Supplement to "Unemployment risk, MPC heterogeneity, and business cycles"

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This article collects the supplemental contents to the main paper. Section A describes the agents' optimality conditions. Section B collects the detrended equilibrium conditions. Section C shows the approximation of the detrended equilibrium conditions. Section D describes data and provides additional estimation results. Section E presents additional simulation results. Section F provides additional impulse responses to the estimated shocks.

Appendix A: Agents' optimality conditions

A.1 *Household*

Patient households' optimization yields

$$C_{H,t}^{-\sigma} = \beta_H \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \frac{\zeta_{t+1}}{\zeta_t} (C_{H,t+1})^{-\sigma} \right].$$

Impatient households' optimization yields

$$C(a,e)^{-\sigma} \geq \beta \mathbb{E}_t \left[\frac{R_t}{\pi_{t+1}} \frac{\zeta_{t+1}}{\zeta_t} C\big(a'(a,e), e'\big)^{-\sigma} \bigg| e \right],$$

with equality if $a'(a,e) > 0$, where

$$C(a,e) = (1-\tau_t)\frac{W_t}{P_t} e + (1-\tau_t) b^u \frac{W_t}{P_t}(1-e) + \frac{R_{t-1}}{\Pi_t} a - a'(a,e)$$

is the optimal consumption for impatient household with liquid asset position a and income state e.

A.2 *Wholesale firms*

The first-order condition with respect to P_t^* is

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\xi_p)^s \left(\frac{1}{\prod_{k=1}^{s} \frac{R_{t+k-1}}{\pi_{t+k}}} \right) \frac{Y_{h,t+s}}{P_{t+s}\eta_{t+s}^p} \left[(1+\eta_{t+s}^p) MC_{t+s} - P_t^* \chi_{t,t+s} \right] = 0.$$

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Given the Calvo assumption, the aggregate price index evolves according to

$$P_t^{-\frac{1}{\eta_t^p}} = (1-\xi_p)\big(P_t^*\big)^{-\frac{1}{\eta_t^p}} + \xi_p\big(\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}P_{t-1}\big)^{-\frac{1}{\eta_t^p}}.$$

A.3 *Intermediate goods firms*

Combining the first-order conditions with respect to v_t and n_t yield

$$\frac{A_t\kappa}{\lambda_t} = \left(\frac{MC_t}{P_t}\right)(1-\alpha)A_t^{1-\alpha}\big(u_t^k K_{t-1}\big)^{\alpha} n_t^{-\alpha} - \frac{W_t}{P_t} + \mathbb{E}_t\left[\frac{1}{R_t/\pi_{t+1}}(1-\rho_{x,t+1})\frac{A_{t+1}\kappa}{\lambda_{t+1}}\right]. \quad \text{(A.1)}$$

The first-order conditions with respect to I_t, K_t, and $u_{k,t}$ are

$$1 = q_t v_t\left[1 - \frac{s''}{2}\left(\frac{I_t}{I_{t-1}}-\mu\right)^2 - s''\frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}}-\mu\right)\right]$$
$$+ s''\mathbb{E}_t\left[\frac{1}{R_t/\pi_{t+1}}q_{t+1}v_{t+1}\frac{I_{t+1}^2}{I_t^2}\left(\frac{I_{t+1}}{I_t}-\mu\right)\right],$$
$$q_t = \mathbb{E}_t\frac{1}{R_t/\pi_{t+1}}\big[(1-\delta)q_{t+1} + r_{t+1}^k u_{k,t+1} - \Psi(u_{k,t+1})\big],$$
$$\Psi'(u_{k,t}) = r_t^k,$$

where $r_t^k \equiv \alpha(\frac{MC_t}{P_t})A_t^{1-\alpha}(u_{k,t}K_{t-1})^{\alpha-1}n_t^{1-\alpha}$.

A.4 *Market clearing and output*

The market clearing condition for liquid assets and the final goods market is

$$(1-\Omega)\int a'(a,e)\,\mathrm{d}\Gamma_t(a,e) + \Omega a_{H,t+1} = B_{t+1}^g, \quad \text{(A.2)}$$

$$(1-\Omega)\int C(a,e)\,\mathrm{d}\Gamma_t(a,e) + \Omega C_{H,t} + I_t + G_t = Y_t - A_t\kappa\mathrm{v}_t - \Psi(u_{k,t})K_{t-1}. \quad \text{(A.3)}$$

Using the demand for wholesale goods (2.4) and intermediate goods firms' technology (2.6), it can be shown that output is

$$\Delta_t^p Y_t = (u_{k,t}K_{t-1})^{\alpha}(A_t n_t)^{1-\alpha} - A_t F,$$

where $\Delta_t^p \equiv \int_0^1 (\frac{P_{h,t}}{P_t})^{-\frac{1+\eta_t^p}{\eta_t^p}}\,\mathrm{d}h$ is a measure of price dispersion across wholesale firms.

Appendix B: Full set of equilibrium conditions

To solve the model, it is necessary to detrend variables that feature a unit root. Let the following variables denote detrended variables:

$$y_t = \frac{Y_t}{A_t}, \qquad x_t = \frac{X_t}{A_t}, \qquad c_{H,t} = \frac{C_{H,t}}{A_t}, \qquad \mu_t = \frac{A_t}{A_{t-1}}, \qquad w_t = \frac{W_t}{P_tA_t}, \qquad p_t^* = \frac{P_t^*}{P_t},$$

$$mc_t = \frac{MC_t}{P_t}, \qquad d_t = \frac{D_t}{P_t A_t}, \qquad a^* = \frac{a}{A_{t-1}}, \qquad i_t = \frac{I_t}{A_t}, \qquad k_t = \frac{K_t}{A_t}.$$

The full set of equilibrium conditions, expressed in terms of detrended variables, is as follows.

Patient households:

$$c_{H,t}^{-\sigma} = \beta_H \mathbb{E}_t\left[\frac{R_t}{\pi_{t+1}} \frac{\zeta_{t+1}}{\zeta_t} (\mu_{t+1} c_{H,t+1})^{-\sigma}\right]. \tag{B.1}$$

Impatient households:

$$c\big(a^*, e\big)^{-\sigma} \geq \beta_L \mathbb{E}_t\left[\frac{R_t}{\pi_{t+1}} \frac{\zeta_{t+1}}{\zeta_t} \big(\mu_{t+1} c\big(a^{*\prime}\big(a^*, e\big), e'\big)\big)^{-\sigma} \Big| e\right], \tag{B.2}$$

with equality if $a^{*\prime}(a^*, e) > 0$, where

$$c\big(a^*, e\big) = (1-\tau) w_t e + (1-\tau) b^u w_t (1-e) + \frac{R_{t-1}}{\Pi_t} \frac{a^*}{\mu_t} - a^{*\prime}\big(a^*, e\big).$$

Dividends:

$$d_t = y_t - w_t n_t - \kappa \mathrm{v}_t - i_t - \Psi(u_{k,t}) \frac{k_{t-1}}{\mu_t}. \tag{B.3}$$

