# Economic consequences of vertical mismatch *

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**Abstract**

We study two first-order economic consequences of vertical mismatch, using a simple (neoclassical) model of under- and over-employment. Individuals of high type can perform both skilled and unskilled jobs, but only a fraction of low-type workers can perform skilled jobs. People have different costs over these jobs. First, we calibrate the model to match US CPS time-series since the 1980s. To control for unobserved heterogeneity, we compute wages based on workers who have switched between skilled and unskilled jobs. We show that changes in educational mismatch has contributed one-sixth as much as skilled-bias technological progress for the rise in the college premium. Second, we calibrate the model to match moments of 50 US states, to measure the output costs of frictions generating mismatch. The cost of frictions is 0.26% of output on average but varies between 0.06% to 0.77% across states. The key variable that explains the output cost of vertical mismatch is not the percentage of mismatched workers but their wage relative to well-matched workers.

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# 1 Introduction

Browse any undergraduate labour economics textbook and you will not find a model of mismatch. This could leave the impression that mismatch is irrelevant or not studied in economics, but both accounts are wrong. There are various dimensions of mismatch, but the main focus in labor statistics and policy circles (OECD, 2018) is on education and skill mismatch, which refers to workers employed in occupations that typically require different education or skills. We call it, more generally, vertical mismatch. Usually, workers are classified as over- or under-employed, depending on whether their education or skill is below or above the occupation average requirement. The OECD reports that, on average across advanced economies, 25% (18%) of workers are under-employed and 22% (7%) are over-employed vis-à-vis their education (skill).[1] The literature on mismatch, reviewed in Section 2, is extensive but, by and large, the models used are complex and less accessible to a general audience. We propose a simple neoclassical model of vertical mismatch to provide model-based measures of economic consequences of mismatch in areas that have attracted much attention in distinct economic literatures: the Tinbergen's race between education and skill-bias technological progress, and the economic costs of mismatch.

The theory, laid out in Section 3, is based on four features. First, individuals of high and low type are endowed with one unit of indivisible labor. Second, a representative firm has jobs which differ on their skill requirements. Individuals of high type are able to perform skilled jobs but also unskilled jobs, with perhaps higher efficiency than workers of low type. Conversely, only a fraction of low-type workers can perform the skilled jobs, albeit with lower efficiency than high-type workers. Third, individuals face an idiosyncratic cost to access these jobs, and optimally sort across them. This cost might prevent some workers from accepting the highest paying job. Fourth, each worker is paid its marginal productivity. This simple variation of Roy (1951) and Borjas (1987) is sufficient to generate a labor market allocation with endogenous under- and over-employment, both responding

[1] See Quintini (2011) and McGowan and Andrews (2015). To clarify the semantics, we refer to "mismatch" as the discrepancy between workers' competencies and those required by the job. "Under-employment" is used here as the under-use of workers' competencies. "Over-employment" refers to the converse. In the application, we focus on a particular dimensions of workers' competencies: education. In OECD reports, this phenomenon is also referred to as over(under)-education. In a companion paper, Garibaldi et al. (2020), we focus on the distinction between education and skill mismatch in OECD economies.

to wage differentials. The agnostic and reduced-form way of introducing the frictions that generate mismatch hampers us from speaking on the sources of mismatch, its welfare costs or normative implications. However, the model is suitable to analyse positive implications of mismatch and several measurement issues. As it is typical with neoclassical models, its main advantages are the simplicity, the transparency of the mechanisms involved, and the clear mapping with the data. The model has six parameters, and each of the six structural parameters is obtained to match exactly the six key moments, including over- and under-employment, and relative wages of well-matched and mismatched workers.

Our first application, examined in Section 4, is to the macroeconomic literature on skill-bias technological progress, that relies on a production function where skilled and unskilled labour are key inputs. When taking it to the data, these are usually matched to workers with and without a college degree. In light of the canonical model, discussed in the Handbook of Labour Economic chapter by Acemoglu and Autor (2011), to rationalize the increase in college premium observed in the US, together with the increase in education attainment, one requires a large improvement in skill-biased technology, in what is referred as the Tinbergen's race. Through the lens of our model, we argue that the fraction of workers with and without college are contaminated measures of relative supply of skilled labour, because they abstract from education mismatch. Using Census data for the US, we find, that between 1970 and 2017, the skilled-unskilled ratio adjusted for mismatch has only increased by 22%, rather than the twofold increase when not adjusted. There are two reasons. First, one third of all college graduates are working in predominantly non-college occupations, a ratio relatively stable over time. As these workers are more productive than workers without college, even in unskilled occupations, they contribute to a higher supply of labour at the low end. Second, about 12% of non-college workers work in predominantly college occupations. This means that, in 1970, many low-educated workers were working in high-skilled occupations.

We can better understand the theory with concrete examples. Take, for instance, *Waiters and Waitresses*, a large occupation that represented 1 million workers in 1970 and doubled by 2017. In 1970, 1% of workers had at least a four-year college degree. In 2017, close to 12% had one. We call them under-employed. When looking at their wages, college workers do have a premium: waiters and waitresses with college earn about 30% more than their non-college counterpart. Similar stories can be told for hairdressers (share of college increased from 0 to 7%), bartenders (share of college increased from 4 to 17%) or door-to-door sales workers, news and street vendors (share of college increased from 2 to 24%), taxi drivers and chauffeurs or telephone operators (share of college increased from 3 to 18%). Also in public-sector occupations like firefighters, the share of college increased from 1 to 22%. In all these occupations, in 2017, there exists a wage premium of having college ranging from 15 to 40%.

Consider now occupations that are majority performed by college graduates, such as *Aircraft Pilots and Flight Engineers*. You might be apprehensive to find that, in 2017, still 23% of them do not have a completed college degree, from 71% in 1970. We call them over-employed. In 1970, these workers earned 10% less than their more educated peers. Now, the wage penalty of not having college is about 40%. Another example is managers, that represent close to 4 million workers in 2017. Still 40% of them do not have a college degree. In 1970, of the 3 million managers, 75% did not. Other similar occupations include managers in marketing, advertising and public relations (share of non-college decreased from 71 to 28), designers (share of non-college decreased from 78 to 41), accountants and auditors (share of non-college decreased from 60 to 0). This also happened in public-sector occupations, like nurses (share of non-college decreased from 83 to 35%). In all these occupations, the current penalty of not having college varies between 30 to 40%, slightly larger than in 1970.

These differences between the average wage of well-matched and mismatched workers are likely to reflect observed and unobserved heterogeneity that we do not consider in our model. Workers that are mismatched are likely to be different from an average worker, with the same level of education. We take this in account when mapping the model to the data. In our calibration, based on CPS data, we estimate wage differences of workers who have switched between skilled and unskilled occupations to identify two key parameters of the model. By examining the wage differences within the context of occupation switches, we emphasize the importance of taking unobserved heterogeneity into account when assessing the wage premium of well-matched workers. This is one of the contributions of our paper, as this has not been done in the existing literature on mismatch, to the best of our knowledge.

When calibrating the model to the US time series, we first show that the measurement of both the relative supply of skills and the college premium affects the estimation of the elasticity of substitution between skilled and unskilled jobs. On the one hand, the relative supply of skills should incorporate the level of mismatched workers and their relative productivity. On the other hand, we can decompose the average college premium into different components related to mismatch. The college premium that is consistent with the model should be measured only using well-matched college and non-college workers. When using these measures, the skilled-unskilled elasticity of substitution is estimated to be 1.7, instead on 2.8 found when estimating using the typical regression of the canonical model. The model with mismatch has built-in substitutability from the perspective of the workers, so from the firm perspective the jobs themselves are less substitutes. We then use our model to decompose the contribution of skill-bias technological progress, increase of education and changes in the mismatch parameters, to the evolution of the observed college premium. Our second result is that changes in the structure of mismatch contribute one-sixth as much to the evolution of the college premium as skill-bias technological progress.

In our second application, examined in Section 5, we use data across US states to quantify the output costs of mismatch. We find that eliminating the frictions that generate education mismatch would raise output by 0.26% on average, with cross-country differences ranging from 0.06 to 0.77% of output. Our main finding is that the key variable that explains the output cost of mismatch is not the percentage of mismatched workers, but their wage relative to well-matched workers. In particular, policy makers should pay attention to the wage loss of an under-employed college graduate with respect to her well-matched counterpart. Economies with sizeable "wage costs of under-employment" suffer from a scarcity of college workers in skilled jobs, thereby they would increase output if they were able to successfully reduce mismatch. Based on regressions with actual and simulated data, a 10 percentage point higher "wage costs of under-employment" is associated with a 0.6 percentage point higher output cost of mismatch. This elasticity is robust to alternative calibrations of the wage cost of under-employment.

Eliminating the frictions behind mismatch would reduce wage inequality, raising unskilled wages by 3.7% and reduce skilled wages by 4.4% across US States.

## 2 Related literature

Mismatch arises in situations where two sides need to match along a particular dimension, but do so incorrectly. Because of the generality of the problem, the concept of mismatch is widespread in economics, which sometimes gives rise to confusions among authors that use the same word to classify different problems. Our paper focus of mismatch between workers and jobs along a dimension of quality, more generally called vertical mismatch because of a natural ranking (over-employment and under-employment). In our application, the dimension of quality is education (education mismatch), but the theory could also be applied to skills, if defined with a natural order, i.e. high and low skills (skill mismatch).

Vertical mismatch is different from horizontal mismatch, in which the dimension of matching does not have a natural order. This is the case, for instance of occupational mismatch (Horváth, 2014) or mismatch of college majors (Robst, 2007). If interpreting skills without a natural order, i.e. numerical skills and social skills, skill mismatch is an example of horizontal mismatch (Guvenen et al., 2020). Furthermore, within the search literature, the word mismatch is often used to denote mismatch unemployment, that arises from the mismatch between unemployed and vacancies, as in Shimer (2007) or Sahin et al. (2014).[2] To add to the confusion, "education mismatch" has been used by some authors to denote the mismatch between the skill of the worker and their education level, in a setting where education is endogenous (Cooper and Liu, 2019, Cervantes and Cooper, 2022), instead of the mismatch between the education and the job (that they refer to as job mismatch).

