Earnings Risk and Heterogeneous Expected Earnings Profiles

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Abstract

We add to the debate about whether models of earnings dynamics should allow for unobservable heterogeneity in expected earnings growth rates by examining implications for a statistic originally proposed to estimate the variance of persistent earnings shocks. While that statistic is unbiased under a common specification, we derive biases that would arise under alternative models and use them to draw inferences about their empirical relevance and to estimate key parameters. Most results cast doubt on substantial heterogeneity in growth rates, though some leave room for a modest role. Estimates of shocks' variance and persistence are more robust.

Keywords: heterogeneous earnings profiles, earnings instability, earnings volatility, risk

JEL codes: D3, J0

1 Introduction

A substantial literature has debated whether empirical specifications of agents' income profiles should include person-specific earnings growth rates. Several papers, perhaps most prominently Baker (1997) and Guvenen (2007, 2009), have argued for "Heterogeneous Income Profile" (HIP) models that feature such profiles, but others find evidence supporting "Restricted Income Profile" (RIP) models that do not (MaCurdy (1982), Abowd and Card (1989), Hryshko (2012)). In addition to its immediate implications, in practice the distinction also leads to different conclusions about the size and persistence of earnings shocks: empirical results from RIP models often imply a major role for permanent earnings shocks, while results from HIP models often find that such shocks are notably smaller and/or less persistent. The choice between the specifications thus has important consequences for many issues, including the interpretation of income inequality and its development over the lifecycle, the relative importance of factors that shape career dynamics (e.g., human capital, job search, promotion and bonuses, etc.), how consumption and savings decisions respond to income shocks, and by extension business cycle dynamics.

This paper aims to provide additional evidence to that debate. While most previous work has investigated by estimating full sets of parameters of the earnings process via a minimum distance estimator, our analysis prioritizes evidence based on variation in statistics intended to measure the variance of potential earnings shocks. This approach is perhaps more natural than it may appear; others including Guvenen (2009) have noted the empirical challenge of distinguishing between predictable earnings growth and persistent changes in income due to shocks, suggesting that efforts to understand them are complementary. The strategy may

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also have some advantage in terms of clarity. A recurring theme has been the fear that some parameters may be misestimated due to biases or restrictions involving other parameters, and such biases may be even more opaque when many parameters are entangled across multiple nonlinear moment conditions. While our approach cannot avoid such complications entirely, it limits their scope because it ultimately leads us to estimate relatively simple specifications that involve only a few parameters that are central to our question.

The statistics we analyze are straightforward to compute, ultimately premised on moment conditions used throughout this literature, and closely related to calculations used in Meghir and Pistaferri (2004). As we shall explain, under a "classic RIP specification" in which persistent shocks have a unit root, means of these statistics are closed-form expressions for those variances. Even under alternate models with HIP and/or autoregression of the persistent shocks, the theoretical predictions of these statistics add only the new parameters and continue to be free of others that could compromise identification. In addition, the empirical strategy readily accommodates variation in risk across and/or within earnings panels, another plausible scenario that might otherwise complicate measurement of heterogeneity in expected earnings growth.

The empirical analysis uses data from the Panel Study of Income Dynamics (PSID) to investigate a series of predictions that could help to distinguish between the RIP and HIP specifications. It begins by revisiting the idea that residual measures of earnings growth would be positively autocorrelated if the empirical specification failed to account for (true) heterogeneity in workers' expected earnings growth (MaCurdy (1982), Abowd and Card (1989)). This would be hard to reconcile with an RIP framework and thus would constitute prima facie evidence against it. Although earlier work has not found such evidence in panels up to 20 years long, proponents of HIP models have argued the proposed autocorrelation may not emerge in realistic data until panels were even longer (Baker (1997), Guvenen (2009)). Here we show that such evidence remains elusive in panels nearly twice as long. While the same arguments prevent us from concluding that expected earnings growth rates are homogeneous, the fact that we still find no contrary evidence over periods roughly as long as typical workers' entire careers suggests the proposed autocorrelation may never be forthcoming.

Our second empirical analysis considers how the statistics we examine vary with the length of the period used to construct them. An algebraic analysis of potentially misspecified RIP and HIP models shows that the statistic is expected to increase in the length of time examined if the specification improperly omits heterogeneous earnings growth, but the opposite relationship would emerge if the specification included unnecessary heterogeneous growth terms. This is conceptually similar to a test proposed by Hryshko (2012), though the details of the empirical procedure differ considerably, and ours can accommodate, e.g., heterogeneity in exposure to risk. The empirical results are both inconsistent with the hypothesis that the RIP model was misspecified and consistent with the HIP model being misspecified. However, the estimates also have the absurd implication that the variance of workers' earnings growth rate is negative.

We then attempt to cure that malady by modifying the specification to allow the persistent stochastic component to autoregress. This variant has been used often in prior work, and it is especially intriguing here because autoregression would cause the statistics we examine to diminish as more periods are used to construct them. Such a dynamic would counter and potentially overwhelm biases caused by improperly assuming that earnings growth is homogeneous, so it provides a plausible explanation for the anomalous estimates. Our final empirical analysis thus aims to measure jointly the autocorrelation parameter and the variance of earnings growth rates. The resulting estimates reject the hypothesis that the persistent component has a unit root, and though the estimated discrepancy is modest, it is large enough to expunge the negative estimated variances. Even so, the revised estimates of that variance remain smaller than many previously cited in support of HIP models and are usually (though not always) statistically insignificant.

The discussion begins by reviewing the theoretical model, explaining our empirical strategy, and introducing the statistics we analyze. Section 3 then explores algebraic implications of misspecification under a "classic" form of an RIP model in which persistent earnings shocks follow a random walk, and Section 4 extends the analysis to allow those shocks to be autoregressive. Section 5 presents the data and preliminary evidence, and Section 6 reports the main empirical results. Section 7 concludes with a brief discussion.

2 Estimating Earnings Risk

2.1 Model of the income process

Analyses of earnings dynamics typically begin by decomposing workers' earnings into three orthogonal parts:

$$y_{it} = F\left(A_{it}\right) + Q_i\left(A_{it}\right) + u_{it}, \tag{1}$$

where y_{it} represents the log earnings of person i in year t. F represents heterogeneity that can be predicted by the econometrician. It is usually specified as a function of each individual i's age or potential experience A_{it} and sometimes other observable factors like demographics, levels of education, or period effects.

The second component, Q_i, is the focus of this paper. It is intended to represent additional heterogeneity at the individual level. In a RIP specification, this is just a person-specific intercept ($Q_i\left(A_{it}\right) = \mu_i$), whereas the HIP specification adds a individual growth rate ($Q_i\left(A_{it}\right) = \mu_i + \theta_i A_{it}$).

The final component, u_{it}, is a deviation from i's own expected age-earnings profile. Seminal empirical studies by MaCurdy (1982) and Abowd and Card (1989) concluded that it could be modeled adequately as

$$\begin{aligned} u_{it} &= \pi_{it} + \nu_{it}, \text{ with} \\ \pi_{it} &= \rho\pi_{i(t-1)} + \eta_{it}. \end{aligned}$$

Here π_{it} is a stock of accumulated persistent shocks $\eta_{i\tau}$ ($\tau \leq t$) to i's earnings process, ρ reflects potential autoregression, and ν_{it} represents shocks that affect i's earnings only temporarily (or perhaps measurement error). It is common to assume η and ν are mean-zero with variances σ_η^2 and σ_ν^2, that they are uncorrelated with one another ($E\left[\eta_{it}\nu_{it'}\right] = 0$ for all t, t'), and that η is uncorrelated over time ($E\left[\eta_{it}\eta_{it'}\right] = 0$). Both MaCurdy and Abowd and Card (as well as some later work) find that $\rho \approx 1$, implying the persistent shocks are effectively permanent, and that ν can be adequately modeled as an MA(1) or MA(2) process. The main significance here is that ν does not exhibit autocorrelation at lags longer than two years, so we assume

$$E\left(\nu_{it} \cdot \nu_{it'}\right) = 0 \text{ for } |t - t'| \geq 3. \tag{2}$$

Some prior evidence on RIP/HIP. MaCurdy (1982) and Abowd and Card (1989) also find no evidence of heterogeneity in earnings profiles, i.e., they do not reject $\operatorname{Var}\theta = 0$. Conversely, later studies premised on RIP models corroborate the idea that π is a random walk; their estimates of ρ typically exceed 0.95 and are not statistically different from 1 (Storesletten, Telmer, and Yaron (2004); Karahan and Ozkan (2012); see also Hryshko (2012)). Since the two properties tend to accompany one another, the pair combine to form a "classic RIP specification:" a model of the earnings process with homogeneous growth rates and earnings shocks that are either permanent or short-lived. The statistics analyzed here are motivated by this specification, and they differ only modestly from those Meghir and Pistaferri (2004) use to estimate it.

Other studies argue for a "classic HIP specification" with heterogeneous growth rates and autocorrelation in the persistent shocks ($\rho < 1$) (Baker (1997); Guvenen (2007, 2009); Guvenen and Smith (2014)).[1] For example, using data from the Panel Study of Income Dynamics (PSID (2017)), Guvenen (2009) estimates $\operatorname{Var}\theta = 0.00038$ and $\rho = 0.821$. Both values are departures from the classic RIP specification, and he shows that the issues are linked, i.e., failure to account for heterogeneity in growth rates biases estimates of ρ toward 1. A similar linkage will arise when we consider models with autoregressive persistent components in Section 4. However, other work has noted assumptions that can bias evidence toward a model with HIP; e.g., Hoffmann (2019) points out that failure to account for life-cycle variation in σ^2_η would bias estimates of $\operatorname{Var}\theta$ upward.

An illustrative alternative. The relationship between σ^2_η and $\operatorname{Var}\theta$ can be further illustrated by considering a variant on the classic RIP model in which $E(\eta_{it}) = \theta_i$, rather than 0, with θ_i varying across individuals. Such a model is nearly equivalent to a HIP model (with $\rho = 1$), but from this perspective θ_i is envisioned as an uncertain forecast of future persistent earnings shocks instead of a steady growth rate.

This form suggests several insights. First, highlights the empirical challenge at the core of the RIP/HIP debate by framing it as the need to distinguish between (a) the cross-sectional variance $\operatorname{Var}\theta$ of agents' expected annual persistent shocks ($\theta_i = E(\eta_i)$) and (b) the variance $\sigma^2_{\eta i}$ of agents' potential shocks around their own expected values. Second, the relationship between those concepts helps to motivate our strategy of inferring the former via statistics originally intended to measure the latter. Third, interpreting θ_i as the mean of a random variable lends credence to the idea that workers may learn their own expected earnings growth rate slowly; Guvenen (2007) and Guvenen and Smith (2014) have considered such a mechanism to account for apparent inconsistencies in the timing of individuals' earnings shocks and consumption responses. More generally, this specification reminds us that forecasts of workers' future (realized) earnings growth rates may remain substantially uncertain even if they have full knowledge of their own θ_i, echoing Guvenen's (2009) comment that heterogeneous income profiles need not be viewed as a stark alternative to risk.

2.2 Measuring the risk of earnings shocks in the classic RIP specification

The statistics we analyze are computed from workers' residual earnings growth between times t and $t+k$,

$$\Omega_i(t, t+k) \equiv u_{i(t+k)} - u_{it}.$$

Even when we consider misspecifications, Ω is intended to represent the true unexpected change in the person's earnings. Under the classic RIP model, this is $\Omega_i(t, t+k) = \nu_{i(t+k)} - \nu_{it} + \sum_{j=1}^{k} \eta_{i(t+j)}$.

Permanent earnings risk. For our purposes, the key prediction is that, for any t and k and when $\min(j, q) \geq 3$, the expected value of a simple calculation equals the variance of permanent earning shocks:

$$E\left[\frac{1}{k}\Omega_i(t, t+k) \cdot \Omega_i(t-j, t+k+q)\right] = \sigma^2_{\eta i}. \tag{3}$$

Note that this is the product of residual earnings growth over two periods: a "short window" from t to $(t+k)$ and an overarching "long window" from $(t-j)$ to $(t+k+q)$. When the ends of those windows are far enough apart, the only shocks correlated across the two windows are the η's that arrive in the short window.

To allow this variance to change over time, Drewianka and Oberg (2019) suggest using the narrowest

[1] Many modifications and alternatives have been estimated as well. See Hoffmann (2019) for a thorough recent review.

possible interval k, so with biennial data they define statistics

$$\gamma_{itjq} \equiv \frac{1}{2}\Omega_i(t, t+2) \cdot \Omega_i(t-j, t+2+q), \tag{4}$$

which have expected value $\frac{1}{2}(\sigma^2_{\eta i(t+1)} + \sigma^2_{\eta i(t+2)})$. Meghir and Pistaferri (2004) previously used a similar strategy with annual data (i.e., $k = 1$) and $j = q = 2$. However, for a given (i, t) it is typically possible to compute many γ_{itjq} by varying the timing and length of the long window. Each of these calculations is, by itself, an extremely noisy (if unbiased) estimator of $\sigma^2_{\eta i(t+1)}$, but many can be combined (e.g., over a population thought to share a common value of σ^2_η) to produce far more precise estimates. For example, Meghir and Pistaferri postulate that σ^2_η is constant within demographic groups, so their moment condition effectively estimates that parameter by the groups' average values of γ_{it22}.[2]

While the empirical analysis below uses all available γ_{itjq} with $\min(j, q) \geq 3$, we should acknowledge that cases with $\min(j, q) = 2$ would also satisfy (3) under the common assumption that ν is an MA(1) process. Since the necessary data are more likely to be available for smaller values of j and q, this would allow us to compute and analyze many more γ statistics. On the other hand, unduly restrictive assumptions about the ν process would introduce significant risks as well. If ν were truly an MA(2) process, but we assumed it was MA(1), then γ_{it2q} and γ_{itj2} would be biased estimators of $\sigma^2_{\eta i}$, even though γ_{itjq} would be unbiased for other values of j or k. Moreover, including those biased γ's could confound estimation of the relationships between γ_{itjq} and $(j + k + 2)$ that are predicted in the discussion below. We have thus opted to maintain the less restrictive assumption (2), but see the Appendix for more evidence on the trade-offs involved.

Temporary earnings shocks. An analogous approach can be used to estimate $\sigma^2_{\nu it}$. If we define

$$\alpha_{itjq} \equiv -\Omega_i(t-j, t) \cdot \Omega_i(t, t+q),$$

the model implies $E[\alpha_{itjq}] = \sigma^2_{\nu it}$ when $\min(j, q) \geq 3$, so our analysis below restricts attention to those cases.

3 Biases from Misspecifing Income Profiles

Much of the interest in the RIP/HIP debate stems from common intuition: models that fail to account for heterogeneous earnings growth are apt to understate differences in workers' predictable earnings and to overstate the risk they face, but models that include unnecessary heterogeneous growth parameters have the opposite bias because those growth parameters are mistakenly identified from realized earnings shocks. The analysis here formalizes that reasoning and derives empirical biases that would arise from misspecification.

