Quantitative Economics 15 (2024), 489–521 1759-7331/20240489

A dynamic model of rational "panic buying"

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This paper analyzes panic buying of storable consumer products accompanied by disasters, using a novel consumer-search theoretic equilibrium model where consumers follow (S, s) inventory policies. We show that, even if consumers are fully rational, an anticipated temporary increase in consumer shopping costs (as well as conventional demand and supply shocks) can trigger an upward spiral of hoarding demand and result in serious shortages. Due to congestion externalities, panic buying leads to the misallocation of storable products and substantial welfare loss. The model is calibrated using survey data and reveals that the timing of recognizing the shopping-cost rise is crucial for the severity of panic buying. Some policy options, such as purchase quotas and future sales-tax reductions, are suggested to mitigate panic buying.

Keywords. Hoarding, panic buying, search frictions, congestion externality, disaster, COVID-19, heterogeneous agents model, mean-field game.

JEL classification. C61, D15, H84.

1. Introduction

Panic buying of necessity goods (such as toilet paper, hygiene products, and canned foods) has occurred historically in anticipation of and in response to various types of emergencies, for example, the recent COVID-19 pandemic.[1] This paper analyzes how

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Authors are listed in alphabetical order. The authors are grateful to Dilip Abreu, Yu Awaya, Jess Benhabib, Jaroslav Borovička, Katarína Borovičková, Ippei Fujiwara, Masao Fukui, Mark Gertler, Masashi Hino, Yasuo Hirose, Hirokazu Ishise, Ryuichiro Izumi, Yuichiro Kamada, Michihiro Kandori, Kazuko Kano, Satoshi Kasamatsu, Kohei Kawaguchi, Keiichiro Kobayashi, Fuhito Kojima, Noritaka Kudoh, Yuhei Miyauchi, Jesse Perla, Michael Peters, Anne-Katrin Roesler, Susumu Sato, Tatsuro Senga, Toshiaki Shoji, Yuta Takahashi, Yuki Teranishi, Gabor Virag, Peifan Wu, Yosuke Yasuda, Yuta Yasui, and participants at Happy Hour Seminar, Canadian Economic Theory Conference 2021, International Workshop in Market Design, Panic Buying Workshop, the 2021 Spring Meeting of the Japanese Economic Association, GSE-OSIPP-ISER Joint Conference in Economics, and the seminars at Keio University, Hitotsubashi University, Osaka University and Hong Kong University of Science and Technology for helpful comments and suggestions. We also thank Kazuma Takakura, Ayano Yago, and Masanori Kobayashi for excellent research assistance. This research is supported by the Hampton Fund Research Grant and JST ERATO GRANT Number JPMJER2301, Japan. All errors are ours.

[1]During the COVID-19 pandemic, English-language media reports collected from 20 countries and regions by Arafat et al. (2020) indicate that there were 214 news reports using the keyphrase "panic buying"

Available at http://qeconomics.org. https://doi.org/10.3982/QE2126

emergencies (i.e., changes in fundamentals) trigger panic buying even in the absence of irrational consumers and misinformation. To this end, we develop a novel dynamic model of a market of storable necessities that takes into account search externalities (Diamond (1982)) and calibrate the model using household survey data. We find that panic buying occurs inevitably when an emergency makes a shopping search more costly. We quantify the welfare cost attributable to panic buying and discover that panic buying becomes much more severe if the emergency is anticipated. Furthermore, we quantitatively evaluate the effectiveness of various policies to curb panic buying.

Our model considers atomistic consumers who consume a storable product at a constant rate, incur holding costs, and face small search frictions in their shopping. Search frictions require consumers to spend time on costly shopping searches. Consequently, consumers who optimally manage their inventory follow (S, s) inventory policies, leading to periodic shopping where they determine the timing and quantity of their purchases based on expected time and costs associated with shopping searches.[2] Products are served on a first-come-first-served basis, and each consumer's shopping decision affects the product availability for other consumers. Specifically, when individuals intensify their hoarding demand, it results in heightened market congestion. This makes it less probable and more time-consuming for other consumers to make a purchase. In essence, even though each consumer behaves rationally, they fail to internalize the effects of *market-congestion externality*. Consequently, while defensive hoarding is optimal for individuals, it leads to an inefficient "panic" in society as a whole.

Using the model, we identify an increase in *shopping costs*, that is, nonpecuniary costs associated with shopping search, as a potential trigger for panic buying. When these shopping costs experience a temporary surge, consumers tend to make larger purchases to reduce the frequency of their shopping trips. This hoarding behavior amplifies market demand and depletes in-store stock. In anticipation of potential stock-outs, consumers rush to secure products before they run out, further exacerbating the scarcity. In this manner, the shopping-cost shock leads to a spiral in which individual consumers, acting in their own self-interest, escalate hoarding out of fear of running out of necessities. As a result, the products become misallocated, with some consumers facing a higher risk of stock-out and spending more time searching, while others incur a higher storage cost due to excessive hoarding. The 2020 toilet paper shortage in the United States, occurring in the absence of supply disruptions or demand increases, is a compelling example of how mobility restrictions during the COVID-19 pandemic raised shopping costs and led to shortages (see Supplemental Appendix C (Noda and Teramoto (2024)) for evidence from Global Mobility Report).[3]

published through May 22, 2020. The majority of the media reporting on panic buying was from the United States (40.7%), the United Kingdom (22%), and India (13.6%).

[2]Several studies, following the works of Caplin (1985), Grossman and Laroque (1990), and Caballero and Engel (1991) have developed models based on the (S, s) inventory policies to analyze the demand for durable or storable consumer products (e.g., Berger and Vavra (2015), Baker, Johnson, and Kueng (2021), for recent studies). However, our model differs from these studies in the nature of the adjustment process. While previous models assume that agents can choose when to adjust their stock, in our model, search frictions prevent consumers from choosing exactly when to adjust their stock.

[3]Keane and Neal (2021) present cross-country evidence of how movement restrictions announced during the COVID-19 pandemic have affected panic buying. They measure the extent of these restrictions using

Notably, our model of panic buying differs from self-fulfilling panic models (such as the classic bank-run model of Diamond and Dybvig (1983)) in that shortages and excessive hoarding only occur in response to adverse fundamental shocks. This property is consistent with reality since panic buying has been observed frequently during emergencies but rarely during normal times.[4] Rather, the structure of our model is similar to the dynamic debt-runs model proposed by He and Xiong (2012) in that a coordination problem exists between consumers acting at different times.[5]

For the quantitative analysis, we present innovative numerical techniques to simulate the dynamic response to a shock. Our model accounts for the heterogeneity of consumers in their stock quantities, and this feature necessitates computing the joint equilibrium dynamics of (i) consumer shopping behavior, (ii) the distribution of consumer stock, and (iii) market conditions. To achieve this, we utilize a continuum of consumers in a continuous-time setting and employ a mean-field game (MFG) representation, customizing the numerical method developed by Achdou, Han, Lasry, Lions, and Moll (2022).[6] Our methods can effectively simulate the dynamic response to a range of shocks, including increases in shopping costs, shifts in preferences, and supply disruptions.

The model is calibrated to align with purchase and inventory behaviors documented in a household survey by Kano (2018), which specifically examines toilet paper consumption, purchase, and inventory. Our quantitative experiments demonstrate that a month-long increase in shopping costs can induce panic buying, revealing two key features.

First, excessive hoarding is more likely to occur when the shock is *anticipated*, and the degree to which consumers recognize the shopping-cost increases in advance has a crucial impact on the severity of panic buying. Specifically, our simulations indicate that (i) the severity of panic buying has a nonmonotonic inverse U-shaped relationship with the time lag between realization and recognition of the shock, and (ii) severe panic buying occurs when consumers recognize the increase about a few weeks before its occurrence.

Second, in our welfare analysis, we distinguish the welfare cost that results from the market-congestion externality and the welfare cost that stems directly from the increased shopping costs. We find that, when severe panic buying occurs, the external-

data on the closure of primary and secondary schools, restrictions on gatherings, encouragement of remote work, limitations on public spaces, and the closure of retail and entertainment businesses.

[4]For example, shortages of essential goods have been observed during the 1962 Cuban missile crisis (George (2003)), the 1973 oil crisis (Malcolm (1974)), the 2008 global rice crisis (Dawe (2010), Hansman, Hong, de Paula, and Singh (2020)), the 2011 Christchurch earthquake (Lauder (2011), Forbes (2017)), the 2011 East Japan earthquake (Hori and Iwamoto (2014)), the 2017 Hurricane Irma (Alvarez (2017)), and Brexit (Coleman, Dhaif, and Oyebode (2022)).

[5]Specifically, search frictions play a significant role in preventing consumers from making simultaneous purchases, and the market-congestion externality causes the dynamic coordination problem and serves to amplify the impact of adverse shocks.

[6]Originally developed for analyzing income and wealth distribution in dynamic general equilibrium models with uninsured idiosyncratic risk, this approach has found extensive application in diverse macroeconomic models with heterogeneous agents (e.g., Kaplan, Moll, and Violante (2018), Ahn, Kaplan, Moll, Winberry, and Wolf (2018), Fernández-Villaverde, Hurtado, and Nuño (2023)).

ity effect of congestion on social welfare is much greater than the direct impact of the underlying shock. In our calibration, the externality exacerbates the welfare impact by more than 5 times.

Using our model and numerical method, we evaluate different policy options for reducing panic buying. Our findings suggest that purchase quota policies, implemented by many stores during the COVID-19 pandemic, can be effective. We also propose tax policies as a potential solution. For instance, announcing a future reduction in sales tax can encourage consumers to delay their purchases and break the spiral of hoarding. Another option of the government is to distribute the product directly to households. This policy alleviates market congestion and enhances the ex ante value of all consumers, even if the government is unable to reach the entire population.

Contributions to the literature Our research contributes to the literature on the purchasing behavior of storable consumption goods and panic buying phenomena. Numerous empirical studies (e.g., Neslin, Henderson, and Quelch (1985), Erdem, Imai, and Keane (2003), Hendel and Nevo (2006a,b, 2013)) emphasize the practical importance of intertemporal demand effects of storable consumption goods. Recently, Keane and Neal (2021) and Prentice, Chen, and Stantic (2020) underscore the relevance of these effects in explaining panic buying, citing that the announcement of government measures to combat the COVID-19 pandemic triggered panic buying. The intertemporal demand effects emphasized in the literature play a crucial role in our model. Amid the heightened attention on panic buying behavior during the COVID-19 pandemic, various models explaining the phenomenon have been developed. For instance, Awaya and Krishna (2021) demonstrate how price flexibility influences panic buying using a two-period model, while Klumpp (2021) develops a consumer-inventory model where stockpiling behaviors accelerate supply shortages. Our model offers three notable advantages compared to theirs: (i) it explicitly considers consumers' decisions on when to go shopping and elucidates how they rush to stores, (ii) our continuous-time model framework can analyze how the market dynamically responds to various fundamental shocks, and (iii) it derives quantitative implications by tying to micro-data evidence.

Our study draws a parallel between panic buying of necessities and bank runs. Specifically, our panic buying model shares similarities with the classical bank-run model developed by Diamond and Dybvig (1983), as the sequential service constraint (Wallace (1988, 1990)) plays a critical role in both.[7] However, our model characterizes panic buying as a phenomenon that amplifies changes in fundamentals due to the coordination problem, akin to the dynamic debt runs of He and Xiong (2012), rather than a self-fulfilling panic.

Distinct from He and Xiong (2012), our model considers the intensive margin (the quantity per purchase), which is essential for analyzing the propagation of shopping-cost shocks. Our model also shares similarities with the bank-runs model developed by Gu (2011) that features herding effects due to information externality. While her model

[7]The literature on banking crises broadly consists of two views: the first view is that crises are based on panics, that is, random events, while the second view argues that crises occur due to poor fundamentals (see, e.g., Allen and Gale (2009)). Our panic-buying model falls into the latter category.

emphasizes the nature that a consumer's decision depends on the other consumers' past actions, our model emphasizes that it depends on other consumers' future actions resulting from search externality. This feature is crucial in explaining why panic buying is more likely to occur under announced emergencies.

