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Estimating macroeconomic models of financial crises: An endogenous regime-switching approach

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We develop a new model of cycles and crises in emerging markets, featuring an occasionally binding borrowing constraint and stochastic volatility, and estimate it with quarterly data for Mexico since 1981. We propose an endogenous regime-switching formulation of the occasionally binding borrowing constraint, develop a general perturbation method to solve the model, and estimate it using Bayesian methods. We find that the model fits the Mexican data well without systematically relying on large shocks, matching the typical stylized facts of emerging market business cycles and Mexico's history of sudden stops in capital flows. We also find that interest rate shocks play a smaller role in driving both cycles and crises than previously found in the literature.

Keywords. Business cycles, Bayesian estimation, endogenous regime-switching, financial crises, Mexico, occasionally binding constraints, sudden stops.

JEL classification. C11, E3, F41, G01.

1. Introduction

The global financial crisis triggered a strong renewed interest in understanding the causes, consequences, and remedies of financial crises. In this context, dynamic

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stochastic general equilibrium (DSGE) models with occasionally binding financial frictions or stochastic volatility proved successful as laboratories for studying the anatomy of both business cycles and crises and exploring optimal policy responses to these dynamics. This success is because occasionally binding financial frictions are mechanisms that create an amplification of regular business cycle dynamics. At the same time, a now large literature has shown that uncertainty modeled as stochastic volatility also contributes to cycles and crises, capturing fat tails and skewness in the data. The structural estimation of these models is important for inference on key parameters governing financial frictions and exogenous processes and the decomposition of important historical episodes, yet it is very challenging.

In this paper, we develop a new model of cycles and crises in emerging markets, featuring an occasionally binding borrowing constraint and stochastic volatility, and estimate it with quarterly data for Mexico since 1981. The paper makes three contributions. First, we propose a new specification of the occasionally binding collateral constraint that permits matching crises of different durations and intensities. Second, we develop a perturbation solution method suitable for solving models like ours in a way that allows for likelihood-based estimation. Third, we apply the proposed framework to investigate sources of business cycles and crises in Mexico since 1981—a case studied most often as a typical emerging market economy.

As a first step, we propose a new formulation of occasionally binding constraint models. As in the traditional specification of such models, our setup has two states or regimes: in the first, limited leverage amplifies regular shocks and gives rise to fire sales and debt-deflation dynamics; in the second, access to financing is unconstrained, and the economy displays regular business cycles. However, in our specification, the transitions between the two regimes depend on a *range* rather than a *unique* level of leverage, with endogenous switching probabilities that are a function of the borrowing capacity of the economy and the multiplier associated with the leverage constraint. This formulation maps the model with an occasionally binding leverage constraint into an endogenous regime-switching model. The paper focuses on a particular constraint and type of crisis, the so-called sudden stop in capital flows, but the proposed specification has broader applicability to other types of occasionally binding constraints and frictions. For example, our proposed approach could be applied to the formulation and estimation of models with housing constraints, downward wage rigidity, or the zero lower bound.

Next, we develop a perturbation-based solution method to solve the endogenous regime-switching model. The perturbation method is fast enough to permit likelihood-based estimation, is scalable to models larger than the one we estimate in this paper, and displays typical levels of accuracy. We also analytically show that approximating the model solution to the second order is necessary and sufficient to capture at least some of the effects of the endogenous transition probabilities on the model's policy functions, including precautionary behavior, and that these effects would be missed by linear approximations or exogenous regime switching models. The second-order solution also allows us to capture the effects of regime shifts in the volatility of exogenous shocks. Again, the solution method that we develop can be applied to a wide range of models with endogenous regime-switching.

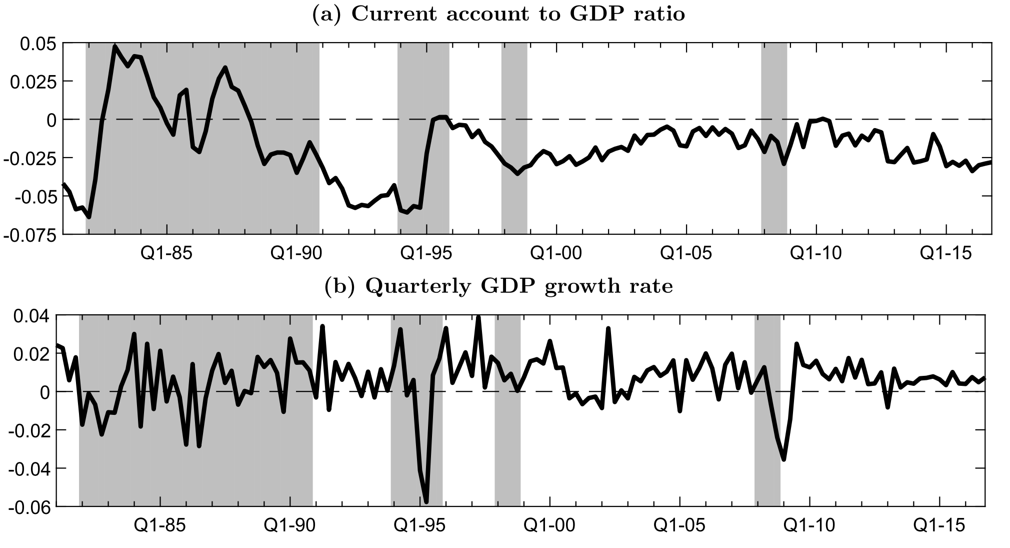

FIGURE 1. Current account and GDP in Mexico, 1981–2016. Note: Panel (a) plots Mexico's current account balance as a share of GDP. Panel (b) shows Mexico's quarterly log-change of real GDP. The light gray regions denote the periods of currency or external debt crisis according to Reinhart and Rogoff (2009), which we call the External Crisis Tally Index. See the Supplementary Appendix (Benigno, Foerster, Otrok, and Rebucci (2024)) for data sources. Sample period 1981:Q1-2016:Q4.

Finally, we apply the framework that we developed to the Bayesian estimation and analysis of Mexico's history of cycles and sudden stop crises since 1981. The Mexican economy is a particularly interesting laboratory because many of the seminal contributions to the literature on business cycles and crises in emerging markets previously studied this case. Figure 1 plots two critical Mexican data macroeconomic series: the current account balance as a share of GDP and the quarterly real GDP growth. The figure also indicates as (gray) shaded areas Mexico's periods of currency and external debt crisis identified in Reinhart and Rogoff (2009). The figure illustrates the regular fluctuations in the data, as well as the multiple episodes of large current account reversals and output growth declines. Large current account reversals and output drops of heterogeneous size and persistence are the two main empirical features commonly associated with sudden stops in capital flows, not only in Mexico but also in many other emerging markets. In this paper, we focus on the challenge of fitting a structural model to Mexico's business cycle and sudden stop history without imposing ad hoc restrictions on the magnitude or persistence of these episodes. The figure also displays the marked shift in the volatility of the economy both before and after the mid-1990s and during certain periods of time, which may not be captured by an occasionally binding borrowing constraint or financial shocks.

Despite the econometric challenges in characterizing data like those shown in Figure 1, our estimated model fits Mexico's business cycles and sudden stop episodes well, without systematically relying on large shocks to explain crises, but instead letting the economic structure of the model explain those events. The inclusion of stochastic volatility helps us to distinguish between the binding constraint and periods of more volatile shocks as drivers of fluctuations. The model produces business cycle statistics that match the second moments of the data and yields evidence that neither productivity nor interest rate shocks are the most important drivers of Mexico's business cycles. Most importantly, our specification of the collateral constraint identifies crisis episodes and dynamics of varying duration and intensity that match the crisis periods identified with a narrative approach in Reinhart and Rogoff (2009).

Related literature

Our paper is connected to several strands of literature. The paper relates to the large literature on the Bayesian estimation of DSGE models (e.g., Schorfheide, 2000, Otrok, 2001, Smets and Wouters, 2007, Iacoviello and Neri, 2010, Bianchi, 2013). We extend that successful approach to models with occasionally binding collateral constraints, which have become the benchmark for normative analysis of macro-prudential optimal policy.

Our paper is closely related to the empirical work in Bocola (2016), where the model is solved with global methods and estimated. However, that estimation exercise is made possible by first estimating the model outside the crisis, and then appending an estimate of the crisis in a second step. While this procedure does not matter for the specific application in Bocola (2016), it is not necessarily applicable more generally. Our approach permits joint estimation of the model inside and outside the crises and is potentially scalable to larger and more complex models, while maintaining a satisfactory level of accuracy relative to global solution methods.

The paper is also closely related to the literature on likelihood-based estimation of Markov switching DSGE models initiated by the seminal contribution of Bianchi (2013), and applied in Bianchi and Ilut (2017) and Bianchi, Ilut, and Schneider (2018). The filter we use in estimation differs in two key respects. First, our regime-switching transition matrix is endogenous. Second, conditional on the regime, we solve the model to the second order. So, we employ the Sigma Point Filter to evaluate the likelihood function in place of the modified Kalman filter in Bianchi (2013).

In the literature on Markov-switching DSGE models, our paper builds on the method proposed by Foerster, Rubio-Ramirez, Waggoner, and Zha (2016), who developed perturbation methods for the solution of exogenous regime-switching models. The perturbation approach that we propose allows for second- and higher-order approximations that go beyond the linear models studied by Davig and Leeper (2007) and Farmer, Daniel, Waggoner, and Zha (2011). In fact, we show that at least a second-order approximation is necessary in order to capture the effects of the endogenous switching.

The paper is naturally related to the literature that focuses on endogenous regime-switching models. Davig and Leeper (2008), Davig, Leeper, and Walker (2010), and Alpanda and Ueberfeldt (2016) all consider endogenous regime-switching but employ global solution methods that hinder likelihood-based estimation. Lind (2014) develops a regime-switching perturbation approach for approximating nonlinear models, but it requires repeatedly refining the points of approximation, and hence, it is not suitable for estimation purposes.

Here, our paper relates closely to OccBin, a set of procedures for the solution of models with occasionally binding constraints, developed in Guerrieri and Iacoviello (2015). OccBin is a certainty equivalent solution method that captures nonlinearities but not precautionary effects, which are a critical feature of models with occasionally binding collateral constraints.[1] A key feature of our approach is to preserve precautionary saving effects, as agents in the model adjust their behavior due to the presence of the constraint even when the constraint does not bind, and vice versa.

The specification of the inequality constraint and the accompanying solution method that we propose can be applied to models with occasionally binding zero-lower bound on interest rates (e.g., Adam and Billi (2007), Aruoba, Cuba-Borda, and Schorfheide (2018), Atkinson, Richter, and Throckmorton (2018)).[2] Existing methods for the estimation of such models may limit scalability due computational costs (Gust, Herbst, Lopez-Salido, and Smith (2017)). The approach we propose is scalable and applicable to large models.

The application of the methodology that we propose relates to the literature on emerging market business cycles, which includes Aguiar and Gopinath (2007), Mendoza (2010), Garcia-Cicco, Pancrazi, and Uribe (2010), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011), among others. Encompassing most shocks previously considered, we consider transitory and permanent technology, preference, expenditure, interest rate, and terms of trade shocks in our analysis. We allow for regime switching in the volatility of shocks as in (Liu, Waggoner, and Zha (2011), Bianchi (2013)) since stochastic volatility has been shown to be an important feature of emerging markets data (Fernandez-Villaverde et al. (2011)). Relative to Mendoza (2010), we provide a Bayesian estimation of the model and consider a wider set of structural shocks. Relative to Garcia-Cicco, Pancrazi, and Uribe (2010), we empirically evaluate the relative importance of interest rate shocks by modeling the amplification induced by financial frictions with an occasionally binding borrowing constraint.

The rest of the paper is organized as follows. Section 2 describes the model and discusses the proposed formulation of the collateral constraint and stochastic volatility. Section 3 presents our perturbation solution method for endogenous regime-switching models. Section 4 describes the Bayesian estimation procedure and reports the estimation results. Section 5 discusses the main empirical results on the analysis of Mexico's business cycle and history of sudden stop crises. Section 6 concludes. Technical details and additional results are reported in the Appendix and in the Supplementary Appendix (Benigno et al. (2024)).

2. The model

The framework is a medium-scale model for the analysis of business cycles and sudden-stop crises in emerging market economies. The model is a small, open production econ-

[1]Cuba-Borda, Guerrieri, Iacoviello, and Zhong (2019) study how the solution method and likelihood misspecification interact and possibly compound each other.

[2]An occasionally binding zero-lower bound is not comparable to the constraint with endogenous collateral value that we estimate in this paper. Indeed, endogenous collateral valuation features different amplification mechanisms and entails additional computational complexities.

omy as in Mendoza (2010) with endogenous labor supply, investment, and an occasionally binding collateral constraint. We consider a large set of shocks as in Garcia-Cicco, Pancrazi, and Uribe (2010), including permanent and temporary productivity shocks (Aguiar and Gopinath (2007)), intertemporal preference, expenditure, interest rate, and terms of trade shocks. The model also features stochastic volatility in all shocks (Fernandez-Villaverde et al. (2011)).

In the rest of this section, we briefly state the optimization problem of the representative household-firm and then discuss the specification of the borrowing constraint and the shock processes, which are the novel features of our model. The derivation of the equilibrium conditions and the formal definition of the competitive equilibrium of the economy are in Appendix A.

2.1 *Preferences, constraints, and shock processes*

There is a representative household-firm that maximizes the following utility function:

$$\mathbb{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{\left(C_t - Z_{t-1} \dfrac{H_t^{\omega}}{\omega} \right)^{1-\rho} - 1}{1-\rho} \right\}, \tag{1}$$

where C_t denotes consumption and H_t the supply of labor. The utility depends on an exogenous and stochastic preference shock d_t, and the permanent technology level Z_{t-1}, which follow the processes specified below.[3] The household-firm chooses consumption, labor, capital K_t, imported intermediate inputs V_t given an exogenous stochastic for their relative price P_t also specified below, and holdings of real one-period international bonds, B_t. Negative values of B_t indicate borrowing from abroad. The household-firm can borrow in international markets by issuing one-period bonds that pay a market or country net interest rate r_t.

The household-firm faces the following budget constraint:

$$C_t + I_t + E_t = Y_t - \phi r_t (W_t H_t + P_t V_t) - \frac{1}{(1+r_t)} B_t + B_{t-1}, \tag{2}$$

where Y_t is gross domestic product (GDP) given by

$$Y_t = A_t K_{t-1}^{\eta} (Z_t H_t)^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t. \tag{3}$$

Here, A_t denotes a stationary, exogenous, and stochastic level of technology, and Z_t is a non-stationary, exogenous, and stochastic level of technology. E_t is an exogenous and stochastic expenditure process possibly interpreted as a fiscal or net export shock as in Garcia-Cicco, Pancrazi, and Uribe (2010). The term $\phi r_t (W_t H_t + P_t V_t)$ describes a working capital constraint, stating that a fraction of the wage and intermediate goods bill must be paid in advance of production with borrowed funds. The relative price of labor and

[3]Scaling hours worked by Z_{t-1} permits obtaining a balanced growth path with GHH preferences (see, e.g., Garcia-Cicco, Pancrazi, and Uribe (2010)).

capital are given by W_t and q_t, respectively, both of which are endogenous but taken as given by the individual household-firm. Capital accumulation depends on investment I_t and is subject to adjustment costs:

$$K_t = (1-\delta)K_{t-1} + I_t - \frac{\iota}{2}\left(\frac{K_t - \Lambda_k K_{t-1}}{K_{t-1}}\right)^2 K_{t-1}, \tag{4}$$

where Λ_k denotes the growth rate of capital along the economy's balanced growth path (BGP).