The recent theoretical literature on education mismatch mainly uses search models. A first group of papers on educational mismatch focus only on under-employment, explaining it within a model of on-the job search model with career mobility, for instance Sicherman and Galor (1998) and Dolado et al. (2009). Chassamboulli (2011) and Barnichon and Zylberberg (2019) study the cyclical dimension of under-employment that arises from workers having higher job-finding rates in lower-ranked jobs. Arseneau and Epstein (2014) measures the

[2] Other relevant papers that study the positive implications of unemployment mismatch are Barnichon and Figura (2017) and Herz and van Rens (2020). Barnichon and Figura (2017) propose a framework in which labor market segmentation and heterogeneity across workers and jobs affect the aggregate matching efficiency and estimates that over the 2008-2012, matching efficiency fell and caused a decline of job-finding rate by 30%. Herz and van Rens (2020) estimate that mismatch unemployment account for 13% of the cyclical variation in unemployment in the US.

welfare cost of eliminating under-employment in a search and matching model. A more theoretical oriented literature focuses on search with multiple applicants (Shimer, 1999, Julien et al., 2000) or bilateral heterogeneity and sorting (Eeckhout and Kircher, 2010) that builds on an earlier literature that emphasised the role of assignment and job characteristics reviewed in Sattinger (1993). Blázquez and Jansen (2008) and Gautier (2002) study a setting with two types of jobs and two types of workers (similar to ours) in a labour market with random search and ex-post bargaining, and discuss its efficiency properties. Another branch of the search literature has focused on issues of adverse selection that can arise if the type of worker is unobservable, and generate some type of mismatch i.e. Guerrieri et al. (2010) or Fernandez-Blanco and Gomes (2017).[3]

Our model abstracts from most of the elements studied within the search and matching framework, namely unemployment, job-finding rates or career promotions, nor it microfounds the frictions behind the mismatch. To compensate, our model offers two advantages relative to this literature. On the one hand, it considers general equilibrium effects of over- and under-employment on marginal productivity and wages. When low-type workers take up skilled jobs, they drive skilled wages down and unskilled wages up, making under-employment more attractive to some high-type workers. Our model suggests that over- and under-employment reinforce each other and shows the importance of accounting for both phenomena simultaneously. This stands in sharp contrast with the search and matching literature that, in general, assumes an exogenous and constant productivity of workers in the different jobs. On the other hand, the simplicity of our Neoclassical model and the transparency of the mechanism contrasts with the complexity of some of the models with explicit microfoundations that sometimes obscures the underlying economic mechanisms. We believe these advantages, together with a direct mapping to the data, make our model a useful addition to an economist's toolkit.

Our approach has similarities with the literature on the causes and costs of misallocation

[3]Although less related to our paper, there was a early literature from the 70s on education choice that was mainly concerned with over-education, defined as acquiring too much education in the context of the return to schooling (Freeman (1975); McGuinness (2006)). More recently, this has been studied in the context of search frictions, for instance by Charlot and Decreuse (2010), or in a macro model with borrowing constraints by Cooper and Liu (2019). In our model, the worker type is exogenous so we are unable to analyse these issues.

(Restuccia and Rogerson, 2017) that grew from the seminal paper by Hsieh et al. (2019) that study the misallocation of talent and occupation choice in a the U.S. While they do not focus on vertical mismatch, Hsieh et al. (2019) propose a simple Roy model of occupation choice that study the general equilibrium effect of misallocation of capital. Further, the paper uses two exogenous parameters to measure frictions to human capital accumulation and discrimination obstacles that are similar in spirit to the parameters on the cost of mismatch highlighted in our model. Third, they estimate the cost of human capital misallocation in the U.S. with a method that is similar to our estimates on the cost of mismatch.

# 3 Model Set up

## Technology and Preferences

Individuals are endowed with 1 unit of indivisible labor and firms require different jobs to produce output. There are two types of individuals. For exposition, we will use the words "high-type" ($h$) and "low-type" ($l$). We make the empirical distinction based on education when taking the model to the data.

The supply of high-type individuals in the economy is indicated with $n$, while the supply of workers of low type is $1 - n$. There are two types of jobs in the economy, that we shall call the "skilled" and the "unskilled" jobs. In what follows, we describe a general production function, that we will assume in the applications to be a CES:

$$Y = F(j_h, j_l) \tag{1}$$

where $j_h(j_l)$ is the number of skilled (unskilled) jobs in efficiency units. Firms produce $Y$ with a constant return technology in different jobs.

A key assumption in our theory concerns the ability of different individuals to perform different jobs. First, high-type workers can perform skilled jobs with one efficiency unit, but they are also able to perform unskilled jobs with efficiency units $\zeta \geq 1$. This assumption is consistent with several findings by previous papers, such as Duncan and Hoffman (1981) on the wage effect of over-education. Barnichon and Zylberberg (2019) also provide evidence that high-educated workers are better paid than low-educated workers hired in the same occupation. As in Barnichon and Zylberberg (2019), we estimate $\zeta$ based on wage data and confirm that $\zeta \geq 1$ holds. We take $\zeta$ as exogenous and changes could be driven, among other things, by the quality of the education system in improving general human capital.

Secondly, low-type workers can perform unskilled jobs with one efficiency unit, but only a fraction $\Theta$ of them can also perform the skilled jobs with lower efficiency units $\chi \leq 1$.[4] The natural asymmetry in the problem between workers with high and low type is captured by $\zeta \geq 1 \geq \chi$ and $\Theta \leq 1$. The lower productivity of low-type workers on skilled jobs is consistent with findings by Heckman et al. (2011) who stress that individuals endowed with the highest (lowest) cognitive abilities and soft skills choose higher (lower) education and sort into skilled (unskilled) jobs. It is then natural to infer that low-type workers in skilled jobs are less able than high-type workers on the same job. The data based on wages of over-employed workers shown later confirms that $\chi$ is less than 1. We expect $\chi$ to fall over time as technology has made jobs more complex and requiring more years of education to be done at the highest standard.

Individual preferences are linear, and the model is static. The wage paid per efficiency unit of the skilled job is indicated with $w_h$ while for the unskilled job is indicated with $w_l$. Workers have heterogeneous cost to perform/access these jobs, and each individual draws a relative cost for both jobs $\epsilon_i^h$ and $\epsilon_i^l$ from distributions with cumulative density $G(.)$, assumed to be a standard Gumbel distribution as in Garibaldi et al. (2021). The cost is a shortcut that captures all possible reasons, other than wages, that push people to accept a job with lower wages, which might also include non-pecuniary costs, preferences, personal circumstances, labour market conditions, housing market, transport policies, or regulation of specific occupations. As such, the model is not equipped to make any normative statement or to think about optimal policies. Workers utility is thus the wage net of costs, so for an individual $i$ earning a wage $w$, its utility is $U_i = w - \nu\epsilon_i$, where $\nu$ summarizes the weight of

[4]This assumption makes the model consistent with the empirical fact that, among non-college graduates there are more well-matched than over-employed workers, while over-employed workers earn more than their well-matched counterparts. Without this assumption, as over-employed workers earn more than their well-matched counterparts, the majority of low-type workers would want to sort into high-type occupations, which is not consistent with the data. $\Theta$ can be interpreted as regulatory elements in some occupations. For instance, requirements on experience or degree in some high-skilled jobs.

these barriers in the individual preferences.

## Sorting by High-Type Workers and Under-employment

The key decision of a high-type worker $i$ concerns the sector in which to supply her indivisible unit of labor. The problem reads

$$U_i^h = Max\{w_h - \nu\epsilon_i^h, \zeta w_l - \nu\epsilon_i^l\} \tag{2}$$

An high-type worker prefers an unskilled job only if $w_h - \nu\epsilon_i^h < \zeta w_l - \nu\epsilon_i^l$, or if $\epsilon_i^h - \epsilon_i^l > \frac{\zeta w_l - w_h}{\nu}$. In contrast, she takes a skilled job if its cost over the unskilled job is not low enough to compensate the wage differential. In other words, the wage differential is a key determinant of under-employment. This simple sorting condition implies that there is an endogenously determined level of under-employment defined as:

$$u = n\left[\frac{e^{\frac{\zeta w_l}{\nu}}}{e^{\frac{\zeta w_l}{\nu}} + e^{\frac{w_h}{\nu}}}\right] \tag{3}$$

This expression follows from the fact that the difference between independent standard Gumbel distributions (extreme type I error) has a logistic distribution. Notice that, as $\nu$ tends to zero, in partial equilibrium, all individuals will prefer the option which offers the highest wage, meaning that either $u = 0$ or $u = 1$ depending on whether $w_h \gtreqless \zeta w_l$.

## Sorting by Low-Type Workers and Over-employment

We assume an asymmetry between the problem of workers of high and low type: only a fraction $\Theta$ of low-type workers receive an opportunity in a skilled job. For a worker $i$ that has an opportunity, the sorting decision reads:

$$U_i^l = Max\{\chi w_h - \nu\epsilon_i^h, w_l - \nu\epsilon_i^l\} \tag{4}$$

The low-type worker takes on the skilled job only if $\chi w_h - \nu\epsilon_i^h > w_l - \nu\epsilon_i^l$, or if $\epsilon_i^h - \epsilon_i^l < \frac{w_l - \chi w_h}{\nu}$. This sorting condition implies that there is an endogenously determined value of over-employment defined as

$$o = (1-n)\Theta\left[\frac{e^{\frac{\chi w_h}{\nu}}}{e^{\frac{\chi w_h}{\nu}} + e^{\frac{w_l}{\nu}}}\right] \tag{5}$$

## Labor Demand and Market Clearing

A representative firm maximises profits taking as given the wage for both jobs. Labor demand is given by two conditions equating wages to marginal productivities:

$$w_h = \frac{\partial F(j_h, j_l)}{\partial j_h} \quad , \qquad w_l = \frac{\partial F(j_h, j_l)}{\partial j_l}. \tag{6}$$

Wages adjust until the demand for jobs is equal to the supply of efficiency units, such that all workers get paid their marginal productivity. Given the different efficiency units, there are four different wages in the economy. $w_h$ and $w_l$ are the wages paid to "well-matched" high- and low- type workers. The education premium of well-matched workers is given by the ratio of the two ($\frac{w_h}{w_l}$). Over-employed workers get $\chi w_h$ and under-employed workers get $\zeta w_l$. The education premium of under-employed workers is given by $\zeta$. This is a crucial variable in the papers by Arseneau and Epstein (2014) and Barnichon and Zylberberg (2019). Market clearing equilibrium implies

$$j_h = (n-u) + \chi o \quad , \qquad j_l = (1-n-o) + \zeta u. \tag{7}$$

## Equilibrium

**Definition 1** *A steady-state equilibrium consists of wages $\{w_h, w_l\}$, skilled and unskilled jobs $\{j_h, j_l\}$, under-employment $\{u\}$, over-employment $\{o\}$, such that*

1. *The representative firm maximizes profits (6).*

2. *High-type workers sort across labour markets according to (2).*

3. *Low-type workers with an opportunity to chose between markets, sort according to (4).*

4. *Markets clear (7).*

We can summarize the model in four equations: under- and over-employment given by equations (3) and (5) respectively and two wage equations (equations (6)) in which the number of skilled $j_h$ and unskilled jobs $j_l$ are replaced by their expressions (equations (7)).

The fact that we do not specify the barriers that generate mismatch does not mean that the model is silent about the drivers of mismatch. The simple setting allows us to understand the role of wage differentials in determining under-employment and over-employment, and the reverse causality from mismatch to wages. An increase wage differentials between skilled and unskilled jobs lowers under-employment and increase over-employment. The model features complementarity between over- and under-employment. Suppose that, in partial equilibrium, over-employment increases, more low-type workers are in complex jobs, the supply of unskilled labor goes down, such that skilled wages fall and unskilled wages increase, thereby reducing the wage differential and increasing under-employment. Conversely, if under-employment increases, it pushes unskilled wages down and skilled wages up, increasing wage differential and, hence raising over-employment.[5]

# 4 Mismatch and Tinbergen's Race

Our first application of the model relates to the macroeconomic literature on skill-bias technological progress, discussed in Acemoglu (2002) and Caselli (2016). The concept of vertical mismatch is completely absent in this literature. By introducing it, our model takes up some features from more recent Ricardian models of labour market. As explained in Acemoglu and Autor (2011), recently, the literature has been moving from the canonical model towards models based on two-sided heterogeneity (building on the work of Autor et al. (2003)). In these models, jobs require different tasks and workers are endowed with different skills, and the output is based on the pairing between tasks and skills. Our theory has similarities with this approach, starting by a clear distinction between workers' quality and job types, but with a key subtle difference. These models assume that workers are always employed in their highest-paying job, where their marginal productivity is the highest.