Structure of the paper The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 defines the equilibrium with rational expectations and describes the stationary equilibrium. Section 4 calibrates the model, and Section 5 studies the dynamic responses to shopping-cost shocks and investigates policy interventions. Section 6 analyzes other shocks that cause panic buying, and Section 7 provides sensitivity analyses and model extensions. Section 8 offers concluding remarks.

2. Model

We consider a consumer-search model for a storable necessity product (e.g., toilet paper) in continuous time with an infinite horizon. Let $t \in [0, \infty)$ index time. In this economy, there is a unit mass of consumers. Nonnegative random variable $k_i(t) \geq 0$ stands for the stock of the product held in the consumer i's private inventory at time t and $G(t, k) = \int_{i \in [0,1]} \mathbb{1}_{\{k_i(t) \leq k\}} \, di$ for $k \geq 0$ is the distribution function of consumers' private stock at time t.

We assume, as in the model in Blanchard (1985), that consumers stochastically exit from the economy at a Poisson rate $\theta > 0$ and a mass θ of new consumers enters per unit of time so that total population size is kept at one. We further assume that the consumers who exit take their stock away and newly entered consumers start with initial stock $k_o > 0$, which is drawn from a (time-invariant) distribution function G_{new} that has a density function g_{new}.[8]

There is a marketplace in which a store sells storable products. The store can hold the product in its warehouse. Let $S(t) \geq 0$ denote the store's stock in the warehouse at time t. The product is replenished to the warehouse at an exogenous rate $s > 0$ every time.

To purchase the product, the consumers have to travel to the marketplace and locate a store. However, due to search frictions, locating it takes time and is costly. In practice, shopping incurs travel costs, costs of acquiring product information, and opportunity costs of the time spent shopping. In this paper, we collectively refer to these costs as shopping costs. The size of shopping costs $c(t)$ is assumed to be common to all consumers while it may change over time. Note that shopping costs in our model are flow costs incurred while conducting a shopping search, rather than one-time fixed costs.

Let $p(t)$ denote the unit market price of the product at time t. We assume that, in the long-run stationary equilibrium, the market price is established so that supply and demand flows are balanced, but this is not the case when the economy is out of the

[8] This exit-and-reentry structure is commonly used as "stabilizing forces" to ensure the existence of a unique stationary distribution (See Supplemental Appendix F for the proof). See, for example, Gabaix, Lasry, Lions, Moll, and Qu (2016, Online Appendix D) for detailed descriptions of other specifications of stabilizing forces.

stationary equilibrium. The intertemporal pricing policy for storable products is complex (see Su (2010), who characterizes an optimal pricing strategy of the monopolistic seller), and modeling the pricing policy in emergency situations is beyond the scope of this study.[9] Thus, we instead treat the price as an exogenous variable.

2.1 *The consumer's problem*

The consumer's stock $k_i(t)$ changes over time as a result of consumption and purchases from the store in the marketplace. As the product is storable, any unconsumed quantity can be set aside for future consumption. Reselling of the product is not allowed.[10] Depreciation of the product is not explicitly considered, since we focus on the short-term behavior of the economy. At every time t, a consumer chooses (i) the flow consumption $x_i(t) \in \mathbb{R}_+$, (ii) whether to do a shopping search $A_i(t) \in \{0, 1\}$, and (iii) how much to buy upon finding available stock at the store, $q_i(t) \in \mathbb{R}_+$.[11]

To purchase the product, a consumer has to engage in a costly shopping search $(A_i(t) = 1)$. We assume that during this search, a consumer locates the store at a Poisson rate of $\alpha > 0$. This Poisson shock structure, within the context of our continuous-time framework, prevents instantaneous purchases by consumers and results in only a very small fraction of consumers having the opportunity to make a purchase within a short time interval. This feature allows us to focus on the coordination problem between consumers who make a purchase at different times, as in the dynamic debt-runs model of He and Xiong (2012).

With the assumptions made above, a mass $\alpha(\int_{i\in[0,1]} A_i(t)\,di)\,dt$ of consumers locates the store within each time interval $[t, t + dt]$. In what follows, we refer to the consumers who arrived at the store as *buyers*. Upon reaching the store, they are served according to the sequential service rule—the buyers are randomly sorted into a queue for purchasing and are allowed to purchase the desired quantity $q_i(t) \geq 0$ in order of the queue *as long as the store's supply lasts*. We emphasize that even if locating a store, she is not necessarily able to make a purchase there, as the store may have depleted its stock before her turn arrives. Let $R(t) \in (0, 1]$ be the fraction of the buyers at time t who are actually able to make a purchase. That is, the individual buyer faces an idiosyncratic event $z_i(t)$ that determines whether the store has supplies or not. Here, $z_i(t)$ is an independent Bernoulli random variable with success probability $R(t)$, that is, $z_i(t) \sim \mathrm{Ber}(R(t))$.

Taken together, a searching consumer faces two types of idiosyncratic risk: (i) whether she can locate a store and (ii) whether, after locating a store, she can make a

[9] Recent evidence suggests that the price dynamics of consumer products during emergency situations differs from those in normal times, partly due to fairness considerations that result in an increased reluctance to raise prices in emergency situations (Cavallo, Cavallo, and Rigobon (2014), Gagnon and López-Salido (2019), Hansman et al. (2020), Cabral and Xu (2021)). See Rotemberg (2005) for a model of price adjustment that incorporates customers' reactions based on fairness considerations.

[10] In their empirical analysis, Hansman et al. (2020) find that hoarding during the 2008 Global Rice Crisis was mostly for the consumer's own use. They argue that this seemed to be the case for hoarding during the COVID-19 pandemic as well, referring to media reports at the time.

[11] Total spending on the product is assumed to be relatively small compared to total expenditures.

purchase there. Accordingly, the time-evolution equation of the consumer's inventory is given by

$$dk_i(t) = -x_i(t)\,dt + A_i(t) \cdot \left[dN_i(t) \cdot z_i(t)\right] \cdot q_i(t), \tag{1}$$

where $N_i(t)$ represents the process of the independent Poisson shock of locating a store, that is, $\text{Prob}(dN_i(t) = 1) = 1 - \mathrm{e}^{-\alpha \cdot dt}$. In the right-hand side of (1), the first term represents consumption, while the second term represents the purchase of the product. Note that $k_i(t)$ is a càdlàg process (right continuous with left limit). When the consumer makes a purchase, the amount of her private stock jumps to $\bar{k}_i(t) = k_i(t^-) + q_i(t)$, where $k_i(t^-) := \lim_{s \uparrow t} k_i(s)$ is the amount of the product she held in her inventory just before making a purchase.

We turn to the decision making faced by the consumers. Each consumer discounts the future at a rate of $\rho > 0$ and seeks to maximize the expected present value of her total payoff $\mathbb{E}[\int_0^\infty \mathrm{e}^{-rs}\, d\pi_i(s)]$, with $r = \rho + \theta$ being the effective time-discount rate.[12]

The instantaneous payoff is given by

$$d\pi_i(t) = \left[u\big(x_i(t)\big) - b_i(t) - A_i(t) \cdot c(t)\right] \cdot dt - \big(A_i(t) \cdot dN_i(t) \cdot z_i(t)\big) \cdot p(t) \cdot q_i(t),$$

where $u(x_i(t))$ is the flow utility from consumption and $b_i(t)$ is the flow cost of storing the product in the private inventory. We posit $b_i(t) = \bar{b} \cdot k_i(t)$ with $\bar{b} > 0$ being the storage cost per unit of the product, the size of which would depend on the cost of storage space, the degree of shrinkage, the foregone interest income, and so on. We use the flow utility function that takes the following form:

$$u\big(x_i(t)\big) = \begin{cases} 0, & x_i(t) \geq 1; \\ -a < 0, & x_i(t) < 1. \end{cases} \tag{2}$$

Considering that the product is a daily necessity and not substitutable, the "need" is highly inelastic: a consumer only needs a unit of the product for a unit of time, but she receives a large disutility $a \gg 0$ if she fails to consume it. We assume that a is sufficiently large, ensuring consumers initiate shopping searches at least when they are out of stock (see Assumption 2 presented in Section 3 for the formal condition).

Given this flow utility function (2), it is clearly optimal to choose the flow consumption $x_i(t) = 1$ whenever the consumer has some stock of the product.[13] Thus,

$$x_i(t) = x\big(k_i(t)\big) = \begin{cases} 1, & k_i(t) > 0; \\ 0, & k_i(t) = 0, \end{cases}$$

and the net flow gain from holding stock k, defined as $h(k) := u(x(k)) - \bar{b} \cdot k$, is concave in k. Accordingly, each consumer uses $k_i(t)$ as an idiosyncratic state variable to decide whether to search and how much to purchase.

[12] Recall that θ is the exogenous exit rate. Here, we assume that the payoff after exiting is zero.

[13] This constant-consumption-rate policy has been widely employed in the literature of dynamic inventory models (e.g., Neslin, Henderson, and Quelch (1985), Hendel and Nevo (2006a)).

2.2 *Aggregate dynamics of store stock*

We turn to the aggregate dynamics (see Supplemental Appendix D for a detailed description of a large-market limit of a finite economy in the continuous-time limit). Let $dD(t) = \int_{i\in[0,1]} A_i(t) \cdot dN_i(t) \cdot q_i(t)\, di$ represent the total amount of the products demanded by the consumers who arrived at the store over the infinitesimal time interval $[t, t+dt]$. Hence, the flow rate of demand at time t, $d(t) := dD(t)/dt$ is explicitly given by

$$d(t) = \alpha\left(\int_{i\in[0,1]} A_i(t) \cdot q_i(t)\, di\right).$$

Recall that only the fraction $R(t)$ of such consumers are able to make a purchase. We refer to $R(t)$ as the *availability* (of the product in the market) at time t. Hence, the total amount of the product actually purchased over $[t, t+dt]$ is $R(t)\, dD(t) = R(t)\, d(t)\, dt$. In this respect, we refer to $d(t)$ as the *potential demand* flow, as distinguished from the amount purchased.

According to the store's selling rules described above, the availability $R(t)$ is determined by the following rationing rule:

$$R(t) = \begin{cases} 1, & S(t) > 0; \\ \min\left\{\dfrac{s}{d(t)}, 1\right\}, & S(t) = 0. \end{cases} \tag{3}$$

This rule shows that rationing (i.e., $R(t) < 1$) occurs if and only if the store is out of stock ($S(t) = 0$) and the potential demand flow exceeds the flow of the store's supply ($d(t) > s$). When rationing occurs, the total amount purchased is limited by the store's supply: $R(t)\, d(t) = s$. Otherwise, the "flow supply" (which is infinity if the store is in stock, $S(t) > 0$) is larger than the flow demand and, therefore, all consumers arriving at that moment confront plenty of stock.

Finally, the time-evolution equation of the store's stock is given as follows:

$$\frac{dS(t)}{dt} = s - R(t)\, d(t), \tag{4}$$

with an initial condition $S(0) = S_o > 0$. That is, the store's stock at time t is the amount of products left unsold by time t. Note that $S(t) \geq 0$ for all $t \geq 0$ since $dS(t) \geq 0$ if $S(t) = 0$.

3. Rational-expectations equilibrium

In this section, we formulate the consumer's optimization problem and the equilibrium dynamics with rational expectations.

3.1 *Consumers' optimization*

Let $\boldsymbol{Y}(t) = (S(t), G(t, k))'$ be the set of endogenous aggregate state variables.[14] Given $\boldsymbol{Y}(t)$, consumers form a belief about the future path of product availability $\{R(\tau)\}_{\tau\geq t}$

[14] Throughout this paper, we do not consider aggregate uncertainty. Thus, take $\boldsymbol{Y}(t)$ to be the deterministic path for a set of endogenous aggregate state variables.