Production function:

$$\Delta_t^p y_t = (u_{k,t} k_{t-1})^{\alpha} n_t^{1-\alpha} - F. \tag{B.4}$$

Price setting of wholesale firms:

$$p_t^* = \frac{h_{1,t}^p}{h_{2,t}^p}, \tag{B.5}$$

$$h_{1,t}^p = \big(1+\eta_t^p\big) mc_t + \xi_p \mu_{t+1} \mathbb{E}_t\left(\frac{1}{R_t/\pi_{t+1}}\right) h_{1,t+1}^p, \tag{B.6}$$

$$h_{2,t}^p = 1 + \xi_p \mu_{t+1} \mathbb{E}_t\left(\frac{1}{R_t/\pi_{t+1}}\right) \frac{\pi_t^{\iota_p} \pi^{1-\iota_p}}{\pi_{t+1}} h_{2,t+1}^p, \tag{B.7}$$

$$1 = (1-\xi_p)\big(p_t^*\big)^{-\frac{1}{\eta_t^p}} + \xi_p \left(\frac{\pi_{t-1}^{\iota_p} \pi^{1-\iota_p}}{\pi_t}\right)^{-\frac{1}{\eta_t^p}}, \tag{B.8}$$

$$\Delta_t^p = (1-\xi_p)\big(p_t^*\big)^{-\frac{1+\eta_t^p}{\eta_t^p}} + \xi_p \left(\frac{\pi_{t-1}^{\iota_p} \pi^{1-\iota_p}}{\pi_t}\right)^{-\frac{1+\eta_t^p}{\eta_t^p}} \Delta_{t-1}^p. \tag{B.9}$$

Intermediate goods firms:

$$\frac{\kappa}{\lambda_t} = mc_t (1-\alpha) \left(\frac{u_t^k k_{t-1}}{n_t}\right)^{\alpha} - w_t + \mathbb{E}_t\left[\frac{\mu_{t+1}}{R_t/\pi_{t+1}} (1-\rho_{x,t+1}) \frac{\kappa}{\lambda_{t+1}}\right], \tag{B.10}$$

$$1 = q_t v_t \left[1 - \frac{s''}{2}\left(\frac{i_t \mu_t}{i_{t-1}} - \mu\right)^2 - s'' \frac{i_t \mu_t}{i_{t-1}} \left(\frac{i_t \mu_t}{i_{t-1}} - \mu\right)\right]$$

$$+ s''\mathbb{E}_t \frac{1}{R_t/\pi_{t+1}} q_{t+1} v_{t+1} \left(\frac{i_{t+1}\mu_{t+1}}{i_t}\right)^2 \left(\frac{i_{t+1}\mu_{t+1}}{i_t} - \mu\right), \tag{B.11}$$

$$q_t = \mathbb{E}_t \frac{1}{R_t/\pi_{t+1}} \big[(1-\delta)q_{t+1} + r^k_{t+1} u_{k,t+1} - \Psi(u_{k,t+1})\big], \tag{B.12}$$

$$\Psi'(u_{k,t}) = r^k_t, \tag{B.13}$$

$$r^k_t = \alpha mc_t \left(\frac{u_{k,t} k_{t-1}}{n_t}\right)^{\alpha-1}, \tag{B.14}$$

$$k_t = v_t i_t \left[1 - \frac{s''}{2}\left(\frac{i_t \mu_t}{i_{t-1}} - \mu\right)^2\right] + (1-\delta)\frac{k_{t-1}}{\mu_t}. \tag{B.15}$$

Wage setting:

$$w_t = \left(\frac{w_{t-1}}{\pi_t \mu_t}\right)^{\iota_w} \left(w\left(\frac{n_t}{n}\right)^{\xi_w}\right)^{1-\iota_w}. \tag{B.16}$$

Labor market flows:

$$n_t = (1-\rho_{x,t}) n_{t-1} + \psi \mathrm{v}_t \left(\frac{\mathrm{v}_t}{\widetilde{u}_t}\right)^{-\gamma}, \tag{B.17}$$

$$\rho_{x,t} = \frac{1}{1 + \exp(\overline{\rho}_x - \widetilde{\rho}_{x,t})}, \tag{B.18}$$

$$\widetilde{u}_t = u_{t-1} + \rho_{x,t} n_{t-1}, \tag{B.19}$$

$$u_t = (1-f_t) u_{t-1} + \rho_{x,t}(1-f_t) n_{t-1}, \tag{B.20}$$

$$u_t = 1 - n_t, \tag{B.21}$$

$$\lambda_t = \overline{M}(\mathrm{v}_t/\widetilde{u}_t)^{-\gamma}. \tag{B.22}$$

Monetary policy:

$$\log\left(\frac{R_t}{R}\right) = \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\phi_\pi \log\left(\frac{\pi_t}{\pi}\right) + \phi_X \log\left(\frac{x_t}{x}\right)\right] + \epsilon^R_t. \tag{B.23}$$

Government budget constraint:

$$\overline{B}^g + \tau_t\big(w_t n_t + b^u w_t u_t\big) = b^u w_t u_t + \left(1 - \frac{1}{g_t}\right) y_t + \frac{R_{t-1}}{\pi_t}\frac{\overline{B}^g}{\mu_t}. \tag{B.24}$$

Market clearing:

$$c_t + i_t + \left(1 - \frac{1}{g_t}\right) y_t = y_t - \kappa \mathrm{v}_t - \Psi(u_{k,t})\frac{k_{t-1}}{\mu_t}. \tag{B.25}$$

Real GDP and aggregate consumption:

$$x_t = c_t + i_t + \left(1 - \frac{1}{g_t}\right) y_t, \tag{B.26}$$

$$c_t = (1-\Omega)\int c\big(a^*, e\big)\,\mathrm{d}\Gamma_t\big(a^*, e\big) + \Omega c_{H,t}. \tag{B.27}$$

Evolution of the distribution for all measurable set $\mathcal{A}$:

$$\Gamma_{t+1}(\mathcal{A}, e') = \sum_{\epsilon} \boldsymbol{\pi}_t(e'|e) \int \mathbf{1}\{a^{*\prime}(a^*, e) \in \mathcal{A}\} \Gamma_t(\mathrm{d}a^*, e), \tag{B.28}$$

where $\boldsymbol{\pi}_t(e'|e)$ is the period t transition probability from e to e'.

Aggregate shocks:

$$\log(1+\eta_t^p) = (1-\rho_{\eta^p})\log(1+\eta^p) + \rho_{\eta^p}\log(1+\eta_{t-1}^p) + \epsilon_t^{\eta^p}, \tag{B.29}$$

$$\log \mu_t = (1-\rho_\mu)\log \mu + \rho_\mu \log \mu_{t-1} + \epsilon_t^\mu, \tag{B.30}$$

$$\log v_t = \rho_v \log v_{t-1} + \epsilon_t^v, \tag{B.31}$$

$$\log \widetilde{A}_t = \rho_{\widetilde{A}} \log \widetilde{A}_{t-1} + \epsilon_t^{\widetilde{A}}, \tag{B.32}$$

$$\log \zeta_t = \rho_\zeta \log \zeta_{t-1} + \epsilon_t^\zeta, \tag{B.33}$$

$$\widetilde{\rho}_{x,t} = \rho_{\rho_x} \widetilde{\rho}_{x,t-1} + \epsilon_t^{\rho_x}, \tag{B.34}$$

$$\log g_t = (1-\rho_g)\log g + \rho_g \log g_{t-1} + \epsilon_t^g. \tag{B.35}$$

Appendix C: Approximated equilibrium

I describe the procedure of discretizing the infinite-dimensional representation of the equilibrium conditions. I follow the procedure presented in the user guide of Winberry (2018), which describes the discretization of the Krusell and Smith (1998) economy. The model in the present paper contains two infinite-dimensional objects: decision rules of impatient households and their distribution over liquid wealth.