[5]While one might suspect the possibility of multiple equilibria, we show in Appendix A that, for a Cobb-Douglas function, given the asymmetries of the model (as long as $\zeta \geq 1 \geq \chi$ does not hold with two equalities), the equilibrium exists and is unique. When using a CES function, we verify this statement numerically.

Instead, we assume that workers face some type of friction or barrier that prevents them to choose the highest-paying job, that generate mismatch. The extent of mismatch is governed by four parameters that have a clear interpretation and can be backed out from basic labour statistics. The sizable share of college workers in unskilled occupations and non-college workers in skilled occupations, that we document for the United States, is naturally interpretable within our model, but can also be rationalized in the Ricardian model.

## 4.1 Setting

We assume a CES production function as in Acemoglu and Autor (2011):

$$Y = \left[ (A_h j_h)^{\frac{\sigma-1}{\sigma}} + (A_l j_l)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{8}$$

where $\sigma$ is the elasticity of substitution between skilled and unskilled jobs, and $A_h$ and $A_l$ are factor-augmenting technologies. The wage per efficiency unit of each job is:

$$w_h = A_h^{\frac{\sigma-1}{\sigma}} \left[ A_h^{\frac{\sigma-1}{\sigma}} + A_l^{\frac{\sigma-1}{\sigma}} \left( \frac{j_l}{j_h} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} \quad , \quad w_l = A_l^{\frac{\sigma-1}{\sigma}} \left[ A_l^{\frac{\sigma-1}{\sigma}} + A_h^{\frac{\sigma-1}{\sigma}} \left( \frac{j_h}{j_l} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} . \tag{9}$$

The ratio of the two measures is the college premium per efficiency units and it is given by:

$$\frac{w_h}{w_l} = \alpha \left( \frac{j_h}{j_l} \right)^{-\frac{1}{\sigma}} \tag{10}$$

where $\alpha = \left( \frac{A_h}{A_l} \right)^{\frac{\sigma-1}{\sigma}}$. Relative to the canonical model, our model of mismatch implies two changes when bringing it to the data. First, the relative supply of skills is no longer only based on the fraction of workers with college ($\frac{j_h}{j_l} = \frac{n}{1-n}$), but it accounts for both the size of mismatch and the relative efficiency ($\frac{j_h}{j_l} = \frac{(n-u)+\chi o}{(1-n-o)+\zeta u}$). Second, the college premium should be measured only for well-matched workers $\frac{w_h}{w_l}$. Notice that the overall college premium (CP) used in the existing literature is computed as follows in our model: this is a ratio of weighted averages involving the four wages (college/non-college, well-matched/mismatched) $CP = \frac{\frac{(n-u)w_h + u\zeta w_l}{n}}{\frac{(1-n-o)w_l + o\chi w_h}{1-n}}$.

## 4.2 Time-Series Calibration

We perform a time-series exercise for the U.S.. We want our measure of mismatch to be consistent with the dichotomic nature of the theoretical model with two groups of workers: high-type and low-type. There is an important debate on the literature about the best methodology to identify mismatched workers (Flisi et al., 2017). Baert et al. (2013) for instance use subjective approaches based on self-reported mismatch. Workers are asked about their feeling about the job match. In contrast, Bauer (2002), among others, rely on objective measures, such as the actual level of education attained by peers working in the same occupation. We follow this last approach and use the workers' education levels within each occupation to infer the education level required for a job. Mismatch occurs when the individual's education level deviates from the majority of types (education) within each occupation.

When taking the model to the data, we use two datasets. First, Census data to categorize occupations, and compute labor stocks as well as average wages. Indeed, as stressed by Acemoglu and Autor (2011), among others, Census provides a substantially large sample, which makes Census better suited for the analysis of occupational employment. Second, to identify $\chi$ and $\zeta$, we use CPS monthly data. We estimate the wage of the *same* worker when well-matched and mismatched. In doing so, we purge the data from observed and unobserved worker heterogeneity.

**Census Data for calculation of labor stocks and occupational analysis.** We use Census IPUMS data (1970, 1980, 1990, and annually between 2000 and 2017). These data sources provide substantially larger samples than either the March or May/ORG surveys. They are better suited for the analysis of educational attainment in occupations.[6] We consider employed workers, working for pay, aged 16 to 80. We define college-educated workers as those who completed at least four years of college education, with $n$ defining their share among workers. We then compute the share of non-college workers within each 4-digit occupation (2010 harmonized occupation coding scheme based on the Census Bureau's 2010

[6]The Census samples comprise 1% of the US population in 1970, and 5% of the population in post-1980 surveys. See Appendix B for further details on the data.

ACS occupation classification scheme, with 493 categories). A college-educated (non-college) worker is classified as under-employed (over-employed) when working in an occupation that is majority non-college (college). A college-educated (non-college) workers working in an occupation that is majority college (non-college) is considered as well-matched.[7]

To verify the quality of the split of occupations across the two groups, we cross-reference it with data from the BLS Occupational Outlook Handbook (OOH) that provides information on entry-level education requirements for 792 occupational profiles. The Handbook does not report any occupational classification code so we use a text matching algorithm to match occupational titles from Census 2017 with BLS occupational names. We successfully match 97% of 2017 Census occupations.[8] Among the occupations categorized as "majority-college" using our measure, 94% of them indeed require "Bachelor's degree", "Master's degree" or "Doctoral or professional degree" according to the BLS.[9] Symmetrically, among the occupations categorized as "majority-non college" using our measure, 96% of them indeed require "High school diploma or equivalent", "No formal educational credential" or "Postsecondary non-degree award."[10]

We calculate weekly earnings from the Census data, as annual wage and salary income divided by the number of weeks worked. We then compute the college premium of well-matched workers.

**CPS data on occupational switchers.** Our baseline calibration uses wage differentials of workers who switch between skilled and unskilled occupations. For that, we use IPUMS monthly CPS data. Since we look at weekly earnings, the data only starts in 1982m1. The time period stops in 2017m12 for the sake of consistency with Census data. As we need to compare wages of different years, we compute real earnings using the Consumer Price Index

[7]We determine the skilled and unskilled occupations based on 2017 data and fix throughout the sample. In 92% of the occupations the definition would be unchanged if we set the baseline in 1970, with only 8% of the occupations have been upgraded in the past 50 years. See Appendix B for further details.

[8]Census includes occupations related to gaming (Gaming Services Workers, Gaming Managers, etc ...), which are not yet documented in BLS OOH.

[9]The discrepancy between the two measures comes from occupations such as "Private Detectives and Investigators" that require "High school diploma or equivalent" according to the BLS, but is actually "majority college" as 56% of Private Detectives and Investigators held a Bachelor's degree or more in 2017. See Appendix C for further details.

[10]The discrepancy comes from occupations such as "Statistical assistants" or "Construction managers" that require "Bachelor's degree" according to the BLS, but is actually "majority non-college" as only 35% of workers in these two categories held a Bachelor's degree or more, in 2017.

for All Urban Consumers: All Items in U.S. City Average, Index 1982-1984=100, Seasonally Adjusted.

We focus on individuals who are employed in the 4th and 8th interviews. We compute yearly transitions by linking the 4th to the 8th interview of CPS respondents. College CPS respondents are identified as switchers when

i. they are well-matched in the 4th interview and under-employed in the 8th interview

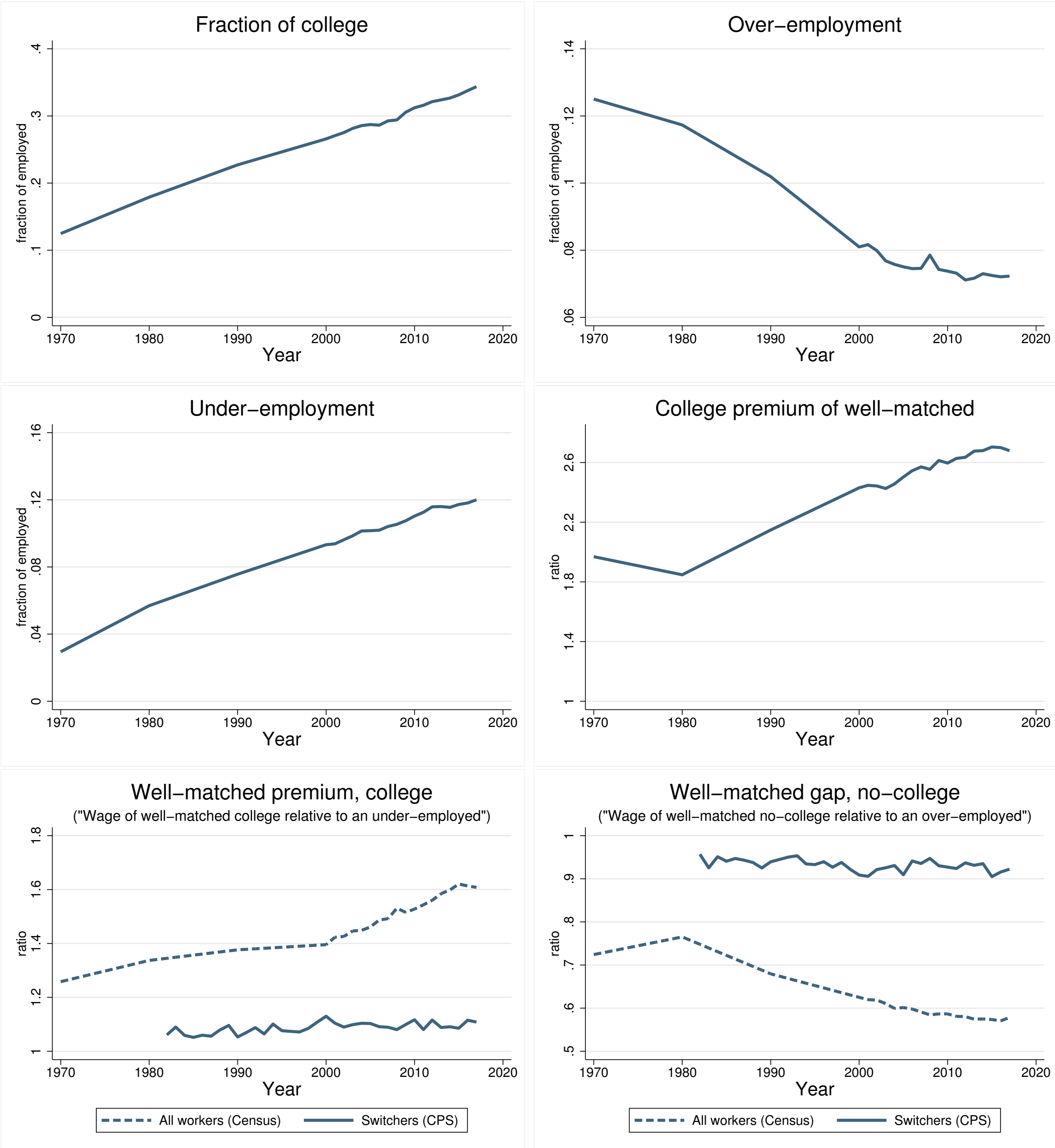
ii. they are under-employed in the 8th interview and well-matched in the 8th interview

For each type of transition, earnings of each well-matched worker can be compared to earnings when the very same worker is under-employed. The well-matched wage premium can be computed for each college CPS switcher. The data suggests that, on average, workers who start as mismatched and become well-matched one year later are as numerous as workers doing the reverse transition. The well-matched wage premium of college workers is then the mean of well-matched wage premium of workers with transition i. and their counterparts with transition ii. We repeat the procedure on non-college CPS respondents (See Appendix D for further details).