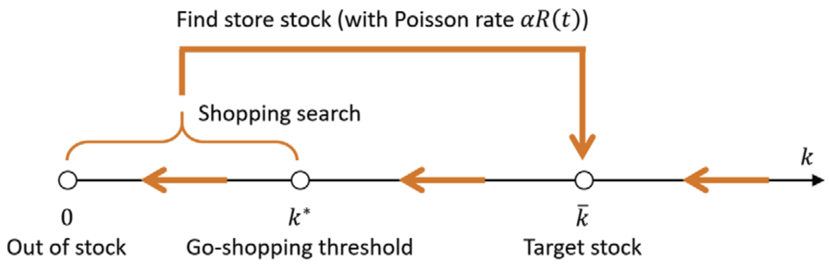


FIGURE 1. The dynamics of a consumer's stock k, which is characterized by the go-shopping threshold k^* and the target stock $\bar{k}$. For every $k > 0$, the rate of consumption is 1.

using the belief functions Γ_Y and Γ_R: $\dot{\boldsymbol{Y}}(t) = \Gamma_Y(\boldsymbol{Y}(t))$ and $R(t) = \Gamma_R(\boldsymbol{Y}(t))$. Let $V(\boldsymbol{Y}(t), k_i(t))$ be the value function for a consumer who has stock $k_i(t)$ at time t. The consumer's problem can be formulated as the following optimal stopping-time problem:

$$V\big(\boldsymbol{Y}(t), k_i(t)\big) = \sup_{T\geq 0} \mathbb{E}\bigg[\int_t^{t+T} \mathrm{e}^{-r(s-t)} h\big(k_i(s)\big)\, ds + \mathrm{e}^{-rT} V^*\big(\boldsymbol{Y}(t+T), k_i(t+T)\big)\bigg], \tag{5}$$

where $k_i(\tau) = k_i(t) - \int_t^{\tau} x(k_i(s))\, ds$ for $\tau \in [t, t+T)$ and $V^*(\boldsymbol{Y}(\tau), k_i(\tau))$ is the expected value of conducting a shopping search at time $\tau \geq t$.[15] V^* satisfies the Hamilton–Jacobi–Bellman (HJB) equation:

$$\begin{aligned} rV^*\big(\boldsymbol{Y}(\tau), k_i(\tau)\big) = {} & h\big(k_i(\tau)\big) - c(\tau) + \alpha R(\tau)\big[V^A\big(\boldsymbol{Y}(\tau), k_i(\tau)\big) - V^*\big(\boldsymbol{Y}(\tau), k_i(\tau)\big)\big] \\ & + \frac{\partial V^*\big(\boldsymbol{Y}(\tau), k_i(\tau)\big)}{\partial \boldsymbol{Y}} \dot{\boldsymbol{Y}}(\tau) - \frac{\partial V^*\big(\boldsymbol{Y}(\tau), k_i(\tau)\big)}{\partial k} x\big(k_i(\tau)\big), \end{aligned}$$

where $R(\tau)$ is given by the belief functions Γ_Y and Γ_R, and $V^A(\boldsymbol{Y}(\tau), k)$ is the value right after purchasing at time τ:

$$V^A\big(\boldsymbol{Y}(\tau), k\big) = \max_{\bar{k}\geq k} V\big(\boldsymbol{Y}(\tau), \bar{k}\big) - p(\tau) \cdot (\bar{k} - k). \tag{6}$$

The optimal stopping-time problem (5) induces the *optimal stopping-time policy* $T(\boldsymbol{Y}(t), k)$. We define the *action region* as the set of the stock levels at which the consumer engages in a shopping search: $\mathcal{A}(\boldsymbol{Y}(t)) = \{k \in \mathbb{R}_+ \mid T(\boldsymbol{Y}(t), k) = 0\}$. Since she has a stronger incentive to go shopping as the stock in her inventory gets smaller, the optimal policy exhibits a threshold behavior: $\mathcal{A}(\boldsymbol{Y}(t)) = [0, k^*(\boldsymbol{Y}(t))]$. We refer to k^* as the *go-shopping threshold*.

The maximization problem (6) derives the decision rule on the purchase quantity. Let $\bar{k}(\boldsymbol{Y}(t), k)$ be the solution of (6). It is clear that, if $\bar{k}(\boldsymbol{Y}(t), k) \geq k$, all searching consumers desire to increase their stock to the same level $\bar{k}(\boldsymbol{Y}(t))$ regardless of the current stock $k_i(t)$.[16] In what follows, we refer to $\bar{k}(\boldsymbol{Y}(t))$ as the *target stock*.

[15]See, for example, Stokey (2009) for the formulation of the Bellman equation for optimal stopping-time problems.

[16]The consumers who choose $\bar{k}(\boldsymbol{Y}(t), k) = k$ clearly do not search since there is no gain from searching.

In the end, the consumers' decision rule can be characterized by two variables: the go-shopping threshold $k^*(\boldsymbol{Y}(t))$ and the target stock $\bar{k}(\boldsymbol{Y}(t))$. As illustrated in Figure 1, they engage in a shopping search if and only if their inventory stock is smaller than $k^*(\boldsymbol{Y}(t))$: once they find an open store, they stock up to $\bar{k}(\boldsymbol{Y}(t))$.

3.2 *Law of motion for the aggregate state*

Given the consumers' decision, we derive the (actual) law of motion for the aggregate variables. First, the consumers' optimal strategy induces a mapping Ψ_d from the aggregate state $\boldsymbol{Y}(t)$ to the potential demand $d(t)$ as

$$d(t) = \Psi_d\big(\boldsymbol{Y}(t)\big) := \alpha\left(\int_{k\in[0,k^*(\boldsymbol{Y}(t))]} q\big(\boldsymbol{Y}(t), k\big) g(t, k)\, dk\right),$$

where $q(\boldsymbol{Y}(t), k) := \max\{\bar{k}(\boldsymbol{Y}(t)) - k, 0\}$ represents the optimal purchase quantity and $g(t, \cdot)$ is a generalized probability density function of the distribution function $G(t, \cdot)$.[17] Recall that the availability $R(t)$ is determined by $d(t)$ and $S(t)$ according to (3). Therefore, $R(t)$ can also be written with a mapping Ψ_R as $R(t) = \Psi_R(\boldsymbol{Y}(t))$.

Then, given the consumer's decisions, the Kolmogorov forward (KF) equation for the measure of consumers g can be written as

$$\frac{\partial g(t,k)}{\partial t} = \begin{cases} \dfrac{\partial g(t,k)}{\partial k} x(k) + \theta\big[g_{\text{new}}(k) - g(t,k)\big] - \alpha \Psi_R\big(\boldsymbol{Y}(t)\big) g(t,k) & \text{for } k \in \mathcal{A}\big(\boldsymbol{Y}(t)\big), \\ \dfrac{\partial g(t,k)}{\partial k} x(k) + \theta\big[g_{\text{new}}(k) - g(t,k)\big] & \\ \quad + \alpha \Psi_R\big(\boldsymbol{Y}(t)\big) G\big(t, k^*\big(\boldsymbol{Y}(t)\big)\big)\delta\big(k - \bar{k}\big(\boldsymbol{Y}(t)\big)\big) & \text{for } k \notin \mathcal{A}\big(\boldsymbol{Y}(t)\big). \end{cases}$$

From (4), the law of motion for $S(t)$ can be written as $\dot{S}(t) = (s - \Psi_d(\boldsymbol{Y}(t))\Psi_R(\boldsymbol{Y}(t)))$. Therefore, the law of motion for $\boldsymbol{Y}(t)$ can be written as $\dot{\boldsymbol{Y}}(t) = \Psi_Y(\boldsymbol{Y}(t))$. Accordingly, the consumer's decision rules and the aggregation formulas induce a mapping from the perceived law of motion for the aggregate state variables to an actual law of motion for them.

3.3 *Equilibrium definition*

Definition 1 (Rational-Expectations Equilibrium). A *rational-expectations equilibrium (REE)* is defined by a path of the aggregate state variables $\boldsymbol{Y} = (S, G)$, a perceived law of motion Γ_Y, Γ_R, and the consumer's decision rules $\{k^*, \bar{k}\}$ with associated value functions $\{V, V^*\}$ such that the following conditions hold:

(i) Consumer's optimization: given the consumer's beliefs Γ_R and Γ_Y, the decision rules $\{k^*, \bar{k}\}$ and the value functions $\{V, V^*\}$ solve the consumer's optimization problem.

[17] Note that G may have mass points at the boundary ($k = 0$) or in the interior. Thus, we define a generalized probability density function g that satisfies (i) $\int_{k'\in\mathbb{R}_+} g(t, k')\, dk' = G(t, k)$ and (ii) $g(t, k) = \hat{g}(t, k) + \sum_{i=1,\dots,I} m(t, \kappa_i)\delta(k - \kappa_i)$, where $\hat{g}(t, \cdot)$ is a probability density function (a Lebesgue-integrable real-valued function), $m(t, \kappa_i)$ is the probability mass at $\kappa_i \in \mathbb{R}_+$, and $\delta(\cdot)$ is the Dirac delta function.

(ii) Aggregates are determined by individual actions and the aggregate state variables: $d(t) = \Psi_d(\boldsymbol{Y}(t))$, $R(t) = \Psi_R(\boldsymbol{Y}(t))$ and $\dot{\boldsymbol{Y}}(t) = \Psi_Y(\boldsymbol{Y}(t))$, for all $\boldsymbol{Y}(t)$.

(iii) Consumers' beliefs are rational expectations: $\Gamma_Y = \Psi_Y$ and $\Gamma_R = \Psi_R$.

In an REE, given the path of exogenous variables $\{c(t), p(t)\}_{t\geq 0}$, consumers make optimal decisions based on the perceived law of motion, and the perceived law of motion is consistent with the actual one.

3.4 *Stationary equilibrium*

As a benchmark of "normal times," we first look at the situation where all exogenous variables—the flow shopping costs and the price—are constant permanently, that is, $c(t) = c > 0$ and $p(t) = p$ for all t. We say that an REE is *stationary* if the endogenous variables are also constant. The formal definition is as follows.

Definition 2 (Stationary Equilibrium). For $c(t) = c$, $p(t) = p$ for all t, an REE is *stationary* if the consumers' policies and the distribution of consumers' stock are time invariant, that is, $k^*(t) = k_o^*$, $\bar{k}(t) = \bar{k}_o$, $G(t, k) = G_o(k)$ for all t and k.

We consider a situation where consumers can easily purchase the product during normal times, as the flow supply s is sufficiently large to meet the flow demand (we will specify the exact value of s later). Given the flow supply is abundant, consumers rationally expect that the product is fully available all the time, that is, $R(t) = 1$ for all t.

Given the belief of $R = 1$, each consumer follows a time-invariant policy with k_o^* and $\bar{k}_o$ that solve the Hamilton–Jacobi–Bellman variational inequality (HJBVI, henceforth):

$$rV_o(k) = \max\bigl\{h(k) - V_o'(k)x(k), rV_o^*(k)\bigr\}, \tag{7}$$

where V^* satisfies: $rV_o^*(k) = h(k) - c - V_o^{*'}(k)x(k) + \alpha[\max_{q>0}\ (V_o(k+q) - pq) - V_o^*(k)]$.[18] Furthermore, the following lemma ensures that, when the consumers' policies are time-invariant, the distribution of consumers' stock $G(t, k)$ converges to a unique time-invariant distribution $G_o(k)$, that is, $G(t, k) \to G_o(k)$ as $t \to \infty$.