All exogenous processes have stochastic volatility, depending on a regime indicator $s_t^\sigma \in \{H, L\}$, where H and L signify a high and low volatility regime, respectively, as in Liu, Waggoner, and Zha (2011) or Bianchi (2013), among others. The preference process follows

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d\left(s_t^\sigma\right)\varepsilon_{d,t}. \tag{5}$$

The stationary technology process follows

$$\log A_t = (1-\rho_A)\log A^* + \rho_A \log A_{t-1} + \sigma_A\left(s_t^\sigma\right)\varepsilon_{A,t}. \tag{6}$$

The permanent technology process follows

$$\Delta \log Z_t = (1-\rho_z)\log Z^* + \rho_z \Delta \log Z_{t-1} + \sigma_z\left(s_t^\sigma\right)\varepsilon_{z,t}. \tag{7}$$

The interest rate process follows

$$r_t = (1-\rho_r)r^* + \rho_r r_{t-1} + \sigma_r\left(s_t^\sigma\right)\varepsilon_{r,t}. \tag{8}$$

The process for the relative price of intermediate goods follows

$$\log P_t = (1-\rho_P)\log P^* + \rho_P \log P_{t-1} + \sigma_P\left(s_t^\sigma\right)\varepsilon_{P,t}. \tag{9}$$

Finally, the process for the exogenous component of expenditure follows $E_t = e_t/Z_{t-1}$, where

$$\log e_t = (1-\rho_e)\log e^* + \rho_e \log e_{t-1} + \sigma_e\left(s_t^\sigma\right)\varepsilon_{e,t}. \tag{10}$$

The starred variables and the ρ coefficients denote the unconditional mean values and the persistence parameters of these processes, respectively. The ε are assumed i.i.d. $N(0, 1)$ innovations, and the σ parameters control the size of their variances. The transition matrix of the volatility regimes, $\mathbb{P}^\sigma$, is exogenous and given by

$$\mathbb{P}^\sigma = \begin{bmatrix} P_{LL}^\sigma & 1-P_{LL}^\sigma \\ 1-P_{HH}^\sigma & P_{HH}^\sigma \end{bmatrix},$$

where $P_{ij}^\sigma = \Pr(s_{t+1}^\sigma = j | s_{t+1}^\sigma = i)$.

2.2 *The occasionally binding borrowing constraint: An endogenous regime-switching specification*

As in typical models with occasionally binding inequality constraints, the economy fluctuates between two states or regimes. In one state, denoted $s_t^c = 1$ and called the binding or constrained regime, the following constraint on total borrowing binds strictly:

$$\frac{1}{(1+r_t)} B_t - \phi(1+r_t)(W_t H_t + P_t V_t) = -\kappa q_t K_t, \tag{11}$$

with λ_t denoting the corresponding multiplier. Total debt includes borrowing for consumption smoothing plus working capital for the purchase of intermediate inputs and labor for production. Constrained working capital limits the supply response of the economy to shocks in the binding regime. In the other state, denoted $s_t^c = 0$ and called the nonbinding or unconstrained regime, the borrowing limit is slack and $\lambda_t = 0$, and the only constraint is the natural debt limit.

Given these two regimes, which represent the occasionally binding nature of the constraint, we characterize the transition between them stochastically in the sense that, for given values of capital, bond holding, and exogenous processes, there is an endogenous probability of switching between the two regimes. This formulation contrasts with the deterministic relationship between leverage and the regime for given values of endogenous and exogenous state variables in a typical occasionally binding specification. In particular, we assume that the probabilities of switching from one regime to the other follow a logistic function of a restricted subset of the endogenous state variables in the model.[4]

Define the “borrowing cushion,” B_t^*, as the distance of actual borrowing from the debt limit:

$$B_t^* = \frac{1}{(1+r_t)} B_t - \phi(1+r_t)(W_t H_t + P_t V_t) + \kappa q_t K_t, \tag{12}$$

so that when B_t^* is small, total borrowing and leverage are high relative to the value of the collateral. We then assume that the transition from the nonbinding to the binding regime depends on $\tilde{B}_t^* = B_t^*/Z_{t-1}$ according to

$$\Pr\big(s_{t+1}^c = 1 | s_t^c = 0, \tilde{B}_t^*\big) = \frac{\exp\big(-\gamma_0 \tilde{B}_t^*\big)}{1 + \exp\big(-\gamma_0 \tilde{B}_t^*\big)}. \tag{13}$$

Thus, the likelihood that the constraint binds in the *following* period depends on the size of the borrowing cushion in the *current* period.[5] The parameter γ_0 controls

[4] This is similar to the logistic specification in Bocola (2016), where the economy switches between default and nondefault states. Kumhof, Rancière, and Winant (2015) also use a logistic function to model the transition to the default regime in the context of a positive analysis of the relationship between financial crises and inequality. Davig, Leeper, and Walker (2010) and Bi and Traum (2014) use a similar logistic formulation to study the macroeconomic consequences of fiscal limits.

[5] Appendix C shows that this timing difference with respect to a model with a traditional inequality specification of the occasionally borrowing constraint does not affect the first and second moments of the economy.

the steepness of the logistic function, determining the sensitivity of the probability of switching regime to the size of the borrowing cushion. When γ_0 is positive, the probability of switching to the binding regime increases as the cushion declines. The critical difference between our stochastic and the traditional specification of the occasionally borrowing constraint is that the borrowing cushion could be negative but the economy could remain in the nonbinding regime if $s_t^c = 1$ has not been drawn.

Similarly, when the constraint binds, the transition probability to the nonbinding regime is a logistic function of the Lagrange multiplier, $\tilde{\lambda}_t = \lambda_t Z_{t-1}^{-\rho}$:

$$\Pr\left(s_{t+1}^c = 0 | s_t^c = 1, \tilde{\lambda}_t\right) = \frac{\exp(-\gamma_1 \tilde{\lambda}_t)}{1 + \exp(-\gamma_1 \tilde{\lambda}_t)}. \tag{14}$$

Therefore, the probability of switching back from the constrained to the unconstrained regime depends on the shadow value of the economy's desired borrowing relative to the limit set by the collateral constraint. As in the case of a switch from the unconstrained to the constrained regime, the parameter γ_1 affects the sensitivity of this probability to the value of the multiplier. When γ_1 is positive, the probability of exiting the binding regime increases as the multiplier declines. As in the nonbinding regime, for certain draws from the logistic, it is possible that the desired level of borrowing is less than the level forced upon the economy by the binding constraint and this will manifest itself with a negative collateral constraint multiplier.[6]

Putting equations (13) and (14) together, the resulting endogenous regime-switching model has the following transition matrix:

$$\mathbb{P}_t^c = \begin{bmatrix} 1 - \dfrac{\exp\left(-\gamma_0 \tilde{B}_t^*\right)}{1 + \exp\left(-\gamma_0 \tilde{B}_t^*\right)} & \dfrac{\exp\left(-\gamma_0 \tilde{B}_t^*\right)}{1 + \exp\left(-\gamma_0 \tilde{B}_t^*\right)} \\ \dfrac{\exp(-\gamma_1 \tilde{\lambda}_t)}{1 + \exp(-\gamma_1 \tilde{\lambda}_t)} & 1 - \dfrac{\exp(-\gamma_1 \tilde{\lambda}_t)}{1 + \exp(-\gamma_1 \tilde{\lambda}_t)} \end{bmatrix}. \tag{15}$$

Agents in the model have full information and rational expectations about these transition probabilities, which are affected by all endogenous variables in the model in general equilibrium.

The combined regimes in volatility (s_t^σ) and in the borrowing constraint (s_t^c) is denoted by $s_t = s_t^c \times s_t^\sigma$. Since the two sets of transition probabilities are independent of one another—switching in volatility regimes is independent of switching in the borrowing constraint regimes—the combined transition matrix is given by $\mathbb{P}_t = \mathbb{P}_t^c \otimes \mathbb{P}^\sigma$.

2.3 *Remarks on the endogenous regime-switching specification*

Some remarks are useful here on how our stochastic formulation of the borrowing constraint works and differs relative to the typical inequality formulation used in the literature.

[6]By construction, the transition probabilities are 0.5 when their arguments are zero. This assumption can be relaxed by introducing a constant into the arguments of equations (13)–(14). However, estimating the model by allowing for such a degree of freedom suggests that these parameters were effectively zero. For ease of presentation, we have omitted them from the beginning of the analysis.

First, in our setup, agents in the nonbinding regime know that higher leverage increases the probability of switching to the binding regime and vice versa. Critically, this knowledge preserves the interaction in agents' behavior between regimes and gives rise to precautionary behavior, distinguishing the model from those in which the occasionally binding constraint is approximated with solution methods that eliminate the interactions across regimes as in Guerrieri and Iacoviello (2015). One could certainly consider a wider range of variables governing the regime transition. Our choice, as in Bocola (2016), reflects the need to adopt a parsimonious representation and to choose variables motivated by the sudden stop literature. The proposed specification allows us to identify all parameters of interest and fit the data well with small, well-behaved shocks.

Second, as we already noted, for certain draws of the regime state from the logistic distribution, the borrowing cushion and the multiplier can take negative values. This feature of the model implies that the run-up to or the duration of a financial crisis has a variable duration. As a result, unlike traditional models with inequality borrowing constraints, our model generates crisis episodes of varying duration and can provide data-consistent accounts of crisis dynamics without imposing any additional ad-hoc restrictions on the sudden-stop definition. For example, negative values of the borrowing cushion $\tilde{B}_t^*$ in the nonbinding regime are possible if the probability of a switch to the binding regime is elevated, but the switch has not been drawn yet, prolonging the boom phase and postponing the beginning of the crisis episode. In contrast, the economy can remain in the binding regime even if the multiplier is no longer positive, but the nonbinding regime has not been drawn yet, prolonging the duration of the crisis and contributing to a sluggish recovery. Additionally, as the transition probabilities are endogenous, they are time-varying. In contrast, the exogenous Markov-switching setup (Davig and Leeper (2007), Farmer, Waggoner, and Zha (2011), Bianchi (2013), Foerster et al. (2016)) has a constant probability of transitioning between regimes that is independent of the structural shock realizations and the agent's decisions. As we shall see, this specification fits Mexican data better than an exogenous switching specification.

Additionally, this model feature is consistent with a growing body of microeconomic evidence suggesting that a deterministic specification of the switch between regimes in models with occasionally binding collateral constraints may not accurately capture lending and borrowing behaviors at the household and firm or bank level. For example, Chodorow-Reich and Falato (2017) and Greenwald (2019) show that loan covenants are frequently violated, triggering renegotiation rather than suddenly cutting off borrowers from funding once activated. Campello, Graham, and Harvey (2010) also provide survey information on the behavior of financially constrained firms, and Ivashina and Scharfstein (2010) examine loan level data, showing that firms draw down preexisting credit lines in order to satisfy their liquidity needs to avoid hitting borrowing limits. In emerging market economies, official reserves play a similar liquidity role. Thus, in practice, collateral constraints are binding over a *range* of leverage ratios rather than at any *particular level* as in a model with inequality constraints. This model property is also consistent with the macroeconomic empirical evidence of Jorda, Schularick, and Taylor

(2013), who show that the exact level of leverage at which a crisis occurs varies considerably across crisis episodes.[7]

Third, our stochastic specification of the borrowing constraint nests the deterministic case in which regime switching is triggered with probability 1 by a unique level of leverage. In Appendix C, we show that the policy functions of our model capture the typical debt-deflation mechanism of models with occasionally binding borrowing constraints as Mendoza (2010), while at the same time offering a dramatically improved trade-off between accuracy in terms of Euler equation errors and speed in terms of solution time that allows for Bayesian estimation. Moreover, the simulated second moments are practically indistinguishable from those in Mendoza and Villalvazo (2020).

Fourth and finally, as in other specifications of occasionally binding collateral constraints in the literature (e.g., Bianchi (2011), Schmitt-Grohe and Uribe (2020), Farhi and Werning (2020)), our formulation is not derived as an equilibrium outcome of a fully specified contracting environment. However, it could be derived by generalizing existing contract environments. The traditional inequality formulation of our borrowing constraint,

$$\frac{1}{(1+r_t)}B_t - \phi(1+r_t)(W_tH_t + P_tV_t) \geq -\kappa q_t K_t. \tag{16}$$

can be derived as an implication of incentive compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ of the value of the assets owned by a defaulting borrower, for instance, as discussed in Bianchi and Mendoza (2018). An endogenous transition between the two regimes associated with this constraint could be derived in a standard costly state verification framework (Bernanke and Gertler (1989)). For example, Matsuyama (2007) shows how a borrower's net worth causes endogenous switching between investment projects with different productivity levels, which feeds back into borrowers' net worth. Martin (2008) shows that endogenous regime switches in financial contracts from pooling to separating and vice versa leads to switching in lending standards between a "lax" and a "tight" regime.[8]

3. Solving the endogenous switching model

This section describes our solution method for endogenous regime-switching models. In principle, our model can be solved using global methods as, for example, in Davig, Leeper, and Walker (2010). However, as we discuss in Appendix C, with two endogenous and six exogenous state variables, four regimes, and six exogenous shocks, using a global solution method would prohibit likelihood-based estimation. Instead, we solve the model using a perturbation approach, which allows for an accurate approximation fast enough to permit estimation and is potentially applicable to even larger models.

[7]The notion of "debt intolerance" discussed by Reinhart and Rogoff (2009) and credit surface of Fostel and Geanakoplos (2015) also are consistent with our stochastic specification.

[8]Heterogeneous agent models with financial frictions imposed at the individual level could also be consistent with a stochastic specification of the switching between regimes. For example, Fernandez-Villaverde, Hurtado, and Nuno (2019) study financial frictions and wealth distributions, where endogenous aggregate risk generates an endogenous regime-switching process.

We first describe how to implement occasionally binding constraints represented as endogenous regime switching, the approximation point, how to define a steady state in this setup, and the Taylor-series expansions. We then discuss the importance of approximating at least to a second order in our framework. The competitive equilibrium of the endogenous regime-switching model is defined formally in Appendix A. The derivation of the Taylor-series expansions and other details of the solution method are reported in Appendix B.

3.1 *The regime-switching slackness condition*

A critical step in applying our perturbation method is to ensure that the two variables $\tilde{B}^*_t$ and $\tilde{\lambda}_t$ are zero if the economy is in the relevant regime: $\tilde{\lambda}_t = 0$ in the nonbinding regime, and $\tilde{B}^*_t = 0$ in the binding regime. This step ensures that the slackness condition $\tilde{B}^*_t \tilde{\lambda}_t = 0$ always holds. To implement this condition and be consistent with regime-switching DSGE models in which the *parameters* are the model objects that change state, we define two auxiliary regime-dependent parameters, $\varphi(s^c_t)$ and $\nu(s^c_t)$, such that $\varphi(0) = \nu(0) = 0$, and $\varphi(1) = \nu(1) = 1$.[9] Next, we introduce the following regime-switching slackness condition:

$$\varphi(s^c_t)\tilde{B}^*_{ss} + \nu(s^c_t)(\tilde{B}^*_t - \tilde{B}^*_{ss}) = (1 - \varphi(s^c_t))\tilde{\lambda}_{ss} + (1 - \nu(s^c_t))(\tilde{\lambda}_t - \tilde{\lambda}_{ss}), \tag{17}$$

where $\tilde{B}^*_{ss}$ and $\tilde{\lambda}_{ss}$ are the steady state borrowing cushion and collateral constraint multiplier, respectively, defined more precisely below. It is now easy to see from equation (17) that, as desired, $\tilde{\lambda}_t = 0$ when $s^c_t = 0$, and $\tilde{B}^*_t = 0$ when $s^c_t = 1$, always satisfying the slackness condition $\tilde{B}^*_t \tilde{\lambda}_t = 0$. Yet, given a regime s^c_t, equation (17) remains continuously differentiable for any value of $\tilde{B}^*_t$ or $\tilde{\lambda}_t$.

