \boldsymbol{m}^\top\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^\top M\boldsymbol{x}\right),$$

where

$$\boldsymbol{m} = 0.01\alpha \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} \alpha^2\sigma_z^2 & 0 & -\alpha & 0 \\ 0 & \alpha^2\sigma_z^2 & 0 & -\alpha \\ -\alpha & 0 & 0 & 0 \\ 0 & -\alpha & 0 & 0 \end{bmatrix}.$$

If we restrict $\boldsymbol{x}$ to a multivariate Gaussian distribution with mean $\hat{\boldsymbol{x}} = [\hat{\theta}_1, \hat{\theta}_2, 0, 0]^\mathsf{T}$ and covariance matrix

$$\boldsymbol{\Sigma} = \begin{bmatrix} \boldsymbol{\Sigma}_{\boldsymbol{\theta}} & \boldsymbol{\Sigma}_{\boldsymbol{\theta Y}} \\ \boldsymbol{\Sigma}_{\boldsymbol{\theta Y}}^\mathsf{T} & \boldsymbol{\Sigma}_{\boldsymbol{Y}} \end{bmatrix},$$

expected consumption utility admits a closed form expression. By rearranging terms, the integral can be written as

$$\begin{aligned} \mathsf{E}\left[U(\boldsymbol{x}) \mid \boldsymbol{x} \sim \mathcal{N}(\hat{\boldsymbol{x}}, \Sigma)\right] &= -\int_{\boldsymbol{x}} e^{-1.03\alpha - \boldsymbol{m}^\mathsf{T}\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^\mathsf{T}M\boldsymbol{x}} \frac{1}{(2\pi)^2|\Sigma|^{1/2}} e^{-\frac{1}{2}(\boldsymbol{x}-\hat{\boldsymbol{x}})^\mathsf{T}\Sigma^{-1}(\boldsymbol{x}-\hat{\boldsymbol{x}})} d\boldsymbol{x} \\ &= -\frac{|Q|^{1/2}}{|\Sigma|^{1/2}} e^{-1.03\alpha - \frac{1}{2}\hat{\boldsymbol{x}}^\mathsf{T}\Sigma^{-1}\hat{\boldsymbol{x}} + \frac{1}{2}(\Sigma^{-1}\hat{\boldsymbol{x}}-\boldsymbol{m})^\mathsf{T}Q(\Sigma^{-1}\hat{\boldsymbol{x}}-\boldsymbol{m})} \int_{\boldsymbol{x}} \phi(\boldsymbol{x}) d\boldsymbol{x}, \end{aligned}$$

where $\boldsymbol{\phi}(\boldsymbol{x})$ denotes the probability density function of a multivariate normal distribution with mean $Q(\Sigma^{-1}\hat{\boldsymbol{x}} - \boldsymbol{m})$ and covariance matrix $Q = (\Sigma^{-1} - M)^{-1}$. Since the probability density function integrates to one, we obtain

$$\mathsf{E}\left[U(\boldsymbol{x}) \mid \boldsymbol{x} \sim \mathcal{N}(\hat{\boldsymbol{x}}, \Sigma)\right] = -|I - M\Sigma|^{-1/2} e^{-1.03\alpha - \frac{1}{2}\hat{\boldsymbol{x}}^\mathsf{T}\Sigma^{-1}\hat{\boldsymbol{x}} + \frac{1}{2}(\Sigma^{-1}\hat{\boldsymbol{x}}-\boldsymbol{m})^\mathsf{T}Q(\Sigma^{-1}\hat{\boldsymbol{x}}-\boldsymbol{m})}.$$

The mutual information of a multivariate Gaussian distribution also admits a closed-form solution, $\mathsf{MI}(\boldsymbol{\theta}, \boldsymbol{Y}) = \frac{1}{2}\log\left(\frac{|\boldsymbol{\Sigma}_{\boldsymbol{\theta}}||\boldsymbol{\Sigma}_{\boldsymbol{Y}}|}{|\boldsymbol{\Sigma}|}\right)$, where $|\cdot|$ denotes the matrix determinant and $\boldsymbol{\Sigma}_{\boldsymbol{X}}$ denotes the marginal covariance of the $\boldsymbol{X}$.