The key insight is that misspecification creates systematic relationships between γ (and α) statistics and the lengths of time over which they are computed. This recalls Hryshko's (2012) comparison of system parameters (especially σ^2_η) estimated via the method of moments across periods of different lengths, but there are notable differences. For one thing, our approach accommodates heterogeneity in σ^2_η, whereas some of Hryshko's estimates could (at least in principle) have been influenced by the assumption of homogeneity. Moreover, we can typically compute multiple γ statistics for a given (i, t), so they can serve as checks on one another; this raises our confidence that computations using longer period are not driven by (e.g.) trends in mean levels of risk or the evolution of risk over the life-cycle. We also consider estimates of σ^2_ν, which had

[2] Others (Carroll and Samwick (1997), Saks and Shore (2005), Drewianka (2010)) have posited $\sigma^2(X) = \beta X$ and estimated σ^2_{it} as fitted values from regressions on γ or similar statistics. This is identical to Meghir and Pistaferri's approach when X includes only a single set of dummy variables, but it generalizes the idea to allow σ^2 to vary across multiple factors.

escaped earlier scrutiny We thus view this analysis and the results below as extending earlier evidence.

3.1 Incorrectly omitting heterogeneous profiles

First, suppose the true earnings process has $Q_i(A_{it}) = \mu_i + \theta_i A_{it}$, but we instead estimate a model that restricts $\theta_i = 0$ for all i (i.e., a RIP model). Since $\Omega_i(t, t+k)$ is i's true residual earnings growth from time t to $t+k$, the growth estimated by the misspecified model is instead $\Omega_i(t, t+k) + \theta_i k$.

To find the bias in γ caused by the misspecification, it is convenient to denote the residual growth over the "short window" by $\Omega^s_{it} \equiv \Omega_i(t, t+2)$, residual growth over the "long window" by $\Omega^\tau_{itjq} \equiv \Omega_i(t-j, t+2+q)$, and long window length $\tau \equiv j+2+q$. Then using the misspecified residuals would cause us to compute

$$\begin{aligned} \gamma^*_{itjq} &= \left[\frac{\Omega^s_{it}}{2} + \theta_i\right]\left[\frac{\Omega^\tau_{itjq}}{\tau} + \theta_i\right] \cdot \tau \\ &= \gamma_{itjq} + \theta_i \left[\frac{\Omega^s_{it}}{2} + \frac{\Omega^\tau_{itjq}}{\tau}\right] \cdot \tau + \theta_i^2 \cdot \tau. \end{aligned} \tag{5}$$

The expected value of the middle term of (5) is zero because the model posits that $\Omega_{i\cdot}$ is orthogonal to θ_i. Nonetheless, the final term is necessarily positive and reflects the bias anticipated by the usual intuition.

Perhaps more surprisingly, the final term is also increasing in τ. To test this, we can average γ's with the same τ. While the realized values of the middle terms will not generally be zero, the law of large numbers implies their average value converges to zero. If we define M_x as a sample mean given x, it follows that

$$M\left[\gamma^*_{itjq} \,|\, \tau\right] \longrightarrow E[\gamma_{itjq} \,|\, \tau] + (\operatorname{Var}\theta) \cdot \tau.$$

If $E[\gamma_{itjq} \,|\, \tau]$ did not vary with τ, this leads to a straightforward empirical test: if the model incorrectly excluded heterogeneous trends, then $M\left[\gamma^*_{itjq} \,|\, \tau\right]$ would increase linearly in τ.

One potential objection is that $E[\gamma_{itjq} \,|\, \tau]$ may be correlated with τ across observations (i,t). Prior work has found that σ^2_η varies with observable characteristics (Meghir and Pistaferri (2004), Drewianka and Oberg (2019)), including some (e.g., age, years) that directly affect which combinations (j,q) can be used for a given (i,t). While this would not cause $E[\gamma_{itjq} \,|\, \tau]$ to vary with τ for a given (i,t), it may cause $E[\gamma_{itjq} \,|\, \tau]$ to be correlated with τ across observations (i,t), which could confound the proposed test. We can address that concern by subtracting the mean $M_{it}(\gamma^*_{itjq})$ across all available (j,q) for observation (i,t) in order to remove cross-sectional variation. For any variable x_{itjq}, define $\widetilde{x_{itjq}} \equiv x_{itjq} - M_{it}(x_{itjq})$. Then[3]

$$\widetilde{\gamma^*_{itjq}} = \widetilde{\gamma_{itjq}} + \theta_i \widetilde{\Omega^\tau_{itjq}} + \left[\theta_i \frac{\Omega^s_{it}}{2} + \theta_i^2\right] \cdot \widetilde{\tau_{itjq}}.$$

For each (j,q), the expected values of $\widetilde{\gamma_{itjq}}$, $\theta_i \widetilde{\Omega_i}$, and $\theta_i \Omega^s_{it}$ are all zero. Thus, by the same reasoning as above, for any given $\tau_0 \equiv (j_0 + 2 + q_0)$, the sample mean

$$M\left[\widetilde{\gamma^*_{itjq}} \,|\, \tau_0\right] \rightarrow (\operatorname{Var}\theta) \cdot \left[\tau_0 - M\left[M_{it}(\tau) \,|\, \tau_0\right]\right]. \tag{6}$$

An increasing relationship between that average and τ_0 would thus be an indication that the RIP specification

[3] Note that since $M_{it}(\tau)$ is a constant for each (i,t), $\widetilde{\tau} \equiv \tau - M_{it}(\tau)$ is not identically 0.

was insufficient, regardless of whether σ_η^2 were heterogeneous. We could also estimate $\operatorname{Var}\theta$ by β_1 in

$$\widetilde{\gamma_{itjq}^*} = \beta_0 + \beta_1 \left(\widetilde{j+q}\right). \tag{7}$$

Likewise, using residuals that failed to account for (true) heterogeneity profiles would lead us to compute

$$\alpha_{itjq}^* = \alpha_{itjq} - \theta_i \left[\frac{\Omega_i(t-j,t)}{j} + \frac{\Omega_i(t,t+q)}{q}\right] \cdot jq - \theta_i^2 \cdot jq.$$

Since $E[\theta_i \cdot \Omega_i] = 0$, it follows that $E\left(\alpha_{itjq}^*|i,t\right) = E(\alpha_{itjq}|i,t) - \theta_i^2 M_{it}(jq) < E(\alpha_{itjq}|i,t) = \sigma_{\nu it}^2$ – i.e., errant omission of heterogeneous trends would cause us to underestimate σ_ν^2. As in (6), for each $\tau_0' \equiv (jq)_0$

$$M_{jq}\left[\widetilde{\alpha_{itjq}^*}|\tau_0'\right] \rightarrow -(\operatorname{Var}\theta) \cdot \left[\tau_0' - M\left[M_{it}\left(\tau'\right) \mid \tau_0'\right]\right],$$

again suggesting that $\operatorname{Var}(\theta)$ is estimated by the coefficient β_1 the analogous regression

$$\widetilde{\alpha_{itjq}^*} = \beta_0 + \beta_1 \left(-\widetilde{jq}\right). \tag{8}$$

3.2 Misestimation of heterogenous profiles

Now we investigate the effect of errors in the estimation of individual growth rates. This includes the special case in which there is no heterogeneity ($\theta_i = 0$ for all i), but it also allows for estimation error arising from the short periods over which earnings panels are often observed. Suppose we allow for heterogeneous growth and estimate the $Q_i\left(A_{it}\right)$ component from (1) via the same preliminary regression used to remove the F component, leading to the estimate $\widehat{Q_i} \equiv \widehat{\mu_i} + \widehat{\theta}_i A_{it}$. Since $\widehat{Q_i}$ is orthogonal to u_{it}, this would be reasonable if panels were long enough, but more problematic with short panels. There would still be no harm from mismeasuring μ_i since the error would be removed when the data are differenced, but measurement error in the slope ($\widehat{\theta}_i - \theta_i$) is troublesome because $\widehat{\theta}_i$ (unlike the true θ_i) will be positively correlated with the Ω_i's – indeed, this is the fear that realized shocks will be mistaken for expected growth.

To derive the resulting bias, the analogue to (5) is

$$\begin{aligned}\gamma_{itjq}^{**} &= \left[\frac{\Omega_{it}^s}{2} - \left(\widehat{\theta}_i - \theta_i\right)\right]\left[\frac{\Omega_{itjq}^\tau}{\tau} - \left(\widehat{\theta}_i - \theta_i\right)\right] \cdot \tau \\ &= \gamma_{itjq} + \left[\frac{\Omega_{it}^s}{2} + \frac{\Omega_{itjq}^\tau}{\tau}\right] \cdot \theta_i \cdot \tau - \left[r_{it}^s + r_{itjq}^\tau\right] \cdot \widehat{\theta}_i \cdot \tau + \left[\theta_i^2 - \left(\widehat{\theta}_i\right)^2\right]\tau,\end{aligned} \tag{9}$$

where $r_{it}^s \equiv \left[\Omega_{it}^s/2 + \theta_i - \widehat{\theta}_i\right]$ and $r_{itjq}^\tau \equiv \left[\Omega_{itjq}^\tau/\tau + \theta_i - \widehat{\theta}_i\right]$ are i's estimated residual earnings growth per year over the short and long periods. Since r_{it}^s and r_{itjq}^τ are differences between residuals computed from the same earnings regression used to estimate $\widehat{\theta}_i$, $M_i\left[\left(r_{it}^s + r_{itjq}^\tau\right) \cdot \widehat{\theta}_i\right] = 0$ by construction, and the model again predicts $E\left[\theta_i \cdot \Omega_{itjq}\right] = 0$. Thus, (9) implies

$$E[\gamma_{itjq}^{**}] = \gamma_{itjq} + \left(\theta_i^2 - \widehat{\theta}_i^2\right) \cdot \tau.$$

We thus proceed as above to consider differences $\widetilde{\gamma^{**}_{itjq}} \equiv \gamma^{**}_{itjq} - M_{it}[\gamma^{**}_{itjq}]$. The counterpart to (6) is[4]

$$M\left[\widetilde{\gamma^{**}_{itjq}} \mid \tau_0\right] \to \left[\operatorname{Var}\theta - E\left(\widehat{\theta}^2\right)\right] \cdot \left[\tau_0 - M\left[M_{it}(\tau) \mid \tau_0\right]\right].$$

Thus, if the empirical model unnecessarily included heterogeneous profiles (i.e., when $\operatorname{Var}\theta = 0$), we would observe $M\left[\widetilde{\gamma^{**}_{itjq}}|\tau_0\right]$ to be *decreasing* in τ_0 — the opposite of what we predict if the model had instead wrongly excluded heterogeneous trends. As above, this suggests estimating $\operatorname{Var}\theta - E\left(\widehat{\theta}^2\right)$ by β_1 in

$$\widetilde{\gamma^{**}_{itjq}} = \beta_0 + \beta_1\left(\widetilde{j+q}\right). \tag{10}$$

The estimation error would also cause us to compute

$$\alpha^{**}_{itjq} = \alpha_{itjq} - \left[\frac{\Omega_i(t-j,t)}{j} + \frac{\Omega_i(t,t+q)}{q}\right] \cdot \theta_i \cdot jq + (r_{itj0} + r_{it0q}) \cdot \widehat{\theta}_i \cdot jq + \left[\left(\widehat{\theta}_i\right)^2 - \theta_i^2\right] \cdot jq,$$

where r_{itj0} and r_{it0q} are the estimated residual earnings growth rates in the periods $(t-j,t)$ and $(t,t+q)$. As before, $E[\theta_i \cdot \Omega_i] = 0$ and $E[\widehat{\theta}_i \cdot r] = 0$, so we could again estimate $\operatorname{Var}\theta - E\left(\widehat{\theta}^2\right)$ by $\beta_1$in

$$\widetilde{\alpha^{**}_{itjq}} = \beta_0 + \beta_1\left(-\widetilde{jq}\right). \tag{11}$$

3.3 Unwarranted heterogeneous profiles may also cause severe underestimation of σ^2_η

This analysis also presents an opportunity to investigate the relationship between estimates of the variances of heterogeneous income profiles and permanent shocks, i.e., $\operatorname{Var}\theta$ and σ^2_η. As Guvenen (2007, 2009) notes, empirical studies tend to support heterogeneous profiles or highly persistent shocks ($\rho \approx 1$), but not both. This empirical regularity is intuitively understood to reflect the difficulty in distinguishing between highly persistent shocks and steady growth, and the analysis above confirms that intuition, at least in part. For example, if the true model has RIP ($\operatorname{Var}\theta = 0$), then Equation (9) implies that including unwarranted heterogeneous profiles would cause $\gamma^{**}_{it} < \gamma_{it}$. Since the latter is the moment statistic used to measure σ^2_η, overestimation of $\operatorname{Var}\theta$ would directly attenuate estimates of σ^2_η.

We now show this bias is apt to be severe. We begin by explaining why including unnecessary heterogeneous profiles causes γ^{**} to understate the desired quantity γ. To maintain generality, suppose we were interested in a weighted average of the γ^{**}_{itjq} for a given (i,t): $\gamma^{**}_{it} \equiv \sum_{j,q} \gamma^{**}_{itjq} \cdot \omega_{jq}$, and define ℓ and Ω^ℓ_{it} as the corresponding weighted-averages of $(j+2+q)$ and $\Omega_i(t-j,t+2+q)$. Then

$$\gamma^{**}_{it} = \ell\left[\frac{\Omega^s_{it}}{2} - \widehat{\theta}_i\right]\left[\frac{\Omega^\ell_{it}}{\ell} - \widehat{\theta}_i\right]. \tag{12}$$

Intuitively, this calculation is apt to produce a value near 0 because $\widehat{\theta}_i$ and Ω^ℓ_{it}/ℓ are similar conceptually and likely quantitatively, suggesting $\left[\frac{1}{\ell}\Omega^\ell_{it} - \widehat{\theta}_i\right] \approx 0$. More formally, define $\Delta_i \equiv \{(\tau,\tau') \mid \Omega_i(\tau,\tau') \text{ is observed}\}$,

[4] In principle, one might imagine $E(\widehat{\theta}) = 0$ and thus $E\left(\widehat{\theta}^2\right) = \operatorname{Var}\widehat{\theta}$. However, while the former condition holds by construction in the sample used to estimate the earnings model, it is not guaranteed in the sample of γ statistics because γ_{it} cannot always be computed when y_{it} is available (the calculation also uses both pre- and post-period earnings). In practice, we have found $E\left(\widehat{\theta}^2\right)$ is only slightly larger than $\operatorname{Var}\widehat{\theta}$.

and let $D_{it} \equiv \{d \in \Delta_i \,|\, \tau \leq t-3, \tau' \geq t+5)\}$ be the subset where γ_{itjq} can be computed. Then Subrahmanyam's (1972) Lemma 1 implies a least-squares estimate

$$\widehat{\theta}_i = \sum_{(\tau,\tau')\in\Delta_i} \frac{\Omega_i(\tau,\tau')}{\tau'-\tau} \cdot \omega^a_{\tau\tau'} \ , \tag{13}$$

where weights $\omega^a_{\tau\tau'} \equiv (\tau'-\tau)^2 \,/\, \left[\sum_{(\tau,\tau')\in\Delta_i} (\tau'-\tau)^2\right]$. Likewise,

$$\frac{\Omega^\ell_{it}}{\ell} = \frac{\sum\limits_{(\tau,\tau')\in D_{it}} \Omega_i(\tau,\tau')\,\omega_{\tau\tau'}}{\sum\limits_{(\tau,\tau')\in D_{it}} (\tau'-\tau)\,\omega_{\tau\tau'}} = \sum_{(\tau,\tau')\in D_{it}} \frac{\Omega_i(\tau,\tau')}{\tau'-\tau} \cdot \omega^b_{\tau\tau'} \ ,$$

where $\omega^b_{\tau\tau'} \equiv \omega_{\tau\tau'} \cdot (\tau'-\tau) \,/\, \left[\sum_{(\tau,\tau')\in D_i} \omega_{\tau\tau'} \cdot (\tau'-\tau)\right]$. Thus, $\widehat{\theta}_i$ is a weighted average of i's mean earnings growth across all observed intervals, and Ω^ℓ_{it}/ℓ is a differently-weighted average over a subset of the same numbers (i.e., observed earnings growth over the "long windows") that includes those weighted most heavily in $\widehat{\theta}_i$ (i.e., the longest periods). While not algebraically equivalent, the similarities suggest $\widehat{\theta}_i \approx \Omega^\ell_{it}/\ell$ is a reasonable benchmark, and in that case (12) implies $\gamma^{**}_{it} \approx 0$ regardless of the true value of γ_{it}. In other words, the inclusion of unwarranted heterogeneous profiles would attenuate estimates of σ^2_η, likely severely.