Technically, equation (17) "preserves" information in the perturbation approximation that we introduce in Section 3.3, since, at first order, both variables $\tilde{B}^*_t$ and $\tilde{\lambda}_t$ are constant at zero in the respective regimes. The use of the regime-dependent switching parameters, $\varphi(s^c_t)$ and $\nu(s^c_t)$, follows from the partition principle of Foerster et al. (2016), which separates parameters based upon whether they affect the steady state or not. Intuitively, $\varphi(s^c_t)$ captures the *level* of the economy changing across regimes (e.g., the level of capital is lower when the constraint binds), while $\nu(s^c_t)$ captures the different dynamics across regimes (e.g., the response of the economy to shocks changes when the constraint binds).

3.2 *Defining the steady state*

Given the regime-switching slackness condition (17), we define a steady state as a state in which all shocks have ceased and the regime-switching variables that affect the level

[9]In our model, these parameters coincide with the regime-switching indicator variable s^c_t, but more generally they may not. The notation provides a general formulation of the modified slackness condition that is applicable to other setups. See, for example, the discussion of our stochastic specification in the context of other model settings in Binning and Maih (2017).

of the economy ($\varphi(s_t^c)$) take the *ergodic mean* values associated with the steady-state transition matrix:

$$\mathbb{P}_{ss}^c = \begin{bmatrix} 1 - \dfrac{\exp(-\gamma_0 \tilde{B}_{ss}^*)}{1+\exp(-\gamma_0 \tilde{B}_{ss}^*)} & \dfrac{\exp(-\gamma_0 \tilde{B}_{ss}^*)}{1+\exp(-\gamma_0 \tilde{B}_{ss}^*)} \\ \dfrac{\exp(-\gamma_1 \tilde{\lambda}_{ss})}{1+\exp(-\gamma_1 \tilde{\lambda}_{ss})} & 1 - \dfrac{\exp(-\gamma_1 \tilde{\lambda}_{ss})}{1+\exp(-\gamma_1 \tilde{\lambda}_{ss})} \end{bmatrix}. \tag{18}$$

Since this matrix depends on the steady-state level of the borrowing cushion and the multiplier, $\tilde{B}_{ss}^*$ and $\tilde{\lambda}_{ss}$, which in turn depend on the ergodic mean of the regime-switching parameter $\varphi(s_t^c)$, such a steady state is the solution of a fixed-point problem that is described in more detail in Appendix B.[10]

More specifically, consider the model regime-specific parameters defined above and distinguish between $\varphi(s_t^c)$, which affect the level of the economy, and $\nu(s_t^c)$, which affect only its dynamics with no effects on the steady state. Then denote $\xi = [\xi_0, \xi_1]$ the ergodic vector of $\mathbb{P}_{ss}^c$. Next, apply the partition principle of Foerster et al. (2016) to focus only on parameters that affect the level of the economy, and write the ergodic mean of $\varphi(s_t^c)$, denoted $\bar{\varphi}$, as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1). \tag{19}$$

Defining the steady state as the state in which the auxiliary parameter $\varphi(s_t^c)$ is at its ergodic mean value $\bar{\varphi}$ implies that the approximation point constructed is a weighted average of the steady states of two separate models: one in which only the nonbinding regime occurs and one in which only the binding regime occurs. The distance of our approximation point to each of these two other steady states therefore depends on the frequency of being in each of the two regimes.[11]

3.3 *The solution and its properties*

Equipped with the steady state of the endogenous regime-switching economy, we construct a second-order approximation to the policy functions by taking derivatives of the equilibrium conditions. We relegate details of these derivations to the Appendix B, but here we provide a summary.

[10] Stochastic volatility does not affect the steady state of the model. Therefore, we can focus on regime switching on the borrowing constraint while finding the steady state.

[11] The ergodic mean is a natural candidate perturbation point in models with endogenous state variables where at least one regime has a short expected duration. These two features imply that the ergodic mean is in the area of the state space in which the economy operates most frequently. In the specific case of the model in this paper, debt and capital are slow-moving state variables, and the binding regime tends to be self-limiting—that is, being in the binding regime causes the economy to reduce leverage, and hence, switch back to the nonbinding regime. Thus, the economy will rarely reach the area around the steady state of the "binding regime only." Alternative methods for finding solutions to endogenous regime-switching models, such as Maih (2015) and Barthélemy and Marx (2017), propose using regime-dependent steady states as multiple approximation points. Such a strategy would not be suitable for our purposes because the binding regime steady state is a poor approximation point given that the regime is infrequent and usually of shorter duration than normal cycles of expansions and contractions.

For each regime s_t, the policy functions of our model take the form

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \qquad \mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \tag{20}$$

where $\mathbf{x}_t$ denotes predetermined variables, $\mathbf{y}_t$ non-predetermined variables, $\boldsymbol{\varepsilon}_t$ the set of shocks, and χ a perturbation parameter such that when $\chi = 1$ the fully stochastic model results and when $\chi = 0$ the model reduces to the nonstochastic steady state defined above. Using these functional forms, we can express the equilibrium conditions conditional on regime s_t as

$$\mathbb{F}_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi) = 0. \tag{21}$$

We then stack the regime-dependent conditions for each s_t, denoting the resulting system of equations with $\mathbb{F}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$, and successively differentiate with respect to $(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$, evaluating them at the steady state. The systems

$$\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \qquad \mathbb{F}_{\boldsymbol{\varepsilon}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \qquad \mathbb{F}_{\chi}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0 \tag{22}$$

can then be solved for the unknown coefficients of the first-order Taylor expansion of the policy functions in equation (20).

It is important to note that since we are only approximating the decision rules and not the probabilities in equation (15), the probabilities remain bounded between zero and one. If we had instead directly generated approximations of the probability functions, we would run the risk of approximated probabilities outside the unit interval. Our method avoids this issue by only approximating the decision rules.

A second-order approximation of the policy functions in equation (20) can be found by taking the second derivatives of $\mathbb{F}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$. In the end, we have two sets of matrices: $H^{(1)}_{s_t}$ and $G^{(1)}_{s_t}$, characterizing the first-order coefficients; and $H^{(2)}_{s_t}$ and $G^{(2)}_{s_t}$ characterizing the second-order coefficients. Therefore, the approximated policy functions are

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H^{(1)}_{s_t} S_t + \frac{1}{2} H^{(2)}_{s_t}(S_t \otimes S_t), \tag{23}$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G^{(1)}_{s_t} S_t + \frac{1}{2} G^{(2)}_{s_t}(S_t \otimes S_t), \tag{24}$$

where $S_t = [(\mathbf{x}_{t-1} - \mathbf{x}_{ss})' \ \boldsymbol{\varepsilon}_t' \ 1]'$.

Our perturbation method produces stable approximated policy functions, but does not guarantee the existence or uniqueness of the solution. As we discuss in Appendix B, this limitation is shared with global solution methods of models of occasionally binding constraints that check convergence of the numerical algorithm without guaranteeing existence or uniqueness.[12]

[12]While in some simpler models with collateral constraints it is possible to impose parametric restrictions that rule out multiple equilibria (Schmitt-Grohe and Uribe (2020), Benigno, Chen, Otrok, Rebucci, and Young (2016)), in the case of our model, as in Mendoza (2010) and Bianchi and Mendoza (2018), there are no such restrictions and uniqueness must be verified heuristically. We check the mean-squared stability of the first-order approximation paired with the steady state transition matrix $\mathbb{P}_{ss}$ (Farmer, Waggoner, and Zha (2011), Foerster et al. (2016)), and additionally check for explosive simulations.

Our solution method is fast, and readily scales to handle larger models. In total, we have 20 equilibrium conditions, two endogenous and six exogenous state variables, four regimes, and six shocks. The model is solved in about a second on a standard laptop.[13]

As Appendix C discusses in more detail, the proposed solution method is also accurate. We investigate its accuracy by applying it to the calibrated model in Mendoza and Villalvazo (2020) and comparing our endogenous regime switching specification solved by perturbation with the traditional inequality constraint specification solved with global methods. We compare the first and second moments for all model variables and show that our solution method yields results practically indistinguishable from those obtained from a traditional inequality specification of the borrowing constraint. Specifically, we find Euler equation errors in line with the accuracy of perturbation methods applied to exogenous regime-switching models (Foerster et al. (2016)) and models without regime-switching (Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006)). We also document a solution speed more than 800 times faster than the global method. Indeed, this solution speed gain is what makes likelihood-based estimation of the model feasible.

3.4 *Approximation order and endogenous switching*

Our endogenous regime-switching framework must be solved at least to the second order to capture the effects of the endogenous switching on the policy rules, which include precautionary effects that vary with the state of the economy. If we were to approximate only to the first order, we would not capture the precautionary behavior stemming from rational expectations about the dependency of the probability of a regime change on the borrowing cushion and the multiplier. The following proposition states this result formally.

Proposition 1 (Properties of the approximated solution). *The first-order approximation to the endogenous regime-switching model is identical to the first-order approximate solution of an exogenous regime-switching model in which the transition probabilities are given by the steady-state value of the time-varying transition matrix. A second-order approximation to the endogenous regime-switching model is necessary and sufficient to capture precautionary effects of the endogenous switching.*

Proof. See Appendix B. □

This result is analogous to stating that, in models without regime-switching, a first-order solution is invariant to the size of the shocks, a second-order solution captures

[13] A MATLAB code for the proposed solution algorithm is available on the authors' web pages. For this paper, our computational approach is similar to that in Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2015). We use Mathematica to take symbolic derivatives and export these derivatives to C++. We then integrate them in Matlab using mex files to solve the model for different parameterizations. In principle, we could solve and estimate the model using only C++ or FORTRAN, but the gains in terms of speed would be relatively minor; for larger models, these languages might yield more substantial efficiency gains. Since we use a nonlinear filter, the filtering in estimation, not the model solution, is the most time-consuming computational step.

precautionary behavior, and a third-order solution is needed to capture the effects of stochastic volatility that follows an autoregressive process as in Fernandez-Villaverde, Guerron-Quintana, and Rubio-Ramirez (2015). In our context, since we have exogenous regime changes in volatility, a second-order approximation is sufficient to capture precautionary effects associated with volatility changes (Foerster et al. (2016)). In our setting, the shocks to the volatility processes of a stochastic volatility specification manifest themselves as discrete changes in the volatilities and not as shocks to the processes themselves.

Unfortunately, however, using a second-order approximation with endogenous regime-switching poses additional challenges for estimation purposes. We now turn to our strategy to address these issues.

4. Estimating the endogenous switching model

We estimate the model with a Bayesian full information procedure. The posterior distribution has no analytical solution, and we use Markov-Chain Monte Carlo (MCMC) methods to sample from it. Since the Metropolis–Hastings algorithm we use for sampling is standard, we omit the discussion of this step in our procedure.

A critical challenge in posterior sampling is the evaluation of the likelihood function. We face three difficulties relative to linear DSGE models (e.g., Smets and Wouters (2007)). The first is the nonlinearity induced by the presence of multiple regimes. The second is the need to approximate to the second order. The third is the fact that the transition probabilities are endogenous. Bianchi (2013) develops an algorithm to address the first difficulty. Here, we must deal with the second-order approximation and endogenous probabilities in a tractable manner. One alternative is the Particle Filter (Fernandez-Villaverde and Rubio-Ramirez (2007)). However, the regime switching leads to the discarding of a large number of simulated particles, lowering the accuracy for a given number of particles and greatly increasing the computational cost of reaching accuracy (Doucet, Gordon, and Krishnamurthy (2001)). To address these challenges, we use the Unscented Kalman Filter (UKF) with Sigma Points (Julier and Uhlmann (1999)). The Sigma Point filter has been shown to be an efficient way of estimating regime-switching models (Binning and Maih (2015)). Details of the construction of the state space representation and the filtering for the evaluation of the likelihood are reported in the Supplementary Appendix (Benigno et al. (2024)).

4.1 *Observables, data, and measurement errors*

We estimate the model with quarterly data for GDP growth (gross output less intermediate input payments), consumption growth, investment growth, and intermediate import price growth, as well as the current account-to-GDP ratio, and a measure of the country real interest rate.[14]

[14] See the SA for details on variable definitions and data sources. The country interest rate is constructed following Uribe and Yue (2006), and it is the US 3-Month Treasury Bill minus ex post US CPI inflation rate plus Mexico's EMBI Spread.

As there are six shocks with six observables, we do not necessarily need measurement errors. However, measurement errors in the observation equation improve the performance of the nonlinear filter and account for any actual measurement error in the data. As in Garcia-Cicco, Pancrazi, and Uribe (2010), we limit their variance to 5% of the variance of the observable variables. As a consequence, the model will fit the data relatively closely on average, and model fit or lack thereof will be assessed by checking whether it relies on large shocks to generate crisis episodes and by comparing our model with alternative specifications.

4.2 *Calibrated parameters and prior distributions*

Our objective is to estimate the critical parameters governing dynamics in both the binding and nonbinding regimes. We calibrate a subset of model parameters listed in Table 1 on which we have strong prior information from the existing literature, including particularly Mendoza (2010), who calibrated them based on the stylized facts of Mexico's National Accounts at an annual frequency. Our model is calibrated and estimated at a quarterly frequency. The SA provides details on the calibration of these parameters.

For the estimated parameters, we set two types of prior. The first is directly on the parameters. These priors are listed in Table 2. They are relatively diffuse, only imposing sign restrictions or placing low prior probability on parameter values that generate implausible moments in model simulations. The second type of prior is on a model-implied object. The model has an ergodic mean conditional on the nonbinding regime that depends on the model solution. This mean is the level at which the economy stabilizes without any regime changes or shocks. The borrowing cushion associated with this mean implies a transition probability to the binding regime in equation (13). We set a prior on this model-object that is a Beta distribution with mean 0.01 and variance of 0.05.[15] This prior places very low probability mass on combinations of parameters that imply too frequent transitions to the binding regime. Intuitively, it reflects the belief that, in the absence of shocks, the probability that the constraint becomes binding is very low.

Table 1. Calibrated parameters.

Parameter	Description	Value
β	Discount Factor	0.99156
ρ	Risk Aversion	2
ω	Labor Supply	1.846
η	Capital Share	0.3053
α	Labor Share	0.5927
δ	Depreciation Rate	0.0228
A^*	Mean Technology	1.7455
Z^*	Mean Growth	1.006
P^*	Mean Import Price	1.028
e^*	Mean Expenditure	0.11

[15]Priors on model-implied objects are used and discussed, for example, in Otrok (2001) and Del Negro and Schorfheide (2008).

Table 2. Estimated parameters.