Lemma 1. *For any $\bar{k}$ and k^* such that $\bar{k} \geq k^* \geq 0$, with the exogenous exit $\theta > 0$ and $G_{\text{new}} = G_o$, the distribution of consumers' stock in the stationary equilibrium converges to a unique time-invariant distribution G_o that satisfies the ordinary differential equations*:

$$0 = \begin{cases} g_o'(k)x(k) - \alpha g_o(k) & \text{for } k \in [0, k^*], \\ g_o'(k)x(k) + \alpha G_o(k^*)\delta(k - \bar{k}) & \text{for } k \in [k^*, \bar{k}], \end{cases} \tag{8}$$

where g_o is the generalized probability density function for consumers' stock.

[18]Here, we restrict our attention to the case where V is continuous, and V^* is continuous and differentiable. HJBVI (7) derives the action region $\mathcal{A}_o = [0, k_o^*]$, where $h(k) - V_o'(k)x(k) \leq rV_o^*(k)$ for $k \in \mathcal{A}_o$. The target stock $\bar{k}_o$ is given by $\bar{k}_o = \arg\max_{q'>0} V_o(q') - pq'$.

The proof of Lemma 1 is in Supplemental Appendix F.

Given consumers' policy, k_o^* and $\bar{k}_o$, and the distribution G_o, the flow demand rate is also time invariant: $d(t) = d_o$. If $s \geq d_o$, then the flow supply is always larger than the flow demand and, therefore, $R = 1$ is actually the case.

Below, we show that there exists a unique stationary equilibrium satisfying $R = 1$ under some reasonable parametric assumptions. The first specifies the flow supply rate s.

Assumption 1 (Sufficient Supply). *The flow supply rate coincides with the flow demand rate given $R = 1$, that is, $s = d_o$.*

As mentioned earlier, we assume that $s \geq d_o$ to ensure that shopping during normal times is convenient. If $s > d_o$, the store's inventory will increase over time without bound. Although our model does not directly consider the producer's profit-maximization problem, real-world firms aim to avoid overstocking and balance flow supply with demand in the long run. To simplify our analysis, we assume that flow demand and supply are equal during normal times (i.e., in the stationary equilibrium) instead of modeling the producer's long-run adjustment explicitly. Assuming Assumption 1 holds and a stationary equilibrium is reached, the store's stock will be time-invariant, that is, $S(t) = S_o$ for all t, where $S_o \geq 0$.

The second assumption ensures $k_o^* > 0$, meaning that the consumers are willing to conduct shopping searches when their stock is small:

Assumption 2 (Large Out-of-Stock Disutility). *Let V^N be the value function for the consumers that would be achieved if no control is exercised*:

$$V^N(k) := \int_0^\infty \mathrm{e}^{-rs} h\big(\max\{k - s, 0\}\big)\, ds.$$

The flow disutility from running out of stock, a, is sufficiently large to satisfy

$$\max_{q \geq 0} V^N(q) - pq + \frac{a}{r} > \frac{c}{\alpha}.$$

Assumption 2 requires that consumers absolutely desire to avoid stock-out ($k = 0$). If the disutility is small (e.g., because the product is substitutable), then consumers may optimally choose not to consume it. Such a situation is excluded since we focus on the market of an unsubstitutable necessity product.

The third assumption is that the market is not too frictional.

Assumption 3 (Small Search Friction). *The matching rate α is sufficiently large such that $\alpha p > \bar{b}$.*

Recalling that the matching rate α captures the easiness of shopping during normal times, Assumption 3 implies that shopping is an easy task during normal times.

The following proposition states the characteristics of the stationary equilibrium.

Proposition 1. *Suppose Assumptions 1, 2, and 3 hold. Then there exists a unique stationary REE that satisfies the following properties*:

(i) *Consumers engage in shopping periodically; that is,* $0 < k_o^* < \bar{k}_o < +\infty$ *is satisfied.*

(ii) *The consumer's go-shopping threshold* k_o^* *satisfies* $\alpha[V_o^A(k_o^*) - V_o^*(k_o^*)] = c$.[19]

(iii) *The consumer's target stock* $\bar{k}_o$ *satisfies*

$$\bar{k}_o = k_o^* + \frac{1}{r}\log\left(1 + \frac{A_o\big(1 - \mathrm{e}^{-(\alpha+r)k_o^*}\big) + \mathrm{e}^{-(\alpha+r)k_o^*}a - p}{\dfrac{\bar{b}}{r} + p}\right),$$

with

$$A_o = \frac{\alpha p - \bar{b}}{\alpha + r} > 0.$$

(iv) *The long-run stationary distribution for the consumer's stock follows*:[20]

$$g_o(k) = \begin{cases} \dfrac{\alpha}{1 + \alpha\big(\bar{k}_o - k_o^*\big)}\mathrm{e}^{-\alpha(k_o^* - k)}, & k \in \big(0, k_o^*\big], \\ \dfrac{\alpha}{1 + \alpha\big(\bar{k}_o - k_o^*\big)}, & k \in \big[k_o^*, \bar{k}_o\big], \end{cases}$$

and has a mass point at $k = 0$ *with* $G_o(0) = \frac{\mathrm{e}^{-\alpha k_o^*}}{1+\alpha(\bar{k}_o - k_o^*)}$.

(v) *Shopping searches take* $1/\alpha$ *on average.*

The proof of Proposition 1 and the expressions for the value functions are provided in Supplemental Appendix F.

4. Calibration

The calibration of the model aims to ensure that the household inventory behavior in the stationary equilibrium matches empirical evidence. Table 1 presents the chosen parameter values for the benchmark calibration, where a week is considered as one unit of time. The weekly time-discount rate ρ is set to $0.01/52$, implying an annual discount rate of 1 percent, and the weekly exit rate θ is set to $0.04/52$.[21] To normalize the market price of the product, we set $p = 10$, implying that the cost of a week's worth of toilet paper is 10.[22] We set the per unit storage cost $\bar{b}$ to 1, implying that the marginal cost of

[19] This corresponds to the value matching condition: $V_o(k_o^*) = V_o^*(k_o^*)$. HJBVI (7) implies the smooth pasting condition, or the high contact principle $V_o'(k_o^*) = V_o^{*'}(k_o^*)$ (see Øksendal (2003, Chapter 10)).

[20] It is assumed that the distribution for the new entrant's stock G_{new} equals G_o. In Supplemental Appendix F , we generalize the specification of G_{new}.

[21] We assume $\theta > 0$ for technical reasons as described in Lemma 1. In Section 7, we demonstrate that the model dynamics in the short run are not sensitive to the choice of θ value.

[22] Setting $p = 10$ is solely for normalization purposes. Consequently, scaling down the values of a, c, and $\bar{b}$ by a factor of p produces equivalent results.

Table 1. Parameter values.

Parameters		Value	Reference/Target Statistics	
Parameters calibrated from external sources				
ρ	Weekly discount rate	0.01/52	Annual discount rate of 1%	
θ	Weekly exit rate	0.04/52	Annual replacement rate of 4%	
p	Market price of the product	10.0	Normalization	
$\bar{b}$	Storage cost per unit of the product	1.0	Hendel and Nevo's (2006a) estimate	
S_o	Average store's inventory stock	2.5	60% sales increase in March 2020 (Buchholz (2020))	
Parameters calibrated jointly				
a	Flow disutility from stock-out	1069.23	(i)	2 weeks' stock left at timing of purchase
α	Shopping search intensity	2.29	(ii)	Purchase interval of 4 weeks
c	Flow shopping-search cost	14.63	(iii)	1% households have less than 3 days' stock at the time of purchase

Note: The values of a, α, and c are collectively determined to fulfill the three conditions denoted as (i), (ii), and (iii). These calibration criteria, (i), (ii), and (iii), are grounded in survey data presented by Kano (2018).

storing a unit of toilet paper is 10% of its purchase price, which falls within the range of the estimate of laundry detergent storage cost by Hendel and Nevo (2006a).[23]

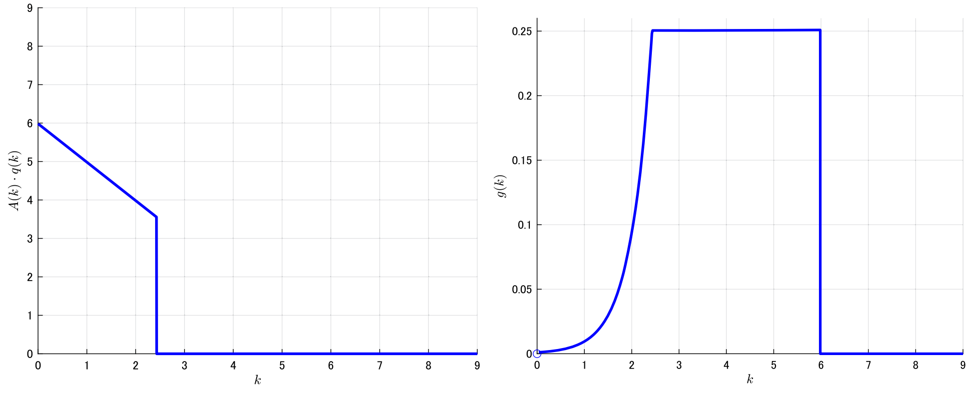
We calibrate the three parameters, a, α, and c, aligning the household inventory behavior with micro-survey evidence on consumer behavior. The parameter a represents the cost associated with not having toilet paper for a week, while α and c govern the degree of search frictions. Although data on household inventory is generally limited, the study by Kano (2018) in Japan offers extensive evidence on the inventory holdings of consumers.[24] We utilize three pieces of evidence from the survey. First, the average inventory of toilet paper at the time of purchase is 9.76 rolls (606.5 m). Considering that the average daily consumption of each household is 0.72 rolls (43.9 m), this implies that households tend to repurchase when they have 13.5 days' worth of stock remaining. Second, the majority of households purchase toilet paper every 3–4 weeks, and the days between purchases often fall in multiples of 7.[25] Third, most households purchase additional toilet paper before exhausting their household inventory.[26] Based on this evi-

[23]Hendel and Nevo (2006a) show that the median purchase price of a 64 oz. bottle of laundry detergent, which is equivalent to about one month's worth of use, is $3.89, whereas if a 128 oz. bottle is purchased instead of a 64 oz. bottle, additional storage costs of approximately $0.20–0.75 are incurred.

[24]The survey was conducted from September to December 2015 on a large-scale consumer database operated by INTAGE, a marketing company in Japan. Notably, this survey included questions about (i) the timing of toilet paper purchases, (ii) the inventory level at the time of purchase, and (iii) daily consumption of toilet paper, making it unique among household surveys.

[25]Such purchasing behaviors have also been observed in US household data. For instance, Hendel and Nevo (2006a) find that the median interval between purchases of laundry detergents is 4 weeks. We also note that, in the survey in Kano (2018), about 5-10 percent of respondents indicated that their purchase interval was less than 1 week, suggesting that such households are in the habit of purchasing small rolls of toilet paper at high frequency, without having a large household inventory. In Supplemental Appendix E.3.3, we provide a model extension that accounts for such household heterogeneity.

[26]Kano (2018) reports that about 93% of survey respondents purchase additional toilet paper even when they have household stocks beyond the rolls currently in use. This suggests that the average household is at very low risk of stock-out, except for the 5–10% of respondents who reported purchasing toilet paper at



(a) Policy for purchase quantity: $A(k) \cdot q_o(k)$ (b) Consumers' distribution: $g_o(k)$

FIGURE 2. An illustration of the stationary equilibrium. *Note*: The horizontal axis represents the amount of the existing consumer's stock. In Figure 2b, the generalized density function g_o has a mass point at $k = 0$ ($G_o(0) = G_o(k_o^*)\mathrm{e}^{-\alpha k_o^*}$).

dence, we target (i) households replenish when they have an average of 2 weeks' worth of stock remaining, (ii) the average purchasing cycle spans 4 weeks, and (iii) the probability that households have less than 3 days' stock at the time of purchase is 1%. This induces $(a, \alpha, c) = (1069.23, 2.29, 14.63)$, implying that the stock-out cost is substantially large and shopping searches are fairly time-consuming and costly.[27]

Our definition of the stationary equilibrium does not determine the level of the store's inventory stock S_o. Thus, we externally set $S_o = 2.5$ to indicate that, during normal times, the store holds inventory stock covering 2.5 weeks or 0.59 months of sales. This implies that if the store's inventory stock is sold out within a particular month, the monthly sales increase by 59%. We chose this value based on the observation that toilet paper sales in the United States in March 2020 increased by about 60% compared to the same month in the previous year (Buchholz (2020)).[28]

In Figure 2, we illustrate the consumer's policy (Figure 2a) and the distribution of the consumer's stock (Figure 2b) in the stationary equilibrium under the parameter values given in Table 1. In the stationary equilibrium, as in our daily life, consumers consume the product in their private inventory at a constant rate (normalized to 1) and

intervals of less than a week, maintaining only a small household inventory, and frequently buying small rolls (see footnote 25).