**Six empirical targets.** Figure 1 shows the six statistics needed to calibrate our model. We use Census to compute $n, u, o, \frac{w_h}{w_l}$ and CPS data on switchers to identify $\frac{w_h}{\zeta w_l}, \frac{w_l}{\chi w_h}$. The increase of the fraction of workers with college has been accompanied by a rise of under-employment $u$ and a decline of over-employment $o$. However, when we analyse them in proportion of the suitable population – the under-employed as a fraction of college workers and over-employed as a fraction of workers without college – they are relative stable. As shown in Figure 2, about 12% of workers without college are working in skilled occupations, a percentage than slightly declined from 14 to 11% over the last five decades. On the other hand, throughout the sample, one third of all college graduates are under-employed. The percentage increased sharply from 24% in 1970 to 32% in 1980, but has been relatively stable since.

The three remaining targets are the relative wages amongst the four groups of workers. We observe three trends over the past five decades. First, based on Census data, the college

Figure 1: Time-Series Calibration Targets

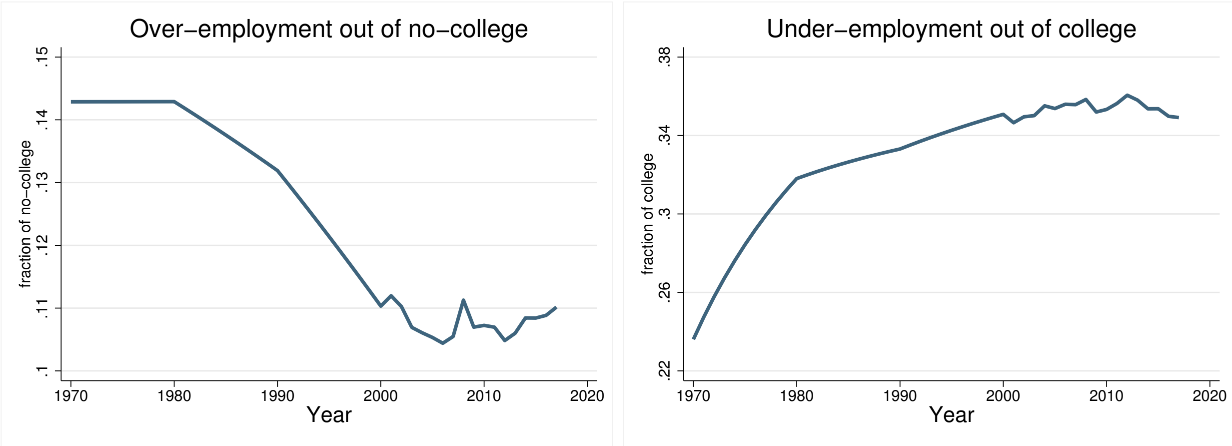



*Note: Calculated from US Census data (1970, 1980, 1990, 2000-2017), and CPS monthly data on occupational switchers (1982-2017). A worker is considered as under-employed (over-employed) if she has a college (non-college) education and working in 4-digit occupations that are majority non-college (college).*

premium of well-matched workers has increased from 80 to 170%.

Second, the cost of being under-employed has increased. When we use Census data, the average well-matched college worker earned 25% more than college workers in unskilled occupations in the 1980s. Nowadays, they earn 60% more. However, these numbers refer to

Figure 2: Over- and Under-Employment as Fraction of Relevant Population




*Note: Calculated from US Census IPUMS data (1970, 1980, 1990, 2000-2017). A worker is considered as under-employed (over-employed) if she is college-educated and working in 4-digit occupations that are majority non-college (college).*

the average workers in each category, which are likely to differ with respect to observable and unobservable characteristics. In our baseline, in order to control for heterogeneity, we focus on the measure of well-matched premium based on CPS switchers. Well-matched college workers earn between 5% to 12% more than if they were under-employed, with a slight positive trend.

Third, the penalty of not having a college degree in skilled occupation has increased by 20% using Census data. Using CPS switchers, the wage gain from over-employment is 6.8% on average, increasing over time from 4% to 10%. The increase is more noticeable when using Census data for the averages.

The difference between the numbers for the averages (from Census) and switchers (from CPS) highlights the role of composition effects. We prefer to control for heterogeneity in our baseline, so our results can be seen as conservative related to the literature that does not control for heterogeneity and usually considers larger productivity differences. We present in section 6 two alternative calibrations using Census: one based on the average wage across categories and the other controlling for observed characteristics using Mincer regression.

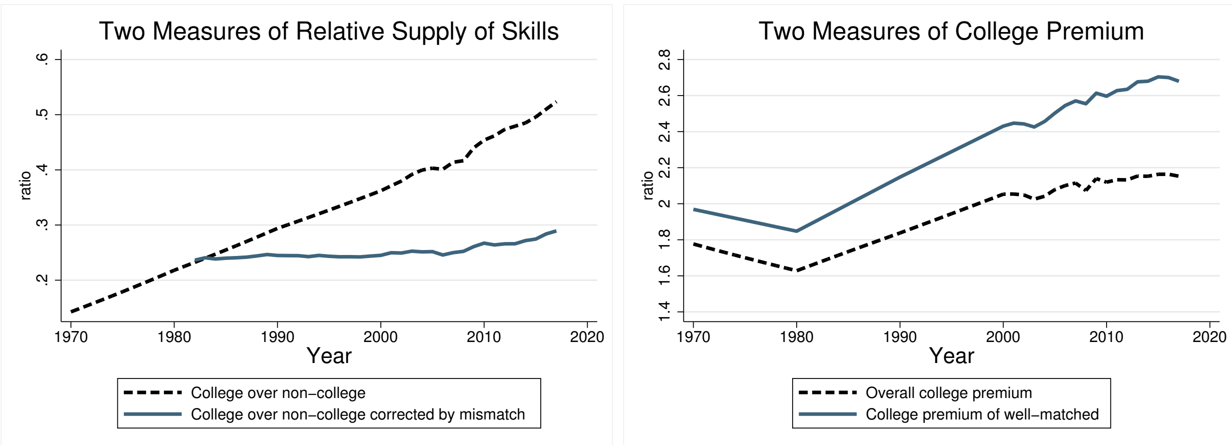
**Identification.** $n$ is given by the fraction of college workers. We then have five parameters $\{\alpha, \zeta, \chi, \Theta, \nu\}$, that are determined for the model to match five targets: $\left\{u, o, \frac{w_h}{w_l}, \frac{w_h}{\zeta w_l}, \frac{w_l}{\chi w_h}\right\}$. The targets are: the fraction of under-employment and over-employment in total employment ($u$ and $o$), the college premium of well-matched workers ($\frac{w_h}{w_l}$), the well-matched premium of college graduates ($\frac{w_h}{\zeta w_l}$) and the well-matched gap of non-college workers ($\frac{w_l}{\chi w_h}$). We calibrate the model independently for each year in the sample. The time-series of the parameters $\{\zeta, \chi\}$ can be backed out directly from the data of relative wages $\left\{\frac{w_h}{w_l}, \frac{w_h}{\zeta w_l}, \frac{w_l}{\chi w_h}\right\}$.

Additionally, we need to estimate $\sigma$, the time-invariant parameter governing the elasticity of substitution. Given that the production function in our model is different, we cannot use the estimates available in the literature. We follow Acemoglu and Autor (2011) and regress the log of the college premium on a constant, a time trend and the log of the relative supply of skills. Using (7), we can calculate $\left(\frac{j_h}{j_l}\right)$ after backing $\{\zeta, \chi\}$.

$$\ln\left(\frac{w_h}{w_l}\right) = \gamma_0 + \gamma_1 t - \frac{1}{\sigma}\ln\left(\frac{j_h}{j_l}\right) \tag{11}$$

Figure 3 compares the adjusted with the unadjusted data. The first striking aspect is that according to the adjusted series, the relative supply of skills in efficiency units has only increased by 22% from the early 80s to 2017, while the unadjusted measure more than doubled in the same period. In the beginning of the sample, the relative supply of skills

Figure 3: Alternative Measures of Relative Supply and College Premium




*Note: Calculated from US Census IPUMS data (1970, 1980, 1990, 2000-2017), and CPS monthly data on occupational switchers (1982-2017). "College over non-college": $\frac{n}{1-n}$ where n is the fraction of college workers. "College over non-college corrected by mismatch": $\frac{jh}{jl} = \frac{(n-u)+\chi o}{(1-n-o)+\zeta u}$ where o (u) denotes over-employment (under-employment). College premium is based on Census weekly earnings. A worker is considered as well-matched if she is college (non-college)-educated and working in 4-digit occupations that are majority college (non-college) "College premium of well-matched workers": ratio of weekly earnings of college versus non-college of well-matched workers. "Overall college premium": ratio of weekly earnings of college versus non-college.*

in efficiency units was higher, when we account for the number on workers without college that were working in skilled jobs. Nowadays, despite the remarkable increase of education, as one third of college graduates work in unskilled occupations, the relative supply of labour in efficiency units is lower than the unadjusted measure suggests. The second adjustment is on the college premium. Without adjusting for mismatch, the overall college premium increased by about 60 percentage points, while when we consider the college premium of well-matched workers, it increased by 80 percentage points.