C.1 *Discretizing the household's decision rules*

Define the conditional expectation function:

$$\chi_t(a^*, e) = \beta_L \mathbb{E}_t\left[\frac{R_t}{\pi_{t+1}} \frac{\zeta_{t+1}}{\zeta_t} \left(\mu_{t+1} c\left(a^{*\prime}(a^*, e), e'\right)\right)^{-\sigma} \middle| e\right].$$

I approximate the conditional expectation function using Chebyshev polynomials

$$\widehat{\chi}_t(a_j^*, e) = \exp\left\{\sum_{j=1}^{n_\chi} \theta_{ej,t} T_j\left(\boldsymbol{\xi}(a_j^*)\right)\right\},$$

where n_χ is the order of approximation and $T_j(\cdot)$ is the jth order Chebyshev polynomial. $\boldsymbol{\xi}(a_j^*) = 2\frac{a_j^* - \underline{a}^*}{\overline{a}^* - \underline{a}^*} - 1 \in [-1, 1]$ is a node on which the Chebyshev polynomials are defined, where $a_j^* \in [\underline{a}^*, \overline{a}^*]$. $\{\theta_{ej,t}\}_{j=1}^{n_\chi}$ are basis coefficients. To construct grids on asset, I create n_χ Chebyshev nodes on the interval $[-1, 1]$ and then use $\boldsymbol{\xi}(a_j^*)$ to obtain asset grids $\{a_j^*\}_{j=1}^{n_\chi}$. Given the approximation of the conditional expectation function, I approximate households' decision rules using collocation, which forces the households' optimality condition to hold exactly on the constructed asset grids. Formally, I solve for $\{\theta_{ej,t}\}_{j=1}^{n_\chi}$ from

$$\exp\left\{\sum_{j=1}^{n_\chi} \theta_{ej,t} T_j\big(\boldsymbol{\xi}(a_j^*)\big)\right\} = \beta_L \mathbb{E}_t\left[\frac{R_t}{\pi_{t+1}} \frac{\zeta_{t+1}}{\zeta_t}\big(\mu_{t+1}\widehat{c}_t\big(\widehat{a}_t^{*\prime}(a_j^*, e), e'\big)\big)^{-\sigma}\Big| e\right],$$

where

$$\widehat{a}_t^{*\prime}(a_j^*, e) = \max\left\{0, (1-\tau)w_t e + (1-\tau)b^u w_t(1-e) + \frac{R_{t-1}}{\Pi_t}\frac{a_j^*}{\mu_t} - \widehat{\chi}_t(a_j^*, e)^{-\frac{1}{\sigma}}\right\},$$

$$\widehat{c}_t(a_j^*, e) = (1-\tau)w_t e + (1-\tau)b^u w_t(1-e) + \frac{R_{t-1}}{\Pi_t}\frac{a_j^*}{\mu_t} - \widehat{a}_t^{*\prime}(a_j^*, e).$$

C.2 *Discretizing the distribution*

I approximate the distribution of households with a parametric function. I first describe the distribution away from the limit and then the mass at the borrowing limit.

Distribution away from the limit The distribution of households over assets $a^* > 0$ is approximated using the probability density function $g_{e,t}(a^*)$,

$$g_{e,t}(a^*) = g_{e,t}^0 \exp\left\{g_{e,t}^1(a^* - m_{e,t}^1) + \sum_{j=2}^{n_g} g_{e,t}^j\left[(a^* - m_{e,t}^1)^j - m_{e,t}^j\right]\right\},$$

where n_g denotes the degree of approximation and $\{m_{e,t}^j\}_{j=1}^{n_g}$ are the centralized moments of the distribution. Given the moments $\{m_{e,t}^j\}_{j=1}^{n_g}$, the coefficients $\{g_{e,t}^{i_m}\}_{i_m=0}^{n_g}$ are determined by the following moment conditions:

$$m_{e,t}^1 = \int a^* g_{e,t}(a^*)\,\mathrm{d}a^*, \qquad m_{e,t}^{i_m} = \int (a^* - m_{e,t}^1)^{i_m} g_{e,t}(a^*)\,\mathrm{d}a^*,$$

for $i_m = 2, \dots, n_g$, and $\int g_{e,t}(a^*)\,\mathrm{d}a^* = 1$. The evolution of moments is

$$\begin{aligned} m_{e,t+1}^1 = \frac{1}{\boldsymbol{\pi}_t(e)}\Bigg[&\sum_{e_{-1}} \boldsymbol{\pi}_{t-1}(e_{-1})(1 - \underline{m}_{e_{-1},t})\boldsymbol{\pi}_{t-1}(e|e_{-1}) \int a_t^{*\prime}(e_{-1}, a) g_{e_{-1},t}(a^*)\,\mathrm{d}a^* \\ &+ \sum_{e_{-1}} \boldsymbol{\pi}_{t-1}(e_{-1})\underline{m}_{e_{-1},t}\boldsymbol{\pi}_{t-1}(e|e_{-1}) a_t^{*\prime}(0, e_{-1})\Bigg], \end{aligned}$$

$$\begin{aligned} m_{e,t+1}^{i_m} = \frac{1}{\boldsymbol{\pi}_t(e)}\Bigg[&\sum_{e_{-1}} \boldsymbol{\pi}_{t-1}(e_{t-1})(1 - \underline{m}_{e_{-1},t})\boldsymbol{\pi}_{t-1}(e|e_{-1}) \\ &\times \int \left[a_t^{*\prime}(e_{-1}, a^*) - m_{e,t+1}^1\right]^{i_m} g_{e_{-1},t}(a^*)\,\mathrm{d}a^* \\ &+ \sum_{e_{-1}} \boldsymbol{\pi}_{t-1}(e_{-1})\underline{m}_{e_{-1},t}\boldsymbol{\pi}_{t-1}(e|e_{-1})\left[a_t^{*\prime}(0, e_{-1}) - m_{e_{-1},t}^1\right]^{i_m}\Bigg], \end{aligned}$$

for $i_m = 2, \dots, n_g$, where $\boldsymbol{\pi}_t(e)$ is the normalizing factor that makes the sum of weights equal to 1. I approximate the integrals using Gauss–Legendre quadrature, which gives nodes $\{a^*_\iota\}_{\iota=1}^{m_g}$ and weights $\{\omega_\iota\}_{\iota=1}^{m_g}$.