Par.	Description	Prior	Posterior Mode	Posterior 5%	Posterior 50%	Posterior 95%
$\bar{r}$	Int. Rate Mean	N(0.0177, 0.005)	0.006	0.005	0.006	0.007
ι	Capital Adj.	N(10, 5)	5.769	5.169	5.769	5.967
ϕ	Working Cap.	U(0, 1)	0.769	0.737	0.769	0.799
κ^*	Leverage	U(0, 1)	0.182	0.171	0.182	0.195
$\log\gamma_0$	Logistic, Enter Binding	U(−20, 20)	2.065	2.033	2.065	2.090
$\log\gamma_1$	Logistic, Exit Binding	U(−20, 20)	4.925	4.900	4.925	4.961
ρ_a	Autocor, TFP	B(0.6, 0.2)	0.982	0.961	0.982	0.988
ρ_z	Autocor, TFP Growth	B(0.6, 0.2)	0.811	0.751	0.811	0.873
ρ_p	Autocor, Imp Price	B(0.6, 0.2)	0.978	0.970	0.978	0.986
ρ_r	Autocor, Int Rate	B(0.6, 0.2)	0.956	0.951	0.956	0.957
ρ_e	Autocor, Expend	B(0.6, 0.2)	0.879	0.848	0.878	0.907
ρ_d	Autocor, Pref	B(0.6, 0.2)	0.882	0.861	0.882	0.904
$\sigma_a(L)$	Low SD, TFP	IG(0.005, 0.01)	0.005	0.004	0.005	0.005
$\sigma_a(H)$	High SD, TFP	IG(0.005, 0.01)	0.012	0.011	0.012	0.012
$\sigma_z(L)$	Low SD, TFP Growth	IG(0.005, 0.01)	0.003	0.001	0.003	0.003
$\sigma_z(H)$	High SD, TFP Growth	IG(0.005, 0.01)	0.010	0.009	0.010	0.010
$\sigma_p(L)$	Low SD, Imp Price	IG(0.05, 0.01)	0.027	0.025	0.027	0.029
$\sigma_p(H)$	High SD, Imp Price	IG(0.05, 0.01)	0.063	0.061	0.063	0.065
$\sigma_r(L)$	Low SD, Int Rate	IG(0.01, 0.025)	0.002	0.001	0.002	0.002
$\sigma_r(H)$	High SD, Int Rate	IG(0.01, 0.025)	0.007	0.006	0.007	0.008
$\sigma_e(L)$	Low SD, Exp	IG(0.5, 0.5)	0.160	0.117	0.160	0.194
$\sigma_e(H)$	High SD, Exp	IG(0.5, 0.5)	0.384	0.322	0.384	0.438
$\sigma_d(L)$	Low SD, Pref	IG(0.05, 0.01)	0.048	0.039	0.049	0.057
$\sigma_d(H)$	High SD, Pref	IG(0.05, 0.01)	0.060	0.053	0.060	0.066
$P\sigma_l$	Prob, Stay Low Vol	B(0.975, 0.025)	0.958	0.937	0.958	0.976
$P\sigma_h$	Prob, Stay High Vol	B(0.975, 0.025)	0.949	0.941	0.949	0.955

Note: This table reports the prior distribution and the posterior moments of the estimated parameters. Priors are Normal (N), Uniform (U), Beta (B), or Inverse Gamma (IG) and show mean and variance, except for the uniform distribution showing the lower and upper bounds. Posterior distributions show mode, along with 5th, 50th, and 95th percentiles of the MCMC posterior draws.

4.3 *Estimated parameters and model fit*

We now discuss the estimated parameters and the model's fit to the data. Table 2 reports the mode, the median, the 5th, and the 95th percentile of the posterior distribution. The parameters have tightly estimated posteriors, so we focus on the posterior modes.

The estimated mean interest rate, with a mode of 0.6% per quarter, is higher than the calibrated value in Mendoza (2010). Since we use data on the interest rate as an observable, this parameter estimate is directly linked to that observable. The observed series exceeds our estimate of 0.6% for the early part of the sample, before gradually declining to a value closer to our estimate. The estimated model interprets the sample interest rate as a persistent process slowly converging to from a higher to a lower mean rate.

The mode of the estimated investment adjustment cost parameter (ι) is 5.769. This parameter critically interacts with the financial friction parameters (working capital, leverage, and logistic function) to determine the dynamics of the model. Its estimated magnitude cannot be compared with the data outside the model. The estimated working capital constraint parameter (ϕ) is plausible, indicating that 77% of the wage and intermediate goods bill must be paid in advance with borrowed funds. This value is substantially higher than the 25.79% calibrated in Mendoza (2010) but lower than the 100% assumed by Neumeyer and Perri (2005) or the 125% used by Uribe and Yue (2006). The estimate is close to the 60% value estimated by Ates and Saffie (2016) using interest payments and production costs from Chilean microeconomic data. The estimated value of the leverage parameter in the borrowing constraint (κ) is 0.18, suggesting that less than a fifth of the value of capital serves as collateral. This value is slightly tighter than the baseline value of 0.20 calibrated in Mendoza (2010), and is at the low end of the 0.15–0.30 range considered in that study.

The posterior modes of the logistic parameters in equations (13) and (14) are 2.065 and 4.925 (in log points), respectively. They are estimated to be in a tight range relative to the very loose priors. Figure 2 illustrates their implications for model dynamics. The figure plots the implied probabilities from equation (13) and (14), evaluated at the posterior mode value of γ_0 and γ_1, together with the estimated ergodic distributions of their arguments, the borrowing cushion $\tilde{B}^*$ and the constraint multiplier $\tilde{\lambda}$.

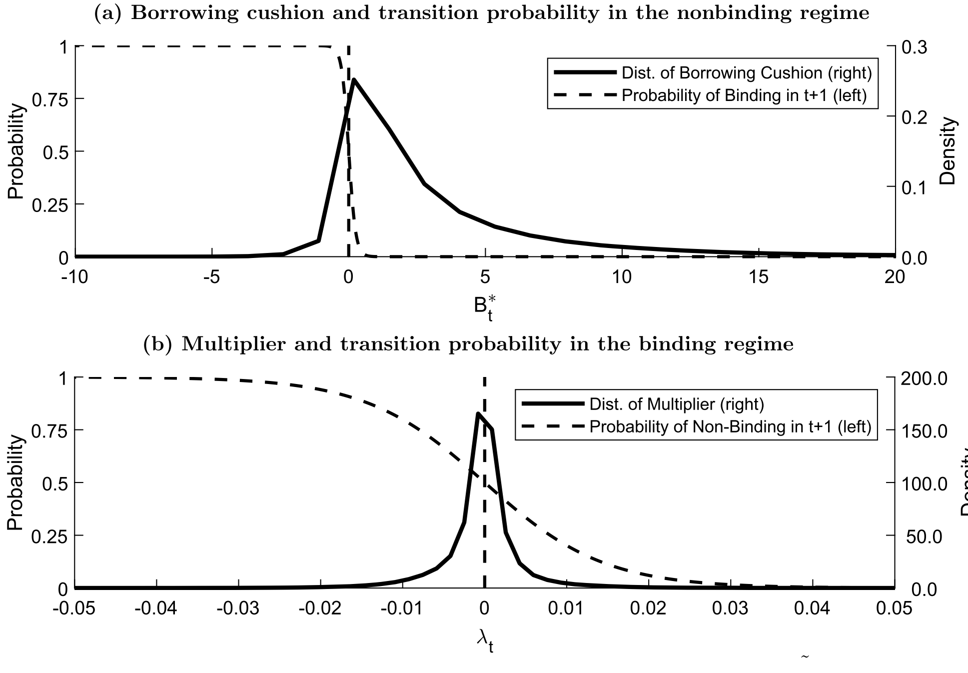

FIGURE 2. The logistic functions and the distributions of their arguments. Note: The top panel shows the model-implied distribution of the borrowing cushion $\tilde{B}^*$ in the nonbinding regime, and the logistic transition function to the binding regime implied by our estimates in of equation (13). The bottom panel shows the model-implied distribution of the multiplier λ in the binding regime, and the transition function to the nonbinding regime as implied by our estimates in equation (14).

The top panel of Figure 2 shows that the bulk of the probability mass is located on the positive side of the ergodic support for this variable, as the economy spends most of its time in the nonbinding regime, above the borrowing limit. As the borrowing cushion declines, the probability of switching to the binding regime increases very rapidly to 1 for small negative values of $\tilde{B}^*$, with a negligible probability mass on larger negative realizations. This implies a relatively quick transition into the binding regime once the borrowing cushion is exhausted as a result of shocks and agents' decisions.

The bottom panel of Figure 2 shows that once the economy is in the binding regime, the ergodic distribution of the multiplier is centered on zero with approximately equal probability on both tails of the support. As $\tilde{\lambda}$ approaches 0 from the positive side of the support, the probability of switching to the nonbinding regime increases only gradually, reaching 1 for λ smaller than -0.02.[16] As we noted earlier, negative values of λ reflect instances in which had the economy been in the nonbinding regime, the borrowing cushion would have been positive, but a switch to the nonbinding regime has not been drawn yet. The estimated values of the logistic function parameters imply that the economy suddenly enters a binding state but exits it only gradually, consistent with the evidence in Cerra and Saxena (2008) and Boissay, Collard, and Smets (2016), among others.

Turning to the exogenous process estimates, all persistent parameters are estimated precisely, with their 95th percentile value below one, except for the temporary productivity shock, whose autocorrelation is very close to one. The estimated volatilities show that the high- and low-volatility regimes are well identified. However, regime change is rare, perhaps reflecting the one-off change in the observable variable behavior after the 1994 Tequila crisis, which is distinctly visible in Figure 3.

The observables used in the estimation are shown in Figure 3, together with the model-implied smoothed series based on the full sample period. The figure also highlights periods of currency or external debt crisis as identified in Reinhart and Rogoff (2009), shown in light gray areas. Since we have assumed a small variance of the measurement errors as a share of the observables', the model tracks the data closely by construction. However, the tracking is consistent throughout the sample period, during both the regular business cycle and the highlighted crisis periods. For example, at the beginning of the 1980s debt crisis and during the 1994–1995 tequila crisis, the data show huge swings in the current account and large drops and rebounds in output, consumption, and investment growth without losing fit. If instead one were to observe a loss of fit during crisis episodes, it would suggest that the model finds it difficult to match the data dynamics during these episodes of critical interest in our empirical analysis.[17]

Likelihood-based estimation permits us to recover the historical shock series that drive the observable variables, which is not possible in a calibrated model. In other words, the estimation not only permits to assess which shocks drive the business cycle in the model, but also which shocks were historically more important in driving specific episodes of crises in Mexico's history.

[16]Values of λ approximately below -0.2, produce a nearly deterministic switch back to the binding regime. The ergodic distribution of λ in the binding regime (Figure 2 b) implies that the probability of exiting that regime exceeds 99% about a quarter of the time.

[17]See the SA for more details.

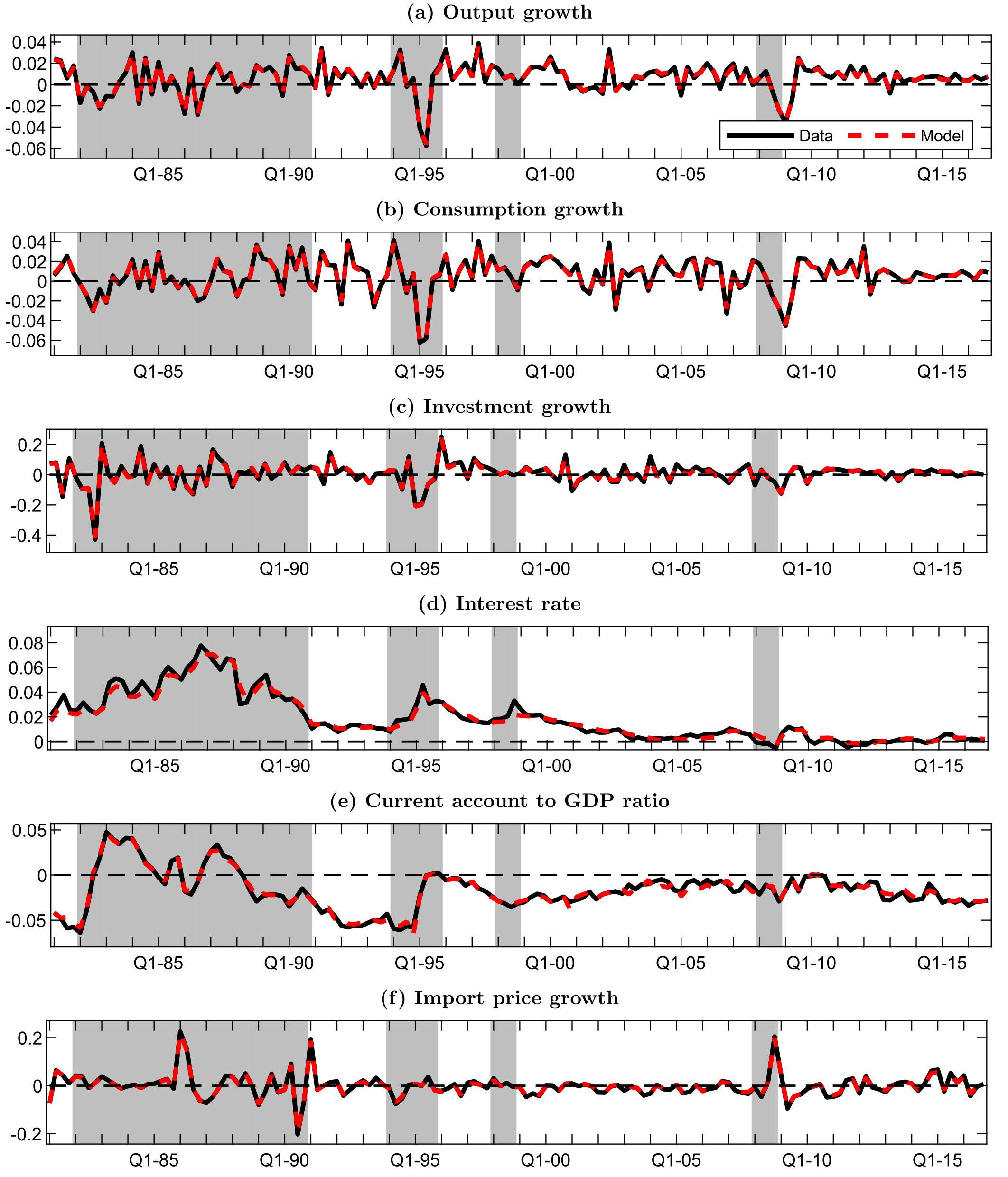

FIGURE 3. Data and model estimates. Note: The figure plots observable variables used in estimation (solid black lines) and fitted values (i.e., model-implied smoothed estimated series based on the full sample, dashed red lines). Light gray areas indicate periods of currency or external debt crisis as identified in Reinhart and Rogoff (2009).