[27]Taken literally, this means spending three days per shopping trip, which might sound too frictional (e.g., Petrosky-Nadeau, Wasmer, and Zeng (2016), who report that the average total shopping time is 40–50 minutes per day in the American Time Use Survey). Our calibration is reasonable, though, if we interpret the search friction in our model to include the mismatch between retail stores' hours of operation and leisure time. According to the aforementioned household survey, many households make purchases on a particular day of the week, which suggests that many households are in the habit of going shopping on their days off. Thus, we interpret that the Poisson arrival rate α also captures variations in shopping cost due to idiosyncratic shocks, which are abstracted for computational reasons.

[28]This choice is roughly consistent with the evidence that the average annual inventory turnover rate of 16.83 for US grocery stores, implying an average inventory period of 21.69 days (365/16.83) (Boigues (2016, Table 3.2)).

start a shopping search when the stock goes down to $k_o^* \approx 2.4$ (weeks). The searching consumer, upon finding a store, purchases the product to stock up to the target stock, $\bar{k}_o \approx 6$. That is, the amount purchased is $q_o(k) = \bar{k}_o - k \approx 6 - k$. Therefore, no consumer has more than $\bar{k}_o$ in stock, and the fraction $G_o(k_o^*)$ of consumers engage in shopping searches every time. The density is flat for the inaction region $k \in [k_o^*, \bar{k}_o]$, while it exponentially increases as k increases for $k \in (0, k_o^*]$ as a result of the Poisson arrival process. Although extremely rare, some consumers unluckily fail to shop and exhaust the stock held at home. With our parameter choice, the share of such stockless consumers ($G_o(k_o^*)\mathrm{e}^{-\alpha k_o^*}$) is around 0.05%. As such, the risk of stock-out is very low in the stationary equilibrium.

5. Dynamic responses to a shopping-cost shock

In this section, we explore the impact of a shock that *temporarily* increases the flow shopping costs $c(t)$. We assume that, until $t = 0$, the economy is on the stationary equilibrium where all the model agents believe that $c(t) = c$ forever, but they are informed of a one-time and deterministic change in $c(t)$ at time $t = 0$, as detailed in Section 5.1. In what follows, $X(t)$ denotes the value of a variable X after t time (weeks) after the awareness of the shock, and $G(t, k)$ denotes the distribution for the consumer's stock at time t with $G(0, k) = G_o(k)$.[29]

5.1 *The fundamental shock and the phases of the emergency*

The path of $c(t)$ is specified by the four parameters $(\bar{c}, T_c^S, T_c^L, T_c^E)$ with $\bar{c} > c$ and $0 \leq T_c^S < T_c^L < T_c^E < \infty$. As illustrated in Figure 3, we consider the following phases.

Predisaster phase ($t < 0$) Consumers, believing that all the exogenous parameters are stationary, that is, $c(t) = c$ and $p(t) = p$ forever, follow the stationary-equilibrium strategy.

Announcement ($t = 0$) At time 0, an event that (will) increase flow shopping costs $c(t)$ is recognized. Consumers immediately change their beliefs about the path of the exogenous variables and the endogenous state variables. In response, they change their shopping behavior at $t = 0$.

Preparation phase ($0 \leq t < T_c^S$) Although consumers know that the flow shopping costs $c(t)$ will increase later, $c(t)$ has not yet increased, that is, $c(t) = c$. Note that, when $T_c^S = 0$, there is no preparation phase, and the restricted-movement phase starts upon announcement.

Restricted-movement phase ($T_c^S \leq t < T_c^L$) Movements are restricted. The flow shopping costs $c(t)$ jumps up to $\bar{c}$ at time T_c^S, and it stays at that level until time T_c^L.

Restriction-lifting phase ($T_c^L \leq t < T_c^E$) The restrictions are gradually relaxed. The flow shopping cost $c(t)$ linearly decreases from the maximum level $\bar{c}$ to the normal level c.

[29] As illustrated in Supplemental Appendix E.1.2, there would be no self-fulfilling panic in our model. Thus, in the absence of any shifts in the model parameters, only $R(t) = 1$ for all t would be rationalized.

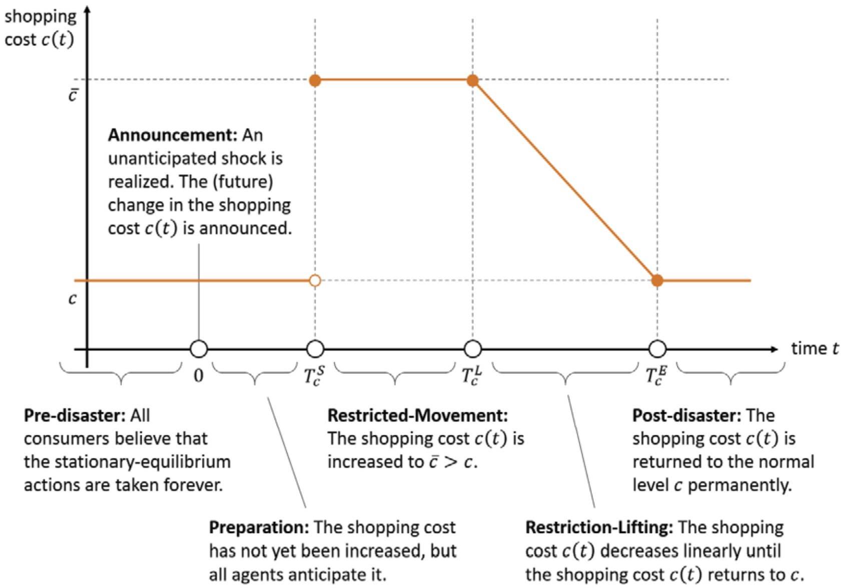


FIGURE 3. An illustration of the phases of the emergency.

Post-disaster phase ($T_c^E \le t$) The "lifting" is completed at time $t = T_c^E$, and then the flow shopping costs $c(t)$ is returned to the normal level c permanently.

To summarize, the dynamics of the flow shopping costs $c(t)$ is given by[30]

$$c(t) = \begin{cases} c, & t \le T_c^S, \\ \bar{c}, & T_c^S \le t \le T_c^L, \\ \bar{c}\left(\dfrac{t - T_c^L}{T_c^E - T_c^L}\right) + c\left(1 - \dfrac{t - T_c^L}{T_c^E - T_c^L}\right), & T_c^L \le t \le T_c^E, \\ c, & T_c^E \le t. \end{cases}$$

Note that shopping-cost shocks have no impact on the consumption and supply of the product in aggregate, and thus, full availability is surely maintained if all consumers keep the stationary equilibrium shopping strategy.

5.2 *Welfare evaluation*

We measure the welfare cost of panic buying by quantifying the degree to which consumers' welfare is decreased due to the disruption of the purchase cycle. To this end, we first define the average *loss* in consumer (annuitized) value from the risk that can be attributed to stochastic shopping searches as follows:

$$\omega\big(V(t,k), G(t,k); \widehat{V}_o(k), \widehat{G}_o(k)\big) = -\left(\int_k rV(t,k)d_k G(t,k) - \int_k r\widehat{V}_o(k)\, d\widehat{G}_o(k)\right),$$

[30]In some simulation scenarios, we also analyze the exogenous shift in sales price $p(t)$.

where $\widehat{V}_o(k)$ and $\widehat{G}_o(k)$ represent the consumer's value function and the distribution of the consumer's stock, respectively, in the hypothetical economy where the shopping search process is *deterministic*. In this economy, consumers can purchase the product with certainty after $1/\alpha$ weeks of costly shopping searches. Thus, each consumer has a constant purchase cycle where they start searching for new stock when $1/\alpha$ weeks of inventory is left, obtain new stock exactly when they run out of the existing stock, and never become stockless.[31]

We define the gross welfare cost, denoted by Ω, as the percentage increase in ω relative to the stationary equilibrium. Formally, we can express it as follows:

$$\Omega = \frac{\omega\big(V(0,k), G(0,k); \widehat{V}_o(k), \widehat{G}_o(k)\big) - \omega\big(V_o(k), G(0,k); \widehat{V}_o(k), \widehat{G}_o(k)\big)}{\omega\big(V_o(k), G(0,k); \widehat{V}_o(k), \widehat{G}_o(k)\big)}. \tag{9}$$

In equation (9), the denominator represents the extent of idiosyncratic risk that consumers face in the stationary equilibrium. On the other hand, the numerator indicates the extent to which consumers suffer from increased shopping costs, which is measured by the average change in their values at time 0.

In our welfare analysis, we disentangle the welfare cost attributable to congestion externality by splitting the gross welfare cost Ω into the direct and indirect effects. The direct effect refers to the welfare cost that consumers would suffer from the shock if the full availability ($R(t) = 1$) were achieved for all $t \geq 0$. It corresponds to the average change in consumers' value in the counterfactual ignoring the supply constraint on the product. We label it $\Omega_{R=1}$, which measures the welfare cost *directly* suffered by consumers from the shock.

The gross welfare cost Ω takes into account the endogenous feedback effect of the market condition, that is, the change in product availability $R(t)$. We can calculate the *indirect* welfare cost suffered from the shock through the endogenous market behavior as $\Omega_{R<1} = \Omega - \Omega_{R=1}$. Note that the equilibrium with $R(t) < 1$ arises from the consumer's selfish purchasing behavior, and $\Omega_{R<1}$ captures the welfare cost attributable to congestion externality.

5.3 *Numerical methods for simulation analysis*

While the stationary equilibrium of the model is obtained in a closed form, we numerically compute the equilibrium transitional dynamics against fundamental shocks. We briefly describe the scheme, with detailed procedures provided in Supplemental Appendix G.

Our numerical method customizes the algorithms developed by Achdou et al. (2022) for solving MFGs efficiently. It consists of two steps. In the first step, we apply the finite difference method to solve the HJBVI equation in (7) and the KF equation in (8), which allows us to obtain the stationary equilibrium in a discretized space.[32] In the second

[31] See Appendix A for the expression of $\widehat{V}_o(k)$ and the associated policy functions.

[32] The advantage of this algorithm lies in its simultaneous solution of the HJBVI and KF equations, leveraging the adjoint relationship between the HJB and KF operators. In this environment, the partial differential equation (7) is reduced to a linear complementarity problem, which we solve using the routines provided on Benjamin Moll's personal website (https://benjaminmoll.com/codes/).

step, we compute the equilibrium transitional dynamics by extending the above algorithm to allow for time-varying aggregate variables. Specifically, we find an REE path of $R(t)$ using an iterative scheme with an initial guess $R(t) = 1$ for all $t \geq 0$. In Supplemental Appendix E.1.1, we present a detailed description of the procedure and illustrate how consumers form their rational beliefs regarding $R(t)$.[33]

5.4 *Simulation results: Benchmark*

In this paper, we refer to the following scenario as the *benchmark* case.