Estimating the equation using the adjusted and unadjusted data gives (t-statistics in parenthesis):

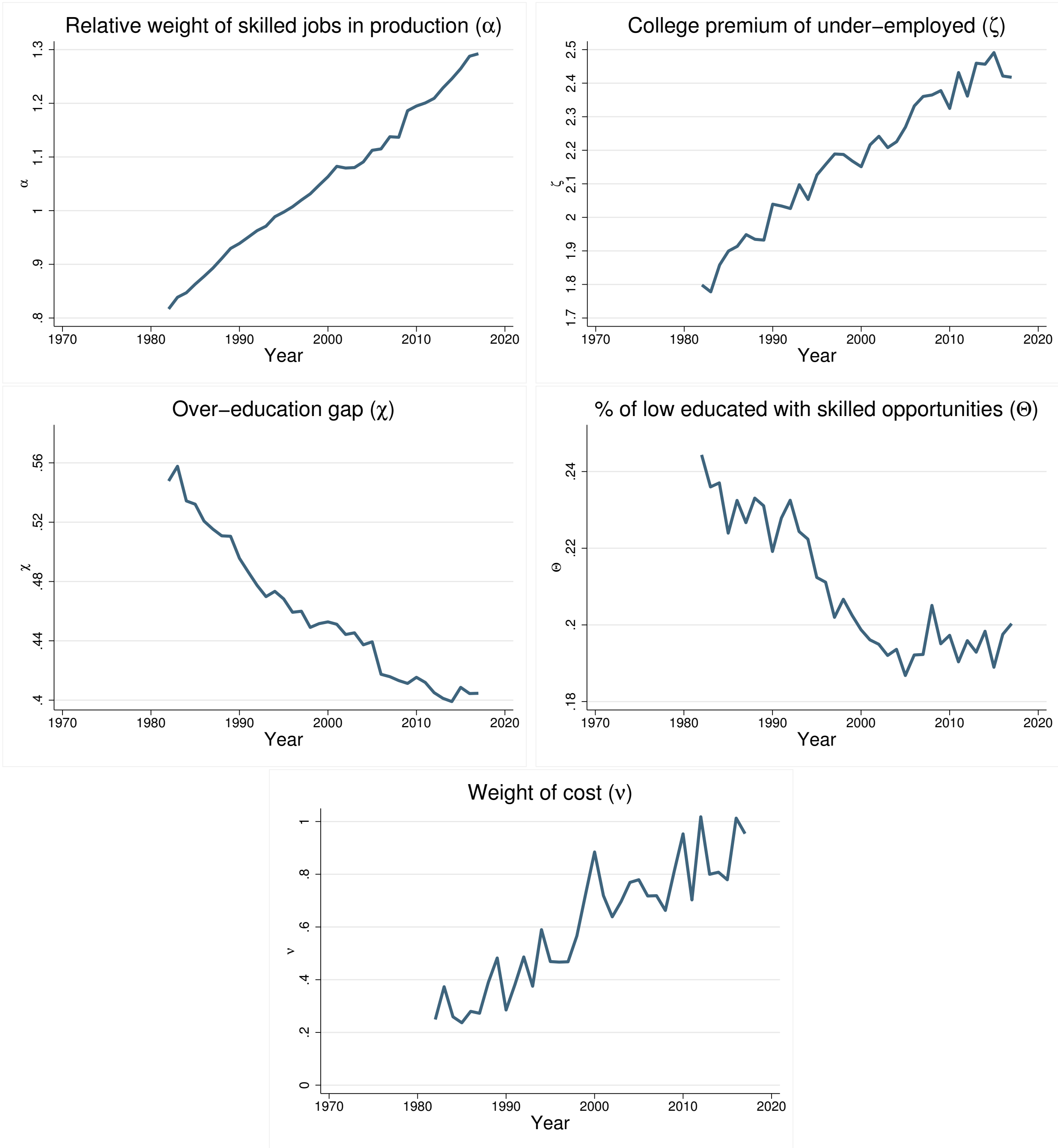
$$\begin{array}{lcccccl} \ln(CP) = & \text{-29.99} & + & 0.015 \times t & - & 0.357 \times \ln\left(\frac{n}{1-n}\right) & \text{Unadjusted } (R^2 = 0.92) \\ & \text{(-7.16)} & & (7.39) & & (-4.26) & \end{array} \tag{12}$$

$$\begin{array}{lcccccl} \ln\left(\frac{w_h}{w_l}\right) = & \text{-24.76} & + & 0.012 \times t & - & 0.588 \times \ln\left(\frac{j_h}{j_l}\right) & \text{Adjusted } (R^2 = 0.99) \\ & \text{(-29.09)} & & (32.87) & & (-7.46) & \end{array} \tag{13}$$

In light of our model, the estimation of the elasticity of substitution is also contaminated when not adjusting the data. When using the unadjusted series, we find a $\hat{\sigma}$ of 2.8 (1/0.357), just below the 2.9 found in Katz and Murphy (1992). Instead, when adjusting the data, we get an estimate of 1.70 (1/0.588), closer to a Cobb Douglas. Our model has build-in substitutability on the side of workers. Given $\sigma$, we can then back out sequentially the remaining parameters: $\{\alpha\}$ to match $\left\{\frac{w_h}{w_l}\right\}$ using equation (10), $\{\nu\}$ to match $\{u\}$ using equation (3), and $\{\Theta\}$ to match $\{o\}$ using equation (5).

The retrieved parameters are shown in Figure 4. Mismatch is driven by four key parameters. The first parameter, $\Theta$, reflects the educational restriction from high occupation, i.e., the restriction that to be designer one needs a bachelors degree. This pushes workers without college out of high-type occupations, which drives down the wages at the bottom and raise them at the top. The parameter has fallen from 0.24 in 1980 to 0.20 by the end of the sample. A second parameter is the efficiency of non-college workers in skilled occupation, $\chi$, as measured by their wages relative to their peers with a college degree. We find that, over the sample, the efficiency units have decreased from 0.55 to 0.40. We interpret

Figure 4: Parameter values




*Note: Estimated to fit empirical targets, given the evolution of* $n$ *and* $\sigma = 1.7$.

this as workers without college having a harder time keeping up with the rise in technology that is making these occupations more complex. A third parameter is the efficiency of college workers in unskill occupations, $\zeta$, as measured by their wages. Over the sample, $\zeta$ has increased from 1.8 to 2.4 (in 1980 college graduates were 80% and now more than twice more productive than non-college workers in unskill occupations). This could reflect the quality of the education system in raising the general level of human capital. The last parameter, $\nu$, reflects the size of the "costs" that might prevent workers from taking the job in which they have a comparative advantage. We should interpret the four-fold increase in the parameter with caution, as we should analyse it in relation to wages. Overall, $\nu$ has increased from 20% of the average wage to 28% of the average wage. It does seem that the barriers preventing workers from accepting better-paying jobs have increased. As we do not take a stance on the source of mismatch, we cannot say whether it arises from inefficient frictions or from efficient sources like preferences.

## 4.3 Decomposing the rise of college premium

We perform several decomposition exercises, shown in Table 1. We start by varying each of the six parameters, keeping the remaining five parameters constant, at their 1980s values. We calculate the changes implied in over-employment, under-employment and the overall college premium. We report the percentage of the actual variation in the variables, explained by each of the parameters individually.

Skilled-bias technological progress ($\alpha$) raised the marginal productivity of skilled jobs, and hence their returns, which discouraged under-employment and encouraged over-employment. This effect was counteracted by the increase in the fraction of college workers $n$. In isolation, increasing $n$ has only lowered the college premium by 7 percentage points. It had sizable positive effects on under-employment and negative effects over-employment, which counteract the standard effect on college premium.

All the remaining four parameters have contributed to increase the college premium. The decrease in $\chi$ and the increase in $\zeta$ had a direct effect lowering the wages of some workers without college (the over-employed) and increasing the wages of the under-employment college graduates, as well as affecting the marginal productivities, raising the supply of unskilled labour and reducing the supply of unskilled labour. Quantitatively, the change in $\zeta$ has a large effect on the college premium. Finally, the increase in the weight of the cost $\nu$ contributed 8% to the rise in the college premium, while the quantitative impact of $\Theta$ was 3%.

When we add the individual contributions of each parameter, the sum is larger than

Table 1: Counterfactual Exercises (Only changing one parameter)

| | ***CPS switchers (1982-2017)*** | | |
|---|---|---|---|
| | Over-employment ($o$) | Under-employment ($u$) | College Premium ($CP$) |
| Variation | -0.04 | 0.06 | 0.70* |
| Percentage explained by | | | |
| $\alpha$ | -131% | -93% | 28% |
| $n$ | 85% | 172% | -7% |
| $\chi$ | 162% | -47% | 22% |
| $\zeta$ | -130% | 104% | 56% |
| $\nu$ | 11% | 23% | 8% |
| $\Theta$ | 41% | -11% | 3% |
| | | | |
| $\alpha, n$ | 22% | 7% | 4% |
| $\chi, \zeta, \nu, \Theta$ | 75% | 42% | 87% |
| | | | |
| **Technology** $(\alpha, \chi)$ | 24% | -102% | 90% |
| **Education** $(n, \zeta)$ | -50% | 289% | 57% |
| **Mismatch** $(\nu, \Theta)$ | 53% | 16% | 14% |

*Own calculations based on model simulations, varying each parameter or combination of parameters while keeping the remaining parameters constant, at their value in the beginning of the sample.* * *The decomposition exercise is based on the average college premium calculated in the model by* $CP = \frac{\frac{(n-u)w_h + u\zeta w_l}{n}}{\frac{(1-n-o)w_l + o\chi w_h}{1-n}}$*, so for the calibration of based on switchers, the implied variation is larger than the observed in the data.*

100%, suggesting there is interdependence between them. In the second block of lines, we change the typical parameters of the Timbergen's race ($\alpha$ and $n$ jointly) fixing the parameters original to our model ($\nu$, $\Theta$, $\zeta$ and $\chi$), and vice-versa. Combined, the new parameters explain the majority of the variation of each variable. One can see the interdependence of the parameters of the model by comparing the joint contributions of $\alpha$ and $n$ for the college premium. When we varied them individually, they contributed 28 and -7% to the variation of the college premium, but, when combined, they contributed only 4%.

Still, we cannot interpret the four new parameters as related to mismatch. We think the decrease in $\chi$ is more related to skilled-bias technology progress that makes workers without education less efficient in high-skilled occupations when compared to college graduates. On the other hand, we think that the increase in $\zeta$ is related to improvements in education that also raise the general level of human capital used in unskilled occupations. The only two parameters related to mismatch are $\nu$ and $\Theta$. The last three lines of Table 1 show this three-way decomposition. Technology explains 90% of the rise of college premium, while the contribution of mismatch is 14%. Hence, changes in the structure of mismatch contribute one-sixth of the evolution of college premium relative to skill-bias technological progress. Education, on the other hand, actually has contributed to an increase in college premium. First, there are leakages, as many college workers work in unskilled occupation. Second, education also increased general human capital so the under-employed workers become better in unskilled occupations too, driving down their wages.

# 5 Output Costs of Mismatch

In this second application, we focus on a question that attracts much interest from policymakers. What are the economic costs of mismatch? Some papers have documented the impact of mismatch on wages or job satisfaction and other correlates of workers' productivity. However, the empirical evidence seems inconclusive as results are sensitive to the nature of the mismatch measure (education, skill or qualification) or lead to ambiguous conclusions. Hence, the empirical literature focus extensively on how measure mismatch in the data, and what measure is more relevant, whether skill or education mismatch (see McGowan and Andrews (2015) and Grunau (2016) for a survey). While we do not contribute to the empirical debate on education vs skill mismatch, we develop an original and simple method to compute the output costs of mismatch, for a cross-section of economies. Direct evidence on the impact of mismatch on the aggregate economy is limited to specific countries using linked employer-employee data, for instance Mahy et al. (2015) for Belgium and Grunau (2016) for Germany. An exception is McGowan and Andrews (2015) who propose a cross-country perspective by looking at the impact of mismatch on labor productivity using industry-level regressions. Our originality lies in proposing a model-based method for assessing the cost of mismatch, thereby taking into account endogenous workers' sorting into jobs and general equilibrium effects on wage premia, which is not taken into account in the current empirical studies. In addition, the cross-state exercise will shed light on the economic interpretation of mismatch parameters.[11]

[11]Indeed, on Figure 4, parameters display trends, which would spuriously correlate with any variable sharing the same trend.