Figure 4 plots the shocks implied by the estimated model in standard deviation units together with a two-standard deviation band. Because the model tracks the observed series closely and evenly over time, the recovered structural shocks are informative about

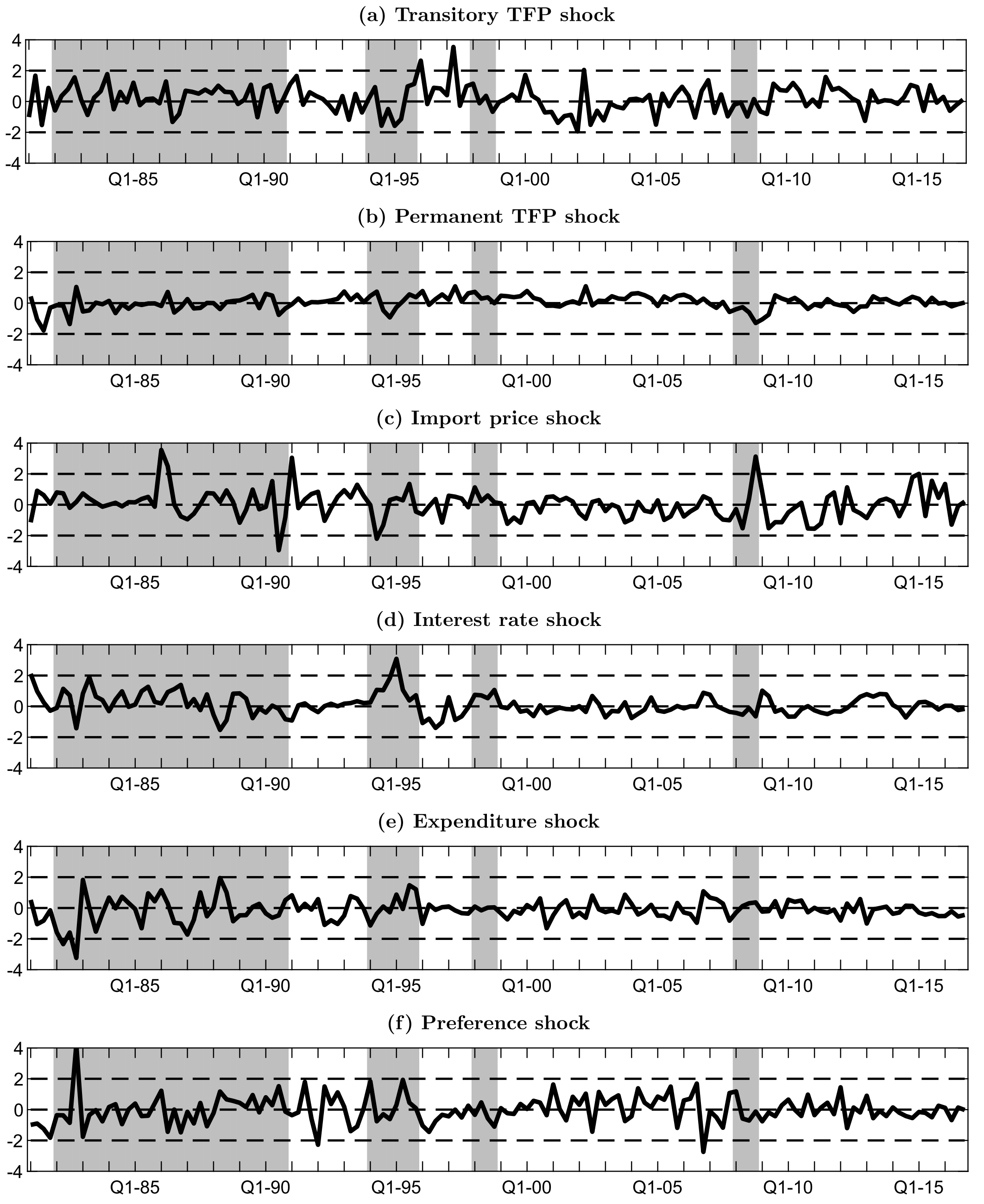

FIGURE 4. Model estimated shocks. Notes: The figure plots the estimated model implied shocks, in standard deviation units, together with a two-standard deviation band (black dashed lines). Light gray areas indicate periods of currency or external debt crisis as identified in Reinhart and Rogoff (2009).

the model mechanisms' ability to drive the outsize movements in the data during crisis periods. If only normally sized shocks are needed, then the model's internal propagation mechanisms must account for the abnormal fluctuations in the data; alternatively, reliance on large, low-probability shocks would cast doubt on the model's ability to capture the crisis dynamics. Well-behaved realized shocks also provide evidence that supports our choice of using the Sigma Point filter.

As we can see from Figure 4, the estimated model fits the data without systematically relying on large shocks, although there are two instances in which large shocks are needed. First, at the beginning of the sample period in 1982:Q3 and Q4, at the peak of the debt crisis, when Mexico devalued the Peso, declared default on its external debt, and nationalized the banking system, very large expenditure and especially preference shocks help to match the current account, which reverted by about 12 percentage points of GDP by the end of that year. Second, an outsize interest rate shock is needed in 1995:Q1, when output dropped by more than 5% in a quarter (the largest change in the sample period), at the end of the Federal Reserve's tightening cycle that started in 1994:Q1, and after the peg was abandoned in December 1994. Two large temporary productivity shocks also match two high growth quarters after the tequila crisis. All other shock realizations are within the two-standard deviation band.[18]

4.4 *Model comparisons*

As a further step in validating our model, Table 3 reports the results of a model comparison exercise using the Schwarz Information Criterion (SIC). We compare our model with a specification with exogenous regime switching and another one without the borrowing constraint. For each of these three models, we also consider a version without stochastic volatility. The SA provides more detail on their estimation and reports model-specific parameter estimates. The statistic reported is the model's posterior density at the posterior mode, adjusted by the Schwarz Information Criteria (SIC) to penalize for additional parameters.[19] A difference of 10 indicates "strong" evidence in favor of the

TABLE 3. Model comparison—Schwarz information criteria.

Model	With Stoch Vol	No Stoch Vol
Endogenous Switching	−4074	−3663
Exogenous Switching	−4039	−3651
No Constraint	−3842	−3465

[18]Three more import price shocks are outside the two-standard deviation error bands: in 2008:Q3, when the oil price reached its historical record high level of $150 per barrel; in 1986:Q1 during the Iran–Iraq War before the oil price collapse later in 1986, and in 1990:Q4 and 1991:Q1 due to the 1991 Iraq War. However, import price shocks are directly implied from the observable series based on commodity terms of trade data.

[19]See Liu, Waggoner, and Zha (2011) for a discussion of the SIC as a goodness-of-fit measure and the challenges of computing marginal data densities in the context of regime switching models, even without considering endogenous switching as in our model.

model with the lower SIC, and a difference of 100 indicates ‘decisive’ evidence (Kass and Raftery (1995)).

The SIC results in Table 3 have three important implications about the fit of our model. First, for all three model specifications, the data prefer the version with stochastic volatility. This result underscores the importance of considering changes in volatility as a driving force in emerging markets, as stressed in Fernandez-Villaverde et al. (2011). Second, the data prefer switching models with a collateral constraint relative to a version without the collateral constraint, where financial frictions take the form of a debt-elastic component on the interest rate. Finally, the data prefer the model version with endogenous switching that we propose to a traditional one in which regime switches are exogenous events with constant transition probabilities. In other words, the model that fits the data best is one in which there is stochastic volatility, regimes based on the state of the collateral constraint (and hence a collateral constraint), and switching between the states of this constraint that is endogenously driven by leverage rather than exogenous probabilities.

5. The anatomy of Mexico’s business cycles and financial crises

In this section, we study Mexico’s history of business cycles and sudden-stop crises through the lens of our estimated model. We first compare moments simulated by the model and in the data and assess the relative importance of different shocks in the business cycle by means of a variance decomposition. We then focus on the model’s fit and the drivers of Mexico’s history of sodden stop crises.

5.1 *Business cycles*

Table 4 compares the data and the simulated second moments of the model, reporting results for output growth, consumption growth, investment growth, the interest rate, trade balance, and the current account.[20]

Table 4. Simulated second moments: data and model.

	Std. Dev.		Relative Std. Dev.		Correlations	
Data Series	Data	Model	Data	Model	Data	Model
GDP Growth	1.41	3.34	1	1	1	1
Cons Growth	1.76	6.44	1.25	1.93	0.73	0.75
Inv Growth	7.57	54.69	5.37	16.38	0.53	0.35
Interest Rate	1.92	1.35	1.36	0.41	−0.11	−0.04
TB/GDP	2.9	9.5	2.0	2.8	0.14	−0.31
CA/GDP	2.2	9.2	1.56	2.76	−0.1	−0.3

Note: The table compares the second moments of the data relative to the moments simulated from the model.

[20] All model-based statistics are based on simulated data from the posterior mode estimates. We generate 100,000 samples of 144 quarters length (the same as our data sample), after a burn-in period of 1000 quarters. We then compute the median values for these 100,000 runs. We use the pruning method in Andreasen, Fernandez-Villaverde, and Rubio-Ramirez (2018) to avoid explosive simulation paths. All reported simulated moments are unconditional rather than conditional on a particular regime.

TABLE 5. Variance decomposition.

Variables/Shocks	Trans TFP	Perm TFP	Imp Price	Int Rate	Expend	Pref.
Output Growth	33.76	22.86	19.33	4.15	15.21	4.69
Cons Growth	18.82	12.12	10.68	6.87	37.06	14.46
Inv Growth	3.62	2.73	2.23	6.45	65.63	19.35
TB/GDP	4.81	3.82	2.69	14.37	57.21	17.1
CA/GDP	4.99	3.99	2.79	9.15	60.89	18.2

Note: For each variable, the decomposition is calculated by setting all shocks to zero except the relevant shock. We then report this variance relative to the sum of all other variances similarly obtained. The variance decomposition sums 100 by construction, but estimates may not add up to 100 exactly due to rounding.

As in other studies, for example, in Mendoza (2010), Garcia-Cicco, Pancrazi, and Uribe (2010), the model has more volatility than the data in absolute terms, especially in the case of investment growth. The model also underestimates the volatility of the country's interest rate process. However, it matches the relative volatilities and the correlations with output very well. Consumption is significantly more volatile than output, while the current account and the interest rate are countercyclical, which are critical stylized facts of emerging market business cycles.[21]

Table 5 reports a variance decomposition for all variables. The table illustrates that all shocks play a significant role, although different shocks matter more for different variables. Productivity and import price shocks and, to a lesser extent, expenditure shocks are the main drivers of output. The main consumption driver is the expenditure shock, followed by the temporary productivity shock. Expenditure and preference shocks are also the main drivers of investment, the trade balance, and the current account. Interest rate shocks appear to play a significant role only for the trade balance and the current account.

These results are most comparable to those estimated in Garcia-Cicco, Pancrazi, and Uribe (2010) because the shocks considered here are the same, except that we also have the import price shock. In our decomposition, interest rate shocks and temporary productivity shocks explain a much smaller share of variance than in Garcia-Cicco, Pancrazi, and Uribe (2010) for all variables. Moreover, Garcia-Cicco, Pancrazi, and Uribe (2010) and Miyamoto and Nguyen (2017) find that the permanent productivity shock is quantitatively unimportant in a model with financial frictions. In contrast, in our model, this shock accounts for more than 20% of the variance of output growth. As we shall see, this shock also play a role in driving crisis dynamics as in Aguiar and Gopinath (2007). The variance share explained by interest rate shocks is also significantly smaller than in Fernandez and Gulan (2015), where these shocks are amplified by a financial accelerator mechanism. In contrast, in Garcia-Cicco, Pancrazi, and Uribe (2010), the financial friction is a debt-elastic country premium, without amplification.

[21] Mexico's trade balance is mildly pro-cyclical in our sample period that ends in 2016, but a countercyclical behavior is typical of emerging market economies, which is what the model is designed to capture.

5.2 *Financial crises*

A defining feature of small open economy models with occasionally binding borrowing constraints is their ability to fit not only the business cycle but also episodes of financial crisis, and particularly the so-called sudden stops in capital flows. So, we now turn to the model-implied reading of Mexico's history of student stops. As a benchmark, we first compare the model implied probability of being in a binding regime or a high volatility state against three episodes of currency and external debt crisis in Mexico's recent history identified in Reinhart and Rogoff (2009): the debt crisis of the 1980s, the 1994–1995 tequila crisis, and the Global Financial Crisis (GFC) in 2008–2009. Next, we investigate their drivers.

5.2.1 *Model validation* The estimated model allows us to *make inferences* on whether the economy is in the binding regime, and thus identify periods of sudden-stop crisis in a model-consistent manner without additional ad hoc restrictions. In the model, the household firm knows the regime, but the estimation procedure does not, and the regime must be inferred based on the information in the data. The estimation results provide a time-varying estimate of the smoothed probability of being in each regime based on the complete information of the sample and a filtered probability based on the information up to time t. Figure 5 plots these two estimates of the probability of being in the binding regime or the high volatility state (solid lines).

The figure shows that our model tracks the debt crisis in the 1980s and the 2008 GFC remarkably well. The model characterizes the debt crisis as a protracted episode (consistent with this crisis being credited for a lost decade in Mexico) in which borrowing is constrained and volatility is high. The filtered probabilities also capture the varying intensity of the crisis.

The model characterizes the beginning of the tequila crisis less sharply but captures its end very precisely. This is because the probabilities of being in the binding regime do not decline in the early 1990s when Mexico's current account deteriorated sharply and persistently before dramatically reverting during 1994:Q4-1995:Q1. However, these probabilities drop to zero very quickly at the end of 1995. In contrast, the smoothed probability of being in a high volatility regime captures well the end of the debt crisis and the beginning of the tequila crisis. Both smoothed and filtered probabilities declined in the run-up to NAFTA in the early 1990s, although they do not increase to one at the peak of the crisis. Therefore, the model fails to identify the early 1990s as a period in which the borrowing constraint is not binding, but correctly characterizes the tequila crisis as a short-lived episode of constrained external borrowing and high volatility.[22]

The model does a very good job of capturing the short-lived nature of the impact of the GFC on Mexico, again characterizing the episode with a spike in the probability of facing a binding borrowing constraint and high volatility. The model does not give any crisis signal matching the external tally index in 1998 at the peak of the Asian crisis, when

[22] Reinhart and Rogoff (2009) identify a protracted banking crisis in Mexico from 1994 to 2000. However, the tequila crisis is widely credited for being the first of a long sequence of V-shaped (i.e., short-lived) external crises in emerging market economies, including for instance the Asian crisis in 1997 and early 1998, and the Russian and Brazilian crises in late 1998.

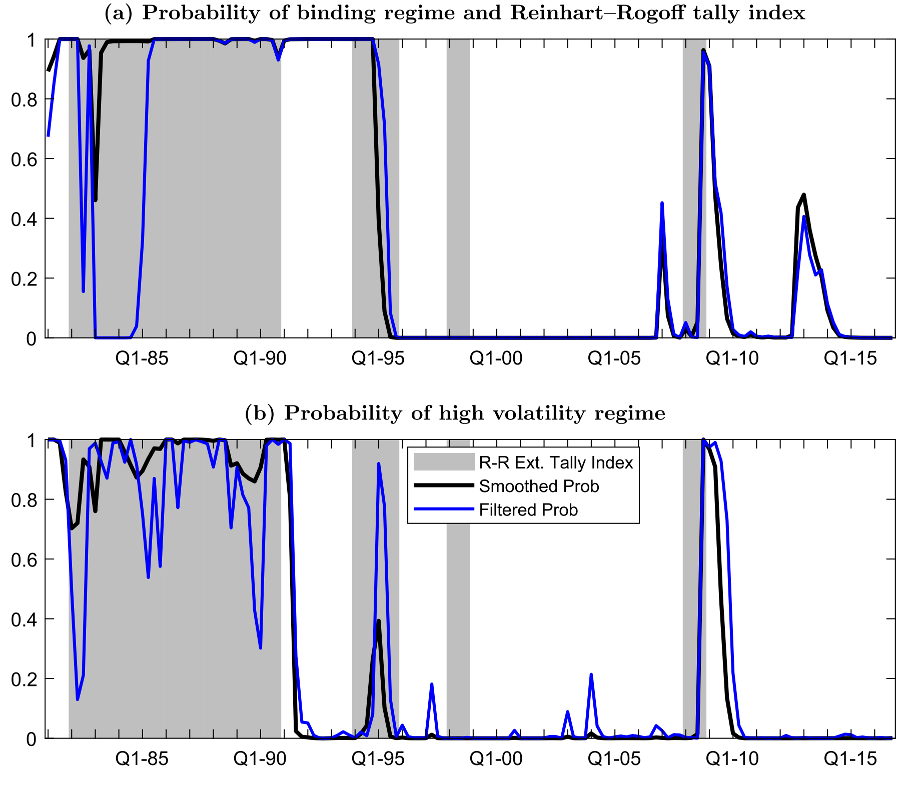

FIGURE 5. Mexico's model-identified crisis episodes.Notes: The darker solid (black) line is the model-implied smoothed probability of being in the binding regime (panel a) or the high-volatility regime (panel b). The lighter solid line (blue) plots the filtered probabilities using information only up to time t. The light gray regions denote the periods of currency or external debt crisis according to Reinhart and Rogoff (2009), which we call the external crisis tally index.

Russia and Brazil experienced default and a huge sudden stops, respectively. However, in general, the model does a remarkable job of identifying and characterizing crises of varying duration and intensity.

5.2.2 *Drivers of Mexico's sudden-stop episodes* We now consider which shocks drove Mexico's history of external crises outlined above. We evaluate the relative importance of shocks that drive the economy before, during, and after these events. This exercise is feasible due to our likelihood-based estimation of the model generating shock sequences, as illustrated in Figure 4, which allows the construction of a historical decomposition.