4 Modifications if persistent shocks are autoregressive

A widely-used variation on this model allows π to be an AR(1) process: $\pi_{it} = \rho\pi_{i(t-1)} + \eta_{it}$ with $\rho \equiv e^{-\delta} < 1$. To be clear, this would not directly affect the biases analyzed above, i.e., those arising from misspecification of the income profiles Q_i, which are orthogonal to π in any event. Even so, falsely restricting $\rho = 1$ may cause us to misinterpret the evidence: the gradual weakening of π_{it} over a subsequent interval would create a steady change in i's wage that could confound identification of income profiles, especially if π were large when i was first observed. Guvenen (2009) used a conceptually similar point to rebut the evidence MaCurdy (1982) and Abowd and Card (1989) had presented in favor of RIP models.

Under this new assumption (but assuming for now that Q_i is correctly specified),

$$\begin{aligned}\gamma_{itjq} = \frac{1}{k}\Bigg\{ &\left[\left(\rho^k - 1\right)\rho^j \pi_{i(t-j)} + \left(\rho^k-1\right)\sum_{a=0}^{j-1}\rho^a\eta_{i(t-a)} + \sum_{a=0}^{k-1}\rho^a\eta_{i(t+k-a)} + \nu_{i(t+k)} - \nu_{it}\right] \\ &\times \left[\left(\rho^{k+q+j}-1\right)\pi_{i(t-j)} + \sum_{a=0}^{k+q+j-1}\rho^a\eta_{i(t+k+q-a)} + \nu_{i(t+k+q)} - \nu_{i(t-j)}\right]\Bigg\}, \end{aligned} \tag{14}$$

where the first term in square brackets is $\Omega_i(t, t+k)$ and the second is $\Omega_i(t-j, t+k+q)$.[5] Many of the terms that arise when the forms are multiplied have mean 0, and we can simplify the expression further by defining $s_i(a,b)$ as the following weighted average of $\sigma^2_{\eta i\tau}$ between times a and b:

$$s_i(t-j+1, t) \equiv \sum_{a=0}^{j-1} \omega^{(j)}_{(t-a)} \sigma^2_{\eta i(t-a)}, \tag{15}$$

[5] Our empirical analysis uses only $k = 2$, but here we keep the more general form to distinguish k from 2's that arise algebraically.

where $\omega_{(t-a)}^{(j)} \equiv \rho^{2a} / \left[\sum_{b=0}^{j-1} \rho^{2b}\right]$. Using that notation, equation (14) implies

$$E[\gamma_{itjq}] = \left[\frac{\left(1-\rho^{k+q+j}\right)\left(1-\rho^{k}\right)\rho^{j}}{k} E\pi_{i(t-j)}^{2}\right] - \left[\frac{\left(1-\rho^{k}\right)\left(1-\rho^{2j}\right)\rho^{k+q}}{k(1-\rho^{2})} \cdot s_i(t-j+1,t)\right] + \left[\frac{\rho^{q}\left(1-\rho^{2k}\right)}{k(1-\rho^{2})} \cdot s_i(t+1,t+k)\right]. \tag{16}$$

Two features warrant brief remarks. First, note that we have not assumed σ_η^2 is constant either within or between panels. While the expression could easily accommodate that restriction, some prior work has found significant cross-sectional variation (e.g., Meghir and Pistaferri (2004)), and life-cycle variation is central to Hoffmann's (2019) analysis, so it seems unwise to foreclose such possibilities prematurely. Second, while one might write $E\pi_{i(t-j)}^2$ in terms of $\sigma_{\eta i\tau}^2$ for $\tau \leq t-j$, we will leave it in the more compact form for now and revisit the issue in the next subsection.

Equation (16) also offers an opportunity to explicate the mechanics of the γ statistics. The three terms reflect the roles of persistent shocks η that arrive during three different periods: respectively, the period up to time $t-j$, the period between $t-j+1$ and t, and the period between $t+1$ and $t+k$ (the "short window"). Shocks (and initial conditions) from the first period produce a persistent component $\pi_{i(t-j)}$ at time $t-j$, which shrinks to $\rho^j \pi_{i(t-j)}$ when the short window begins and contributes $-\left(1-\rho^k\right)\rho^j \pi_{i(t-j)}$ to the person's earnings growth during that period. Likewise it contributes $-\left(1-\rho^{k+q+j}\right)\pi_{i(t-j)}$ to growth during the "long window" (from $t-j$ to $t+k+q$). The expected value of that product (divided by k) is thus the first term in (16). Since shocks that arrive during the second period also arrive by time t, their only effect on earnings growth during the short window reflects the autoregression that occurs. A positive η from that period thus contributes negatively to growth during the short window, even though it contributes positively to growth during the long window – thus leading to the negative second term in (16). Shocks that arrive during the third period are those the statistic has always been intended to capture. Their contributions to earnings growth in the short and long windows are in the same direction, so they contribute positively to $E[\gamma_{itjq}]$ – the third term in (16). The "long window" also includes shocks arriving between $t+k+1$ and $t+k+q$, but they have no effect on $E[\gamma_{itjq}]$ because they arrive after the short window closes.

Since the short window covers a narrow period, it is reasonable to treat $\sigma_{\eta i}^2$ as constant over that range, i.e., $s_i(t+1,t+k) = \sigma_{\eta i(t+1)}^2$. Second-order Taylor approximations of terms involving ρ then imply

$$\begin{aligned} E[\gamma_{itjq}] &\approx \left[\delta^2 E\pi_{i(t-j)}^2 \cdot (j+k+q)\right] - \left[\delta \cdot s_i(t-j+1,t) \cdot j\right] + \left[\sigma_{\eta i(t+1)}^2 - \delta \cdot \sigma_{\eta i(t+1)}^2 \cdot (k+q)\right] \\ &= \sigma_{\eta i(t+1)}^2 \; + \; \left[\delta^2 E\pi_{i(t-j)}^2 - \delta\sigma_{\eta i(t+1)}^2\right] \cdot (j+k+q) \; + \; \delta\left[\sigma_{\eta i(t+1)}^2 - s_i(t-j+1,t)\right] \cdot j. \end{aligned} \tag{17}$$

The second term in (17) poses a significant challenge because it is linear in $(j+q)$. Our earlier analysis indicated the slope (β_1) of γ_{tjq}^* in $(j+q)$ would estimate $\operatorname{Var}\theta$, but here we see that with autoregressive π, that coefficient would estimate $\operatorname{Var}\theta + \delta^2 E\pi^2 - \delta\sigma_\eta^2$. Thus, any given value of β_1 would be consistent with a continuum of values of $(\operatorname{Var}\theta, \delta)$.

The final term in (17) is apt to be much less consequential. Indeed, it would equal 0 if σ_η^2 were constant within panels. Even if σ_η^2 evolved gradually over time and/or the life-cycle, the term is still likely to be small because $s_i(t-j+1,t)$ is a weighted average of the $\sigma_{\eta i\tau}^2$ for the periods immediately before $(t+1)$ and because is multiplied by $\delta \approx 1-\rho$, which is itself apt to be small.

When we consider the possibility of wrongly excluded heterogeneous earnings profiles, $\operatorname{Var}\theta$ would added to the coefficient on $(j+k+q)$ in (17). We had initially hoped the α statistics might provide leverage to break this logjam, but (despite measuring different parameters and being computed differently) α statistics suffer from biases similar to those of the γ statistics. The parallel analysis to (17) yields

$$E[\alpha_{itjq}] \approx \sigma^2_{\nu it} + \left[\delta^2 E\pi^2_{i(t-j)} - \delta s_i(t-j+1,t)\right] \cdot (-jq)\,. \tag{18}$$

Thus, the model predicts the slope coefficient in (18) equals the sum of the two slope coefficients in (17): $\partial E[\alpha_{itjq}]/\partial(-jq) = \partial E[\gamma_{itjq}]/\partial j$. Accordingly, the α statistics offer no additional help, so we shall develop a different empirical strategy below.

4.1 Remarks on the persistent component at time $(t-j)$

Before going further, we pause to discuss the role of the term $\delta^2 \operatorname{Var}\pi_{i(t-j)}$. The specification implies

$$\begin{aligned} \delta^2 E\pi^2_{i(t-j)} &= \delta^2\rho^{2(t-j)}\pi^2_{i0} + \delta^2\left(\frac{1-\rho^{2(t-j)}}{1-\rho^2}\right) s_i(1,t-j) \\ &\approx \rho^{2(t-j)} \cdot \left[\delta^2\pi^2_{i0}\right] + \left(1-\rho^{2(t-j)}\right) \cdot \left[\frac{\delta}{2} \cdot s_i(1,t-j)\right], \end{aligned} \tag{19}$$

where π_{i0} is the worker's initial persistent component, which we take as given. Yet despite the relative simplicity of this expression, there is some ambiguity about how it should be interpreted in practice.

One reason is that it is not clear what constitutes an initial period. It could reasonably correspond to (e.g.) workers' birth, entry into the labor market, or first appearance in the data. The distinction was irrelevant when π was taken to have a unit root; the γ statistics are built from differences in earnings over fixed periods, so they remove all contributions from before that period began. However, when π is autoregressive, differencing does not fully remove such contributions because they are larger at the beginning of the period than at the end – which is why (17) involves $\pi_{i(t-j)}$ at all.

There is also the related problem of what to expect about the initial conditions π_{i0}. While persistent earnings shocks η are often imagined to involve career-related events like promotions, layoffs, occupational migration, or the outcomes of job-searches, π_{i0} could also include early-life factors (e.g., family background or social networks), at least insofar as their effects fade over time. Since this would reflect a qualitatively different source of variation, the magnitude of $\delta\pi^2_{i0}$ may differ markedly from that of $\sigma^2_{\eta i\tau}$ (for any τ).

Further, if initial conditions were quantitatively important, they would act much like heterogeneous earnings profiles. Since their influence would gradually decline, they would create worker-specific earnings trends in the opposite direction of their initial earnings, which is typically what HIP models have found as well. In our view, this should not be viewed as a challenge to HIP models, but rather as a factor that helps to generate them (without precluding others). After all, both "autoregressing initial conditions" and "individual growth rates" are black-box descriptions of heterogeneity in workers' earnings growth rates, and there is little reason to distinguish between them unless they can be attributed to more concrete factors.[6]

Finally, the estimation strategy we develop below requires us to consider the relationship between $E\pi^2_{i(t-j)}$ and $\sigma^2_{\eta i(t+1)}$. While we prefer to remain as agnostic as possible, two benchmark cases will establish bounds on key parameters. First, $E\pi^2_{i(t-j)}$ would be *most* closely related to $\sigma^2_{\eta i(t+1)}$ if $t-j$ were large (so $\rho^{2(t-j)} \rightarrow$

[6] Along similar lines, Guvenen (2009) has investigated the consequences of assuming $\pi_0 = 0$; under realistic parameter values, he shows that relaxing that assumption would have very little quantitative impact on his empirical conclusions.

0) and σ_η^2 were constant within panels (so $s_i(1,t-j) \approx \sigma_{\eta i}^2$). In that case, equation (19) would imply $\delta^2 E\pi_{i(t-j)}^2 \approx \delta\sigma_{\eta i}^2/2$. At an opposite extreme, $E\pi_{i(t-j)}^2$ would be *least* closely related to $\sigma_{\eta i(t+1)}^2$ if $t-j$ were small ($\rho^{2(t-j)} \to 1$) and/or $s_i(1,t-j)$ were uncorrelated with $\sigma_{\eta i(t+1)}^2$ (i.e., if σ_η^2 varied considerably over time within panels). In either of those cases, $\partial E\pi_{i(t-j)}^2/\partial\sigma_{\eta i(t+1)}^2 \approx 0$.

4.2 Estimation strategy

Equation (17) above shows that $E[\gamma_{itjq}]$ is approximately a linear function of $(j+k+q)$ and j, with coefficients that are functions of key elements of the theoretical model. If we imagined all the variances were equal across observations, one might attempt to recover estimates of them via a regression

$$\gamma_{itjq} = \beta_0 + \beta_1 \cdot (j+k+q) + \beta_2 \cdot j + \varepsilon_{itjq}. \tag{20}$$

This is not a straightforward task, however. While β_0 would still estimate σ_η^2, the model now predicts β_1 is a function of σ_η^2, $\operatorname{Var}\theta$, $\operatorname{Var}\pi_0$, and δ. Even if (as suggested above) $\operatorname{Var}\pi_0$ were incorporated into a revised $\operatorname{Var}\theta$ term, there would still be too many parameters to recover from estimates of (β_0, β_1) alone.[7]

The task can be made feasible by allowing heterogeneity in the coefficients in (20), such as if we specify

$$\begin{aligned} \gamma_{itjq} &= \beta_0 + \beta_1 \cdot (j+k+q) + \beta_2 \cdot j + \varepsilon_{itjq}, && \text{with} \\ \varepsilon_{itjq} &= \beta_{0it} + \beta_{1it} \cdot (j+k+q) + e_{itjq}, && \text{and} \\ \begin{pmatrix} \beta_{0it} \\ \beta_{1it} \end{pmatrix} &\sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \phi_{00} & \phi_{10} \\ \phi_{10} & \phi_{11} \end{pmatrix}\right]. \end{aligned} \tag{21}$$

In principle, the theoretical model (see (17)) could also justify heterogeneous β_2, though we argued the term is likely small in any event.[8]

Our strategy for recovering some key parameters begins by noting that the model implies $\beta_0 + \beta_{0it} = \sigma_{\eta i(t+1)}^2$ and $\beta_1 + \beta_{1it} = \theta_i^2 + \delta^2 E\pi_{i(t-j)}^2 - \delta\sigma_{\eta i(t+1)}^2$. If we assume $\partial E\pi_{i(t-j)}^2/\partial\sigma_{\eta i(t+1)}^2 = 0$ (as in the second benchmark case above), it follows that $\phi_{10} = -\delta \operatorname{Var}\left[\sigma_{\eta(t+1)}^2\right]$. The statistic $D \equiv -\phi_{10}/\phi_{00}$ would then estimate δ, and $(\beta_1 + D \cdot \beta_0)$ would estimate $\operatorname{Var}\theta + \delta^2 E \operatorname{Var}\pi$.