## 5.1 Cross-State Calibration

We use the US Census IPUMS data to calculate four targets $\left\{n, u, o, \frac{w_h}{w_l}\right\}$ and CPS monthly data on switchers to identify $\frac{w_h}{\zeta w_l}, \frac{w_l}{\chi w_h}$, for each of the US States. The share of college graduates varies across states from 25% in West Virginia and Arkansas, to more than 40% in Maryland, Colorado, or Massachusetts. The maps in Figure 5 show over-employment as a fraction of workers without college and under-employment as a fraction of college. Over-employment varies from below 11% in Hawaii and Mississippi, to above 15% in Utah, New Hampshire, Colorado, Maryland and Alaska. We observe more variation in under-employment, that ranges from below 30% in Maryland, Virginia, New Mexico or Massachusetts, to more than 40% in Nevada and Hawaii.

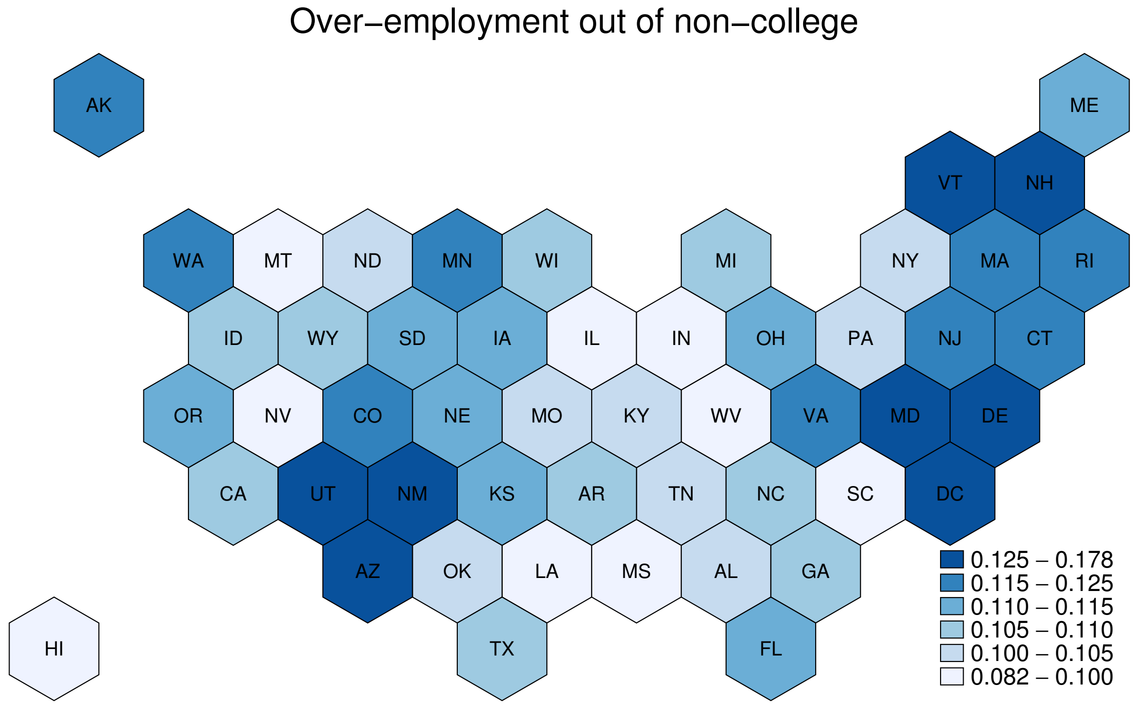
We show the histograms with all the targets in Appendix F. The average college premium of well-matched workers hovers around 140%, ranging from around 80% in Wyoming and Alaska, to close to 200% in California and Connecticut.

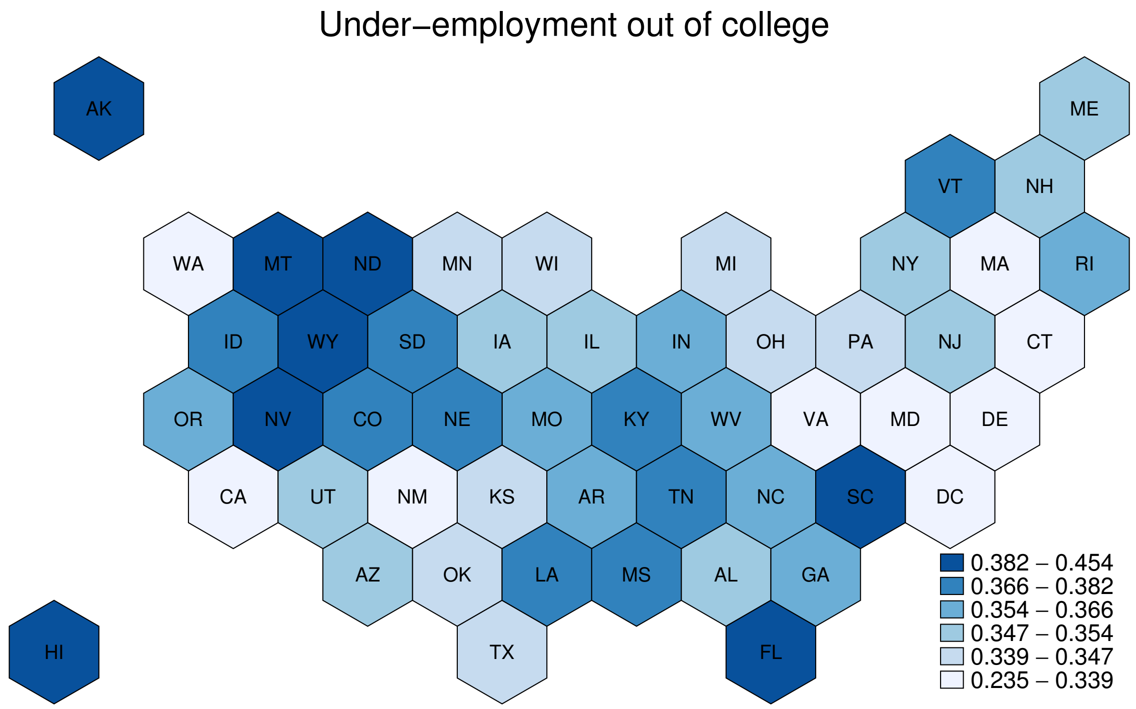
The well-matched premium for college workers varies from 3% in Kansas to 15% in Hawaii. This variable $\frac{w_h}{\zeta w_l}$ is inversely related to the wage loss incurred by an under-employed high-type worker relative to a well-matched counterpart. This is called the "wage cost of under-employment" in Barnichon and Zylberberg (2019). Finally, mismatch of low-type workers lead to a wage gain with respect to their well-matched counterparts : well-matched low-type earn on average around 93% of the wage if they were over-employed workers. It is low in Delaware, Alabama and New Jersey (90%) and the highest in Wyoming (96%). $\frac{w_l}{\chi w_h}$ relates to the "wage gain of over-employment".

We follow the same procedure as in the time series exercise, where the five parameters $\{\alpha, \zeta, \chi, \Theta, \nu\}$, that are determined for the model to match exactly five targets $\left\{u, o, \frac{w_h}{w_l}, \frac{w_h}{\zeta w_l}, \frac{w_l}{\chi w_h}\right\}$, for each of the states, using the value of $\sigma$ estimated in the previous section. $n$ is given by the share of college workers. We show in Appendix F the histograms with the parameter values.

While the interpretation of $\nu$ in time series is not straightforward, the cross-section data allows for a more detailed discussion. To support our view that $\nu$ represents barriers or frictions that prevent workers to accept the highest paying job, we correlate its estimated

Figure 5: Cross-States Over- and Under-Employment as Fraction of Relevant Population




*Note: All calculated from US Census IPUMS data, being an average (1970, 1980, 1990, 2000-2017) for each state. A worker is considered as under-employed (over-employed) if she is college-educated and working in 4-digit occupations that are majority non-college (college).*

values with some external variables that might relate with these frictions (See Appendix F). The parameter is positively associated with the share of families with 2 or more earners (0.18), the fraction of homeowners with a mortgage (0.23), population density (0.44), the log of the median house prices (0.58), median time to work (0.47) and the percentage of people with access to fast broadband (0.38). When all variables are included jointly in a regression, the log of median house prices and the median time to work are significant at 5%. This means that $\nu$ seems related to aspects of housing and commuting in the state.

Furthermore, while in our model $\nu$ and $\Theta$ determine the level of mismatch and relative wages, they could be related with other economic variables outside the model, like wage dispersion, unemployment and employment rates. Higher $\nu$ is associated with higher standard deviation of college workers' wage (0.84) and their unemployment rate (0.42), but less associated with the standard deviation of wages on non-college workers (0.37) and their unemployment rate (0.08). This is consistent with our model's results. In addition, while $\Theta$ is not associated with the employment rate of college graduates (0.05), it shows a positive correlation with the employment rate of non-college graduates (0.34), which is also consistent with our model.

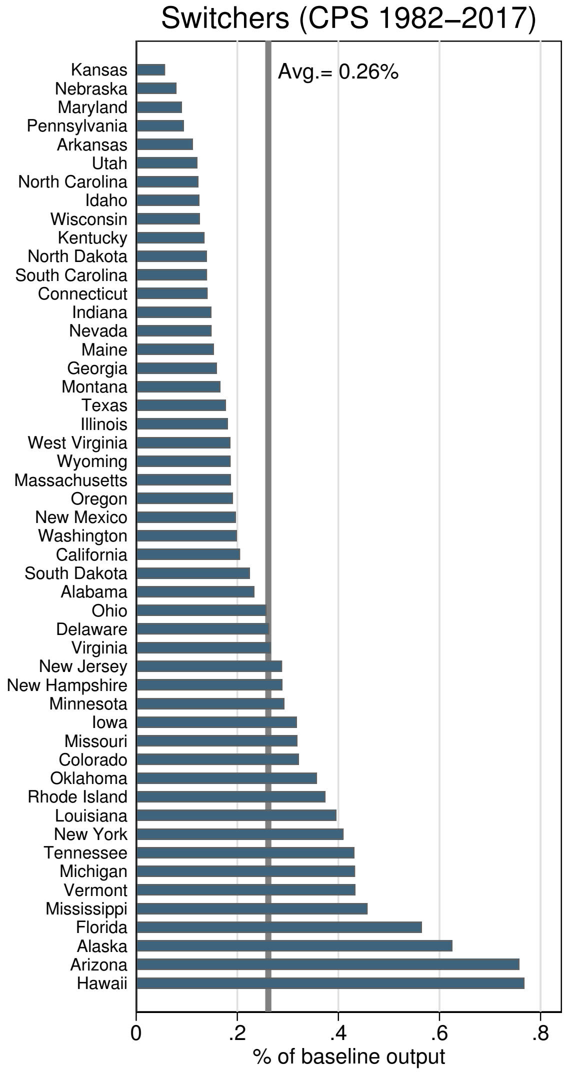
## 5.2 Output gains of eliminating frictions

The main exercise is to quantify what are the gains in terms of output of eliminating the frictions generating mismatch ($\nu = 0$). The results are shown in Figure 6. The output gains are on average 0.26%, but vary substantially across states, with low output costs of mismatch of 0.06% in Kansas, to output costs hovering around 0.77% in Hawaii and Arizona.[12]

For the sake of illustrating our results, let us have a closer look at Kansas and Arizona. One might think that the prevalence of under-employment *per se* are good predictors of output costs. Under-employment in Kansas is 0.12 and in Arizona 0.11, representing in both states 33% of all college graduates. The extent of under-employment is rather similar across the two states while the output costs in Arizona are more than ten times larger than in Kansas. So the extent of mismatch is not a primary driver for output costs.