Mass at the limit Let $\underline{m}_{e,t}$ denote the fraction of households with employment status e at the borrowing constraint. The evolution of the mass at the borrowing limit is

$$\underline{m}_{e,t+1} = \frac{1}{\boldsymbol{\pi}_t(e)}\Bigg[\sum_{e_{-1}} \boldsymbol{\pi}_{t-1}(e_{-1})(1-\hat{m}_{e_{-1},t})\boldsymbol{\pi}_{t-1}(e|e_{-1})\mathbf{1}\{a^{*\prime}_t(a^*, e_{-1}) = 0\} g_{e_{-1},t}(a^*)\,\mathrm{d}a^*$$
$$+ \sum_{e_{-1}} \boldsymbol{\pi}_{t-1}(e_{-1})\hat{m}_{e_{-1},t}\boldsymbol{\pi}_{t-1}(e|e_{-1})\mathbf{1}\{a'_t(0, e_{-1}) = 0\}\Bigg].$$

C.3 *Definition of approximated equilibrium*

Given the discretized households' decision rules and wealth distribution, we are ready to define the approximated equilibrium of the model. The approximated equilibrium is a sequence of $\{\{\{\theta_{ej,t}\}_{j=1}^{n_\chi}\}_e, \{\{g^{i_m}_{e,t}\}_{i_m=1}^{n_g}\}_e, \{\boldsymbol{\pi}_t(e'|e)\}_{e,e'}, \{\boldsymbol{\pi}_t(e)\}_e, \{\{m^{i_m}_{e,t}\}_{i_m=1}^{n_g}\}_e, \{\underline{m}_{e,t}\}_e, c_t,$ $c_{H,t}, i_t, k_t, u_{k,t}, d_t, y_t, x_t, u_t, \widetilde{u}_t, n_t, \lambda_t, f_t, \mathrm{v}_t, \rho_{x,t}, R_t, \pi_t, p^*_t, h^p_{1,t}, h^p_{2,t}, \Delta^p_t, mc_t, r^k_t, q_t, w_t, \tau_t,$ $\eta^p_t, \mu_t, \upsilon_t, \widetilde{A}_t, \zeta_t, \widetilde{\rho}_{x,t}\}_{t=0}^\infty$ that satisfies

$$\exp\Bigg\{\sum_{j=1}^{n_\chi} \theta_{ej,t} T_j\big(\xi(a^*_j)\big)\Bigg\}$$
$$= \beta_L \mathbb{E}_t\Bigg[\frac{R_t}{\pi_{t+1}}\frac{\xi_{t+1}}{\xi_t}\sum_{e'} \boldsymbol{\pi}(e'|e)\big(\mu_{t+1}c_{t+1}\big(\hat{a}'_t(a^*_j, e), e'\big)\big)^{-\sigma}\Bigg], \tag{C.1}$$

$$m^1_{e,t} = \sum_{\iota=1}^{m_g} a^*_\iota g_{e,t}(a^*_\iota), \tag{C.2}$$

$$m^{i_m}_{e,t} = \sum_{\iota=1}^{m_g} (a^*_\iota - m^1_{e,t})^{i_m} g_{e,t}(a^*_\iota), \tag{C.3}$$

$$m^1_{e,t+1} = \frac{1}{\boldsymbol{\pi}_{t-1}(e)}\Bigg[\sum_{e_{-1}} (1-\underline{m}_{e_{-1},t})\boldsymbol{\pi}_{t-1}(e_{-1})\boldsymbol{\pi}_{t-1}(e|e_{-1})$$
$$\times \sum_{\iota=1}^{m_g} \omega_\iota a^{*\prime}_t(a^*_\iota, e_{-1}) g_{e_{-1},t}(a^*_\iota)$$
$$+ \sum_{e_{-1}} \underline{m}_{e_{-1},t}\boldsymbol{\pi}_{t-1}(e_{-1})\boldsymbol{\pi}_{t-1}(e|e_{-1})a^{*\prime}_t(0, e_{-1})\Bigg], \tag{C.4}$$

$$m^{i_m}_{e,t+1} = \frac{1}{\boldsymbol{\pi}_t(e)}\Bigg[\sum_{e_{-1}} (1-\underline{m}_{e_{-1},t})\boldsymbol{\pi}_{t-1}(e_{-1})\boldsymbol{\pi}_{t-1}(e|e_{-1})$$

$$\times \sum_{\iota=1}^{m_g} \omega_\iota \left[a_t^{*\prime}\left(a_\iota^*, e_{-1}\right) - m_{e,t+1}^1\right]^{i_m} g_{e_{-1},t}\left(a_\iota^*\right)$$

$$+ \sum_{e_{-1}} \underline{m}_{e_{-1},t} \boldsymbol{\pi}_{t-1}(e_{-1}) \boldsymbol{\pi}_t(e|e_{-1}) \left[a_t^{*\prime}(0, e_{-1}) - m_{e,t+1}^1\right]^{i_m} \Bigg], \tag{C.5}$$

$$\underline{m}_{e,t+1} = \frac{1}{\boldsymbol{\pi}_t(e)} \Bigg[\sum_{e_{-1}} (1 - \underline{m}_{e_{-1},t}) \boldsymbol{\pi}_{t-1}(e_{-1}) \boldsymbol{\pi}_{t-1}(e|e_{-1})$$

$$\times \sum_{\iota=1}^{m_g} \omega_\iota \mathbf{1}\left\{a_t^{*\prime}\left(a_\iota^*, e_{-1}\right) = 0\right\} g_{e_{-1},t}\left(a_\iota^*\right)$$

$$+ \sum_{e_{-1}} \underline{m}_{e_{-1},t} \boldsymbol{\pi}_{t-1}(e_{-1}) \boldsymbol{\pi}_{t-1}(e|e_{-1}) \mathbf{1}\left\{a_t^{*\prime}(0, e_{-1}) = 0\right\} \Bigg], \tag{C.6}$$

$$\boldsymbol{\pi}_{t-1}(0|0) = 1 - f_t, \tag{C.7}$$

$$\boldsymbol{\pi}_{t-1}(1|0) = f_t, \tag{C.8}$$

$$\boldsymbol{\pi}_{t-1}(0|1) = \rho_{x,t}(1 - f_t), \tag{C.9}$$

$$\boldsymbol{\pi}_{t-1}(1|1) = 1 - \rho_{x,t}(1 - f_t), \tag{C.10}$$

$$\boldsymbol{\pi}_t(0) = u_t, \tag{C.11}$$

$$\boldsymbol{\pi}_t(1) = n_t, \tag{C.12}$$

(B.1), (B.3)–(B.27), and (B.29)–(B.35). $\{a_j\}_{j=1}^{n_\chi}$ are Chebyshev nodes, and $\{a_\iota\}_{\iota=1}^{m_g}$ are quadrature nodes. I set $n_\chi = 25$, $n_g = 4$, and $m_g = 20$.

C.4 *Approximation quality*

In this subsection, I investigate whether the steady-state distribution and consumption policy functions using the Winberry method approximate those computed from the histogram-based method proposed by Reiter (2009) and Young (2010). I then investigate whether changing the degree of approximation in the Winberry method affects the responses of aggregate variables.

Figure C.1 plots the stationary distribution and consumption policy functions under the Winberry and histogram-based method. For both methods, I calibrate the replacement rate b^u and the impatient households' discount factor β_L to match the average quarterly MPC and employed–unemployed consumption difference described in the main text. The figure shows that the $n_g = 4$ degree approximation in the Winberry method captures the histogram-based distribution and consumption policy functions quite well.