More generally, if $\partial E\pi_{i(t-j)}^2/\partial\sigma_{\eta i(t+1)}^2 \equiv \kappa$, then $\phi_{10} = -\,\delta \operatorname{Var}\sigma_{\eta i(t+1)}^2 \cdot [1-\delta\kappa]$ and $D = \delta\,[1-\delta\kappa]$. The benchmark cases considered above implied $\kappa \in [0, 1/2\delta]$, so D and $2D$ are lower and upper bounds on δ. Moreover, $(\beta_{1it} + D \cdot \beta_{0it})$ still computes the component of β_{1it} that is orthogonal to β_{0it}. Its aggregate analogue $(\beta_1 + D \cdot \beta_0)$ thus forms an upper bound on $\operatorname{Var}\theta$:

$$E\left(\beta_1 + D \cdot \beta_0\right) \equiv \operatorname{Var}\theta + \delta^2\left[E \operatorname{Var}\pi - \kappa \cdot E\sigma_\eta^2\right] \equiv E\left(\theta_*^2\right).$$

One potential objection is that realized persistent shocks η are unique for each (i,t), so the observed data could only reflect the squared realized shock $\overline{\eta_{i(t+1)}^2} \equiv \left[\rho^2\eta_{i(t+1)}^2 + \eta_{i(t+2)}^2\right]/(1-\rho^2)$, rather than the variance $\sigma_{\eta i(t+1)}^2$ of potential shocks. However, this point does not fundamentally challenge the identification strategy. The key theoretical prediction about the relationship between ϕ_{10} and ϕ_{00} arises algebraically

[7] If all variances were truly homogeneous, the model would imply $s = \sigma_\eta^2$ and thus $\beta_2 = 0$, but we have retained the $\beta_2 \cdot j$ term here because it will reappear when that assumption is relaxed. Even so, our discussion shall largely ignore that term because our empirical estimates will ultimately indicate that it is small and statistically insignificant.

[8] Note that the distributional assumption essentially posits that *variances* $\sigma_{\eta it}^2$ are normally distributed across (i,t), not that *shocks* η are normal for any given (i,t). Previous work (see, e.g., Guvenen et al (2023)) has often rejected a normal distribution for η, but that is immaterial here because we make no distributional assumptions about η itself.

because the approximation of the final term in (16) contributes to both β_{0it} and β_{1it}, and that relationship will hold even if observed $\overline{\eta^2_{i(t+1)}}$ differ substantially from $\sigma^2_{\eta i(t+1)}$. The same insight provides motivation for specification (21) as well: it may be appropriate even if workers truly faced homogeneous risk ($\sigma^2_{\eta i} = \sigma^2_{\eta}$) because differences in realized outcomes would still cause heterogeneity in $(\beta_{0it}, \beta_{1it})$. Moreover, since $\overline{\eta^2_{i(t+1)}}$ would likely predict $\pi^2_{i(t-j)}$ poorly ($\kappa \approx 0$), this consideration suggests D may be closer to δ than $\delta/2$.

5 Data

The data we shall use to examine these ideas is a sample of men from the 1970-2015 waves of the Panel Study of Income Dynamics (PSID (2017)) who were between 22 and 69 years old when they were surveyed. We consider men rather than women both because some variables are reported more regularly for men and because men are more likely to be strongly attached to the labor force. To avoid mistaking planned transitions for unexpected shocks, we drop observations in which respondents report their main activity as "student" or when the man exits the labor force within the next two years. Where available, we have used the version of the PSID that was recoded for use by the Cross-National Equivalent File (CNEF (2013); see Burkhauser et al. (2000)) in order to reduce the risk of errors and ensure consistency across waves.[9]

Most of the analysis includes men whose families originally came from the Survey of Economic Opportunity (SEO; see Hill (1992)), an oversample of households that were impoverished in the mid-1960s. While it has been more common to exclude these observations, they have been used in some major studies in the literature, including Abowd and Card (1989) and Meghir and Pistaferri (2004). In addition to increasing sample sizes (including a four-fold increase in the number of non-white men), we imagined that including this sample would increase heterogeneity in earnings profiles; since we shall find little heterogeneity anyhow, this can be viewed as part of an *a fortiori* argument. On the other hand, the SEO may well be more representative than one might expect. Drewianka (2010) shows that race-specific distributions of several key variables (earnings, hours, education, etc.) are quite similar in the SEO and the main PSID sample, and Meghir and Pistaferri (2004) note that most remaining discrepancies between those samples are removed when data are differenced (as in the calculations we make). Regardless, we will also present some results that exclude these observations to show that the conclusions are robust.

The variable of interest is the men's log earnings, which we analyze on annual, weekly, and hourly bases. One motive for considering all three measures is the idea that they can cross-validate evidence of heterogeneity in expected earnings growth. Since it is hard to imagine practical conditions that would cause steady long-term *trends* in individuals' hours worked,[10] we would expect heterogeneity in workers' expected hourly earnings growth to be mirrored in their weekly and annual hours as well, and major discrepancies would raise some concern. Each measure is computed in 2015 dollars (using the Consumer Price Index for All Urban Consumers (2017)), and the minimum real-valued top code for annual earnings ($359,117) is imposed across all survey waves so that changes in top codes will not be mistaken for large earnings shocks. We impose slightly different criteria for including these observations, the most important being that annual earnings (but not weekly or hourly earnings) are dropped when the individual reports being out of the labor force at the time of the survey, since their annual earnings were likely affected by their withdrawal. We also

[9] One notable exception is that we have ensured a one-to-one relationship between respondents' ages and years. The original data contains some inconsistencies, so we calculated workers' apparent birth years (minimum (year - reported age) across waves), then used (year - birth year) as their age. In addition, we assigned workers to racial groups using an algorithm that favors self-reports and uses consistent categorization across waves; the main effect is that more people are identified as Hispanic.

[10] In contrast, it is easy to imagine why an individual's hours or weeks worked might change either temporarily (e.g., unemployment) or persistently (e.g., transitions to new jobs with different schedules or seasonality).

drop annual earnings reports of less than \$500, weekly wage data for those with fewer than 5 weeks worked or who earned less than \$100/week, and hourly wages that are less than \$3/hour or based on fewer than 200 annual hours.

Since the analysis uses a statistic (γ) computed from four earnings observations for the same worker, the matter of sample sizes is more complex than usual. The sample described above includes 141,686 observations on men's annual earnings, but only 114,739 of those observations come from men for whom it is ever possible to create a γ statistic for any measure of earnings. The rest can still be used to estimate aggregate age-earnings profiles (F from equation (1)), but they play no further role in the analysis. Further, γ_{it} can be computed for at least one measure of earnings for only 65,347 person-years; the remaining observations for those workers can help to compute γ in the role of $u_{i(t-j)}$, $u_{i(t+k)}$, or $u_{i(t+k+q)}$, but not as u_{it} because the data set does not include observations that could serve in the other roles.

This illustrates an implicit sample inclusion rule caused by the formula for the γ statistic. While we do not formally require men to appear in the sample for a lengthy period, a similar effect arises from the data requirements to compute a γ statistic. Since we use $k = 2$ and need j and $q \geq 3$ to satisfy condition (2), the worker's earnings must observed at least four times over at least 9 years to compute even one γ, or at least 11 years after the PSID became biennial in 1997.[11] Admittedly, men who participate in the labor force and the survey so steadily may differ somewhat from those who cannot, especially if the risk of earnings shocks is correlated (perhaps causally) with non-participation and/or attrition, so this requirement may reduce the sample's cross-sectional representativeness. Still, we have aimed not to restrict the sample beyond what is required for feasibility, and in many ways our sample is more inclusive than those in most prior studies. For example, most studies only consider men who are continuously employed through a series of consecutive survey waves (often ten or more), or at least who report earnings in a large number of years, and some add further restrictions on things like their race or marital status.

Among the 7,486 men for whom any γ can be computed, we are able to compute an average of about 400 unique γ_{tjq} statistics per measure of earnings over 8.6 survey years t. Because the γ statistics require several years of data, the annual earnings of such men are observed in an average of 15.3 survey waves (which become biennial starting in 1997), starting at an average age of 28.7 (25.4 for those who entered the sample after the first wave), and ending at an average age of 49. The data requirements also limit the extent of sample attrition at younger ages. Among those men, 33 percent continue to report earnings in the final wave, and nearly as many others (32 percent of the total) are last observed after age 50.

Table I reports summary statistics for all earnings observations for those 7,486 men, as well as some demographic characteristics. The average age at which their earnings are observed is 38.6, and a slight majority were born between 1944 and 1960, with the remainder split roughly equally before and after that era. The sample appears to be moderately more educated than the actual population, though it is difficult to quantify precisely given the sharp increase in education over this period. Black men comprise a quarter of the sample due to the inclusion of the SEO data, but only seven percent when the SEO is excluded. Asians and Hispanics are underrepresented relative to the current U.S. population because the PSID households were chosen in the mid-1960s (prior to subsequent waves of immigration), though the PSID did add a special "immigrant sample" in 1997 that accounts for 2 percent of our observations.

[TABLE I ABOUT HERE]

[11] In principle, prior to 1997 one could make an analogous computation with $k = 1$, call it $\gamma^1_{itjq} \equiv \Omega_i(t, t+1) \cdot \Omega_i(t-j, t+1+q)$. However, this would have little impact on our analysis because our γ statistics (see (4)) are just simple averages of two of those alternate calculations with equally wide "long windows": $\gamma_{itjq} = \left(\gamma^1_{itj(q+1)} + \gamma^1_{i(t+1)(j+1)q}\right)/2$.

Table I: Sample Statistics

Dependent Variables	Mean	SD
log earnings/year	10.74	0.81
log earnings/week	6.96	0.67
log earnings/hour	3.15	0.63
Independent Variables	**Mean**	**SD**
Age	38.6	10.9
Years of Education	13.4	2.6
Race	**Number**	**Share**
White	74,550	65.0
Black	30,189	26.3
Hispanic	5,871	5.1
Native American	2,481	2.2
Asian	970	0.8
Other	678	0.6
Years of Education		
Less than 12	14,765	12.9
12	38,397	33.5
13-15	28,032	24.4
16	16,794	14.6
More than 16	16,751	14.6
Sample		
Main	77,810	67.8
SEO	36,929	32.2
Birth Cohort		
Before 1944	29,441	25.7
1944-1952	31,951	27.8
1953-1960	29,595	25.8
1961 and after	23,752	20.7
Total Observations	114,739	
Unique Individuals	7,486	

Notes: These statistics include all observations reporting non-zero next-year earnings for men for whom at least one γ statistic can be calculated. The data come from the 1970-2015 waves of the PSID, subject to the sample inclusion criteria described in the text. Years of education is the maximum ever reported.

5.1 Construction of Residual Earnings Growth

We employ two different strategies to construct the earnings growth residuals that are used to compute the γ and α statistics. Both begin by computing residuals u_{it} from regressions intended to capture both the $F(A_{it})$ and $Q_i(A_{it})$ components of the earnings model (1):

$$y_{it} = F(A_{it}) + Q_i(A_{it}) + u_{it} \ ,$$

where F is a standard linear age-earnings profile. In each strategy, we estimate both RIP and HIP versions of Q_i, which are respectively μ_i and $\mu_i + \theta_i A_{it}$. The procedure is estimated via a mixed effects regression in which the components $Q_i(A_{it})$ are treated as random effects; in the RIP version, this is just a standard panel regression with individual random effects, and the HIP version allows μ_i and θ_i to be correlated. Worker i's residual earnings growth is then computed as $\Omega_i(t, t+j) = u_{i(t+j)} - u_{it}$.

The difference between the two strategies involves the specification of the F component. One strategy uses a relatively simple specification: just sets of dummy variables for each age, year, and census division. None of these variables would remove any variation in individuals' earnings growth rates, so the idea is to create a benchmark set of results that is favorable to finding heterogeneity in income profiles.

The second specification adds three features to the simpler specification to remove trends in workers' earnings with familiar origins, the goal being to provide a rough indication of their effect on estimates of heterogeneity in earnings growth rates. First, the specification addresses long-term trends in returns to education by adding linear time trends in five different maximum levels of men's education: less than 12 years, 12 years, 13-15 years, 16 years, and more than 16 years. Second, in recognition of long-term geographic shifts in economic activity, the specification adds a linear time trend for each census division. Finally, and perhaps most importantly, it removes variation associated with demographic differences in age-earnings profiles by adding a quartic in age for each of 32 groups defined by combinations of workers' broadly-defined race (white or non-white), birth cohort (pre-1944, 1944-1952, 1953-1960, post-1960), and maximum level of education (less than 12, 12, 13-15, and 16 or more years), as well as a 33rd quartic that allows the trend for workers with more than 16 years of schooling to differ from that of workers from the same group who have exactly 16 years.[12] Since these traits are constant within panels, each man always belongs to the same group; we have at least 1,000 annual earnings observations from at least 100 men in each group.

5.2 Description of earnings growth and earnings risk statistics

Table II summarizes the impact of these specifications on estimates of residual earnings growth. Each panel reports standard deviations of estimated raw and residual earnings growth over periods of a fixed length; the first panel examines growth over biennia, the second considers growth over six-year periods, and the last one reports growth over decades. Over the shortest period, estimates of residual earnings growth are robust to both specifications and to the inclusion of heterogeneous profiles. The standard deviation of raw biennial annual earnings growth $(y_{i(t+2)} - y_{it})$ is 0.62, and the standard deviation of residual growth is still 0.61 even under the extended specification with HIP. Regardless of whether we use annual, weekly, or hourly earnings, the correlation between any two of these measures of earnings growth across person-years is at least 0.985.

Over longer periods (e.g., 6 or 10 years), we still find little difference between the simpler and extended specifications, but the variance of residual growth does become more sensitive to the heterogeneous profiles,

[12] We would have preferred simply to create a separate post-graduate educational group for each combination of birth cohort and race, but sample sizes would have been insufficient for some combinations, especially in older cohorts and among non-whites.

e.g., including them reduces the standard deviation of residual growth over decades by about 10 percent. While this difference may reflect a substantial role for heterogeneous profiles, it is also consistent with more mechanical explanations, e.g., regardless of their nature, differences in earnings over longer periods simply have greater algebraic influence on estimated growth rates.[13] We are also surprised that the difference between residual variances of the simpler and extended specifications remains small even over these longer periods, as we had expected these well-known long-term trends in the wage structure to account for a larger share of the heterogeneity in individuals' earnings growth rates estimated under the simpler specification.

[TABLE II ABOUT HERE]

The top panel of Table III presents the variances of the estimated earnings growth rates $\widehat{\theta}$ from the HIP specifications. The three rows reflect weighting schemes that differ because the panels are unbalanced: the first treats each worker as a single unit, the second counts each $\widehat{\theta_i}$ once for each year in which a γ_{it} statistic can be computed, and the third row counts each once per computable γ_{itjq}. Despite the fact that more data is available for some workers than others, these variances differ only moderately. For example, when using annual earnings data and the extended specification, the weighting schemes deliver variances of 0.00037, 0.00050, and 0.00044, respectively. This similarity arises because estimates $\widehat{\theta_i}$ are only mildly correlated with either the number of years for which an individual's γ_{it} can be computed (correlations of -0.02 to 0.01 for all three measures of earnings in either specification) or the number of available γ_{itjq} (0.05-0.09). In short, these values do not appear sensitive to issues related to attrition or data availability. They are also within the range of values found in studies supportive of heterogeneous profiles; e.g., the estimated values for Guvenen's (2009) preferred sample and his broadest sample are respectively 0.00038 and 0.00043.