[12] By setting $\nu = 0$, we have a set of corner solutions, that depend on whether $\chi\zeta \gtrless 1$ and the ratio of relative wages. In most states, eliminating the frictions $\nu = 0$ does not eliminate under-employment $u$ but equates $w_h = \zeta w_l$.

Figure 6: Output costs of frictions ($\nu = 0$)




*Note: Model simulations. The graph plots the percentage variation of output relative to baseline when setting $\nu = 0$.*

In order to confirm this intuition, we run univariate regression of output costs on either the fraction of college graduates or the extent of mismatch. Besides using the 50 US states, we also construct a dataset of simulated data. For each parameter we give nine equally distributed numbers between the minimum and the maximum of our set. We simulate a fictitious economy for each possible combination of parameters. We only keep simulations for which the five empirical targets are between the minimum and the maximum of our set of 50 states. In total, we have more than 7,000 observations.

Using both the actual and simulated data, we regress output costs on each observable

Table 2: Regression of output cost of mismatch on observable variables

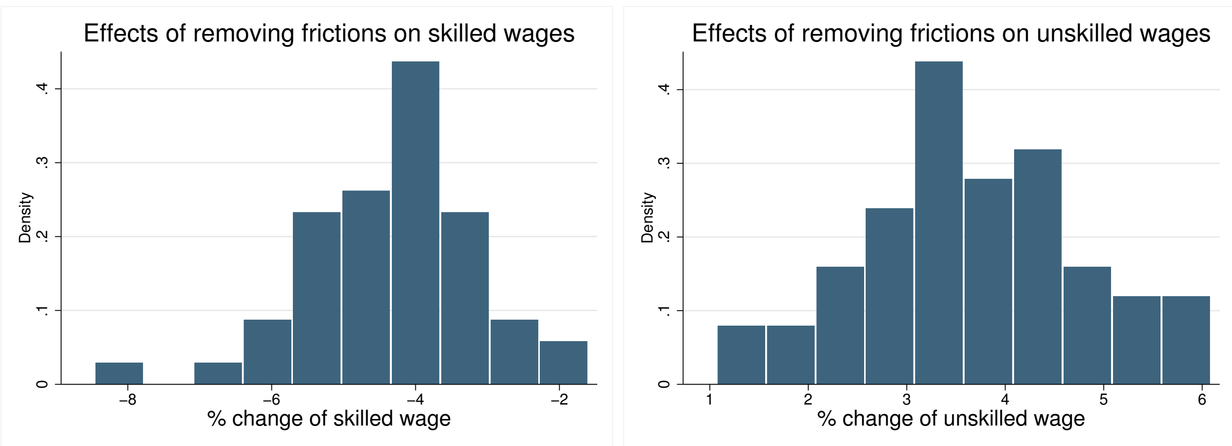
| | ***Cross-States*** | | ***Simulated Data*** | |
|---|---|---|---|---|
| **Variable** | **coef. (t-stat)** | **[R-squared]** | **coef. (t-stat)** | **[R-squared]** |
| $n$ | 0.18 (0.36) | [0.003] | -0.33 (-15.84) | [ 0.034] |
| $o$ | 1.06 (0.30) | [0.002] | 0.54 (3.38) | [0.002] |
| $u$ | 2.43 (1.51) | [0.046] | -0.680 (-6.01) | [0.005] |
| $\frac{w_h}{w_l}$ | -0.11 (-1.22) | [0.030] | -0.06 (-12.80) | [0.023] |
| $\frac{w_h}{\zeta w_l}$ | **5.56** (10.14) | [0.681] | 4.40 (146.52) | [0.753] |
| $\frac{w_l}{\chi w_h}$ | 1.45 (0.95) | [0.018] | 0.63 (5.82) | [0.005] |
| *Obs.* | | (50) | | (7,052) |

*Note: The table shows coefficient, t-statistic and R-squared of the univariate regression of the model-based output costs of mismatch on all observable variables in turn. $n$ share of high-type workers in employment. $u$ and $o$, share of under-employed and over-employed workers, respectively, in total employment. $\frac{w_h}{w_l}$ high-type wage premium of well-matched workers. $\frac{w_h}{\zeta w_l}$ well-matched premium of high-type workers relative to under-employed. $\frac{w_l}{\chi w_h}$ well-matched loss of low-type workers relative to over-employed.*

variables used in the calibration (Table 2) in turn. None of the variables is significant, except the wage loss of under-employment. Output costs are not driven by the percentage of mismatched workers per se.

The wage premium of well-matched college workers alone accounts for 68% of output costs variance (75% using simulated data). In other words, U.S. States, such as Arizona, with wage losses of under-employment of more than 15%, would benefit more from eliminating mismatch than states, such as Kansas, with more moderate wage cost of under-employment

Figure 7: Effects of Removing Frictions of Skilled an Unskilled Wages




*Note: Model simulations. The graph plots the percentage variation of skilled and unskilled wages ($w^h$) and ($w^l$) relative to baseline when setting $\nu = 0$.*

(of around 5%) would not. The magnitude of the coefficient, quite similar in both regressions in Table 2, suggests that a 10 percentage point increase in the wage cost of under-employment is associated with a cost of 0.6% of output. It is easier to understand why this variable is so related to the welfare costs if we compute the derivative of output with respect to under-employment: $\frac{\partial Y}{\partial u} = -w_h + \zeta w_l$. This partial derivative tells us that reducing under-employment at the margin would raise output by $w_h - \zeta w_l$, which dividing by $\zeta w_l$ is exactly the wage loss of under-employment. Given our calibration, reducing under-employment, at the margin, always raises output.

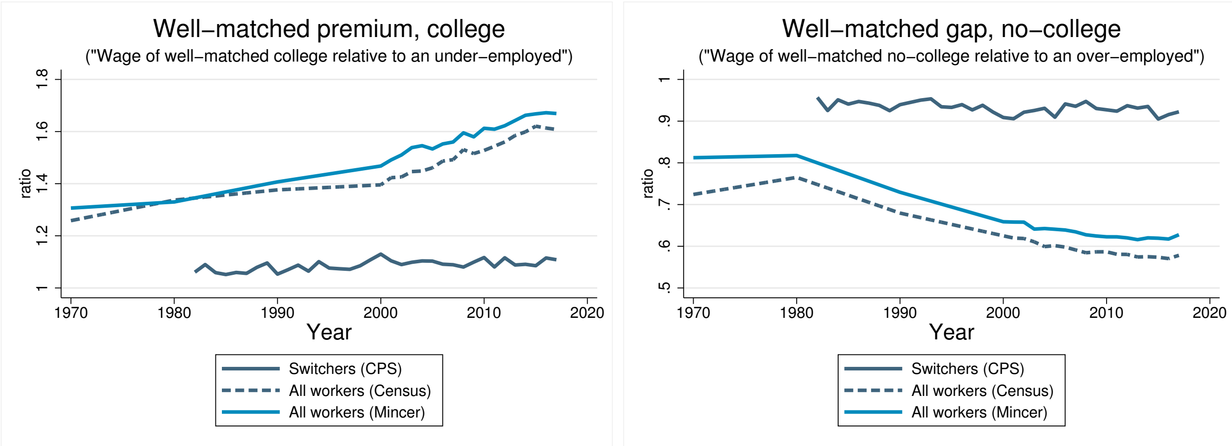
Policy makers in states, such as Arizona, should be concerned that too few educated workers are actually working on well-matched jobs. This scarcity of labor away from more productive jobs is very likely to be costly in terms of output. In contrast, states, such as Kansas, with more moderate wage cost of under-employment should not be concerned about high-type workers taking up unskilled jobs because this does not create too much scarcity in the market of skilled jobs.

Finally, our model has predictions on the effect of mismatch on wage dispersion. In all states, eliminating the frictions raises the number of high-type jobs while lowering the number of low-type jobs, which results in lower wage inequality in the zero-frictions economy compared with the baseline model. Figure 7 shows that removing the frictions would imply a reduction in $w^h$ by 4.4%, and an increase in $w^l$ by 3.7%. The overall college premium would go down by 8 percentage points, on average across states.

# 6 Two Alternative Calibrations

In the baseline calibration, we calculate the relative wages comparing the same workers in the two categories, which lead to a wage loss of under-employment of about 8% between the early 80s and 2017. This estimate is low compared to the standard in the literature. For instance, Barnichon and Zylberberg (2019) report a wage loss of under-employment of 40% on average, and 28% with individual controls, based on regressions of wages of new hires. As we show that the wage loss of under-employment is key in driving output costs, we report in this section the results with two alternative calibrations to mimic their approach. In the first,

Figure 8: Comparison of Relative Wages Across Alternative Estimations



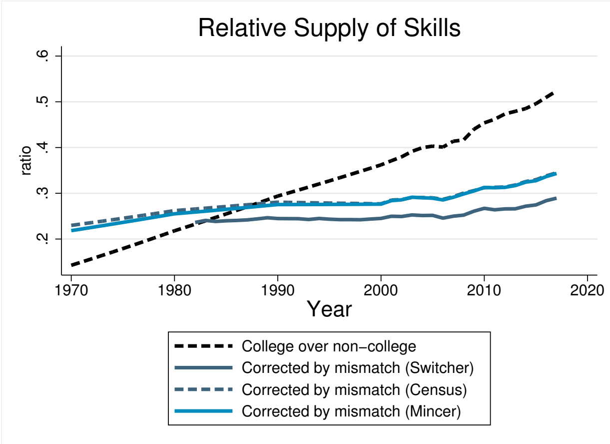

*Note: All calculated from US Census data (1970, 1980, 1990, 2000-2017). A worker is considered as under-employed (over-employed) if she has a college (non-college) education and working in 4-digit occupations that are majority non-college (college). Wage from Census real weekly earnings.*

we look at the average relative wages of mismatched and well-matched workers using Census data. In the second, instead of calculating simple averages of wages, we regress log wages on observed characteristics (race, gender, age, age$^2$) as well as a dummy that equals 1 when the worker is well-matched, 0 otherwise. The estimated coefficient on this well-matched dummy in the Mincer regression provides an estimate for the well-matched premium using microdata, after controlling for individual characteristics. Appendix E provides the technical details and Figure 8 compares the relative wages measured with the different methods.

The wage loss of under-employment, under these alternative calibrations, are closer to Barnichon and Zylberberg (2019)'s estimates: an average well-matched college worker earn 40% more than the mismatched counterpart. The well-matched premium is slightly higher when controlling for individual characteristics (45%).

We reproduce the calibration procedure and all exercises on time-series and across states. These are shown in Appendix G. Figure 9 reports the relative supply of skills under the baseline and alternative calibrations. The quantitative implications for the evolution of the relative supply of skills, when corrected by mismatch, are the same. We use these variables to re-estimate $\sigma$.