Table C.1 reports the standard deviations of selected variables under different degrees of approximation n_g for the Winberry method. The table shows that the aggregate dynamics are largely unaffected by the degree of approximation as long as the same average MPCs and employed–unemployed consumption differences are targeted across

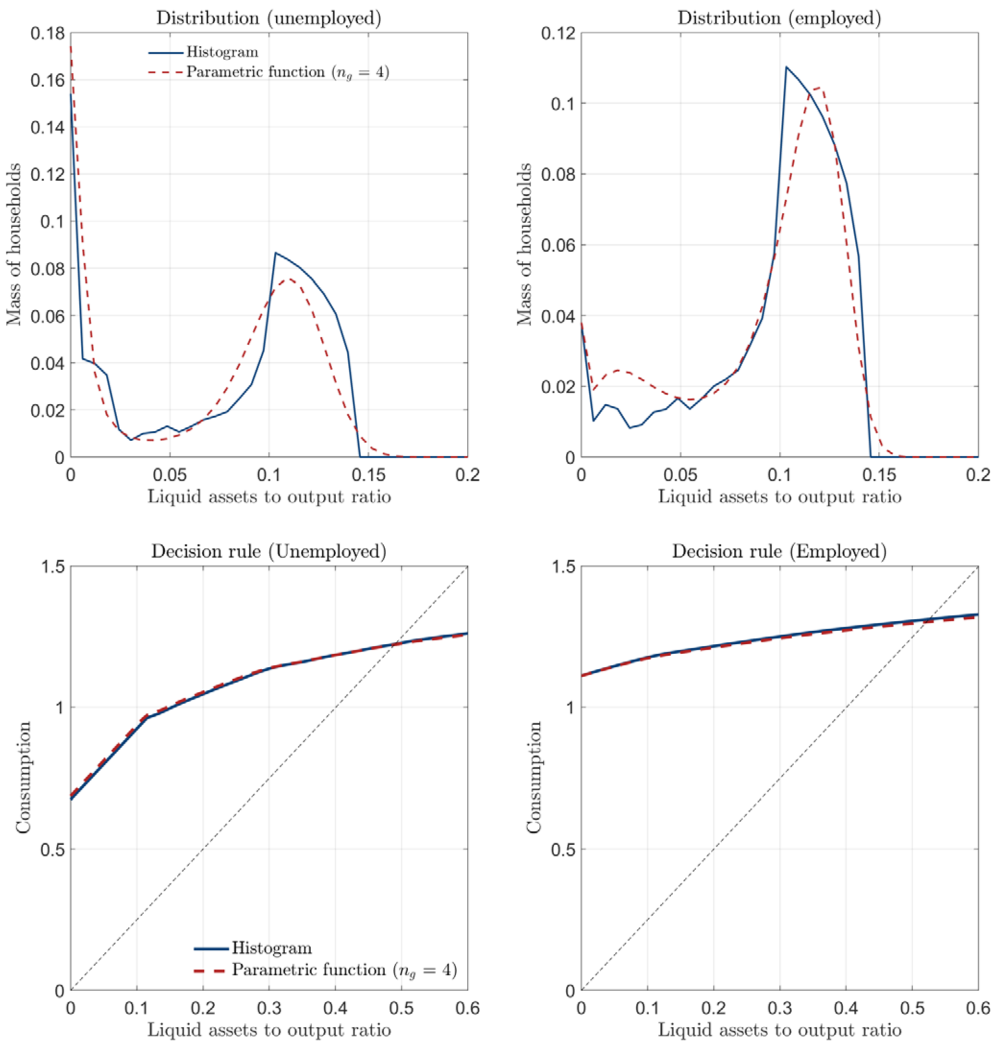


FIGURE C.1. Stationary wealth distribution and decision rules: histogram vs. parametric function. *Note*: n_g is the order of the polynomial used in the parametric function, which captures the distribution of impatient households over liquid wealth.

TABLE C.1. Aggregate volatility under different degrees of approximation.

Degree of Approximation	Standard Deviation				
	C Growth	I Growth	P Inflation	Job-Find. Rate	Job-Loss Rate
$n_g = 2$	1.409	1.803	0.721	5.787	1.191
$n_g = 3$	1.412	1.802	0.719	5.788	1.191
$n_g = 4$	1.415	1.792	0.721	5.802	1.195

Note: Standard deviations of selected observable variables from the estimated HANK model under different degrees of approximation for the Winberry method. $n_g = 4$ degree approximation is used in the main text.

different degrees of approximation. Winberry (2018) also finds that the aggregate dynamics barely change across different degrees of approximation in a lumpy investment model.

Appendix D: Data and further estimation results

D.1 *Data description*

Data for estimation The following series, from 1964Q2 to 2008Q3, are used for estimation. These are from the St. Louis Fed FRED database, and mnemonics are in parentheses. In order to construct real per-capita values, GDP deflator (GDPDEF) and civilian noninstitutional population over 16 (CNP16OV) are used.

- *Consumption growth*: The growth rate of real per-capita consumption, which is the sum of personal consumption expenditures on nondurables goods (PCND) and services (PCESV) divided by the GDP deflator and by population.
- *Investment growth*: The growth rate of real per-capita investment, which is the sum of gross private domestic investment (GPDI) and personal consumption expenditures for durable goods (PCDG) divided by the GDP deflator and by population.
- *Government purchase growth*: The growth rate of real per-capita government purchases, which are government consumption expenditures and gross investment (GCE) divided by the GDP deflator and by population.
- *Price inflation*: The growth rate of the GDP deflator (GDPDEF).
- *Wage inflation*: The growth rate of average hourly earnings of production and nonsupervisory employees (AHETPI).
- *Interest rate*: Quarterly average of the effective federal funds rate (FEDFUNDS).
- *Job-finding and job-loss rates*: Quarterly labor market transition probabilities constructed by Challe, Matheron, Ragot, and Rubio-Ramírez (2017). First, using deseasonalized monthly data on employment (BLS series LNS12000000), unemployment (LNS13000000), and short-run unemployment (LNS13008396), they construct monthly series for the unemployment and short-run unemployment rates. They then construct the monthly job-finding and job-loss rates following the approach of Shimer (2005). Finally, they construct monthly transition matrices across employment statuses and then multiply those matrices over the three consecutive months of each quarter to obtain quarterly transition probabilities.

Treatment prior to estimation For job-finding and job-loss rates, I remove a low-frequency trend as in Shimer (2005) and Challe et al. (2017). To remove a trend, I use a filtering method proposed by Hamilton (2018). According to his method, the trend

TABLE D.1. Volatility of labor market variables: HANK vs. data.

Model/Data	Standard Deviation			
	Job-Find. Rate	Job-Loss Rate	Unemployment Rate	Vacancies
HANK	5.80	1.19	1.58	22.81
Data	5.02	0.88	1.23	20.16

Note: See Appendix D.1 for the treatment of the data.

of time series is the fitted value from a regression of that series on its 4 lagged values, back-shifted by 2 years, and a constant. The remaining series are demeaned prior to estimation.

Additional data The following series are used in producing Table 6, Table D.1, and Figure D.1.