Benchmark simulation At time $t = 0$, there is an announcement that movement restriction will be implemented in 1 week (i.e., $T_c^s = 1$). During the restricted-movement phase, shopping costs increase by $(\bar{c} - c)/c = 500\%$. The restricted-movement phase will continue for 6 weeks ($T_c^L - T_c^S = 6$) and then will be lifted in phases over 3 weeks ($T_c^E - T_c^L = 3$). The market price is fixed at p.[34]

Figure 4 displays the paths of selected variables in the equilibrium dynamics for benchmark simulation: the evolution of the search cost $c(t)$ in the top-left chart ("Shock Process"); the path of the availability $R(t)$ in the top-middle chart ("$R(t)$: Availability"); the paths of the target stock $\bar{k}(t)$ and the go-shopping threshold $k^*(t)$—the key variables that characterize the consumers' optimal strategy—in the top-right chart ("Consumer's Policy"); the path of the fraction of consumers engaging in a shopping search, $100 \cdot G(t, k^*(t))$, in the lower-left chart ("Searching Consumers (%)"); the paths of the fraction of *hoarders* who have a larger stock than the maximum level held in the stationary equilibrium, $100 \cdot (1 - G(t, \bar{k}_o))$, and the fraction of *stockless* consumers who run out of stock, $100 \cdot G(t, 0)$ in the lower-middle chart ("Misallocation (%)");[35] the path of the quantity of in-store stock $S(t)$ in the lower-right chart ("$S(t)$: Store's Stock").

Figure 4 clearly shows that panic buying begins when the announcement is made at $t = 0$: the target stock $\bar{k}(t)$ jumps from 6.0 to 11.5 for avoiding shopping during the

[33]The iterative process begins with the most optimistic guess about the availability, and thus, the equilibrium dynamics shown as a simulation result is unique and stable given the cognitive hierarchy starting from $R(t) = 1$ for all $t \geq 0$.

[34]One reason for considering the fixed price here is that price controls have been widely used in emergency situations. The first state law prohibiting price gouging in the United States was enacted in New York in 1979 in response to rising winter heating oil prices in 1978–1979; these measures were subsequently adopted by other states (Bae (2009), Giberson (2011)). During the COVID-19 pandemic, 42 US states activated some form of price-gouging regulations that restricted retailers from charging exorbitant prices on consumer products. Among the 42 states, eight states (Alaska, Delaware, Maryland, Minnesota, Montana, New Mexico, Ohio, and Washington) did not have price-gouging regulations before the pandemic but newly introduced the regulations under their COVID-19 emergency declarations (Chakraborti and Roberts (2023)). In addition, some recent empirical studies have shown that fairness considerations lead to a reluctance to raise prices in times of emergency (e.g., Cavallo, Cavallo, and Rigobon (2014), Gagnon and López-Salido (2019), Hansman et al. (2020), Cabral and Xu (2021)), in line with Akerlof's (1980) theory of social norms and the suggestions from Kahneman, Knetsch, and Thaler's (1986) questionnaire study.

[35]We select the two variables because they are the moments relevant for the efficiency of the allocation. The welfare loss becomes larger as hoarders and stockless consumers increase because hoarders bear unusually high storage costs and stockless consumers suffer large disutility from stock-out ($a = 1069.23$). Stockless consumers exist even in stationary equilibrium, albeit in very small numbers.

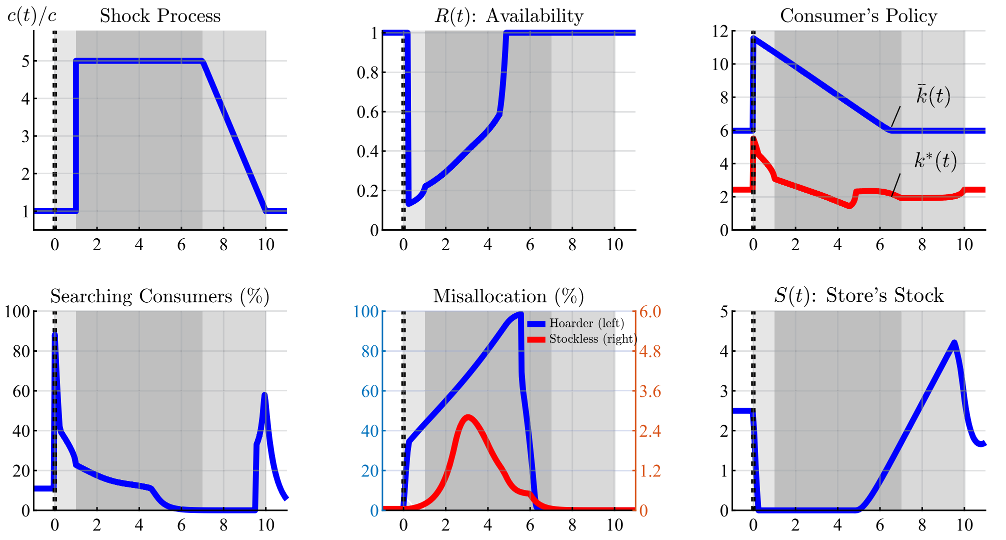


Figure 4. Benchmark: $S_0 = 2.5$, $(\bar{c} - c)/c = 5$, $T_c^S = 1$, $T_c^L - T_c^S = 6$, and $T_c^E - T_c^L = 3$. *Note*: In all charts, the horizontal axis (t) represents the number of weeks after the announcement ($t = 0$). The background color of the graph area illustrates the phase of the emergency: the predisaster phase (white), the preparation phase (light gray), the restricted-movement phase (dark gray), the restriction-lifting phase (medium gray), and the post-disaster phase (white).

restricted-movement phase, and the go-shopping threshold $k^*(t)$ jumps from 2.4 to 5.5 for reducing the risk of running out of stock. This sharply increases the fraction of searching consumers from the predisaster level of 10.9 to 88.1%. The increased market demand rapidly reduces and depletes the store's stock. As a result, the availability $R(t)$ decreases to less than 0.15 at its worst. This dynamic response is in line with the observation that many regions suffered from panic buying immediately after the *announcement*, rather than the implementation, of movement restrictions during the COVID-19 pandemic (Keane and Neal (2021)).[36]

Worse, the low availability persists in the restricted-movement phase (e.g., $R(t) < 0.33$ continues for 2.1 weeks) because the initial stockpiling demand is so large that many consumers who start searching during the preparation phase cannot finish their shopping by the end of the phase. Such consumers are desperate to shop, even bearing higher shopping costs. As a result, more and more consumers experience stock-out at home and the fraction of stockless consumers reaches 2.8%, which is more than 50 times the normal level. At the same time, those who have purchased the product overstock it against shortages, resulting in incurring extra storage costs. In sum, the product is misallocated, with some incurring higher storage costs and others facing increased risk of stock-out at home and conducting shopping searches at higher costs, which causes substantial welfare costs.

[36] In New York City, an epicenter of COVID-19 infections, the state's first case was confirmed on March 1, a state of emergency was declared on March 7, and strict movement restrictions were imposed after March 15. The toilet paper shortage already became serious during the week ending March 14 (Wallace (2020)).

TABLE 2. Welfare costs of a shopping-cost shock: benchmark.

	Gross Welfare Cost	Direct Welfare Cost	Indirect Welfare Cost
Benchmark	$\Omega = 5.05$ (%)	$\Omega_{R=1} = 0.85$ (%)	$\Omega_{R<1} = 4.20$ (%)

Note: See Section 5.2 for the definitions of Ω, $\Omega_{R=1}$, and $\Omega_{R<1}$.

As shown in Table 2, the welfare cost of the shopping-cost shock is large: $\Omega = 5.05\%$, meaning costs (including nonpecuniary costs) associated with shopping searches, storage, and stock-out are 5% higher than in normal times. We emphasize that the large share is attributable to the endogenous market-congestion effect rather than to increased shopping costs due to the exogenous shock: $\Omega_{R=1} = 0.85\%$; $\Omega_{R<1} = 4.20\%$. This result implies that the market-congestion effect amplifies the disaster damage considerably.

In sum, the shopping-cost shock leads to a severe and persistent shortage, even though the shock itself has no impact on either aggregate consumption or aggregate supply of the product. In other words, as there are sufficient resources to meet consumption in the aggregate, full availability would be maintained if all consumers behaved as in the stationary equilibrium. Nevertheless, in the decentralized equilibrium, defensive hoarding by selfish consumers takes the product excessively from the market.

5.5 *Timing of awareness: Anticipated versus unanticipated shock*

In this section, we examine how the length of the preparation phase T_c^S influences the severity of the panic. In practice, the length depends on the forecastability of the emergency. For example, the landfall of a major hurricane can be forecast in advance, while earthquakes, massive blackouts, and terrorist attacks are virtually unpredictable. Furthermore, when government policies cause a shopping-cost shock, the length varies depending on the announcement's timing. For example, at the onset of the global spread of COVID-19, many governments placed movement restrictions on their residents after announcing their implementation in advance. Thus, in the context of the 2020 toilet paper shortage, the simulations below examine the roles played by the timing of awareness of a future shopping-costs increase to shortages.

Figure 5 displays how the welfare cost of a shopping-cost increase varies depending on the timing of awareness. It clearly shows that (i) anticipated shopping-cost increases are more likely to have a greater impact on social welfare than unanticipated increases, and (ii) a shopping-cost increase with 2 week-long grace period tends to trigger the most severe shortages.

In Figure 6, we compare the dynamic responses to a shopping-cost increase that is totally unanticipated ($T_c^S = 0$) and anticipated 1 week in advance ($T_c^S = 1$). In the case where shopping costs rise immediately, while consumers increase their target stock $\bar{k}(t)$ for stockpiling, they do not increase the go-shopping threshold $k^*(t)$ (see the top-right chart) because shopping costs are already higher when becoming aware of the shock. Therefore, the number of searching consumers does not increase upon announcement

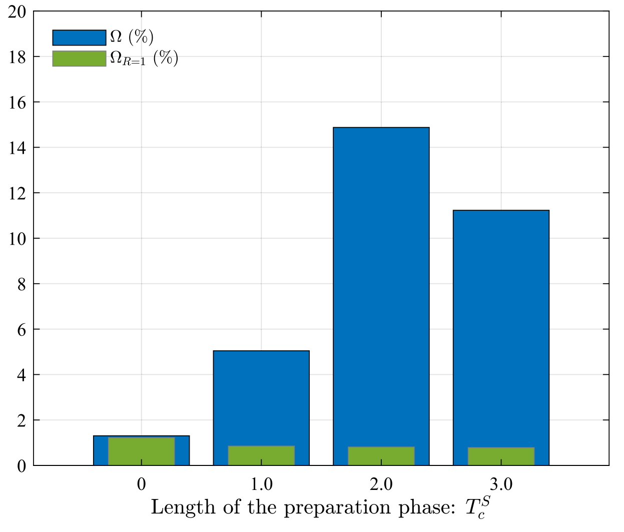


FIGURE 5. Welfare cost of a shopping-cost shock. *Note*: $T_c^S = 1.0$ corresponds to the benchmark. See Section 5.2 for the definitions of Ω and $\Omega_{R=1}$.