[TABLE III ABOUT HERE]

The rest of the table summarizes the γ and α statistics created from the earnings residuals; recall that they estimate σ_η^2 and σ_ν^2 under the classic RIP specification, but would be biased under more elaborate models. The γ calculations from the RIP models are in the range of 0.01 to 0.02, which is similar to or moderately smaller than estimates from other studies - e.g., Hryshko's (2012) estimates are around 0.015, Meghir and Pistaferri's (2004) are around 0.03, and Guvenen (2009) finds values ranging from 0.01 to 0.03 across several specification. Our α statistics are an order of magnitude larger. It is more difficult to compare this to prior estimates based on the components of an underlying MA process, but it would not be surprising if ours remain larger given that we consider a more inclusive sample. In addition, when we construct residuals from an earnings regression with HIP, the mean γ decreases sharply and there is a corresponding increase in the mean α, consistent with our predictions from Section 3.

There is essentially zero correlation between mean values of either γ (always between -0.03 and 0) or α (-0.07 to -0.02) and either the number of years or the number of (j, q) for which γ_{it} can be computed. This is further evidence that our conclusions are unlikely to be biased by selection on the basis of how long the man appears in the sample.

[13] This insight can be seen in the weights associated with equation (13) above, which are increasing in the square of the period length. Estimated individual growth rates thus account for more of the variation in growth over longer periods because they are more heavily influenced by pairs of observations spanning longer periods.

Table II: Standard Deviations of Raw or Residual Earnings Growth, by Specification, Period, and Earnings Measure

Earnings Measure	N	Raw	Simpler Model RIP	Simpler Model HIP	Extended Model RIP	Extended Model HIP
			A. Growth over 2 years			
Annual	108,073	0.62	0.62	0.61	0.61	0.61
Weekly	107,582	0.48	0.47	0.47	0.47	0.47
Hourly	107,788	0.47	0.47	0.47	0.47	0.47
			B. Growth over 6 years			
Annual	74,487	0.73	0.71	0.67	0.70	0.67
Weekly	74,759	0.58	0.56	0.54	0.56	0.53
Hourly	74,837	0.55	0.54	0.52	0.54	0.52
			C. Growth over 10 years			
Annual	50,794	0.79	0.76	0.69	0.75	0.68
Weekly	51,308	0.65	0.61	0.55	0.60	0.55
Hourly	51,305	0.60	0.59	0.53	0.58	0.53

Table III: Estimates from Models of Earnings

	Simpler Specification			Extended Specification		
A. Variances of estimated earnings growth rates (Var θ-hat) from HIP model of earnings, x10,000						
Weighting of observations	annual	weekly	hourly	annual	weekly	hourly
Individuals (panels weighted equally)	4.59	3.03	2.15	3.70	2.30	1.61
Person-years for which γ can be computed	5.92	4.17	3.15	5.01	3.43	2.55
Each available γ_{itjq}	4.94	3.73	2.94	4.40	3.24	2.50
B. Mean values of computed γ statistics, x100						
			1. From RIP model			
Weighting of observations	annual	weekly	hourly	annual	weekly	hourly
Individuals	2.26	1.63	1.23	1.99	1.45	1.06
Person-years	1.93	1.47	1.19	1.73	1.33	1.07
Each available γ_{itjq}	1.43	1.19	1.02	1.29	1.08	0.91
			2. From HIP model			
Weighting of observations	annual	weekly	hourly	annual	weekly	hourly
Individuals	0.08	0.27	0.16	0.09	0.28	0.15
Person-years	0.30	0.36	0.27	0.29	0.35	0.27
Each available γ_{itjq}	0.29	0.30	0.24	0.26	0.28	0.23
Mean number of γ_{itjq} per individual i	404	415	417			
Mean number of γ_{itjq} per (i,t)	47	48	48			
C. Mean values of computed α statistics, x100						
			1. From RIP model			
Weighting of observations	annual	weekly	hourly	annual	weekly	hourly
Individuals	22.67	12.71	12.34	23.32	13.18	12.67
Person-years	18.96	11.28	11.10	19.47	11.68	11.42
Each available α_{itjq}	16.76	10.44	10.22	17.08	10.72	10.50
			2. From HIP model			
Weighting of observations	annual	weekly	hourly	annual	weekly	hourly
Individuals	28.58	16.33	15.10	28.41	16.20	14.96
Person-years	24.28	14.80	13.86	24.14	14.71	13.77
Each available α_{itjq}	21.52	14.06	13.18	21.34	13.90	13.07

Notes: The first panel reports variances of estimated person-specific earnings growth rates from regressions with heterogeneous earnings profiles. The next two panels summarize the mean values of γ and α statistics computed using differenced residuals from those earnings regressions; if the baseline model were correct, these would respectively estimate the variances of permanent and temporary shocks (η and ν). Values differ across rows within each column due to heterogeneity in the number of times individuals appear: the top first line weights individuals equally (one obervation per worker), the second weights workers based on the number of years for which a γ statistic can be computed (one observation per person-year), and the last weights individuals based on the number of available combinations (t,j,q). For annual earnings, 2.96 million γ_{itjq} statistics can be computed for 62,720 person-years (i,t) by 7,328 workers (i), as well as 4.18 million α_{itjq} statistics for 79,970 (i,t) by 8,599 workers (i).

6 Empirical Evidence on Heterogeneous Earnings Profiles

6.1 Autocorrelation of residual earnings

As we focus more specifically on the existence of heterogenous income profiles, we begin by briefly examining autocorrelations in biennial earnings growth. Intuitively, one might expect a positive correlation if we incorrectly restricted expected earnings growth to be homogeneous, since the omitted individual growth parameters would cause workers' earnings to drift in the same direction in all periods (MaCurdy (1982), Abowd and Card (1989)). However, Guvenen (2009) shows this reasoning is only relevant over long periods: even if the true model featured heterogeneous earnings growth, under plausible parameters residual earnings growth $\Omega_i(t, t+2)$ over one biennium may not be significantly correlated with $\Omega_i(t+k, t+2+k)$ until k is 20 years or more. Still, considering that our earnings data cover a 45-year period (20 years longer than Guvenen's data), it is not unreasonable to expect a misspecified RIP model to reveal positive autocorrelation over at least the longest available periods, and in any event it would be difficult to reconcile a substantial positive autocorrelation with an RIP model.

Figure 1 thus depicts the pairwise autocorrelations of raw and residual biennial earnings growth, for each measure of earnings over each length of time k, for both the simple and extended models under specifications with both RIP and HIP. It is not surprising that each series begins with a substantial negative correlation — u_{it} contributes positively to $\Omega_i(t-2, t)$ and negatively to $\Omega_i(t, t+2)$ — so we include those points mainly to illustrate the scale. Apart from that algebraically-predictable relationship, all 15 series presented in Panel A depict very small correlations (in the range of $(-0.07, 0.05)$) between residual earnings growth over even the most distantly separated biennia, with starting dates up to 36 years apart. Only a handful of these correlations are statistically significant, those that are (even ignoring those starting less than 6 years apart) are more often negative than positive.[14] There is also no sign of a trend in the period of separation k.

[FIGURE 1 ABOUT HERE]

One potential explanation for the small correlations is that trends in expected earnings are difficult to detect over short periods. Panels B and C of Figure 1 thus plot autocorrelations for annual earnings over 6- and 10-year periods, rather than biennia. (Graphs for weekly and hourly earnings are similar.) Even with the longer periods, both the raw data and the residuals from the RIP models again show no evidence of positive autocorrelation across any non-overlapping periods. While there is a slight upward trend across the gap in time, the pattern is not monotonic, many correlations are negative, and they always remains less than 0.1. Estimated autocorrelations from the HIP models are larger in magnitude, particularly for periods closer in time (smaller k), but they are *negative*; since the autocorrelations were approximately zero in the raw data, this is consistent with having overfit those individual trends. In short, nothing in these graphs contradicts an RIP specification, nor lends any support to an HIP specification.

6.2 Comparison of earnings risk statistics by "long window" length

One key prediction from Section 3 is that misspecifying a RIP model, when the true model has HIP, would cause the mean computed risk statistic $M[\gamma^*_{itjq}|\, \tau]$ to be increasing in $\tau = (j+q+2)$. In contrast, the analysis

[14] Confidence intervals are omitted to avoid further cluttering the graph, but they are intially quite narrow and remain reasonably tight even at the longest periods, where the fewest observations are available. For example, for the series on raw annual earnings growth, the 95 percent confidence interval is $(-0.008, 0.009)$ for biennia beginning 6 years apart (55,137 observed pairs), and it widens to $(-0.086, 0.094)$ for the longest interval shown (36 years apart; 475 pairs).

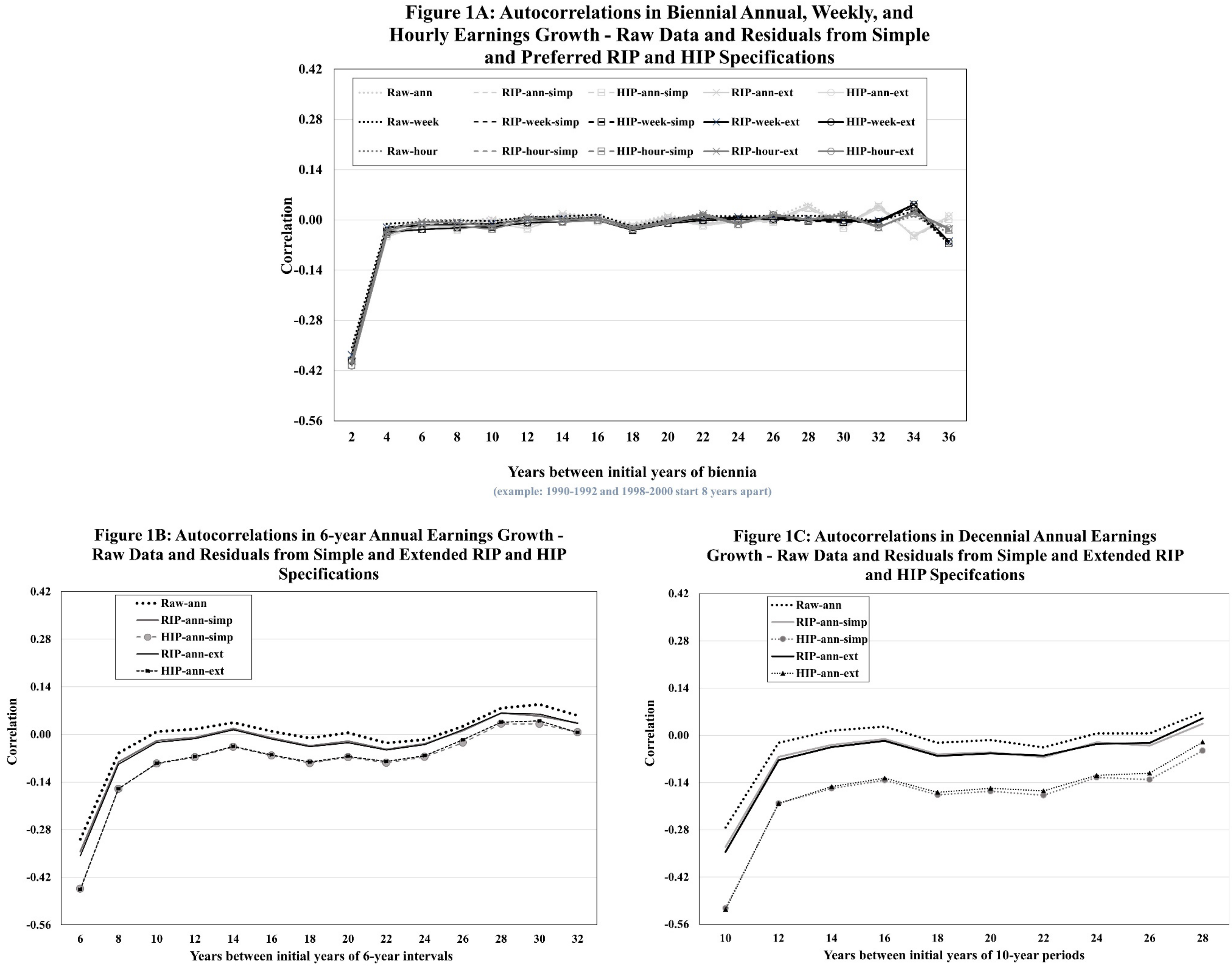


Figure 1: Autocorrelations between men's earnings growth over periods of varying length. Except for periods that share a common endpoint (e.g. $(t-2,t)$ and $(t,t+2)$), there is little evidence of autocorrelation, even over long intervals separated by large gaps.

showed the analogous relationship would be decreasing if a HIP model were used when a RIP model would have sufficed. If the classic RIP specification were the true model, there should be no relationship between $M[\gamma^*_{itjq} | \tau]$ and τ, since all γ statistics for a given (i,t) are just different estimates of the same variance.

Figure 2 presents tests based on those predictions. For each τ, the upper panel plots $M[\gamma_{itjq} | \tau]$ under both the RIP (dotted lines) and HIP (solid lines) specifications, while the lower panel plots analogous series in which both γ and τ are demeaned at the (i,t) level (i.e., $\widetilde{\gamma_{itjq}}$ and $\widetilde{\tau_{itjq}}$). To better illustrate the estimates' sensitivity to τ, all series are scaled by their global means (e.g., the upper panel lines are $M[\gamma_{itjq} | \tau] \,/\, M[\gamma_{itjq}]$). The top panels report results for all three measures of earnings under the simpler specification of the expected earnings process, but for clarity the lower panel reports results only for annual earnings. Graphs are similar if we instead use the other measures of earnings.

[FIGURE 2 ABOUT HERE]

In both cases, the lines corresponding to the HIP models show that mean γ statistics (whether recentered or not) are sharply decreasing in τ, consistent with the prediction for a specification that unnecessarily allows for heterogeneous expected earnings growth. In contrast, the lines for the RIP models are only slightly decreasing, approaching the flat relationship predicted expected under the classic RIP specification and at odds with the increasing relationship predicted if the true model had HIP.

Figure 3 thus plots the empirical relationship between $\widetilde{\alpha_{itjq}}/M[\alpha_{itjq}]$ and $\widetilde{jq}$ for annual earnings data. The patterns are again consistent with an RIP specification, but not with an HIP specification. As derived in Section 3, when the true model has HIP, statistics α^*_{itjq} constructed from a misspecified RIP model would tend to decrease linearly in the product jq, and statistics α^{**}_{itjq} computed using a unnecessary HIP would tend to increase linearly in jq. The latter prediction is consistent with Figure 3: $\widetilde{\alpha_{itjq}}$ computed using HIP is indeed increasing in $\widetilde{jq}$, and a simple linear model accounts for 94 percent of the variation in the data. In contrast, while $\widetilde{\alpha_{itjq}}$ is also increasing under the RIP specification, that pattern is inconsistent with the wrongful omission of heterogeneous trends, and a simple linear fit has less explanatory power (R^2=0.70).