- *Government debt*: Market value of privately held federal debt from the Dallas Fed website. The quarterly series is constructed by averaging the monthly series of each quarter.
- *Unemployment rate*: Using deseasonalized monthly data on employment (BLS series LNS12000000) and unemployment (LNS13000000) level, the unemployment rate is computed as the unemployment level divided by the sum of employment and

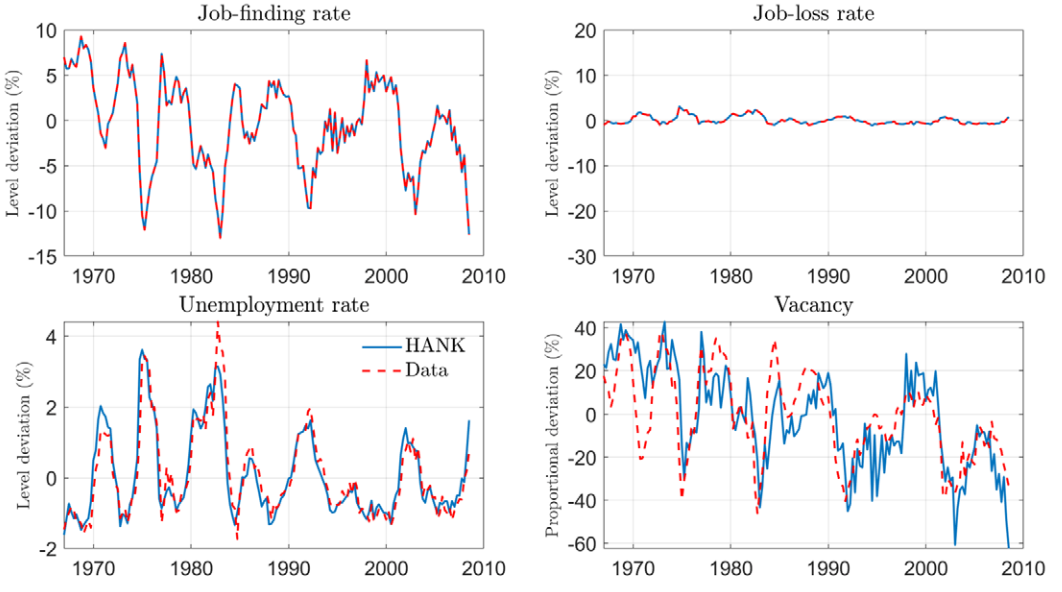


FIGURE D.1. Historical labor market variables: HANK vs. data. *Note*: For the employment, job-finding, and job-loss rates, the y-axis represents the level deviation from the mean (steady state). For vacancies, the y-axis represents the proportional deviation from the mean (steady state). See Appendix D.1 for the treatment of the data.

unemployment level. The quarterly series is constructed by averaging the monthly series of each quarter, and then the Hamilton (2018) filter is used to remove a trend.

- *Vacancies*: The monthly composite help-wanted index constructed by Barnichon (2010). The quarterly series is constructed by averaging the monthly series of each quarter. The log is taken, and then the Hamilton (2018) filter is used to remove a trend.

D.2 *Convergence diagnostics*

TABLE D.2. Geweke (1992) convergence diagnostics.

Parameter	Description	P-value	
		HANK	RANK
s''	Invest. adjustment cost	0.239	0.269
ψ	Capital utilization cost	0.653	0.542
ξ_p	Price stickiness	0.538	0.812
ι_p	Price indexation	0.072	0.608
ξ_w	Wage flexibility	0.664	0.486
ι_w	Wage indexation	0.906	0.278
ρ_R	Taylor rule: smoothing	0.967	0.617
ϕ_π	Taylor rule: inflation	0.595	0.344
ϕ_X	Taylor rule: GDP	0.825	0.582
ρ_{η_p}	Auto. price markup	0.499	0.741
ρ_μ	Auto. nonstat. tech.	0.315	0.448
ρ_υ	Auto. MEI	0.123	0.601
ρ_ζ	Auto. preference	0.436	0.258
ρ_{ρ_x}	Auto. job-separation	0.462	0.837
ρ_g	Auto. gov. purchase	0.162	0.291
$\rho_{\tilde{A}}$	Auto. stat. tech.	0.209	0.415
σ_{η_p}	Std price markup	0.350	0.869
σ_μ	Std nonstat. tech.	0.514	0.235
σ_υ	Std MEI	0.209	0.278
σ_R	Std mon. policy	0.296	0.434
σ_ζ	Std preference	0.331	0.202
σ_{ρ_x}	Std job-separation	0.546	0.983
σ_g	Std gov. purchase	0.674	0.485
$\sigma_{\tilde{A}}$	Std stat. tech.	0.409	0.053
σ_w	Std wage measurement	0.637	0.129

Note: P-values are computed based on the 15% Taper window of Newey–West standard errors. After the burn in, 280,000 draws are used to compute the test statistics. For all parameters, one cannot reject the equality of the means of the first 10% and the last 50% of a Markov chain at the 5% significance level.

Appendix E: Additional simulation

E.1 *Labor market variables*

As is well known from Shimer (2005), calibrated search and matching models with Nash bargaining generates volatile real wages. This feature causes the unemployment rates from these models to become less volatile compared to the data. Given that I do not use the data on the unemployment rate in estimation, one might wonder how well the estimated HANK model predicts the labor market data, including the unemployment rate.

Table D.1 reports the standard deviations of the selected labor market variables computed using the estimated HANK model and their empirical standard deviations. Notably, all labor market variables, including the unemployment rate, from the model are more volatile than the data, thus avoiding the Shimer (2005) puzzle. Several factors lead the estimated HANK model not to face the puzzle. First, precautionary motives and high MPCs, absent in Shimer (2005), cause consumption to respond more sensitively to aggregate shocks. Second, although the degree of nominal rigidity is estimated to be lower than in RANK, it is higher than in Shimer (2005). In this environment, precautionary motives and high MPCs interact with nominal rigidity, generating volatile output and unemployment rate through the Keynesian multiplier.

An alternative way to assess the performance of the estimated HANK model in predicting labor market variables is to use the sequences of shocks obtained by matching the model-based data to the empirical data over the sample period. One can feed these shocks back into the model and compare the model-based and empirical data. Because the job-finding and job-loss rates are used to estimate the model and construct shocks, the model-based job-finding and job-loss rates are exactly equal to their empirical counterpart, as shown in Figure D.1. As the unemployment rate and vacancies are not used in constructing shocks, the model-based unemployment rate and vacancies do not coincide with their empirical counterpart. However, the discrepancy between the model and the data appears small, implying that the estimated model predicts the unemployment rate and vacancies over the sample period quite well.

E.2 *Preference shocks*

Figure E.1 portrays the impulse responses to a positive preference shock in three models: HANK, RANK, and the constant risk model. In RANK, a positive preference shock makes all households more impatient, causing all households to consume more and save less. The resulting rise in the real interest rate depresses investment, leading to a decrease in GDP except for the first two periods. In HANK, the magnitude of consumption responses is much smaller than in RANK. This is because, for households close to the borrowing limit, there are almost no savings to use for consumption. Hence, their consumption is barely affected even if they become more impatient. Only the households that are far away from the borrowing limit respond sensitively to a preference shock, causing mild responses of aggregate consumption, investment, and GDP. The responses of all