Many counterfactuals are possible, and there is no standard for this class of models, so here we present results from one possible approach. The multiple sources of nonlinearity in the model, the endogenous regime-switching, and the second-order approximation, pose a challenge for computing historical decompositions. The task is further complicated by the fact that shocks have nonlinear effects not only on the endogenous variables but also on the realization of the regimes in subsequent periods. To address these issues, rather than decomposing the change in individual endogenous variables, we compute a summary measure, namely the importance of each shock in terms of its contribution to the likelihood.

Specifically, for each shock and time period, we counterfactually recalculate the model likelihood, evaluated at the posterior mode, turning one shock off at the time while leaving all other shocks at their estimated values, and then repeat the calculation for all six shocks and all time periods. Denoting with LL the maximized log-likelihood, and letting $CLL_{i,t}$ denote the counterfactual log likelihood when $\varepsilon_{i,t} = 0$, the measure is $\Lambda_{i,t} = \frac{LL - CLL_{i,t}}{\sum_j LL - CLL_{j,t}}$, which measures the relative importance of $\varepsilon_{i,t}$ in period t. By construction, the percentages in Figure 6 add up to one across the shocks for each period.

First, consider the 1980s debt crisis. Productivity and import price shocks have above-average relative importance in 1981, when the oil price starts to decline, and the economy is slowing. The preference shock, the import price shock, and then the expenditure shock, in this sequence, drive the current account reversal in 1982, possibly reflecting delayed fiscal adjustment, the decline in the real oil price from its peak in 1979, and import and fiscal contraction typically associated with a sudden stop and its aftermath. Temporary and permanent productivity shocks and expenditure shocks are more important in the second phase of the crisis, when the economy stagnates and the fiscal consolidation and the structural reforms imposed by the IMF and the World Bank adjustment programs take hold. Interest rate shocks seem more important in the mid-1985 and in 1987.

Next, consider the tequila Crisis. Before the crisis, expenditure shocks play an important role in 1992 at the expense of all other shocks. Permanent productivity and preference shocks are the most important in 1993, possibly capturing the anticipation effects associated with the signing of NAFTA. The importance of the preference shock increases again during 1994, but less than in 1993. The peak of the crisis in 1994:Q4 and 1995:Q1 is captured by a permanent productivity shock. In the aftermath of the crisis, all shocks matter, with small increases in the importance of permanent productivity shocks.

Lastly, consider the 1998 period of sudden stops in other emerging markets and the GFC episode. In 1998, no shock has a disproportionate impact on the likelihood. However, we can see a slight increase in the importance of expenditure shocks at the expense of import price shocks. In the run-up to the GFC, preference (2006:Q4) and expenditure (2007:Q1) shocks have a lower importance, while productivity, import price shocks, and interest rate shocks are of greater importance than average. During the crisis itself, no shock plays a disproportionate role, and all shocks seem to contribute roughly equally. In the aftermath of the crisis, again, all shocks have similar importance through the end of the sample period.

These results show that while all shocks have comparable average importance from a likelihood perspective, as the mean differences are very small, some have higher weights than others before or during periods of sudden stop in Mexico's recent history. However,

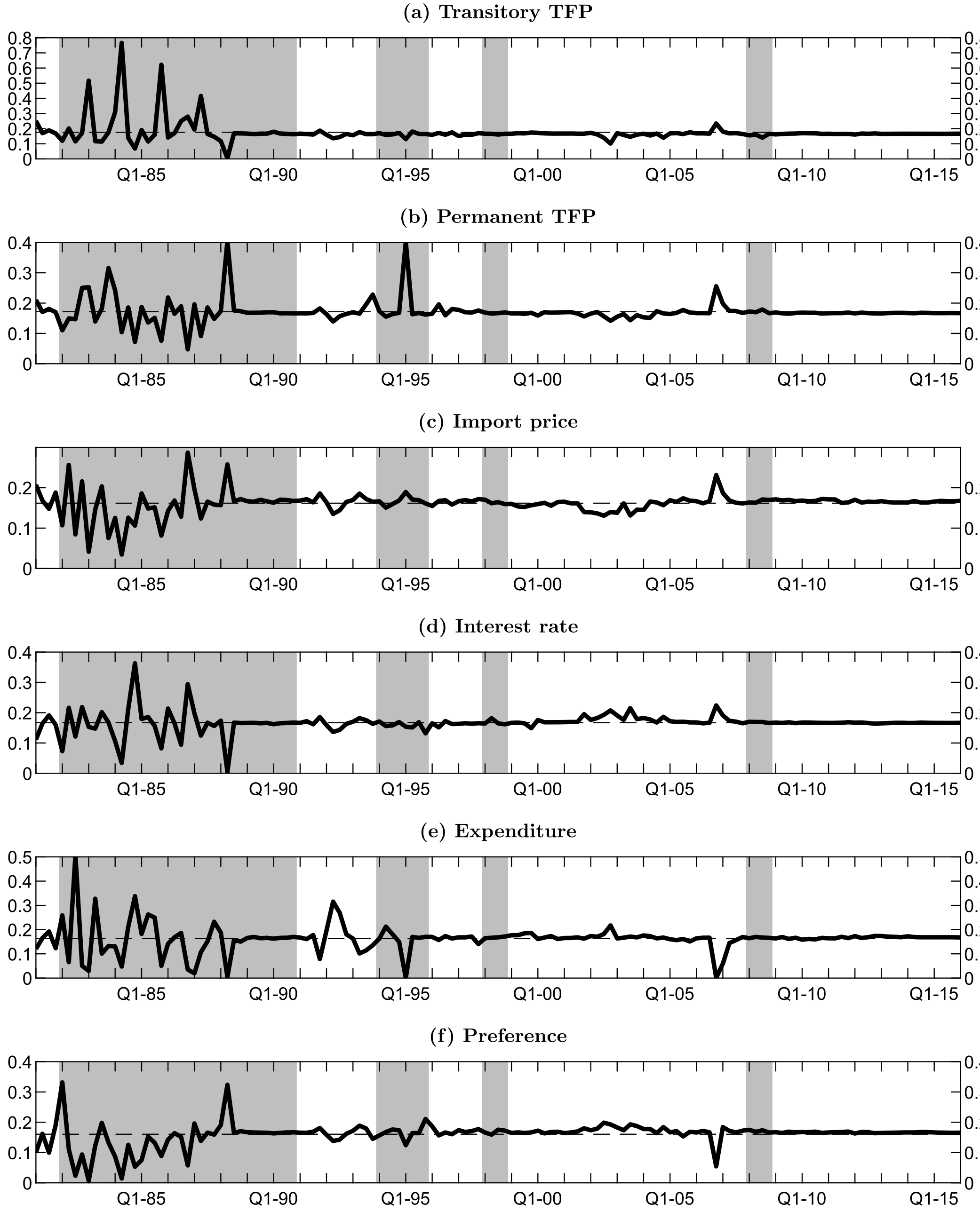

Figure 6. Historical importance of shocks. Notes: The figure plots a measure of relative contribution to the likelihood for each shock in each quarter, computed by setting the given shock to zero in the given quarter (solid thick black lines) as described in the text. The figure also plots the average shock contribution over the full sample (dashed thin black lines). The gray areas denote the currency and external debt crisis periods identified by Reinhart and Rogoff (2009).

and importantly, the few shock realizations that are outside the two-standard deviation error band in Figure 4 are not those that are most important from a likelihood perspective. In other words, they are not those that drive the likelihood before or during crisis periods.

6. Conclusions

In this paper, we propose a new approach to specifying and solving DSGE models with occasionally binding constraints and stochastic volatility that is suitable for structural estimation using full information methods. We then obtain estimates of critical model parameters and conduct likelihood-based inference and counterfactual experiments. The critical step in our approach is to specify the occasionally binding nature of the borrowing constraint stochastically, so that the formulation can be mapped into an endogenous regime-switching model.

We apply this new framework to Mexico's history of cycles and crises by estimating the model with Bayesian methods on quarterly data since 1981. We find that the estimated model fits Mexico's business cycle and crisis episodes well without relying on large shocks. We also find that critical parameter of the estimates differ from values previously used in the literature, and that interest rate shocks may matter less than previously estimated in the literature. Finally, we also show that our estimated model identifies sudden stops of varying duration and intensity, without imposing ad hoc restrictions on their amplitude or duration. The specific shocks that drive these episodes are found in line with commonly accepted narratives of Mexico's history of financial crises.

We regard the theoretical analysis of contracting environments in which the economy switches endogenously but stochastically between lending regimes, consistent with growing microeconomic evidence of covenant contract violations and financing cut-off periods of varying intensity and duration, as an important area of future research.

Appendix A: Model and equilibrium definition

This Appendix derives the equilibrium conditions and defines a competitive equilibrium of the model.

A.1 *Derivation of equilibrium conditions*

The household-firm maximizes the utility function

$$\mathbb{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \left\{ d_t \beta^t \frac{\left(C_t - Z_{t-1} \frac{H_t^{\omega}}{\omega} \right)^{1-\rho} - 1}{1-\rho} \right\}, \tag{A.1}$$

subject to

$$C_t + I_t + E_t = A_t K_{t-1}^{\eta} (Z_t H_t)^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t - \phi r_t (W_t H_t + P_t V_t) - \frac{1}{(1+r_t)} B_t + B_{t-1}. \tag{A.2}$$

The preference shock follows

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}. \tag{A.3}$$

Transitory technology follows

$$\log A_t = (1-\rho_a) \log \bar{A} + \rho_a \log A_{t-1} + \sigma_a \varepsilon_{a,t}, \tag{A.4}$$

while permanent technology follows

$$\Delta \log Z_t = (1-\rho_z) \log \bar{Z} + \rho_z \Delta \log Z_{t-1} + \sigma_z \varepsilon_{z,t}. \tag{A.5}$$

Total value added or GDP is given by

$$Y_t = A_t K_{t-1}^{\eta} (Z_t H_t)^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t. \tag{A.6}$$

Capital accumulates according to

$$K_t = (1-\delta) K_{t-1} + I_t - \frac{\iota}{2} \left(\frac{K_t - \Lambda_k K_{t-1}}{K_{t-1}} \right)^2 K_{t-1}, \tag{A.7}$$

where I_t denotes investment and Λ_k the growth rate of capital along the balanced growth path. The expenditure process E_t follows

$$\log e_t = (1-\rho_e) \log \bar{e} + \rho_e \log e_{t-1} + \sigma_e \varepsilon_{e,t}, \tag{A.8}$$

where

$$e_t = E_t / Z_{t-1}. \tag{A.9}$$

In the binding regime, the collateral constraint is given by

$$\frac{1}{(1+r_t)} B_t - \phi (1+r_t)(W_t H_t + P_t V_t) = -\kappa q_t K_t, \tag{A.10}$$

with the corresponding multiplier denoted λ_t. In the nonbinding regime, the collateral constraint disappears, and the multiplier is $\lambda_t = 0$. The constraint is implemented by defining

$$B_t^* = \frac{1}{(1+r_t)} B_t - \phi (1+r_t)(W_t H_t + P_t V_t) + \kappa q_t K_t \tag{A.11}$$

and using the regime-switching slackness condition

$$\varphi(s_t) B_{ss}^* + \nu(s_t)\big(B_t^* - B_{ss}^*\big) = \big(1 - \varphi(s_t)\big) \lambda_{ss} + \big(1 - \nu(s_t)\big)(\lambda_t - \lambda_{ss}) \tag{A.12}$$

The household-firm maximizes the following Lagrangian:

$$
\begin{aligned}
L = \mathbb{E}_0 \sum_{t=0}^{\infty} d_t \beta^t \frac{\left(C_t - Z_{t-1}\dfrac{H_t^{\omega}}{\omega}\right)^{1-\rho} - 1}{1-\rho} \\
+ \sum_{t=0}^{\infty} \beta^t \mu_t \left[\begin{array}{c} A_t K_{t-1}^{\eta} (Z_t H_t)^{\alpha} V_t^{1-\alpha-\eta} - P_t V_t - \phi r_t (W_t H_t + P_t V_t) \\ -\dfrac{1}{(1+r_t)} B_t + B_{t-1} \\ -C_t - K_t + (1-\delta) K_{t-1} - \dfrac{\iota}{2}\left(\dfrac{K_t - \Lambda_k K_{t-1}}{K_{t-1}}\right)^2 K_{t-1} - E_t \end{array}\right] \\
+ \sum_{t=0}^{\infty} \beta^t \lambda_t \left[\frac{1}{(1+r_t)} B_t - \phi (1+r_t)(W_t H_t + P_t V_t) + \kappa q_t K_t\right].
\end{aligned}
$$

The first-order conditions are

$$
C_t : d_t \left(C_t - Z_{t-1}\frac{H_t^{\omega}}{\omega}\right)^{-\rho} - \mu_t = 0; \tag{A.13}
$$

$$
P_t : \mu_t \big((1-\alpha-\eta) A_t K_{t-1}^{\eta} (Z_t H_t)^{\alpha} V_t^{-\alpha-\eta} - P_t - \phi r_t P_t\big) - \lambda_t \phi (1+r_t) P_t = 0; \tag{A.14}
$$

$$
H_t : \left[\begin{array}{c} -d_t \left(C_t - Z_{t-1}\dfrac{H_t^{\omega}}{\omega}\right)^{-\rho} Z_{t-1} H_t^{\omega-1} \\ + \mu_t \big(\alpha A_t K_{t-1}^{\eta} Z_t^{\alpha} H_t^{\alpha-1} V_t^{1-\alpha-\eta} - \phi r_t W_t\big) - \lambda_t \phi (1+r_t) W_t \end{array}\right] = 0; \tag{A.15}
$$

$$
B_t : -\frac{\mu_t}{1+r_t} + \beta \mu_{t+1} + \frac{\lambda_t}{1+r_t} = 0; \tag{A.16}
$$

$$
K_t : \left[\begin{array}{c} \mu_t \left(-1 - \iota\left(\dfrac{K_t - \Lambda_k K_{t-1}}{K_{t-1}}\right)\right) + \lambda_t \kappa q_t \\ + \beta \mu_{t+1} \left(\begin{array}{c} \eta A_{t+1} K_t^{\eta-1} (Z_{t+1} H_{t+1})^{\alpha} V_{t+1}^{1-\alpha-\eta} + 1 - \delta \\ + \iota \Lambda_k \left(\dfrac{K_{t+1} - \Lambda_k K_t}{K_t}\right) + \dfrac{\iota}{2}\left(\dfrac{K_{t+1} - \Lambda_k K_t}{K_t}\right)^2 \end{array}\right) \end{array}\right] = 0. \tag{A.17}
$$

Combining and simplifying these expressions produces

$$
d_t \left(C_t - Z_{t-1}\frac{H_t^{\omega}}{\omega}\right)^{-\rho} = \mu_t; \tag{A.18}
$$

$$
(1-\alpha-\eta) A_t K_{t-1}^{\eta} (Z_t H_t)^{\alpha} V_t^{-\alpha-\eta} = P_t \left(1 + \phi r_t + \frac{\lambda_t}{\mu_t} \phi (1+r_t)\right); \tag{A.19}
$$

$$
\begin{aligned}
\alpha A_t K_{t-1}^{\eta} Z_t^{\alpha} H_t^{\alpha-1} V_t^{1-\alpha-\eta} = \phi W_t \left(r_t + \frac{\lambda_t}{\mu_t}(1+r_t)\right) \\
+ Z_{t-1} H_t^{\omega-1};
\end{aligned} \tag{A.20}
$$

$$
\mu_t = \lambda_t + \beta (1+r_t) \mathbb{E}_t \mu_{t+1}; \tag{A.21}
$$

$$\beta \mathbb{E}_t \mu_{t+1} \begin{pmatrix} \eta A_{t+1} K_t^{\eta-1} (Z_{t+1} H_{t+1})^{\alpha} V_{t+1}^{1-\alpha-\eta} \\ + 1 - \delta \\ + \iota \Lambda_k \left(\dfrac{K_{t+1} - \Lambda_k K_t}{K_t} \right) \\ + \dfrac{\iota}{2} \left(\dfrac{K_{t+1} - \Lambda_k K_t}{K_t} \right)^2 \end{pmatrix} = \mu_t \left(1 + \iota \left(\frac{K_t - \Lambda_k K_{t-1}}{K_{t-1}} \right) \right) - \lambda_t \kappa q_t. \tag{A.22}$$

Factor prices must satisfy $W_t = -\frac{\partial \mathbb{U}/\partial H_t}{\partial \mathbb{U}/\partial C_t}$ and $q_t = \partial I_t / \partial K_t$, which means

$$W_t = Z_{t-1} H_t^{\omega-1}, \tag{A.23}$$

$$q_t = 1 + \iota \left(\frac{K_t - \Lambda_k K_{t-1}}{K_{t-1}} \right). \tag{A.24}$$

The interest rate is an exogenous process given by

$$r_t^* = (1 - \rho_r) \bar{r}^* + \rho_r r_{t-1}^* + \sigma_r \varepsilon_{r,t}, \tag{A.25}$$

with the possibility of a debt-premium (only in an alternative model specification, in the baseline $\psi = 0$) specified as follows:

$$r_t = r_t^* + \psi \left(\exp(\bar{b} - B_t / Z_{t-1}) - 1 \right). \tag{A.26}$$

The external financing premium on debt is

$$EFPD_t = \frac{\lambda_t}{\beta \mathbb{E}_t \mu_{t+1}}. \tag{A.27}$$

A.2 *Competitive equilibrium*

A competitive equilibrium of our economy is a sequence of quantities $\{K_t, B_t, C_t, H_t, V_t, I_t, A_t, Z_t, d_t, E_t, B_t^*\}$ and prices $\{P_t, r_t^*, r_t, q_t, w_t, \mu_t, \lambda_t\}$ that, given the 5 exogenous processes (A.3), (A.4), (A.5), (A.8), (A.25), satisfies the first-order conditions for the representative household-firm (A.18)–(A.22), the market price equations (A.23)–(A.24), the market clearing conditions (A.2)–(A.7), the debt cushion definition (A.11), regime-switching slackness condition (A.12), and the equation for the interest rate (A.26).