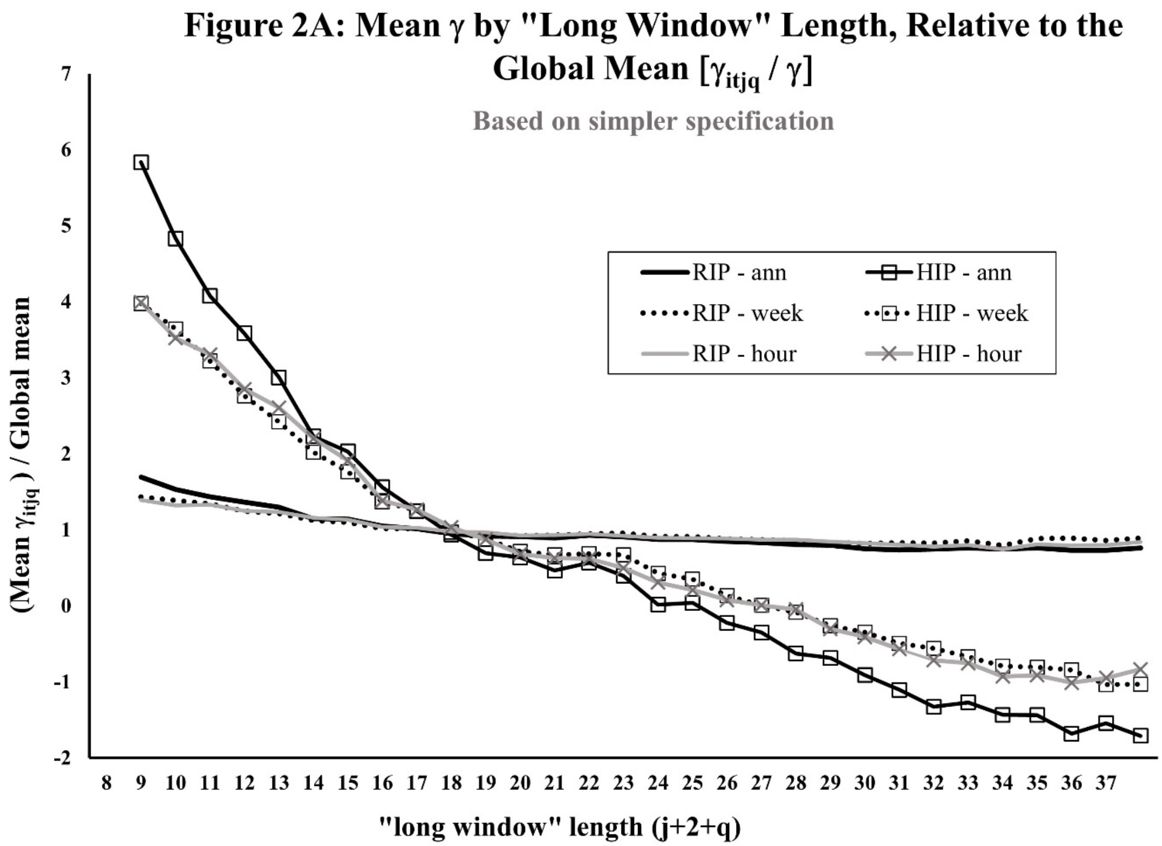
[FIGURE 3 ABOUT HERE]

6.3 Estimates assuming π is a random walk

While these graphs test some qualitative predictions of the specifications we have considered, the same framework also offers opportunities to quantify the variance of individuals' earnings growth rates. The top panel of Table IV presents estimates from the regressions specified by (7) and (8). They measure slopes akin those of the "RIP" lines in Figures 2B and 3, except that here the $\widetilde{\gamma_{itjq}}$ and $\widetilde{\alpha_{itjq}}$ statistics are neither aggregated nor scaled by global means. As explained in Section 3, these slopes would represent the variance of individual earnings growth rates ($\operatorname{Var}\theta$) under a model in which persistent earnings shocks have a unit root. However, each of the estimates in the upper panel of the table is negative and statistically significant, which would obviously be impossible if the coefficient truly represented a variance. Still, just as in Figures 2 and 3, nothing here supports a role for heterogeneous profiles.

[TABLE IV ABOUT HERE]

The lower panel presents analogous estimates of $\left[E\left(\widehat{\theta}^2\right) - \operatorname{Var}\theta\right]$ from regressions (10) and (11). The table also reports the actual mean squared growth rates $E\left(\widehat{\theta}^2\right)$ for the same sample. This is invariably smaller than the regression coefficient, which at face value would imply $\operatorname{Var}\theta < 0$.



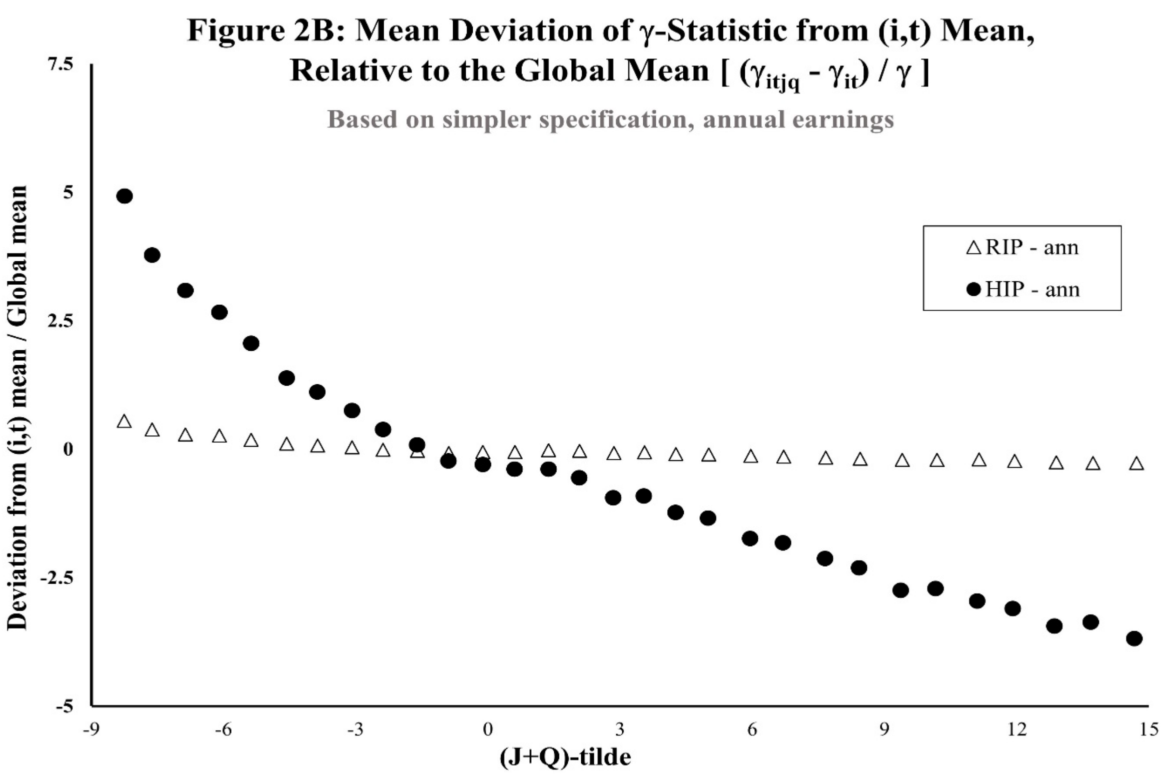


Figure 2: Relationships between statistics intended to measure the variance of annual persistent earnings shocks and the lengths of the periods used to calculate those statistics. In a properly specified models, the slopes of these lines should be 0. If the only misspecification reflects wrongly excluded individual earnings growth rates, the slopes of these lines would measure the variation in individuals' baseline earnings growth rates.

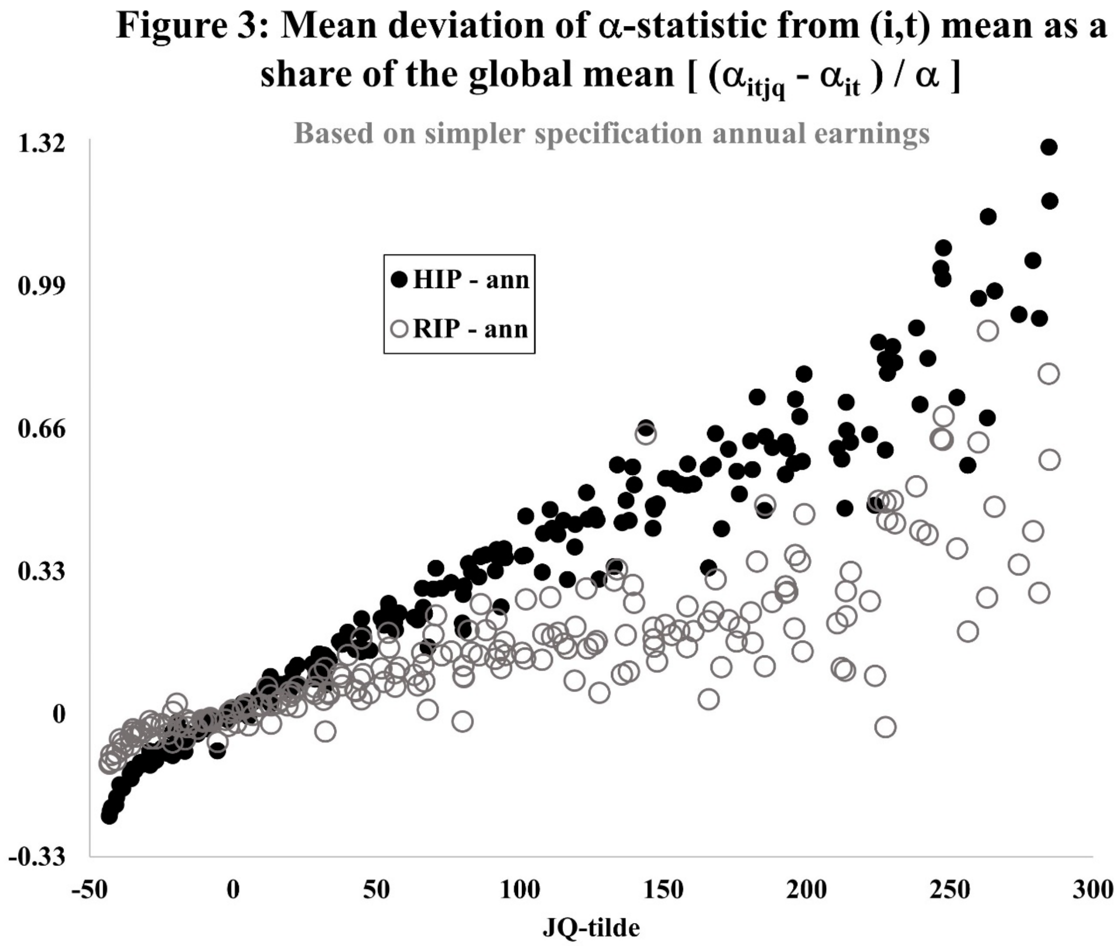


Figure 3: Relationships between statistics intended to measure the variance of annual transitory earnings shocks and the lengths of the periods used to calculate those statistics. If the model were properly specified, the slopes of these lines should be 0. If the only misspecification related to the presence of individual expected earnings growth rates, the slopes of these lines should be (-1) times the variance of annual temporary earnings shocks.

Table IV: Estimated Variances of Expected and Estimated Earnings Growth Rates

Based on a model in which persistent earnings shocks are permament (unit root)

A. Implied Variance of Expected Earnings Growth Rates (Var θ), x10,000

Based on biases in estimate from a RIP model if the true model has HIP

	Simpler Specification						Extended specification					
Earnings measure:	Annual		Weekly		Hourly		Annual		Weekly		Hourly	
	Est	SE	Est	SE	Est	SE	Est	SE	Est	SE	Est	SE
1. Based on γ_{itjq}:												
(j+q)	-3.76	0.61	-2.80	0.39	-2.27	0.36	-4.06	0.60	-3.03	0.37	-2.55	0.36
2. Based on α_{itjq}:												
(-jq)	-2.68	0.50	-1.99	0.38	-1.55	0.33	-2.60	0.45	-1.92	0.33	-1.60	0.29

B. Implied Overestimation of Variance of Earnings Growth Rates [E(θ-hat)^2 - Var θ], x10,000

Based on biases in estimates from a HIP model if the true model has RIP

	Simpler Specification						Extended specification					
Earnings measure:	Annual		Weekly		Hourly		Annual		Weekly		Hourly	
	Est	SE	Est	SE	Est	SE	Est	SE	Est	SE	Est	SE
1. Based on γ_{itjq}:												
-(j+q)	9.23	0.60	7.03	0.37	5.74	0.34	8.94	0.58	6.79	0.36	5.56	0.33
2. Based on α_{itjq}:												
(jq)	8.14	0.47	6.09	0.34	4.64	0.29	7.49	0.42	5.53	0.29	4.25	0.25
Actual mean (θ-hat)^2:												
γ_{itjq} sample	5.13		3.90		3.06		4.44		3.28		2.52	
α_{itjq} sample	5.04		3.89		3.04		4.36		3.26		2.50	
Number of γ_{itjq}	2,962,137		3,036,171		3,043,408							
Number of α_{itjq}	4,181,338		4,284,591		4,292,022							

Notes: The reported values are the estimated coefficients and standard errors from the regressions proposed in the text (equations 7 and 8 for Panel A, equations 10 and 11 for Panel B), multiplied by 10,000 for readability, with standard errors clustered by individuals. As explained in the text, each coefficient in the top panel estimates the variance of (true) individual earnings growth rates (Var θ) if heterogeneous income profiles have been falsely omitted and persistent shocks (π) follow a random walk. Those in the lower panel estimate the difference between the variances of the estimated and actual individual earnings growth rates (E(θ-hat)^2 - Var θ) in a model that includes heterogeneous income profiles, again assuming that persistent shocks follow a random walk. For comparison, the last lines present the actual mean of the squared individual earnings growth rates that were estimated for the same observations; they differ slightly across the γ and α data because α statistics can be computed in some cases when γ statistics cannot.

The comparison between the simpler and extended specifications is also a bit curious. It is not surprising the mean value of $\widehat{\theta}^2$ is always larger under the simpler specification, since the between-group variation in growth rates adds to $E\left(\widehat{\theta}^2\right)$ in that specification but not in the extended specification. The estimates in the lower panel of Table IV also indicate that $E\left(\widehat{\theta}^2\right) - \operatorname{Var}\theta$ is larger in the simpler specification, whereas we might have expected the opposite. Intuitively, the fear that $E\left(\widehat{\theta}^2\right)$ may overstate $\operatorname{Var}\theta$ imagines that random variation in realized earnings growth rates may be falsely attributed to heterogeneity in expected growth rates. To the extent that between-group variation in realized earnings growth rates represents actual differences in expected growth rates, removing that variation from the data would reduce the signal-to-noise ratio, suggesting $E\left(\widehat{\theta}^2\right) - \operatorname{Var}\theta$ would be larger when we use the extended specification.