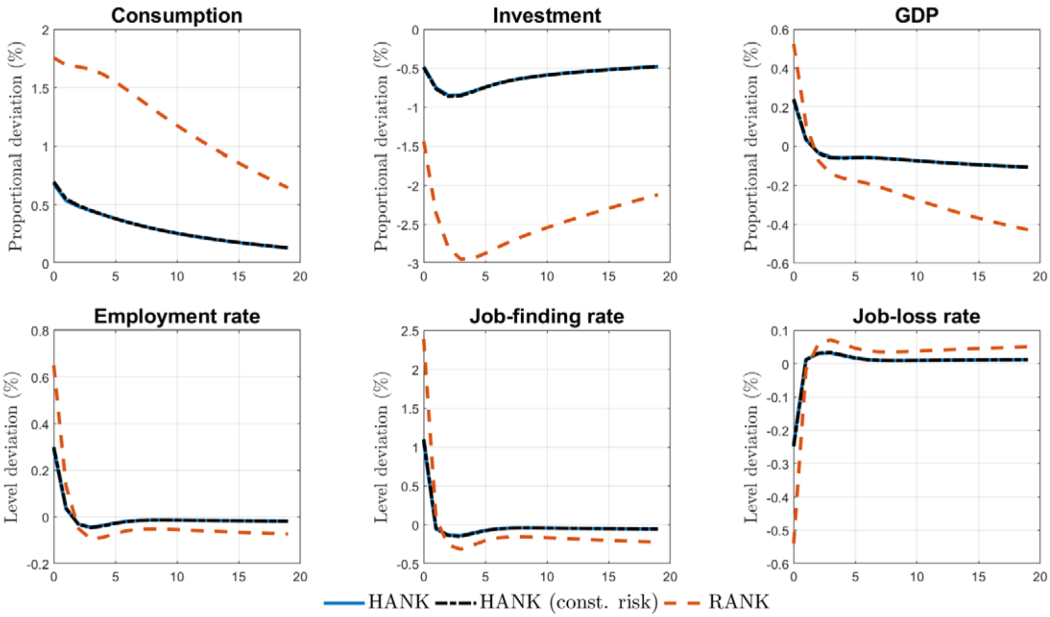


FIGURE E.1. IRFs to a positive preference shock. *Note*: The x-axis measures the time horizon in quarters. For the employment, job-finding, and job-loss rates, the y-axis represents the level deviation from a steady state. For the other variables, the y-axis represents the proportional deviation from a steady state.

variables in the constant risk benchmark appear to be very close to those in HANK, confirming that the effect of precautionary motives from unemployment risk in propagating preference shocks is minimal.

Small aggregate consumption responses in HANK conditional on preference shocks is the main reason for the consumption and GDP volatility in RANK to be substantially higher than in HANK. To confirm this visually, I simulate RANK by feeding all shocks recovered from HANK. As discussed in Section 5, these shocks are constructed by matching the observable variables predicted from HANK to the actual data over the sample period. Figure E.2 compares the historical path of selected variables in HANK, RANK, and the constant risk model subject to these shocks. Note that, in Figure E.2, consumption, the job-finding rate, and the job-loss rate predicted from HANK are exactly equal to the data by construction. GDP, which is not used in estimation and in constructing shocks, is produced from the model by using the definition of GDP, that is, $X_t \equiv C_t + I_t + G_t$. While the model-produced log-differences of GDP and consumption, that is, $\Delta \log X_t$ and $\Delta \log C_t$, are stationary, the model-produced log GDP and log consumption are not. Accordingly, in order to focus on cyclical movements, I present the detrended log GDP and log consumption, using the Hamilton (2018) filter, in the figure. Clearly, consumption and GDP in RANK are substantially more volatile than in HANK because of preference shocks. The difference between the GDP path in HANK and RANK in Figure E.2 is greater than in Figure 2, because preference shocks are excluded when producing the latter figure.

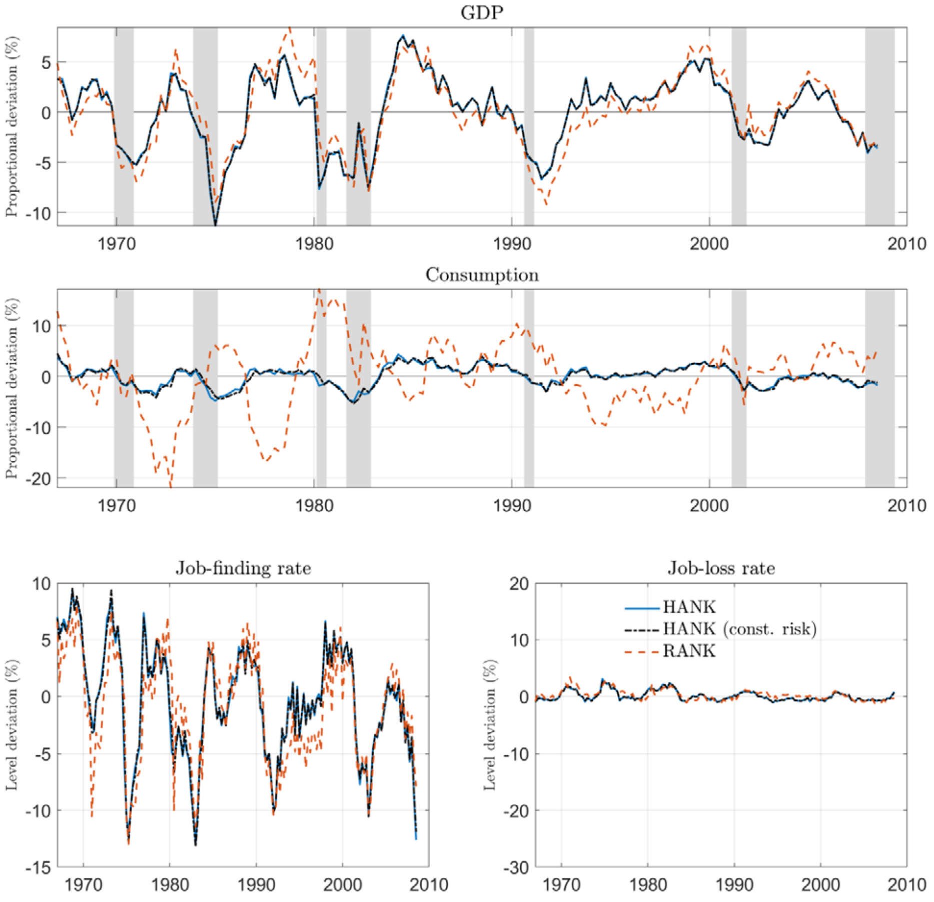
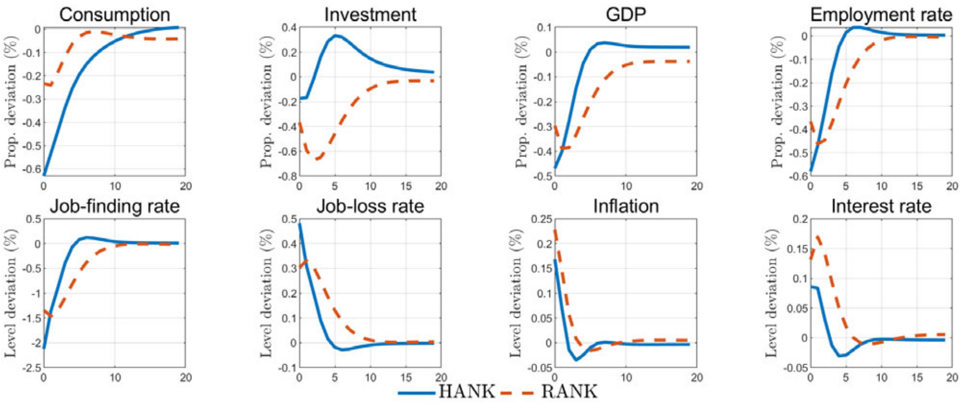
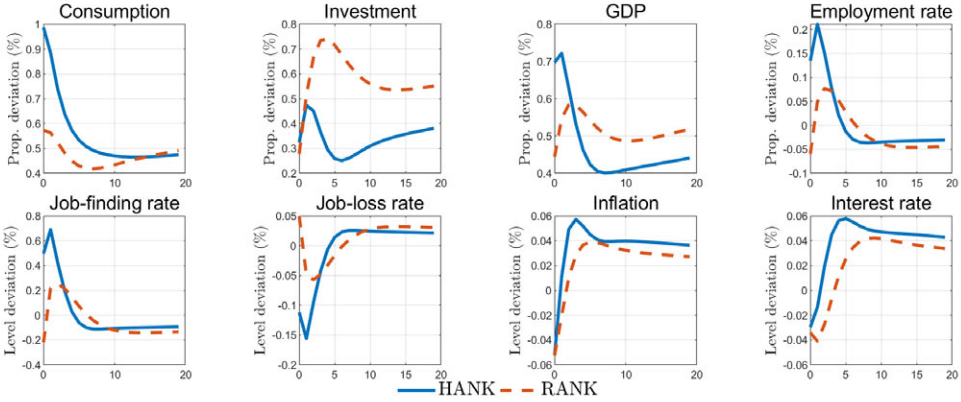