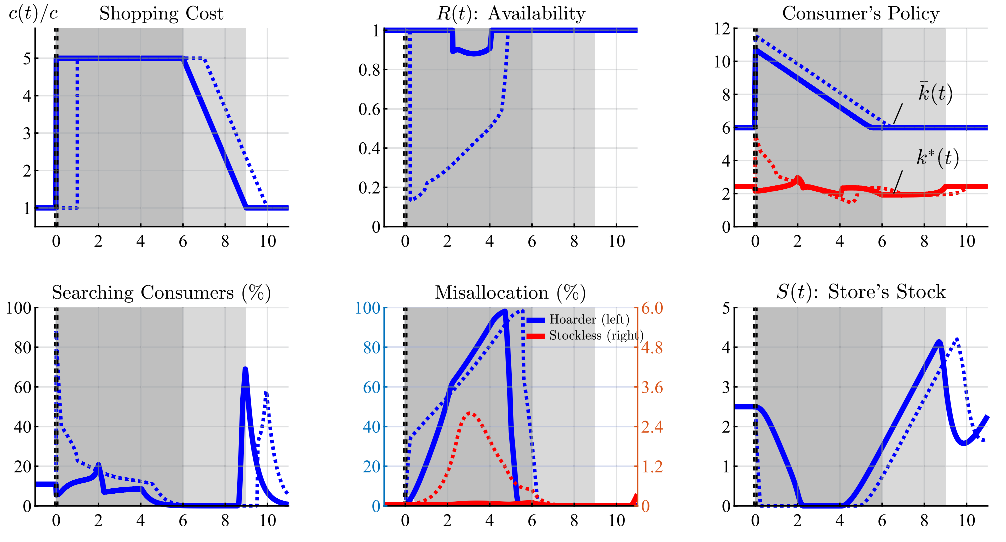


FIGURE 6. Unanticipated shopping-cost increase: $T_c^S = 0$. *Note*: The horizontal axis represents the number of weeks after the announcement. The dotted lines show the results for the benchmark case ($T_c^S = 1$).

(see the bottom-left chart), which leads to a slower decline in the store's stock $S(t)$ and a higher availability $R(t)$ than in the benchmark case.

This implies that the very existence of a preparation phase plays an important role in amplifying panic buying. The preparation phase (as in the benchmark) lures consumers to shop before shopping costs rise, causing a concentration of demand. By contrast, if there is no preparation phase, it is too late to rush to the market, resulting in a mild increase in market demand.[37]

Figure 5 also indicates that the extended duration of the preparation phase (e.g., $T_c^S = 3$) mitigates the impact of the shock. In this case, unlike the benchmark case, many consumers do not rush to the market right after hearing the news because they find that stockpiling too far in advance would be too costly for storage. Thus, depending on the initial stock of private inventory, consumers react differently to the news: Some purchase a lot just before the shopping costs increase, while others shop after the shortage of the product is nearly eliminated. In this manner, the extended grace period could mitigate the concentration of timing of purchases, resulting in a less severe shortage.

5.6 *Policy interventions*

We discuss policy options for curbing panic buying. Given that we have demonstrated that panic buying is an upward spiral of hoarding demand, it is natural to infer that breaking the spiral is essential to curb panic buying. We evaluate the performance of the three different policies: quotas on purchases, short-term sales-tax change, and non-market distribution.

5.6.1 *Quotas on purchases* Limiting the quantity purchased is one of the common measures implemented in many cases of shortages. In practice, when faced with a sudden increase in demand, grocery stores often limit the number of items that can be purchased by each shopper. Below, assuming that such a quota is perfectly enforceable, we evaluate the performance of the following policy.

Policy simulation (Purchase Quota). Consumers are not allowed to purchase more than 4 units, that is, $q_i(t) \leq 4$, for the first 5 weeks $t \in [0, 5]$.

Under the purchase restriction policy, consumers are allowed to purchase only up to four units, and those who want to purchase more must start a shopping search again. Therefore, as can be seen from Figure 7, although the measure of searching consumers increases in response to the awareness of rising future shopping costs, they cannot stock up to the (privately) optimal level $\bar{k}(t)$ in the presence of the purchase quota.[38] As a result, a serious shortage is prevented, which reduces welfare costs of the shopping-cost shock: $\Omega = 1.40\%$. Furthermore, in Supplemental Appendix E.1.3.3, we confirm that a quota policy makes all consumers better off in the sense that it increases the consumer's value at $t = 0$, $V(0, k)$, for all k.

[37] It is somewhat difficult to find historical evidence for instances in which panic buying did not occur. Nevertheless, no serious panic buying was reported in London after the July 7, 2005, bombings (Burney and Jones (2005)), or in New York City after the September 11 attacks, even though these terrorist attacks restricted the daily lives of the residents there. Considering that the restrictions on movement due to these disasters were totally unanticipated, the response to an unanticipated shopping-cost shock is in line with these experiences.

[38] Note that the purchase-quota $q_i(t)$ is binding for most of the cases because the average purchasing cycle of 4 weeks is one of our calibration targets, and consumers want to purchase more when the shock arrives.

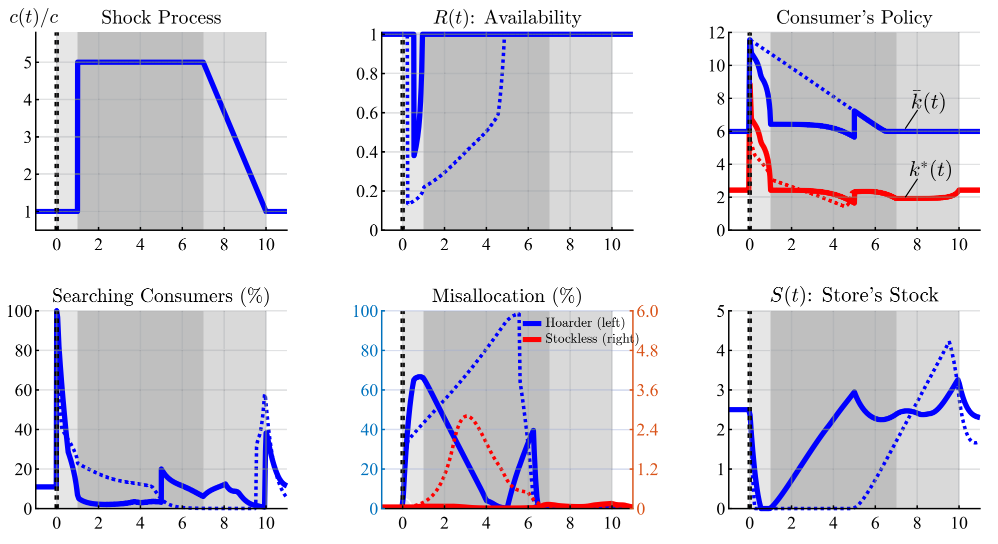


FIGURE 7. Purchase quota: $q_i(t) \leq 4$ for $t \in [0, T_c^L] = [0, 5]$. *Note*: The horizontal axis represents the number of weeks after the announcement. The dotted lines show the results for the benchmark case.

Enforcing the purchase-quota policy in practice might not be fully feasible as consumers can potentially violate the quotas without being tracked for their purchase history. For instance, upon finding a product in a store, consumers can repeatedly approach the cash register as new customers to buy an excessive amount, even if the store has set limits on the quantity allowed to be purchased by a single consumer at a time.

5.6.2 *Tax policies* We delve into the policy interventions that can be implemented via sales taxes. It is reasonable to anticipate that raising the sales tax rate would discourage consumers from purchasing excessive quantities, thus preventing excessive hoarding. Nonetheless, as detailed in Supplemental Appendix E.1.3.1, we find that the efficacy of a sales tax hike heavily relies on the timing of implementation. Specifically, any delay in its implementation would limit its effectiveness, as even a few days' delay encourages consumers to buy before the tax hike. Given the impracticality of an immediate tax hike at least as of this writing, we contend that a short-term sales tax hike is not a potent strategy to mitigate panic buying.[39]

Instead of an immediate tax hike, we propose a future tax *reduction*, which may spread out the timing of purchases by incentivizing consumers to wait until the tax rate is lowered. Concretely, we simulate the case where the government announces at $t = 0$ that it will implement a month-long sales-tax cut in 3 weeks as follows.

POLICY SIMULATION (Future Sales-Tax Reduction). The government announces at $t = 0$ that it will reduce sales-tax rate by τ percent for 4 weeks in 3 weeks. The after-tax price

[39] The growth of e-commerce retail sales has altered consumer shopping habits and supply chain efficacy, which could have contributed to panic buying during the COVID-19 pandemic (Nielsen Holdings PLC (2020)). The wider adoption of e-commerce may enable flexible modifications to the sales tax rate in the future.

TABLE 3. Future sales-tax reduction.

Sales-Tax Reduction (%)	Shortages (Weeks)			Welfare (%)	
τ	$R(t) < 0.5$	$R(t) < 0.33$	$R(t) < 0.15$	Gross (Ω_{tax})	Tax Revenue (GR)
Benchmark ($\tau = 0$)	3.72	2.12	0.31	5.05	0
8.0	3.85	2.09	0.30	5.08	−1.87
12.0	3.91	2.08	0.30	5.11	−2.83
16.0	3.98	2.07	0.29	5.16	−3.83
20.0	0.0	0.0	0.0	1.13	−9.64

Note: $R(t) < 0.5$, $R(t) < 0.33$, and $R(t) < 0.15$ indicate the duration (weeks) for which low product availability (less than 50%, 33%, and 15%, respectively) persists.

is given by[40]

$$\hat{p}(t) = \begin{cases} \left(1 - \dfrac{\tau}{100}\right) \cdot p & \text{if } t \in [3, 7]; \\ p & \text{otherwise.} \end{cases}$$

Let $GR = \int_0^\infty \mathrm{e}^{-rt}[\hat{p}(t) - p(t)]R(t)\,d(t)\,dt$ denote the present value of government revenues, evaluated at $t = 0$. Taking into account the decreased government revenues due to the tax reduction, the overall social welfare cost of the shopping-cost shock is evaluated by

$$\Omega_{\text{tax}} = \frac{\left[\omega\left(V(0, k), G(0, k); \widehat{V}_o(k), \widehat{G}_o(k)\right) - rGR\right] - \omega\left(V_o(k), G(0, k); \widehat{V}_o(k), \widehat{G}_o(k)\right)}{\omega\left(V_o(k), G(0, k); \widehat{V}_o(k), \widehat{G}_o(k)\right)}.$$

Table 3 presents the relationship between the percentage of the sales-tax reduction, the duration of shortages, and social welfare. It shows that announcing future sales-tax reductions is successful in mitigating shortages and improving social welfare, *provided that the tax reduction is substantial enough*. Our calibration suggests that the government must decrease the sales tax rate by a minimum of approximately 20%. Failure to do so would not resolve shortages, and the government would only incur a loss in tax revenues, resulting in an increase in the overall social welfare cost Ω_{tax}.

5.6.3 *Nonmarket distribution* Governmental rationing of basic necessities was implemented during the COVID-19 pandemic in some countries.[41] In Supplemental Appendix E.1.3.2, we explore policy simulations wherein the government purchases the product from the market at the market price and distributes it to consumers.[42] Our simulations show that the policy is effective even if the government cannot distribute the

[40] In this simulation, the market price is fixed at $p(t) = p$ for all times.

[41] In Taiwan, the government distributed face masks by allowing each resident to purchase two masks in a week in February 2020 (see https://www.nhi.gov.tw/english/Content_List.aspx?n=022B9D97EF66C076 for the detail of the rationing system). In Japan, government-sponsored face masks were mailed to each household in April 2020.

[42] We assume that the government cannot target specific consumers in urgent need. This limitation arises due to the unavailability of information on individual consumers' stock levels (k) or their behavior, such as whether they are searching or not.

product to the entire population. In other words, by accommodating only a portion of the population through nonmarket distribution, the government can improve the welfare of the entire population. This is because the nonmarket distribution discourages rationed consumers from rushing to the market, mitigating the market congestion. Consequently, all consumers, including those who were not rationed, can easily purchase products. Additionally, we demonstrate that even with a lag between government purchase and distribution, nonmarket distribution could be effective during times of disaster.[43]

6. Other sources of panic buying

This section examines other shocks that may cause panic buying. Although we have concentrated on shocks that increase shopping costs as the primary cause of panic buying even when there are no resource shortages, the rise in shopping costs is not necessarily the *sole* reason for panic buying. In practice, we have also observed instances of panic buying that were most likely caused by a lack of resources, such as the scarcity of face masks during the initial wave of the COVID-19 outbreak.