It is also notable that the mean values of $\widehat{\theta}^2$ shown in Table IV fall within the range of earlier estimates of $\operatorname{Var}\theta$. Depending on the specification, the mean $\widehat{\theta}^2$ for annual earnings is on the order of 0.00044 to 0.000051, whereas Haider (2001) estimates $\operatorname{Var}\theta$ for annual earnings to be 0.00041 (SE=0.00012), Guvenen (2009) reports three estimates for his broadest sample ranging from 0.00038 (0.00008) to 0.00055 (0.00013), Guvenen and Smith's (2014) benchmark model implies a value of 0.00031, several specifications estimated by Baker (1997) yield values near 0.00080, and his largest estimate is more than twice as large. Haider's estimate of $\operatorname{Var}\theta$ for hourly earnings (0.00043, SE=0.00012) is also larger than any of the respective means of $\widehat{\theta}^2$ in Table IV, all of which are between 0.00025 and 0.00031.

One potential implication is that this similarity lends credibility to the estimated earnings growth rates $\widehat{\theta}$, and conversely the mean $\widehat{\theta}^2$ provides a reasonable estimate of $\operatorname{Var}\theta$. However, we are more inclined to emphasize the strong *a priori* belief that mean $\widehat{\theta}^2$ is inherently prone to overestimate $\operatorname{Var}\theta$ given that it is derived from short panels. From that perspective, the fact that estimates of $\operatorname{Var}\theta$ in the existing literature are often at least as large raises the concern that they may be biased upwards as well.

Regardless, we view the results in Table IV less as a refutation of heterogeneous profiles than of the assumptions underlying the interpretations above. The anomalous results would be easier to understand if we relaxed the assumption that the persistent process has a unit root, however. Although here we have interpreted the relationship between $\widetilde{\gamma^*_{itjq}}$ and $\widetilde{(j+q)}$ as a measure of $\operatorname{Var}\theta$, Section 4 established that when π autoregresses at a constant rate $\rho \equiv \exp(-\delta)$, that relationship would instead be $\operatorname{Var}\theta + \delta^2 \operatorname{Var}\pi_{i(t-j)} - \delta\sigma^2_{\eta i(t+1)}$. Since prior estimates of $\sigma^2_{\eta i(t+1)}$ (and our mean γ statistics) are roughly two orders of magnitude larger than the estimated slopes from the upper panel of Table IV, the results above could be rationalized by even a very small value of δ. It could also explain the seemingly odd comparisons between the simpler and extended specifications in Table IV if σ^2_η and $\operatorname{Var}\theta$ were sufficiently positively correlated across demographic groups. This is again plausible: in our data, the correlation between the groups' mean values of γ and $\widehat{\theta}^2$(their intended analogues) are greater than 0.67 for all three measures of earnings.

6.4 Estimates assuming π is autoregressive

We thus turn attention to a model in which the persistent process π is autoregressive. In section 4.2 we developed a strategy to estimate such models via random coefficient regressions like (21):

$$\gamma_{itjq} = \beta_0 + \beta_1 \cdot (j + k + q) + \beta_2 \cdot j + \varepsilon_{itjq}.$$

There we proposed a simple specification in which β_0 and β_1 varied across (i, t). We shall also estimate some specifications in which those coefficients vary only by i. In principle, the model could also justify them varying by (i,t,j), although recall that we expect $\beta_2 \approx 0$ in any event (see page 10).[15]

Table V presents empirical estimates from a series of variations on this strategy. Three key parameters are reported: the mean value of $\sigma^2_{\eta i(t+1)}$, D, and $E\left(\theta^2_*\right)$. Recall that our earlier discussion suggested that D and $2D$ are lower and upper bounds on δ and that $E\left(\theta^2_*\right)$ is an upper bound on $\operatorname{Var}\theta$.

[TABLE V ABOUT HERE]

Orientation. As in Table IV, the left half of Table V reports results derived from the simpler specification of the predictable component (F) of the earnings process, while the right half is based on the extended specification. The samples used in the top half of the table include all available observations, whereas those in the bottom half exclude those from the Survey of Economic Opportunity. Most of our analysis has used the broader sample, and it does seem advantageous to work with a more diverse sample considering that our estimation strategy depends on heterogeneity, yet most prior work (with some exceptions) has excluded SEO observations because they are not representative of the broader population. The lower half of the table thus fulfills the twin goals of testing our estimates' robustness and facilitating comparisons with earlier findings.

Within each quadrant, the table reports five different sets of estimates. The first set comes from estimating equations (21) with β_0 and β_1 allowed to vary across workers i. Although the table does not report its estimate, the specification also includes j as a regressor with a fixed (i.e., homogeneous) coefficient β_2. The hypothesis that $\beta_2 \approx 0$ is rejected in several cases in this first set, but it is never rejected under any of the other specifications in which β_2 can be estimated, with p-values exceeding 0.3 in nearly every case.

The second set of estimates differs only in that β_2 is also allowed to vary across workers. The table shows this makes little practical difference, except that estimates of D and $E\left(\theta^2_*\right)$ become somewhat smaller and notably less precise. The third set limits attention to γ statistics with $j=4$ in order to quash the concern that β_1 may vary by j as well; while any specific value would have sufficed, $j=4$ is an ideal choice because it is both the most common value and the smallest value that can be used with all waves of the survey, leaving nearly 400,000 γ statistics for each regression.[16] Finally, the last two models are analogues to the first and third (respectively), but with (β_0, β_1) allowed to vary by (i,t) rather than just by i.

We view these last two specifications as preferred estimates. For one thing, they accord more closely with evidence from several prior studies that $\sigma^2_{\eta it}$ evolves over the life-cycle. However, the more important point (explained in Section 4.2) is that the estimation strategy relies on heterogeneity in squared realized shocks η_{it} (rather than in *ex ante* risk), which are unique for each (i,t).[17] Since the first three models effectively pool all shocks for each i, they may obscure the systematic relationships used to identify D and $E\left(\theta^2_*\right)$, thus raising the risk of attenuation.

We are hesitant to choose between those last two specifications, however. The advantage of the final model is that it addresses the concern that β_{1it} may also vary by j, but this is apt to be a minor issue because the variation in j would reflect only updating averages of $\sigma^2_{\eta i}$,[18] and it comes at the cost of reduced sample size. As we discuss below, these specifications lead to somewhat different conclusions about $E(\theta^2_*)$. At the other extreme, the third specification seems to suffer from the most shortcomings, and it is the only one that continues to yield negative estimates of $E(\theta^2_*)$.

[15] Sample size is also a significant constraint. We can compute at most 5 γ statistics for the majority of (i,t,j), versus nearly 50 for the average (i,t) and more than 400 for the average i (see Table III.B).

[16] While this restriction precludes including j as a separate regressor, this is of little concern given that we expect $\beta_2 \approx 0$ anyway.

[17] In practice, ϕ_{00} $(= \operatorname{Var}\left[\sigma^2_\eta\right])$ is roughly 5-10 times larger when β_0 is allowed to vary by (i,t) rather than by i alone.

[18] Equations 17 and 19 imply $\beta_{1it(j+1)} - \beta_{1itj} < (\delta/2)\left[s_i(1,t-j-1) - s_i(1,t-j)\right]$, where s_i is the intertermporal average of $\sigma^2_{\eta i\tau}$ defined in (15). This upper bound is likely quite small, both because $(\delta/2)$ is and because s_i presumably evolves slowly.

Table V: Estimated Variances of Persistent Earnings Shocks and Individual Earnings Growth Rates

Based on a model with an autoregressive persistent component

	Simpler specification										Extended specification									
	$E(\sigma_\eta^2)$ x 100			D			$E(\theta_*^2)$ x 10,000				$E(\sigma_\eta^2)$ x 100			D			$E(\theta_*^2)$ x 10,000			
		95% C.I.			95% C.I.			95% C.I.		P:		95% C.I.			95% C.I.			95% C.I.		P:
	Est	low	high	Est	low	high	Est	low	high	Est = 0	Est	low	high	Est	low	high	Est	low	high	Est = 0
A. Using all observations																				
1. Model identified from variation in β_0 and β_1 across panels																				
Annual	3.1	2.6	3.6	0.031	0.025	0.036	1.9	-0.1	4.0	0.06	2.9	2.5	3.4	0.031	0.026	0.037	1.1	-0.7	2.9	0.23
Weekly	2.0	1.8	2.2	0.030	0.026	0.034	1.1	0.0	2.3	0.05	1.9	1.7	2.1	0.031	0.028	0.035	0.5	-0.5	1.6	0.33
Hourly	1.6	1.4	1.7	0.030	0.025	0.034	1.6	0.7	2.6	0.00	1.5	1.3	1.6	0.031	0.028	0.035	1.1	0.3	2.0	0.01
2. Model identified from variation in β_0, β_1, and β_2 across panels																				
Annual	3.1	2.6	3.6	0.025	0.013	0.036	1.8	-2.3	5.8	0.39	2.9	2.4	3.4	0.028	0.016	0.041	1.8	-2.1	5.6	0.37
Weekly	2.0	1.8	2.2	0.019	0.002	0.035	0.2	-3.3	3.8	0.90	1.9	1.7	2.1	0.022	0.006	0.038	-0.2	-3.4	3.0	0.90
Hourly	1.6	1.4	1.7	0.021	0.010	0.033	0.7	-1.1	2.6	0.45	1.5	1.3	1.6	0.026	0.015	0.037	0.7	-1.0	2.4	0.45
3. Model identified from variation in β_0 and β_1 across panels - using only observations with j=4																				
Annual	2.8	2.4	3.2	0.002	-0.009	0.012	-3.6	-6.5	-0.6	0.02	2.7	2.3	3.1	0.002	-0.009	0.012	-4.2	-7.0	-1.4	0.00
Weekly	1.9	1.7	2.1	0.008	-0.001	0.017	-1.1	-2.8	0.6	0.21	1.9	1.6	2.1	0.008	-0.001	0.017	-1.8	-3.4	-0.2	0.03
Hourly	1.4	1.2	1.6	0.005	-0.004	0.015	-0.7	-2.1	0.7	0.34	1.4	1.2	1.6	0.008	-0.001	0.017	-0.9	-2.2	0.4	0.16
4. Model identified from variation in β_0 and β_1 across (i,t)																				
Annual	2.7	2.4	3.0	0.031	0.028	0.034	2.6	1.0	4.1	0.00	2.6	2.3	2.9	0.031	0.029	0.034	2.4	0.8	3.9	0.00
Weekly	2.0	1.9	2.2	0.030	0.028	0.032	2.0	1.1	2.9	0.00	2.0	1.8	2.1	0.030	0.028	0.032	1.7	0.8	2.6	0.00
Hourly	1.6	1.4	1.7	0.031	0.029	0.033	2.3	1.5	3.1	0.00	1.5	1.4	1.7	0.032	0.030	0.034	2.1	1.3	2.9	0.00
5. Model identified from variation in β_0 and β_1 across (i,t) - using only observations with j=4																				
Annual	2.7	2.4	3.1	0.027	0.020	0.034	1.8	-0.1	3.7	0.07	2.6	2.3	2.9	0.028	0.021	0.035	1.5	-0.3	3.3	0.11
Weekly	2.0	1.8	2.2	0.022	0.018	0.027	0.9	0.0	1.7	0.06	1.9	1.7	2.1	0.023	0.019	0.027	0.6	-0.2	1.5	0.13
Hourly	1.6	1.4	1.7	0.023	0.019	0.027	1.0	0.4	1.7	0.00	1.5	1.3	1.7	0.024	0.020	0.028	0.8	0.1	1.5	0.02
B. Excluding observations from SEO																				
1. Model identified from variation in β_0 and β_1 across panels																				
Annual	2.9	2.4	3.4	0.027	0.020	0.033	1.5	-0.6	3.5	0.16	2.8	2.3	3.2	0.028	0.021	0.034	0.6	-1.3	2.6	0.52
Weekly	2.2	1.9	2.5	0.029	0.024	0.033	0.7	-0.6	2.0	0.30	2.0	1.8	2.3	0.029	0.025	0.033	-0.1	-1.2	1.1	0.91
Hourly	1.7	1.5	2.0	0.029	0.024	0.034	1.4	0.2	2.5	0.02	1.6	1.4	1.8	0.031	0.026	0.035	0.9	-0.2	1.9	0.10
2. Model identified from variation in β_0, β_1, and β_2 across panels																				
Annual	3.0	2.5	3.5	0.018	0.000	0.036	0.7	-4.7	6.0	0.81	2.8	2.3	3.3	0.022	0.003	0.041	0.6	-4.4	5.7	0.80
Weekly	2.2	1.9	2.5	0.020	0.001	0.039	-0.2	-4.5	4.2	0.93	2.0	1.8	2.3	0.023	0.005	0.041	-0.7	-4.6	3.1	0.71
Hourly	1.7	1.5	2.0	0.020	0.009	0.031	0.2	-1.8	2.2	0.85	1.6	1.4	1.8	0.024	0.013	0.035	0.2	-1.7	2.0	0.87
3. Model identified from variation in β_0 and β_1 across panels - using only observations with j=4																				
Annual	2.7	2.2	3.1	0.005	-0.007	0.017	-2.4	-5.4	0.7	0.13	2.5	2.1	3.0	0.005	-0.006	0.016	-3.2	-6.1	-0.2	0.04
Weekly	2.1	1.8	2.4	0.012	0.004	0.020	-0.7	-2.4	1.0	0.39	2.0	1.7	2.3	0.011	0.002	0.019	-1.8	-3.4	-0.1	0.04
Hourly	1.6	1.4	1.8	0.011	0.002	0.019	-0.3	-1.8	1.1	0.66	1.5	1.3	1.8	0.012	0.004	0.021	-0.7	-2.1	0.6	0.28
4. Model identified from variation in β_0 and β_1 across (i,t)																				
Annual	2.6	2.3	2.9	0.028	0.025	0.031	2.2	0.6	3.9	0.01	2.5	2.2	2.8	0.028	0.025	0.031	2.2	0.5	3.9	0.01
Weekly	2.1	1.9	2.4	0.028	0.026	0.030	2.0	0.9	3.0	0.00	2.1	1.8	2.3	0.028	0.026	0.030	1.7	0.7	2.7	0.00
Hourly	1.7	1.5	1.9	0.029	0.027	0.032	2.1	1.2	3.1	0.00	1.6	1.5	1.8	0.030	0.028	0.032	2.0	1.1	2.9	0.00
5. Model identified from variation in β_0 and β_1 across (i,t) - using only observations with j=4																				
Annual	2.6	2.2	3.0	0.026	0.016	0.036	1.9	-0.5	4.2	0.13	2.5	2.1	2.8	0.027	0.017	0.036	1.5	-0.6	3.7	0.16
Weekly	2.1	1.9	2.4	0.021	0.017	0.026	0.9	-0.1	1.9	0.08	2.1	1.8	2.3	0.022	0.018	0.027	0.6	-0.3	1.6	0.18
Hourly	1.7	1.5	1.9	0.023	0.018	0.027	1.0	0.3	1.8	0.01	1.6	1.4	1.8	0.023	0.019	0.028	0.7	0.0	1.5	0.04

Notes: Each line reports calculations constructed from estimates of regressions of the form $\gamma_{itjq} = \beta_0 + \beta_1{*}(j+k+q) + \beta_2{*}j$, with some or all coefficients (as specified in the table) treated as random effects that may be correlated. As explained in the test, these calculations are interpreted via a model in which the earnings process has an AR(1) persistent component (π). The first column estimates the average variance of persisitent earnings shocks ($\sigma_{\eta}^{2}{}_{(t+1)}$). The second set of columns reports a value constructed from the covariance matrix of the random effects: $D = -\,\mathrm{Cov}(\beta_0, \beta_1) / \mathrm{Var}(\beta_0)$. D and 2D are lower and upper bounds on δ (= 1 - ρ). The last set of columns estimates the variance of heterogeneous earnings profiles plus any other components that are orthogonal to $\sigma_{\eta}^{2}{}_{(t+1)}$. Full sample sizes for panel A are identical to the numbers of γ_{itjq} reported in Table IV (approx. 3 million in each case), and about 20 percent smaller for panel B (approx. 2.3-2.4 million); when restricted to j=4, sample sizes are about 380,000 for panel A and 290,000 for panel B. Standard errors are clustered by panels.