Figure 9: Comparison of Relative Supply, Alternative Calibrations




*Note: All calculated from US Census data (1970, 1980, 1990, 2000-2017). "College over non-college": $\frac{n}{1-n}$ where $n$ is the fraction of college workers. "College over non-college corrected by mismatch": $\frac{jh}{jl} = \frac{(n-u)+\chi o}{(1-n-o)+\zeta u}$ where $o$ ($u$) denotes over-employment (under-employment).*

$$\ln\left(\frac{w_h}{w_l}\right) = \underset{(-17.78)}{-29.36} + \underset{(19.21)}{0.015 \times t} - \underset{(-7.61)}{0.853 \times \ln\left(\frac{j_h}{j_l}\right)} \quad \text{Adjusted-Census } (R^2 = 0.98) \tag{14}$$

$$\ln\left(\frac{w_h}{w_l}\right) = \underset{(-19.88)}{-32.62} + \underset{(21.22)}{0.016 \times t} - \underset{(-9.63)}{0.952 \times \ln\left(\frac{j_h}{j_l}\right)} \quad \text{Adjusted-Mincer } (R^2 = 0.98) \tag{15}$$

Using the Census data, based on the average wage difference, the estimated elasticity of substitution is 1.17 (1/0.853). When we use the relative wage data estimated with Mincer regressions, it is 1.05 (1/0.952) even closer to a Cobb-Douglas.

Larger wage differences require more frictions in the economy. As a result, the role of $\nu$ and $\Theta$ is amplified. In this calibration, $\nu$ has more than quadrupled, representing than a 100% of the average wage by the end of the sample. Frictions $\nu$ and $\Theta$ contribute half as much as skilled-biased technological progress for the rise in the college premium.

The output costs of mismatch is similar between Census data (2.55%) and Mincer estimates (2.56%), much larger than the baseline estimated costs. This is not surprising, given that, under the alternative calibrations, there is a much larger well-matched premium for college workers, and we just saw in the previous section that the output costs are related to the wage cost of under-employment. Interestingly, in both calibrations, we find an elasticity of output cost of mismatch to wage loss of under-employment of around 0.07, close to the value found in the baseline calibration.

# 7 Conclusion and Discussion

We propose a model-based measure of the economic implications of vertical mismatch, using a simple theory of under- and over-employment, based on a variation of a Roy sorting mechanism in an otherwise Neoclassical model. Despite its simplicity, the model highlights one particular determinant of mismatch - the wage differential amongst alternatives - and illustrates clearly the mechanism of reverse causality from mismatch to wage differentials, a mechanism that has been overlooked in the literature. In addition, this simple model proposes a direct mapping with the data, which provides clear theoretical baselines to the quantification of the economic consequences of mismatch. We show the versatility of the model with two exercises.

In our first exercise, we examine the impact of mismatch on the evolution of the college premium in the US, and find that it contributed one-sixth as much as skill-bias technological progress. The deeper implication is that economists should take seriously the role of mismatch. Often, they advise for more investments in education without realizing the leakages that arise through mismatch.

In our second illustration, we match key moments of 50 U.S. States. The output costs of mismatch lies between 0.06 and 0.77% across states. The key variable that explains the output cost of mismatch is not the percentage of mismatched workers but their wage relative to well-matched workers. In particular, output cost of mismatch is larger in economies with higher wage cost of under-employment. While the costs depends on the way the wage cost of under employment is estimated, the elasticity is robust at around 0.06.

We show that the output costs are determined by wage differences between well-matched vs. mismatched college workers, independent of the size of mismatch. If there were no wage differences, the output cost would be zero. Quantitatively, a 10-percentage point increase in the wage cost of under-employment is associated with a cost of 0.6% of output. This finding stresses the importance of estimating these wage differences properly. As our baseline presents low wage cost of under-employment relative to the literature, we see it as a lower bound for the output costs of mismatch. On the other hand, the alternative calibrations use a higher estimate of the wage cost of underemployment, more in line with the literature. Hence, we interpret the output costs of mismatch of 2.5% as an upper bound.

# References


Acemoglu, D. (2002): "Technical Change, Inequality, and the Labor Market," *Journal of Economic Literature*, 40, 7–72.

Acemoglu, D. and D. Autor (2011): "Chapter 12 - Skills, Tasks and Technologies: Implications for Employment and Earnings," Elsevier, vol. 4 of *Handbook of Labor Economics*, 1043 – 1171.

Arseneau, D. and B. Epstein (2014): "The Welfare Costs of Skill-Mismatch Employment," Finance and Economics Discussion Series 2014-042, Washington: Board of Governors of the Federal Reserve System.

Autor, D. H., F. Levy, and R. J. Murnane (2003): "The Skill Content of Recent Technological Change: An Empirical Exploration*," *The Quarterly Journal of Economics*, 118, 1279–1333.

Baert, S., B. Cockx, and D. Verhaest (2013): "Overeducation at the start of the career: Stepping stone or trap?" *Labour Economics*, 25, 123–140.

Barnichon, R. and A. Figura (2017): "Labor Market Heterogeneity and the Aggregate Matching Function: The Causes and Costs of Misallocation," *American Economic Journal: Macroeconomics*, 31, 222–249.

Barnichon, R. and Y. Zylberberg (2019): "Underemployment and the Trickle-Down of Unemployment," *American Economic Journal: Macroeconomics*, 11, 40–78.

BAUER, T. (2002): "Education mismatch and wages: A panel analysis," *Economics of Education Review*, 21, 221–229.

BLÁZQUEZ, M. AND M. JANSEN (2008): "Search, mismatch and unemployment," *European Economic Review*, 52, 498 – 526.

BORJAS, G. J. (1987): "Self Selection and the Earnings of Immigrants," *The American Economic Review*, 77, 531–553.

CASELLI, F. (2016): *Technology Differences over Space and Time*, Princeton University Press.

CERVANTES, C. V. AND R. COOPER (2022): "Labor market implications of education mismatch," *European Economic Review*, 148, 104179.

CHARLOT, O. AND B. DECREUSE (2010): "Over-education for the rich, under-education for the poor: A search-theoretic microfoundation," *Labour Economics*, 17, 886 – 896.

CHASSAMBOULLI, A. (2011): "Cyclical Upgrading of Labor and Employment Differences across Skill Groups," *The B.E. Journal of Macroeconomics*, 11, 1–42.

COOPER, R. AND H. LIU (2019): "Mismatch in Human Capital Accumulation," *International Economic Review*, 60, 1291–1328.

DOLADO, J. J., M. JANSEN, AND J. F. JIMENO. (2009): "On-the-Job Search in a Matching Model with Heterogeneous Jobs and Workers," *Economic Journal*, 119, 200–228.

DUNCAN, G. AND S. HOFFMAN (1981): "The incidence and wage effects of overeducation," *Economics of Education Review*, 1, 75–86.

EECKHOUT, J. AND P. KIRCHER (2010): "Sorting and Decentralized Price Competition," *Econometrica*, 78, 539–574.

FERNANDEZ-BLANCO, J. AND P. GOMES (2017): "Unobserved Heterogeneity, Exit Rates, and Re-Employment Wages," *The Scandinavian Journal of Economics*, 119, 375–404.

Flisi, S., V. Goglio, E. C. Meroni, M. Rodrigues, and E. Vera-Toscano (2017): "Measuring Occupational Mismatch: Overeducation and Overskill in Europe:Evidence from PIAAC," *Social Indicators Research*, 131, 1211–1249.

Freeman, R. B. (1975): *The Overeducated American*, New York: Academic Press.

Garibaldi, P., P. M. Gomes, and T. Sopraseuth (2020): "Output Costs of Education and Skill Mismatch," IZA Discussion Papers 12974, Institute of Labor Economics (IZA).

——— (2021): "Public Employment Redux," *Journal of Government and Economics*, 1.

Gautier, P. A. (2002): "Unemployment and Search Externalities in a Model with Heterogeneous Jobs and Workers," *Economica*, 69, 21–40.

Grunau, P. (2016): "The impact of overeducated and undereducated workers on establishment-level productivity: First evidence for Germany," *International Journal of Manpower*, 37, 372–392.

Guerrieri, V., R. Shimer, and R. Wright (2010): "Adverse Selection in Competitive Search Equilibrium," *Econometrica*, 78, 1823–1862.

Guvenen, F., B. Kuruscu, S. Tanaka, and D. Wiczer (2020): "Multidimensional Skill Mismatch," *American Economic Journal: Macroeconomics*, 12, 210–44.

Heckman, J., J. Humphries, S. Urzua, and G. Veramendi (2011): "The Effects of Educational Choices on Labor Market, Health, and Social Outcomes," Working Paper Series 2011-002, University of Chicago.

Herz, B. and T. van Rens (2020): "Accounting for Mismatch Unemployment," *Journal of the European Economic Association*, 18, 1619–1654.

Horváth, G. (2014): "Occupational mismatch and social networks," *Journal of Economic Behavior §Organization*, 106, 442 – 468.

Hsieh, C.-T., E. Hurs, C. L. Jones, and P. J. Klenow (2019): "The Allocation of Talent and U.S. Economic Growth," *Econometrica*, 87, 1439–1474.

Julien, B., J. Kennes, and I. King (2000): “Bidding for Labor,” *Review of Economic Dynamics*, 3, 619–49.

Katz, L. F. and K. M. Murphy (1992): “Changes in Relative Wages, 1963-1987: Supply and Demand Factors,” *The Quarterly Journal of Economics*, 107, 35–78.

Mahy, B., F. Rycx, and G. Vermeylen (2015): “Educational Mismatch and Firm Productivity: Do Skills, Technology and Uncertainty Matter?” IZA Discussion Paper 8885.

McGowan, M. A. and D. Andrews (2015): “Labour Market Mismatch and Labour Productivity: Evidence from PIAAC Data,” OECD Economics Department Working Papers 1209, OECD Publishing.

McGuinness, S. (2006): “Overeducation in the Labour Market,” *Journal of Economic Surveys*, 20, 387–418.

Quintini, G. (2011): “Right for the job: over-qualified or under-skilled?” OECD Social, Employment and Migration Working Papers 120, Organisation for Economic Co-operation and Development.

Restuccia, D. and R. Rogerson (2017): “The Causes and Costs of Misallocation,” *Journal of Economic Perspectives*, 31, 151–74.

Robst, J. (2007): “Education and job match: The relatedness of college major and work,” *Economics of Education Review*, 26, 397 – 407.

Roy, A. D. (1951): “Some Thoughts On The Distribution Of Earnings,” *Oxford Economic Papers*, 3, 135–146.

Sahin, A., J. Song, G. Topa, and G. Violante (2014): “Mismatch Unemployment,” *The American Economic Review*, 104, 3529–3564.

Sattinger, M. (1993): “Assignment models of the distribution of earnings,” *Journal of Economic Literature*, 31, 101–122.

Shimer, R. (1999): "Job Auctions." *https://pdfs.semanticscholar.org/2fb6/b8bbcb4287266b4f06ae30dc826a75dc1dc0.pdf.*

——— (2007): "Mismatch," *American Economic Review*, 97, 1074–1101.

Sicherman, N. and O. Galor (1998): "A Theory of Career Mobility," *Journal of Political Economy*, 169–192.