In Supplemental Appendix E.2, we examine the impact of consumption shocks on panic buying. Unlike shopping-cost shocks, which increase shopping costs, a consumption shock results from an exogenous shift in consumer preferences that leads to a temporary increase in the instantaneous consumption rate. Our simulations show that a consumption shock can also trigger panic buying, regardless of whether it is anticipated or not, because it creates a shortage of resources, leading consumers to compete for scarce products. In other words, awareness of the shock's timing is less critical in driving panic buying in the case of consumption shocks.

Likewise, our model framework can analyze panic buying driven by various shocks and examine how the characteristics of panic buying differ depending on its sources. Although we cannot list all possible scenarios due to space limitations, policymakers can use our approach to evaluate and prepare for the impact of a wide range of disasters.

7. Robustness

Finally, we present sensitivity analyses and extensions. In Appendix B, we explore the robustness of our quantification, comparing simulation behavior at different parameterization schemes. Our results, presented in Table B.1, show that the model dynamics for shopping-cost shocks and the magnitude of the market-congestion externality's amplification effect remain mostly unchanged regardless of the values of θ (exit rate) and $\bar{b}$ (per unit storage cost), as long as the three parameters, a, α, and c, are calibrated to match our calibration targets.

In Supplemental Appendix E.3.1, we investigate how the magnitude of the shopping-cost increase affects the model's results. Our analysis reveals a nonlinear S-shaped relationship between the size of the shopping-cost increase ($\bar{c}$) and the resulting welfare cost

[43] Despite criticisms regarding unequal distribution and slow delivery of government-sponsored face masks in Japan (Eguchi, Kamizawa, and Okazaki (2020)), our findings suggest that this policy may have played a role in mitigating panic buying.

(Ω). Specifically, we observe a significant increase in welfare costs when the shopping-cost increase surpasses a certain threshold. Our model's insights corroborate empirical observations that panic buying is infrequent, yet once it occurs, it quickly escalate into a critical situation. Consequently, policymakers should consider implementing policies to prevent panic buying in disaster situations, especially if they involve prolonged periods of restricted mobility, as shopping costs can exceed the threshold.

In Supplemental Appendix E.3.2, we investigate the effects of price changes in contrast to the simulations with a fixed price. In particular, we allow the market price to increase at a constant rate in response to an increase in demand. Our findings demonstrate that inflation exacerbates the impact of shopping-cost shocks, as expectations of future price rises encourage consumers to buy storable products early and stockpile even more. This is consistent with the occurrence of inflation-driven shortages during the 2008 global rice crisis (Hansman et al. (2020)).

In Supplemental Appendix E.3.3, we extend our model by incorporating heterogeneity in the degree of product market friction faced by consumers. Although we target households that, on average, maintain 2 weeks' worth of stock at the time of purchase, we acknowledge the significant diversity in purchasing behaviors among households. Notably, Kano (2018) documents that many households make purchases with as little as a week's inventory, suggesting that they have the habit of buying storable and nonstorable products simultaneously.[44] To examine how the presence of these frequent shoppers affects the aggregate dynamics, we incorporate heterogeneous households into our model.[45] Our simulation results indicate that these consumers maintain lower levels of private inventory during normal times and are more susceptible to stock depletion when goods become scarce, which prompts increased hoarding behavior.

8. Concluding remarks

This paper has theoretically studied the panic buying of storable consumer products, which has repeatedly occurred in times of disaster. We developed a dynamic consumer search model of the market for storable daily necessities and numerical methods for the model simulations to demonstrate how panic buying initiates, spreads, and reaches its peak, as well as how it negatively impacts consumers. By using our model, we provided a plausible explanation for the worldwide scarcity of consumer products observed during the COVID-19 pandemic, along with some policy recommendations. We highlight the following results.

(i) Panic buying can arise even when all consumers are fully rational, there is no misinformation, and the disaster does not impact the consumption or production of the products. When shopping costs temporarily increases, the demand for hoarding is amplified, resulting in excessive congestion in the retail market, which in turn, spurs additional defensive hoarding.

[44] See her Figure 2a.

[45] In this model extension, we do not integrate multiple goods, such as storable and nonstorable goods, into our model because very large changes are necessary to fully incorporate them into the model. In particular, it is challenging to deal with the optimal stopping-time problem in a multivariate setting. We leave the formal analysis of the multiple-good model to future research.

(ii) The presence of a market-congestion externality exacerbates shortages, leading to an inefficient allocation of storable products among consumers, with some hoarding excessive amounts and others going without.

(iii) The severity of panic buying is heavily influenced by the timing of recognizing the shopping-cost increase. If the increase is predicted at the last minute, there is a high risk of severe panic buying. Therefore, it is essential for governments to make timely policy announcements and implement purchasing regulations that prevent consumers from concentrating their purchases over a short period of time.

Our framework possesses broad applicability across various scenarios and is amenable to policy analysis. By adjusting the parameters that determine the market structure and the characteristics of the underlying shocks, we can investigate the effects of different types of shocks, such as natural disasters, wars, and terms-of-trade shocks, on markets for various consumer products, including food, fuel, hygiene products, and medicine.

Appendix A: Deterministic shopping-search model

We provide the expressions for $\widehat{V}_o(k)$, which represents the consumers' value function in the stationary equilibrium of the deterministic shopping-search model. In this model, a shopping search takes a fixed duration of $1/\alpha$ weeks, meaning that each consumer is guaranteed to make a purchase after $1/\alpha$ weeks of initiating their shopping process.

We begin by demonstrating that every consumer starts a shopping search at $\hat{k}_o^* = 1/\alpha$. First, it is obvious that consumers with $k \in (1/\alpha, \infty)$ do not engage in shopping searches. If they were to do so, there would exist products that are purchased and stored at a cost but remain unconsumed. The problem faced by a consumer with $k = 1/\alpha$ is as follows:

$$\sup_{\tau \in [0,k)} \left[-\bar{b} \int_0^k \mathrm{e}^{-rs}(k-s)\, ds - a \int_k^{\frac{1}{\alpha}+\tau} \mathrm{e}^{-rs}\, ds + \mathrm{e}^{-r\tau} \left(\mathrm{e}^{-\frac{r}{\alpha}} \widehat{V}^A(0) - \underbrace{\frac{c}{r}\left(1 - \mathrm{e}^{-\frac{r}{\alpha}}\right)}_{\to\, c/\alpha \text{ as } r \to 0} \right) \right],$$

where τ is the time she start a shopping search and $\widehat{V}^A(k)$ is the value right after making a purchase with stock k: $\widehat{V}^A(k) = \sup_{q \geq 0} \widehat{V}_o(k+q) - pq = \sup_{k' \geq k} \widehat{V}_o(k') - pk' + pk$. Note that, $\widehat{V}^A(k) \leq 0$ by construction of the consumer's problem. Thus, as long as $a > 0$, she chooses $\tau^* = 0$, so she starts a shopping search at $\hat{k}_o^* = 1/\alpha$ not to run out the inventory.

Hence, we can write the value function as follows:

$$\widehat{V}_o(k) = \begin{cases} -\bar{b} \int_0^k \mathrm{e}^{-rs}(k-s)\, ds - c \int_0^k \mathrm{e}^{-rs}\, ds + \mathrm{e}^{-rk} \widehat{V}^A(0) & \text{for } k \in [0, 1/\alpha], \\ -\bar{b} \int_0^{k-1/\alpha} \mathrm{e}^{-rs}(k-s)\, ds + \mathrm{e}^{-r(k-1/\alpha)} \widehat{V}_o(1/\alpha) & \text{for } k > 1/\alpha. \end{cases}$$

Using $\widehat{V}_o(0) = \widehat{V}^A(0)$, we have

$$\widehat{V}_o(k) = \begin{cases} -\frac{\bar{b}}{r}\left[k - \frac{1}{r}\left(1 - \mathrm{e}^{-rk}\right)\right] - \frac{c}{r}\left(1 - \mathrm{e}^{-rk}\right) + \mathrm{e}^{-rk}\widehat{V}_o(0) & \text{for } k \in [0, 1/\alpha], \\ -\frac{\bar{b}}{r}\left[k - \frac{1}{r}\left(1 - \mathrm{e}^{-rk}\right)\right] - \mathrm{e}^{-r(k-1/\alpha)}\left[\frac{c}{r}\left(1 - \mathrm{e}^{-\frac{r}{\alpha}}\right)\right] + \mathrm{e}^{-rk}\widehat{V}_o(0) & \text{for } k > 1/\alpha. \end{cases}$$

Thus, for $k > 1/\alpha$,

$$\widehat{V}_o'(k) = -\mathrm{e}^{-rk}\left(r\widehat{V}_o(0) - B\right) - \frac{\bar{b}}{r} \quad \text{with } B = r\mathrm{e}^{\frac{r}{\alpha}}\left[\frac{c}{r}\left(1 - \mathrm{e}^{-\frac{r}{\alpha}}\right)\right] + \frac{\bar{b}}{r} > 0,$$

which implies that $\widehat{V}_o'(k)$ is continuous and monotonically decreasing in k with

$$\lim_{k\to\infty} \widehat{V}_o'(k) = -\bar{b}/r < 0.$$

With the parametric assumption ensuring $\widehat{V}_o'(1/\alpha) > p$, $\widehat{V}_o(0) = \sup_{k\geq 0} \widehat{V}_o(k) - pk$ has an interior solution such that

$$\widehat{V}_o'(\hat{\hat{k}}) = -\mathrm{e}^{-r\hat{\hat{k}}}\left(r\widehat{V}_o(0) - B\right) - \frac{\bar{b}}{r} = p \quad \Longrightarrow \quad \hat{\hat{k}} = \frac{1}{r}\log\left(\frac{B - r\widehat{V}_o(0)}{p + \bar{b}/r}\right),$$

where

$$\widehat{V}_o(0) = \widehat{V}_o(\hat{\hat{k}}) - p\hat{\hat{k}}.$$

Appendix B: Sensitivity analysis

Table B.1 shows that the model dynamics for shopping-cost shocks and the magnitude of the market-congestion externality's amplification effect remain mostly unchanged regardless of the values of θ (exit rate) and $\bar{b}$ (per unit storage cost), as long as the three parameters, a, α, and c, are calibrated to match our calibration targets.

Table B.1. Sensitivity analysis.

Parameters					Shortages (Weeks)		Welfare (%)		
θ	a	$\bar{b}$	α	c	$R(t) < 0.5$	$R(t) < 0.33$	Gross (Ω)	Direct ($\Omega_{R=1}$)	Indirect ($\Omega_{R<1}$)
Benchmark									
0.04	1069.23	1.0	2.29	14.63	3.72	2.12	5.05	0.85	4.20
Exit rate θ									
0.03	1069.23	1.0	2.29	14.66	3.69	2.09	3.99	0.68	3.31
0.05	1076.92	1.0	2.29	14.76	3.69	2.09	6.00	1.02	4.98
Per unit storage cost $\bar{b}$									
0.04	553.51	0.5	2.29	7.44	3.68	2.08	5.03	0.86	4.18
0.04	2145.48	2.0	2.29	29.33	3.69	2.09	4.98	0.85	4.14

Note: The values for a, α and c are chosen to be $k_o^* - 1/\alpha = 2.0$, $\bar{k}_o - k_o^* = 4.0$, and $G_o(3/7)/G_o(k_o) = 0.01$. $R(t) < 0.5$ and $R(t) < 0.33$ indicate the duration (weeks) for which low product availability (less than 50% and 33%, respectively) persists. See Section 5.2 for the definitions of Ω, $\Omega_{R=1}$, and $\Omega_{R<1}$.

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Co-editor Morten Ravn handled this manuscript.

Manuscript received 27 March, 2022; final version accepted 6 January, 2024; available online 12 January, 2024.

The replication package for this paper is available at https://doi.org/10.5281/zenodo.10455236. The Journal checked the data and codes included in the package for their ability to reproduce the results in the paper and approved online appendices.