Estimates of risk and persistence. Substantively, the most striking pattern may be the robustness of estimates of mean σ_η^2. For annual earnings, the estimated value is always near 0.03, with reasonably narrow confidence intervals. This consistency is not surprising considering that the parameter is the intercept of regression (21), so it is effectively set to align the mean predicted value of the dependent variable with the mean observed value. Such values are consistent with estimates from prior work as well.[19] When using the full set of observations (top half of the table), estimates of σ_η^2 are about half as large for hourly earnings, with values for weekly earnings between the two, presumably reflecting the role of persistent shocks in time worked (e.g., associated with job changes). Point estimates change mildly when the SEO observations are excluded, falling slightly for annual earnings but rising slightly for weekly and hourly earnings; we conjecture the difference reflects the SEO men being more likely than others to experience persistent shocks in the form of time worked rather than wage rates, but in any event the differences are small and statistically insignificant.

Estimates of D are robust as well. For four of the five specifications, all the point estimates (even across measures of earnings) are between 0.018 and 0.033, implying $\delta \in [0.018, 0.066]$. This evidence is consistent with a mild rate of autoregression, but none of it affirms the larger values found in some prior studies with HIP models. If anything, our point estimates align with magnitudes from studies supporting a classic RIP framework, except that here nearly all confidence intervals exclude 0.

Estimates of heterogeneity in expected earnings growth. Perhaps the most encouraging result in Table V is that (apart from the exception noted above) we no longer find significant negative values for $E\left(\theta_*^2\right)$ – even our modest estimates of δ resolve the disconcerting evidence from Table IV. In most specifications, the estimates decrease as we move from the simpler specification to the extended one, but this is not surprising given that the latter includes controls for education and other factors commonly suspected of causing heterogeneity in earnings growth rates. If anything, the modest size of the reduction again suggests that such heterogeneity arises mainly from other factors. Similarly, the estimates do not change very much when the SEO observations are excluded.

It is less clear whether the estimates are significantly positive, however. The strongest support for the HIP model come from the fourth sets of estimates of $E\left(\theta_*^2\right)$, which is one of our two co-preferred specifications. Those estimates are invariably positive and statistically significant, and all point estimates fall within a relatively narrow range (0.00017 to 0.00026) that overlaps with at least some prior estimates cited in favor of the HIP model and the value (0.000225) used in Guvenen et al's (2023) calibration. On the other hand, as noted above, the largest prior estimates were still 50-150 percent larger than our maximal estimate.

Moreover, estimates from the other specifications in Table V are far less corroborative. Despite being estimated via an identical specification on a subset of the same observations, the point estimates from the other co-preferred model (the fifth) are notably smaller and often statistically insignificant. Estimates from the remaining specifications are even less supportive. We also note that (apart from those from the fourth model), estimates of $E\left(\theta_*^2\right)$ based on workers' annual earnings are substantially larger than those based on hourly or weekly measures; earlier we argued that such differences are suspicious because they appear to require heterogeneous long-term trends in workers' hours worked. Thus, while we do find some credible evidence of heterogeneity in expected earnings growth, that conclusion appears uncomfortably fragile, and in any event there is little indication that such heterogeneity is more than moderate.

Indeed, the most striking finding in Table V is arguably that our largest estimates of $E\left(\theta_*^2\right)$ are still two orders of magnitude smaller than than estimates of mean σ_η^2. As a consequence, future earnings would

[19] This has not been a contentious parameter in the literature. Similar values emerge in, e.g., Meghir and Pistaferri (2004), Guvenen (2009), Hryshko (2012), Guvenen and Smith (2014), and several studies cited by Guvenen (2007).

remain substantially uncertain even for workers with relatively large expected earnings growth rates. For the set of results with the largest point estimate for Var θ (panel A, 4th set, simpler model), the variance of individuals' potential annual earnings growth rates exceeds $E\left(\theta_*^2\right)$ until $t \approx 40$,[20] implying that only around 5% of workers have sufficiently positive θ to ensure less than a 5% chance of experiencing below-average realized earnings growth over their entire careers. Thus, even if workers knew their own θ's perfectly, their incomes would be only modestly more predictable than if they did not. To the extent they must learn their own θ's via experience, this evidence also supports Guvenen's (2007) argument that such learning is apt to be slow and that workers would effectively view θ as a risky component of their earnings process.

7 Concluding Remarks

To summarize, this paper has revisited the debate between RIP and HIP models through the lens of statistics we called γ. These statistics are products of workers' residual earnings growths over a short period and over an encompassing period with greater length ℓ, so they effectively measure the extent to which changes in their earnings during the shorter period persist over the longer period. Under a "classic RIP" framework, without HIP or autoregression of persistent shocks, the expected value of γ is just the variance of the persistent earnings innovations, and it should not vary with ℓ. In contrast, differences in workers' expected earnings growth (HIP) would cause the covariance of their short- and longer-term earnings growth to rise when evaluated over a longer period, while the gradual erosion of persistent shocks would cause that covariance to fall in ℓ – and faster so when the initial shock was larger.

Since we find a negative empirical relationship between γ and ℓ, it is difficult to avoid the conclusion that persistent earnings shocks autoregress. Our estimates of the autoregression parameter ρ invariably fall between 0.93 and 0.98 and (unlike similar estimates from several previous studies) are statistically distinct from a unit root in almost all variations we consider. Such magnitudes would fully account for the negative relationship between γ and ℓ as well.

Nonetheless, even after accounting for that autoregression, the remaining relationship between γ and ℓ still leaves room for at most mild heterogeneity in expected earnings growth. Our estimates of that variance are often statistically insignificant and modest in size compared to some prior estimates cited in favor of HIP models, though admittedly the evidence is not one-sided. The estimates that lend the strongest support for HIP come from specifications that seem more appropriate than some others, and they do decrease moderately when we control for non-controversial covariates of earnings growth like education. There is thus at least some evidence for the common belief that some workers may rationally be more optimistic than others about their future earnings growth, even if the magnitudes and fragility of the most supportive estimates do not strike us as especially impressive or conclusive.

Regardless, the more important point may be that even the largest estimated variance in expected earnings growth rates indicates that such advantages are obscured by the much greater variance of persistent earnings shocks. As noted above, those estimates would still imply that few workers could be confident that their realized remaining lifetime earnings growth rate would be greater or smaller than average, even if they were fully aware that their expected earnings growth rate differed substantially from the population average – and of course they would face even greater initial uncertainty if they needed to learn their expected earnings growth rate via experience, as posited by Guvenen (2007). This suggests their behavioral responses to their

[20] This is a mild underestimate because it ignores temporary shocks ν. The variance of workers' average annual growth rate over t years $(1/t)\log(y_{it}/y_{i0})$ is then $\sigma_\eta^2\left(1-\rho^{2t}\right)/t^2\left(1-\rho^2\right)$, which under the estimated parameters equals $E\left(\theta_*^2\right)$ at $t=40$.

idiosyncratic expected earnings growth would occur mainly after their earnings became realized, except perhaps toward the end of the lifecycle. It thus appears that accounting for potential HIP would provide at best modest benefit to research focused on individuals' exposure to earnings risk and its consequences for their choices, although its value may well be more evident in studies focusing on dynamics in distributions across larger populations, e.g., as a source of variation in inequality over the lifecycle.

Finally, from a methodological perspective, these results also vindicate the use of γ statistics to measure the risk of persistent earnings shocks. In principle, heterogeneous income profiles could have biased upwards the estimated risk of persistent shocks, but our results should alleviate such fears. While there does appear to be some benefit in accounting for the autoregression of those shocks, such adjustments are both relatively easy to implement and relatively modest in effect. In our view, this is an encouraging methodological conclusion because such strategies have a notable history (since at least Meghir and Pistaferri (2004)) and involve fairly simple calculations that can be readily disaggregated to accommodate variation in risk across workers with different characteristics.

Appendix: Robustness to restricted specification of ν

The analysis above is based on all available γ_{itjq} with $\min(j,q) \geq 3$; under that restriction, $E\gamma$ would not involve transitory shocks ν that could be modeled as an MA(2) process. However, as noted earlier, much of the previous literature specifies ν as MA(1), and in that case transitory shocks would not contribute to $E\gamma_{it2q}$ or $E\gamma_{itj2}$ either. This would allow more γ statistics to be computed and used in the analysis.

Table VI presents some evidence on the implications. The left side of panel A shows how restrictions on $\min(j,q)$ affect the number of available γ statistics. While there are slight differences depending on the measure of earnings used, allowing cases with $\min(j,q) = 2$ invariably enables more than a million new γ's to be constructed, raising the total number from approximately 3 million to 4 million.

[TABLE VI ABOUT HERE]

The right-hand side of panel A illustrates the danger in including those observations. For one thing, the mean of the γ statistics rises about 20 percent. For example, when γ is computed using annual earnings and the simpler specification (the first line of the table), the mean γ is 0.017 when those incremental observations are included, versus 0.014 when they are not. This is not too surprising in a qualitative sense – the evidence above consistently found that mean γ is decreasing in $(j+q)$, so it stands to reason that mean γ would rise when we add cases with $j = 2$ or $q = 2$. However, when we stratify the γ statistics by exact values of $\min(j,q)$, it appears there is a discrete jump at $\min(j,q) = 2$. For instance, in the first line of the table, mean γ rises from 0.018 to 0.019 when we reduce $\min(j,q)$ from 4 to 3, but it leaps to 0.024 for $\min(j,q) = 2$. While hardly conclusive, such a pattern is consistent with the fear that transitory shocks contribute to $E\gamma_{it2q}$ or $E\gamma_{itj2}$, but not to $E\gamma_{itjq}$ for other values of j and q. Of course, that is the very concern that led us to restrict attention to cases with $\min(j,q) \geq 3$.

Panel B of Table VI then uses the less restrictive sample (i.e., with $\min(j,q) \geq 2$) to re-estimate the preferred specifications from Table V (models 4 and 5 in panel A). As one might anticipate given the increase in mean γ, estimates of $E(\sigma^2_\eta)$ are larger here than the comparable estimates from Table V, and estimates of D are substantially similar to their predecessors. However, the new estimates of $E\left(\theta^2_*\right)$ offer less support than before for heterogeneous earnings profiles: the estimates from model 4 are now moderately

Table VI: Comparing restrictions on min(j,q)

A. Sample sizes and means of gamma statistics, by restrictions on j and q

	Number of gamma statistics					Mean				
	All	min(j,q) > 2	min(j,q) = 2	min(j,q) = 3	min(j,q) = 4	All	min(j,q) >= 3	min(j,q) = 2	min(j,q) = 3	min(j,q) = 4
1. Simpler specification										
annual	4,032,353	2,962,137	1,070,216	628,429	675,281	0.017	0.014	0.024	0.019	0.018
weekly	4,120,684	3,036,171	1,084,513	635,979	689,307	0.014	0.012	0.018	0.014	0.015
hourly	4,130,700	3,043,408	1,087,292	639,404	690,391	0.012	0.010	0.015	0.012	0.012
2. Extended specification										
annual	4,032,353	2,962,137	1,070,216	628,429	675,281	0.016	0.013	0.023	0.017	0.016
weekly	4,120,684	3,036,171	1,084,513	635,979	689,307	0.013	0.011	0.017	0.013	0.014
hourly	4,130,700	3,043,408	1,087,292	639,404	690,391	0.011	0.009	0.014	0.011	0.011

B. Estimated parameters from preferred specifications (comparable to specifications 4 and 5 from Table V.A)

	$E(\sigma_\eta^2)$ x 100			D			$E(\theta_*^2)$ x 10,000			
		95% C.I.			95% C.I.			95% C.I.		P:
	Est	low	high	Est	low	high	Est	low	high	Est = 0
4. Model identified from variation in β_0 and β_1 across (i,t)										
a. Simpler specification										
annual	3.4	3.1	3.6	0.030	0.028	0.033	2.5	1.1	3.9	0.00
weekly	2.3	2.2	2.5	0.029	0.027	0.031	2.0	1.2	2.9	0.00
hourly	1.9	1.8	2.0	0.030	0.028	0.032	2.1	1.4	2.9	0.00
b. Extended specification										
annual	3.2	3.0	3.5	0.031	0.028	0.034	2.2	0.9	3.6	0.00
weekly	2.2	2.1	2.4	0.029	0.027	0.031	1.6	0.8	2.4	0.00
hourly	1.8	1.7	2.0	0.031	0.029	0.033	1.9	1.2	2.6	0.00
5. Model identified from variation in β_0 and β_1 across (i,t) - using only observations with j=4										
a. Simpler specification										
annual	3.1	2.8	3.4	0.026	0.018	0.033	0.4	-1.7	2.6	0.69
weekly	2.3	2.1	2.5	0.023	0.020	0.027	0.7	-0.2	1.6	0.14
hourly	1.8	1.6	1.9	0.022	0.018	0.026	0.4	-0.3	1.2	0.24
b. Extended specification										
annual	3.0	2.7	3.3	0.026	0.019	0.033	0.2	-1.9	2.2	0.88
weekly	2.2	2.0	2.4	0.024	0.021	0.028	0.5	-0.4	1.3	0.31
hourly	1.7	1.5	1.9	0.023	0.019	0.027	0.2	-0.5	0.9	0.57

Notes: Panel A presents the number of gamma statistics that can be computed under different restrictions on parameters j and q, as well as their associated means. The first column includes all available gamma statistics with j and q >1, while the second column requires min(j,q)>2; the former is only suitable if the transitory shock process is MA(1), whereas the latter (which corresponds to the specification in the main text) can also accommodate an MA(2) transitory process. The third column reflects the gammas that are included in the first column but not the second, and the remaining columns report estimates from the next smallest values of min(j,q). Panel B presents estimates of the key parameters that were estimated in the preferred specifications of Table V (panel A); the only difference is that these estimates are based on all gamma statistcs with min(j,q)>1 rather than min(j,q)>2. Sample sizes for specification 4 are shown in Panel A (first column), while the sample sizes for specification 5 are approximately 440,000.

smaller, though still statistically significant, whereas those from model 5 are now substantially smaller and have lost any hint of statistical significance.

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