Exploiting the symmetry of the problem, we use a nonlinear solver with five de-

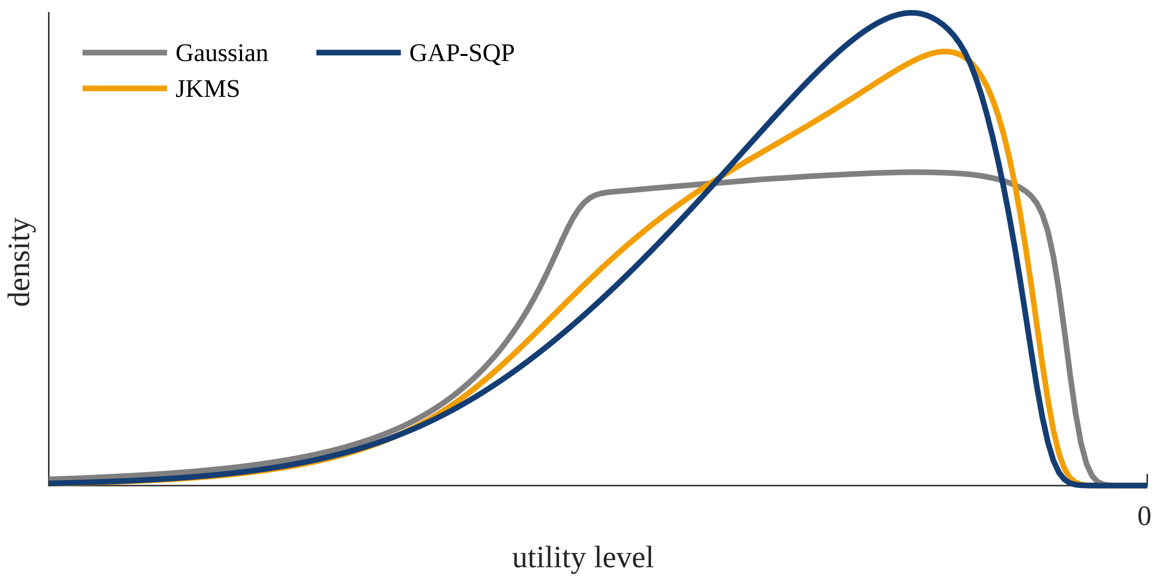

Figure 11: Payoff distribution across algorithm estimates, smoothed with a kernel density estimate with bandwidth 0.01.

grees of freedom (one for the mean $\hat{\boldsymbol{x}} = (\hat{\theta}, \hat{\theta}, 0, 0)$, and two each two for the bisymmetric covariance matrices $\Sigma_{\boldsymbol{\theta}}$ and $\Sigma_{\boldsymbol{\theta Y}}$), we obtain the best Gaussian solution, with

$$\hat{\boldsymbol{x}} \approx \begin{bmatrix} 19.0375 \\ 19.0375 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \Sigma \approx \begin{bmatrix} 25.11^2 & 0 & 0.3313 & 0 \\ 0 & 25.11^2 & 0 & 0.3313 \\ 0.3313 & 0 & 0.02^2 & 0 \\ 0 & 0.3313 & 0 & 0.02^2 \end{bmatrix}.$$

Additional Figures. For comparison purposes, Figure 11 plots the statewise payoff distribution $U(\boldsymbol{\theta}, \boldsymbol{Y}) - \lambda \mathsf{MI}$, assuming $(\boldsymbol{\theta}, \boldsymbol{Y})$ is distributed according to the numeric solution of GAP-SQP (blue), JKMS (orange), or the optimal Gaussian solution (gray), and the information cost MI is borne unconditionally. We explicitly compute joint probabilities for the two discrete solutions. For the optimal Gaussian, we use Monte-Carlo methods and sample 100 million draws from the distribution. We then plot weighted kernel density estimates with a bandwidth of 0.01.