Appendix B: Perturbation solution method

This Appendix provides details about two aspects of the solution method: (1) the definition of, and solution for, the steady state of the endogenous regime-switching economy; and (2) the perturbation method that generates second-order Taylor expansions to the solution of the economy around the steady state. For notational simplicity, we focus on a version of the model without volatility switching.

B.1 *Regime switching equilibrium*

Write the equilibrium conditions as

$$\mathbb{E}_t f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{x}_t, \mathbf{x}_{t-1}, \chi \boldsymbol{\varepsilon}_{t+1}, \boldsymbol{\varepsilon}_t, \boldsymbol{\theta}_{t+1}, \boldsymbol{\theta}_t) = 0. \tag{B.1}$$

Here, $\mathbf{y}_t$ denotes the non-predetermined variables, $\mathbf{x}_t$ predetermined variables, $\boldsymbol{\varepsilon}_t$ the exogenous shocks, $\boldsymbol{\theta}_t$ the regime-switching parameters, and χ the perturbation parameter. In general, the regime-switching parameters are partitioned into those that affect the steady state, $\boldsymbol{\theta}_{1,t}$, and those that do not, $\boldsymbol{\theta}_{2,t}$.[23] In the case of our specific application, the partition is

$$\boldsymbol{\theta}_{1,t} = \big[\varphi(s_t)\big] \qquad \boldsymbol{\theta}_{2,t} = \big[\nu(s_t)\big]. \tag{B.2}$$

In order to solve the model, we assume the functional forms

$$\boldsymbol{\theta}_{1,t+1} = \bar{\boldsymbol{\theta}}_1 + \chi \hat{\boldsymbol{\theta}}_1(s_{t+1}), \qquad \boldsymbol{\theta}_{1,t} = \bar{\boldsymbol{\theta}}_1 + \chi \hat{\boldsymbol{\theta}}_1(s_t), \tag{B.3}$$

$$\boldsymbol{\theta}_{2,t+1} = \boldsymbol{\theta}_2(s_{t+1}), \qquad \boldsymbol{\theta}_{2,t} = \boldsymbol{\theta}_2(s_t), \tag{B.4}$$

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \tag{B.5}$$

$$\mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \qquad \mathbf{y}_{t+1} = g_{s_{t+1}}(\mathbf{x}_t, \chi \boldsymbol{\varepsilon}_{t+1}, \chi) \tag{B.6}$$

and

$$\mathbb{P}_{s_t, s_{t+1}, t} = \boldsymbol{\pi}_{s_t, s_{t+1}}(\mathbf{y}_t). \tag{B.7}$$

Now, substituting these functional forms in the equilibrium conditions and being more explicit about the expectation operator, given $(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$ and s_t, we have

$$F_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi) = \int \sum_{s'=0}^{1} \boldsymbol{\pi}_{s_t, s'}\big(g_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi)\big) \times f \begin{pmatrix} g_{s_{t+1}}\big(h_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \chi \boldsymbol{\varepsilon}', \chi\big), \\ g_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \\ h_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \\ \mathbf{x}_{t-1}, \chi \boldsymbol{\varepsilon}', \boldsymbol{\varepsilon}_t, \\ \bar{\boldsymbol{\theta}} + \chi \hat{\boldsymbol{\theta}}(s'), \bar{\boldsymbol{\theta}} + \chi \hat{\boldsymbol{\theta}}(s_t) \end{pmatrix} d\boldsymbol{\mu}\boldsymbol{\varepsilon}', \tag{B.8}$$

where $d\boldsymbol{\mu}\boldsymbol{\varepsilon}'$ denotes the joint pdf of the shocks.

Finally, stacking all conditions by regime yields

$$\mathbb{F}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi) = \begin{bmatrix} F_{s_t=0}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi) \\ F_{s_t=1}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi) \end{bmatrix} = 0. \tag{B.9}$$

[23] In the version with volatility switches, these parameters belong to the second set.

B.2 *Steady-state definition and solution*

The model has two features that make defining a steady state challenging. First, as it is common in a regime-switching framework, some structural parameters may be switching. In the case of our application, there is only one switching parameter that affects the steady state, $\varphi(s_t)$. Nonetheless, in principle, one could allow for regime switching also in the parameters of the exogenous processes, $A^*(s_t)$ and $P^*(s_t)$, or the structural parameter $\kappa^*(s_t)$, which would affect the level of the economy and the steady-state calculations. Following Foerster et al. (2016), we define the steady state in terms of the ergodic means of these parameters across regimes. To define the steady state, we set $\boldsymbol{\varepsilon}_t = 0$ and $\chi = 0$, which implies that the steady state is given by

$$f\big(\mathbf{y}_{ss}, \mathbf{y}_{ss}, \mathbf{x}_{ss}, \mathbf{x}_{ss}, 0, 0, \bar{\boldsymbol{\theta}}_{\mathbf{1}}, \boldsymbol{\theta}_2\big(s'\big), \bar{\boldsymbol{\theta}}_1, \boldsymbol{\theta}_2(s)\big) = 0 \tag{B.10}$$

for all s', s.

In our case, the transition matrix evaluated at steady-state $\mathbb{P}_{ss}$ is endogenous, since it depends on variables that in turn depend on the steady-state value of the transition matrix. To find a solution for the steady state (balanced growth path), we proceed in two steps. First, we assume the steady-state transition matrix is known and solve for all the steady- state prices and quantities. Second, we use the steady-state values of the borrowing cushion $\tilde{B}^*_{ss}$ and multiplier λ_{ss} from step 1 to update the steady-state transition matrix. We then iterate to convergence.

Step 1: Solve steady state using a given steady-state transition matrix. First, assume that the steady-state transition matrix at iteration i, $\mathbb{P}^{(i)}_{ss}$, is known. Next, let $\xi = [\xi_0, \xi_1]$ denote the ergodic vector of $\mathbb{P}^{(i)}_{ss}$. Then, as noted in the paper, define the ergodic means of the switching parameters as

$$\bar{\varphi} = \xi_0 \varphi(0) + \xi_1 \varphi(1).$$

The steady state of the regime-switching economy depends on these ergodic means, and we can now solve for the steady states of all variables.

Step 2: Updating the transition matrix. Step 1 yields the variables $\tilde{B}^*_{ss}$ and λ_{ss}, and hence, provides a new value of the transition matrix for iteration $i+1$:

$$\mathbb{P}^{(i+1)}_{ss} = \begin{bmatrix} p_{00,ss} & p_{01,ss} \\ p_{10,ss} & p_{11,ss} \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{\exp\big(-\gamma_0 \tilde{B}^*_{ss}\big)}{1 + \exp\big(-\gamma_0 \tilde{B}^*_{ss}\big)} & \dfrac{\exp\big(-\gamma_0 \tilde{B}^*_{ss}\big)}{1 + \exp\big(-\gamma_0 \tilde{B}^*_{ss}\big)} \\ \dfrac{\exp(-\gamma_1 \lambda_{ss})}{1 + \exp(-\gamma_1 \lambda_{ss})} & 1 - \dfrac{\exp(-\gamma_1 \lambda_{ss})}{1 + \exp(-\gamma_1 \lambda_{ss})} \end{bmatrix}, \tag{B.11}$$

which can be checked against the guess in Step 1. We then iterate to convergence until

$$\big\| \mathbb{P}^{(i+1)}_{ss} - \mathbb{P}^{(i)}_{ss} \big\| < \textit{tolerance},$$

where in our application we use a tolerance of 10^{-10}.

B.3 *Generating approximations*

To compute a second-order approximation to the endogenous regime-switching model solution, we largely follow Foerster et al. (2016), adapting to the case with endogenous probabilities.

We take the stacked equilibrium conditions $\mathbb{F}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$, and differentiate with respect to $(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$. The first-order derivative with respect to $\mathbf{x}_{t-1}$ produces a polynomial system denoted

$$\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0. \tag{B.12}$$

In Foerster et al. (2016), when the transition probabilities are exogenous and fixed, this system needs to be solved via Gröbner bases, which finds all possible solutions in order to check them for stability. The relevant stability concept is mean square stability (MSS), which requires the expectation of first and second moments to be finite (see Costa, Fragoso, and Marques (2005)). In our case with endogenous probabilities, the check for MSS is not applicable, so we focus on finding a single solution and ignore the possibility of multiple solutions or indeterminacy, a common simplification in the regime-switching literature with and without endogenous switching (e.g., Farmer, Waggoner, and Zha (2011), Foerster (2015), Maih (2015), Lind (2014)). This simplification is also common to global solution methods of models with occasionally binding constraints, where numerical methods converge to a given solution but do not guarantee uniqueness of that solution. Instead, the focus typically is on checking robustness of the solution to initial conditions. While in some simpler models with collateral constraints it is possible to impose parametric restrictions that rule out multiple equilibria (Schmitt-Grohe and Uribe (2020), Benigno et al. (2016)), in the case of our model, as in Mendoza (2010) and Bianchi and Mendoza (2018), there are no such restrictions and uniqueness must be verified numerically.

To find a model solution, we guess a set of policy functions for regime $s_t = 1$, which reduces the equilibrium conditions $\mathbb{F}_{\mathbf{x}}(\mathbf{x}_{ss}, \mathbf{0}, 0; s_t = 0)$ to a fixed-regime eigenvalue problem, and solve for the policy functions for $s_t = 0$. Then, using this initial solution as a guess, we solve for regime $s_t = 0$ under the fixed-regime eigenvalue problem, and iterate to convergence. After solving the iterative eigenvalue problem, the remaining systems to solve are

$$\mathbb{F}_{\boldsymbol{\varepsilon}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \tag{B.13}$$

$$\mathbb{F}_{\chi}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \tag{B.14}$$

and the second-order systems of the form

$$\mathbb{F}_{\mathbf{i},\mathbf{j}}(\mathbf{x}_{ss}, \mathbf{0}, 0) = 0, \quad \mathbf{i}, \mathbf{j} \in \{\mathbf{x}, \boldsymbol{\varepsilon}, \chi\}. \tag{B.15}$$

Recalling now that the decision rules have the form

$$\mathbf{x}_t = h_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \tag{B.16}$$

$$\mathbf{y}_t = g_{s_t}(\mathbf{x}_{t-1}, \boldsymbol{\varepsilon}_t, \chi), \tag{B.17}$$

the second-order approximation are

$$\mathbf{x}_t \approx \mathbf{x}_{ss} + H_{s_t}^{(1)} S_t + \frac{1}{2} H_{s_t}^{(2)} (S_t \otimes S_t), \tag{B.18}$$

$$\mathbf{y}_t \approx \mathbf{y}_{ss} + G_{s_t}^{(1)} S_t + \frac{1}{2} G_{s_t}^{(2)} (S_t \otimes S_t) \tag{B.19}$$

where $S_t = [(\mathbf{x}_{t-1} - \mathbf{x}_{ss})' \ \boldsymbol{\varepsilon}_t' \ 1]'$, with $\mathbf{x}_{ss}$ denoting the value of the steady-state variables.

B.4 *Proof of Proposition 1 (properties of the approximated solution)*

To prove Proposition 1, take the first-order derivatives of (B.9) with respect to its arguments, evaluated at the steady state. This yields

$$\begin{aligned} \mathbb{F}_{\mathbf{x},s_t}(\mathbf{x}_{ss}, \mathbf{0}, 0) &= \sum_{s'} \pi_{s_t,s',y}(\mathbf{y}_{ss}) g_{\mathbf{x},s_t} f_{ss}(s', s_t) \\ &\quad + \sum_{s'} \pi_{s_t,s'}(\mathbf{y}_{ss}) \begin{bmatrix} f_{\mathbf{y}_{t+1}}(s', s_t) g_{\mathbf{x},s'} h_{\mathbf{x},s_t} + f_{\mathbf{y}_t}(s', s_t) g_{\mathbf{x},s_t} \\ + f_{\mathbf{x}_t}(s', s_t) h_{\mathbf{x},s_t} + f_{\mathbf{x}_{t-1}}(s', s_t) \end{bmatrix}, \end{aligned} \tag{B.20}$$

$$\begin{aligned} \mathbb{F}_{\boldsymbol{\varepsilon},s_t}(\mathbf{x}_{ss}, \mathbf{0}, 0) &= \sum_{s'} \pi_{s_t,s',y}(\mathbf{y}_{ss}) g_{\boldsymbol{\varepsilon},s_t} f_{ss}(s', s_t) \\ &\quad + \sum_{s'} \pi_{s_t,s'}(\mathbf{y}_{ss}) \begin{bmatrix} f_{\mathbf{y}_{t+1}}(s', s_t) g_{\mathbf{x},s'} h_{\boldsymbol{\varepsilon},s_t} + f_{\mathbf{y}_t}(s', s_t) g_{\boldsymbol{\varepsilon},s_t} \\ + f_{\mathbf{x}_t}(s', s_t) h_{\boldsymbol{\varepsilon},s_t} + f_{\boldsymbol{\varepsilon}_t}(s', s_t) \end{bmatrix} \end{aligned} \tag{B.21}$$

and

$$\begin{aligned} \mathbb{F}_{\chi,s_t}(\mathbf{x}_{ss}, \mathbf{0}, 0) &= \sum_{s'} \pi_{s_t,s',y}(\mathbf{y}_{ss}) g_{\chi,s_t} f_{ss}(s', s_t) \\ &\quad + \sum_{s'} \pi_{s_t,s'}(\mathbf{y}_{ss}) \begin{bmatrix} f_{\mathbf{y}_{t+1}}(s', s_t) g_{\mathbf{x},s'} h_{\chi,s_t} + f_{\mathbf{y}_t}(s', s_t) g_{\chi,s_t} \\ + f_{\mathbf{x}_t}(s', s_t) h_{\chi,s_t} \\ + f_{\boldsymbol{\theta}_{t+1}}(s', s_t) \hat{\boldsymbol{\theta}}(s_{t+1}) + f_{\boldsymbol{\theta}_t}(s', s_t) \hat{\boldsymbol{\theta}}(s_t) \end{bmatrix}. \end{aligned} \tag{B.22}$$

Note now that, by definition of a steady state, $f_{ss}(s', s_t) = 0$, and so the first term of each of these expressions equals zero. Hence, we are left with the expressions for the exogenous transition probabilities as in Foerster et al. (2016), given by $\mathbb{P}_{ss} = \pi_{s_t,s'}(\mathbf{y}_{ss})$.

To prove the second part of the proposition, namely that endogenous regime-switching shows up at second order, it suffices to show that the second derivatives of the system with respect to $\mathbf{x_{t-1}}$ are dependent on the derivatives of the probability functions. We include the full set of the second-order derivatives in the SA. Now note that the derivatives have the following form:

$$[F_{s,xx}]^a_{b,c} = \mathbf{A} + \sum_{s'} \left(\sum_j \pi_{s,s',y_{j,t}} g^j_{s,x_b} \right) \mathbf{B} + \sum_{s'} \left(\sum_j \pi_{s,s',y_{j,t}} g^j_{s,x_c} \right) \mathbf{C}, \tag{B.23}$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are expressions include scalars and unknowns. It is now evident that this equation is a function of $\pi_{s,s',y_{j,t}}$, meaning that second-order coefficients of the decision-rules are functions of the change in the probabilities. QED.

Appendix C: Solution accuracy and comparison with traditional inequality specification

To gauge the accuracy and speed of our solution method and to compare the endogenous regime switching specification of the borrowing constraint relative to the traditional inequality one, we compare a suitably modified and calibrated version of our model to Mendoza (2010) solved with the FiPIt method of Mendoza and Villalvazo (2020).[24] To do so, we calibrate our model at an annual frequency and retain the interest rate, productivity, and intermediate input price shocks, as in Mendoza and Villalvazo (2020), and drop the expenditure and preference shocks and the permanent productivity shock. We then compare Euler equation errors and solution speed, simulated model first and second moments, ergodic distributions, and decision rules for bond holding and capital.

In the endogenous regime switching model, we calibrate all common model parameters as in Mendoza and Villalvazo (2020). The logistic function parameters are specific to our model. We set $\gamma_0 = \gamma_1 = 30$ to match the probability of a positive multiplier on the borrowing constraint. Both Mendoza (2010) and Mendoza and Villalvazo (2020) use finite-state Markov processes, while we use discrete-time autoregressive processes over a continuous support. We set these parameters so that the unconditional moments of the two sets of processes coincide.

Table C.1 reports the statistics on the errors in the Euler equation and the computation time. The table illustrates a stark trade-off between speed and accuracy. Our method is about 800 times faster than the FiPIt, with log-10 absolute Euler equation errors that are 1–3 times larger than FiPIt. The size of our Euler equation errors is in line with the values typically found when solving exogenous regime-switching models with perturbation methods (Foerster et al. (2016)) and models without regime switching (Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006)). To put these numbers in perspective, the implied accuracy differences represent only a dollar error per 1000 dollars of consumption, which is very small in absolute terms by the standards in the literature.

Table C.2 compares the first and second moments and the probability of a positive multiplier. To size the differences between FiPIt and our model, as a reference, we also

Table C.1. Solution accuracy and speed.

	FiPIt	Mendoza (2010)	End. Switch.
Euler Equation Errors ($\log_{10}$ units)			
Bond – Mean	−6.27	na	−2.92
Bond – Max	−1.56	na	−1.61
Capital – Mean	−7.04	na	−3.61
Capital – Max	−6.68	na	−2.41
Computing Time (seconds)	810	na	1.00

[24]See Binning and Maih (2017) for an analysis of the properties of our solution method applied to other structural models, such as the zero lower bound, in which they found a high degree of accuracy.

Table C.2. First and second moments.

	FiPIt	Mendoza (2010)	End. Switch.
Means			
gdp	393.619	388.339	393.168
c	274.123	267.857	268.826
inv	67.481	65.802	67.136
nx/gdp (%)	1.5	2.4	−0.3
k	765.171	747.709	763.154
b/gdp (%)	1.3	−10.4	−17.9
q	1	1	1
lev (%)	−10.3	−15.9	−19.4
v	42.617	41.949	42.600
wc	76.658	75.455	76.598
Standard deviations (%)			
gdp	3.94	3.85	3.81
c	4.03	3.69	3.628
inv	13.33	13.45	10.884
nx/gdp	2.94	2.58	1.307
k	4.49	4.31	4.185
b/gdp	19.62	8.9	2.154
q	3.2	3.23	2.568
lev	9.22	4.07	8.560
v	5.89	5.84	5.87
wc	4.35	4.26	4.21
Correlations with with gdp			
gdp	1	1	1
c	0.842	0.931	0.984
inv	0.641	0.641	0.748
nx/gdp	−0.117	−0.184	−0.047
k	0.761	0.744	0.849
b/gdp	−0.12	−0.298	−0.172
q	0.387	0.406	0.478
lev	−0.111	0.258	−0.076
v	0.832	0.823	0.832
wc	0.994	0.987	0.994
Autocorrelations			
gdp	0.825	0.815	0.815
c	0.83	0.766	0.804
inv	0.501	0.483	0.441
nx/gdp	0.601	0.447	0.246
k	0.962	0.963	0.975
b/gdp	0.99	0.087	0.779
q	0.447	0.428	0.350
lev	0.992	0.04	0.864
v	0.777	0.764	0.774
wc	0.801	0.777	0.784
Sudden-stop statistics (%)			
Prob. positive multiplier	2.6	na	3.615

report the simulated second moments from the original Mendoza (2010). The comparison shows that our specification of the occasionally binding borrowing constraint essentially produces the same first and second moments as the FiPIt, with differences that are very small relative to the gaps between the FiPIt and Mendoza (2010). Two exceptions are the net foreign asset position as a share of GDP (b/GDP) and the net export-to-GDP ratio (nx/GDP). The ergodic mean of b/GDP is about -17.9% in our model, while it is small and positive in the FiPIt model (1.3%).[25] In Mendoza (2010), this simulated moment is about -10%. Mexico's net foreign asset position averaged -37% of GDP and was never positive from 1970 to 2015. Therefore, our model generates an ergodic mean of debt significantly closer to the average in the data compared to the FiPIt.

The second discrepancy is nx/GDP, which is negative -0.3% in our model, while it is positive 1.5% in the FiPIt, and 2.4% in Mendoza (2010). The FiPIt generates a counterfactual net foreign asset position with a positive net export-to-GDP ratio that should be negative in the ergodic distribution if the bond position is positive. Our model generates a data-consistent debt position with a small negative ergodic trade surplus. In contrast, in Mendoza (2010), the ergodic trade balance and debt proportions have the opposite sign as one would expect. Our model also generates a less volatile bond position and trade balance relative to FiIPt, which in turn generates more volatility than the original Mendoza (2010). A result that could be due to the mechanics of our specification of the borrowing constraint.

Figure C.1 plots the ergodic distributions of debt (panel a) and capital (panel b), together with their joint distribution as in Figure 1 of Mendoza and Villalvazo (2020). The debt distribution of our model has little or no density outside the $[-100, 0]$ interval of the support. In contrast, the FiPIt distribution has a significant probability mass on positive and large values, which is counterfactual as we discussed above. Instead, the ergodic distribution of capital has essentially the same support in the two models. The joint distribution reflects these differences.

The policy functions of our model solved with perturbation methods illustrate its ability to capture the occasionally binding nature of the borrowing constraint consistent with Proposition 1. Figure C.2 plots the decision rules for capital, bonds, and the borrowing cushion in our model, in both the nonbinding and the binding regimes. These rules are evaluated at the steady-state values of the technology, interest rate, and intermediate input price processes, covering a rectangular support associated with the joint ergodic distribution of capital and debt. Therefore, the decision rules presented here are conditional on no shocks. Importantly, note here that the actual behavior of the model also depends on the specific values of the exogenous states. In our framework, endogenous states, exogenous processes, and shocks all have continuous support; while FiPIt's use of a discretized state space means that a finite set of states are repeatedly visited. This fact is crucial because the ergodic sets are influenced by the underlying processes. In other words, the portions of the distribution's support that the economy visits depend

[25] All model and solution variants reported in Mendoza and Villalvazo (2020) also have a positive level of ergodic debt.

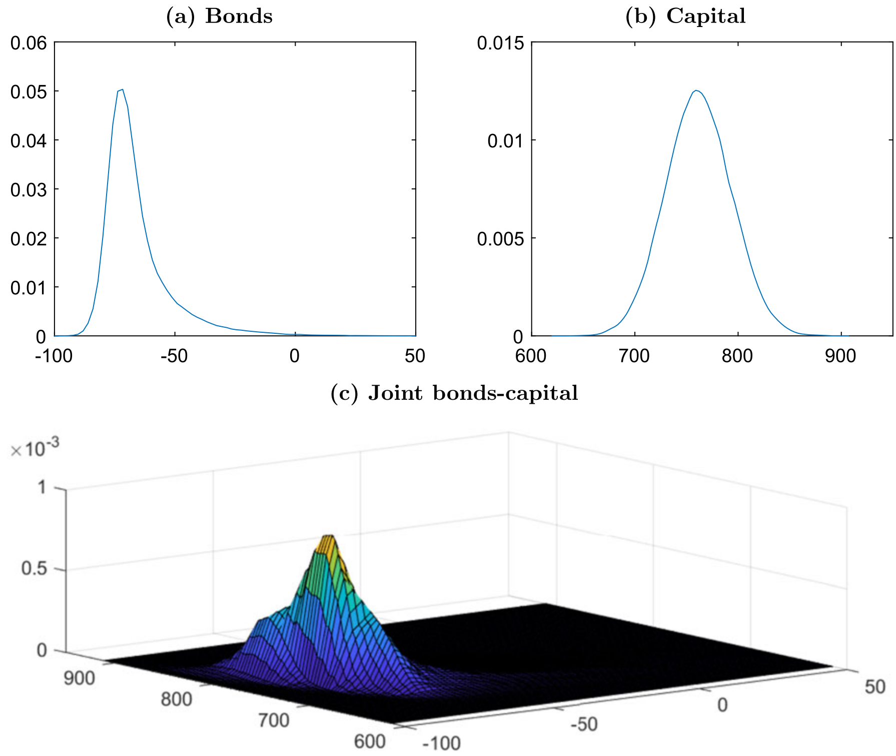

Figure C.1. Endogenous switching model's ergodic distributions. Notes: The figure plots the simulated ergodic distribution of the bond and capital in the endogenous switching model calibrated as described in the Appendix text.

on the specific values of the exogenous processes. In contrast, the decision rules of traditional occasionally binding models are functions of the entire state space, encompassing shocks, previous period values of the processes, and endogenous states.

First, consider the behavior of the economy in the nonbinding regime and recall that, in our framework, the regime assignment is predetermined relative to the shocks and the agent's decision. This means that, by construction, shocks cannot push the economy into the binding region within the same period. The curvature in the nonbinding regime reflects how likely and how severely the credit constraint may be expected to bind at $t+1$ when it does not bind at time t. In the nonbinding regime, the capital decision rule is upward-sloping in the current period debt (Figure C.2a). As the economy borrows more, investment, the price of capital, and collateral value increase. However, the impact of higher borrowing on the borrowing cushion is stronger than the impact on the collateral value. Consequently, the borrowing cushion monotonically decreases as the current period debt increases (Figure C.2e). As we move toward the tail of the ergodic distribution, the second-order approximation introduces some curvature, and the slopes of the policy functions change sign. However, these regions of the distribution are seldom visited. Indeed, when we numerically evaluate the probability of encountering derivatives with the wrong signs, we find that the upward-sloping behavior occurs only

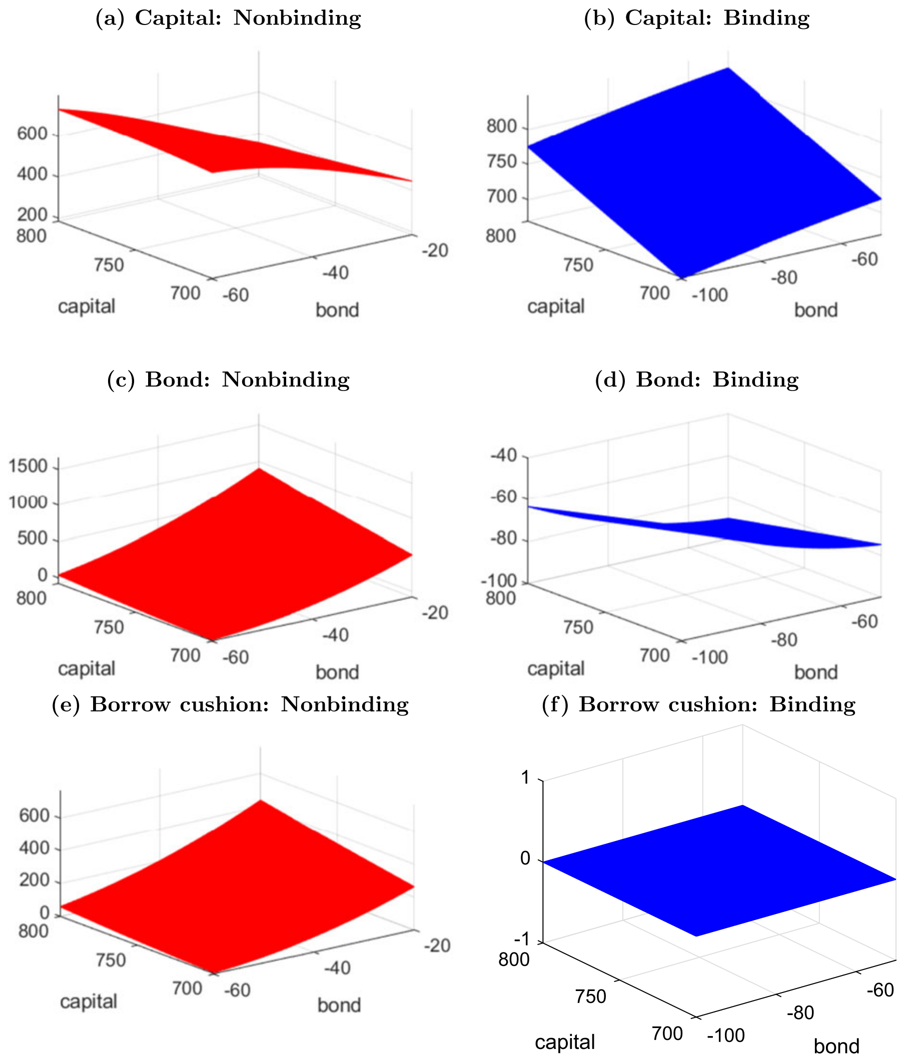

Figure C.2. Endogenous switching model's decision rules across regimes. Notes: The figure plots the bond and capital decision rules in the endogenous switching model calibrated as discussed in the Appendix text, by regime, evaluated at the steady-state value of the technology, interest rate, and intermediate input price processes.

in 1% of the ergodic set. This result simply highlights that, when solving by perturbation methods, the decision rules can behave oddly in areas not frequently visited.

Second, consider the behavior of the economy in the binding regime when the borrowing cushion, by definition, is at the steady-state values of the processes. In the binding regime, the economy displays typical debt deflation dynamics. The capital decision rule is downward-sloping. The debt decision rule is downward sloping at low levels of debt, but is upward sloping at higher levels of debt because the approximation loses accuracy as we move toward the edges of the ergodic support.

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