FIGURE E.2. Counterfactual dynamics in response to all shocks. *Note*: For GDP and consumption, the y-axis represents the proportional deviation from the trend by applying the Hamilton filter. For the job-finding and job-loss rates, the y-axis represents the level deviation from the mean (steady state).

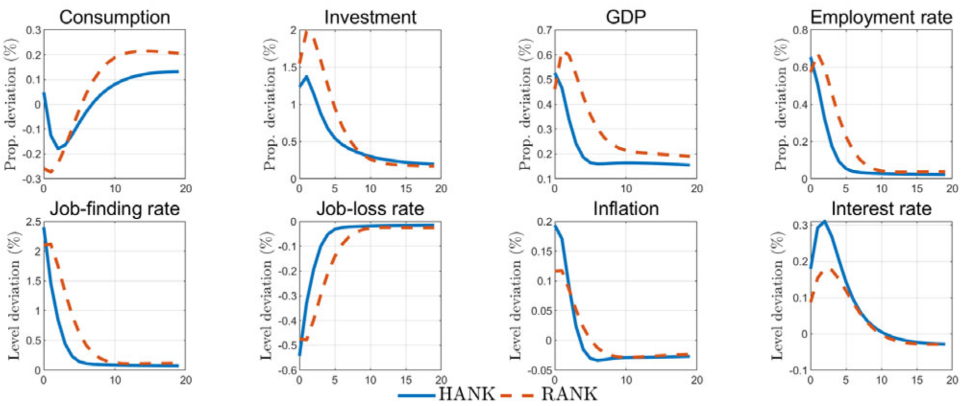
APPENDIX F: ADDITIONAL IMPULSE RESPONSES



(a) IRFs to a price markup shock

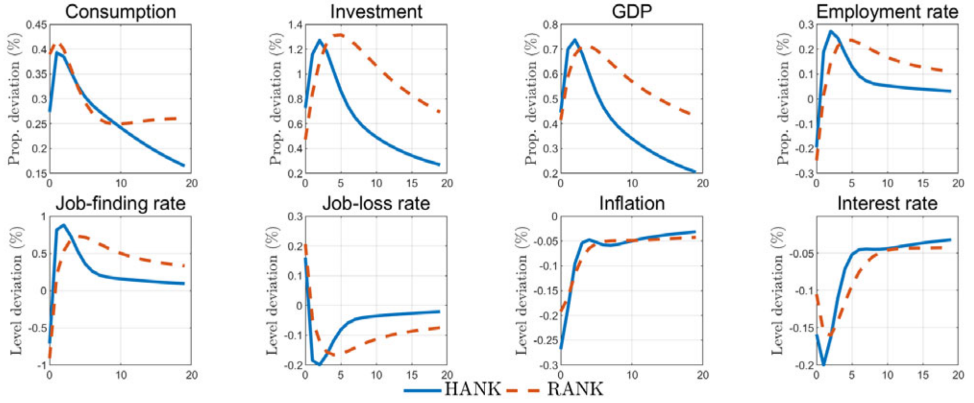


(b) IRFs to a nonstationary technology shock

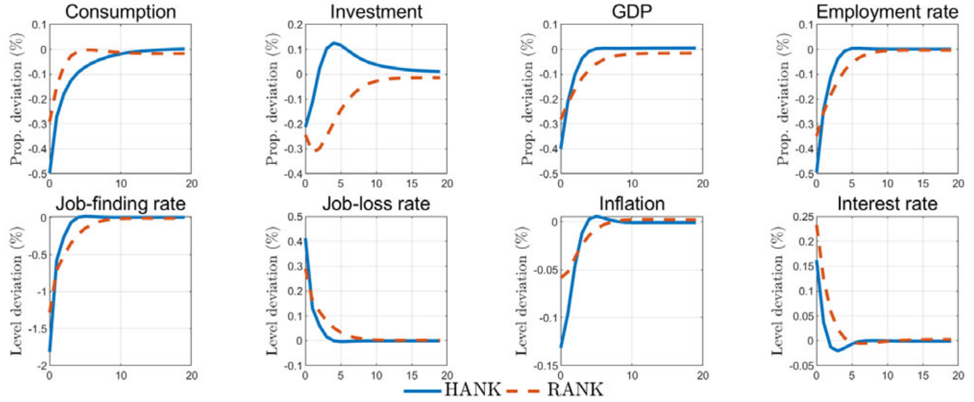


(c) IRFs to a marginal efficiency of investment shock

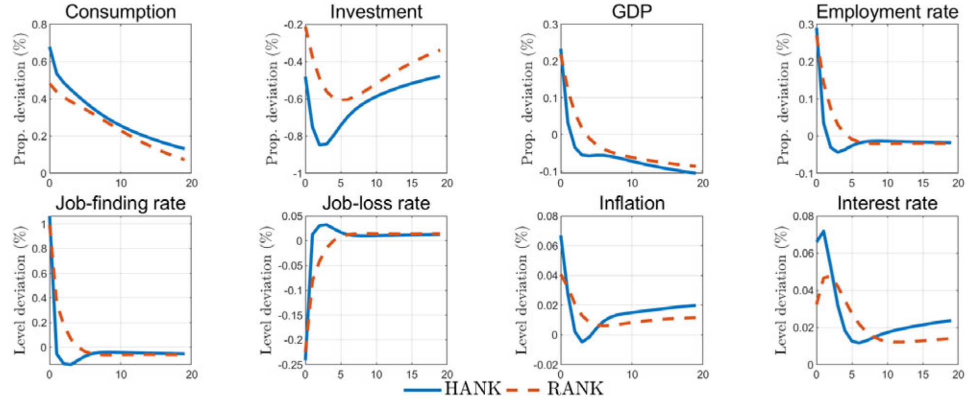
FIGURE F.1. IRFs to markup, nonstationary technology, and marginal efficiency of investment shocks.



(a) IRFs to a stationary technology shock

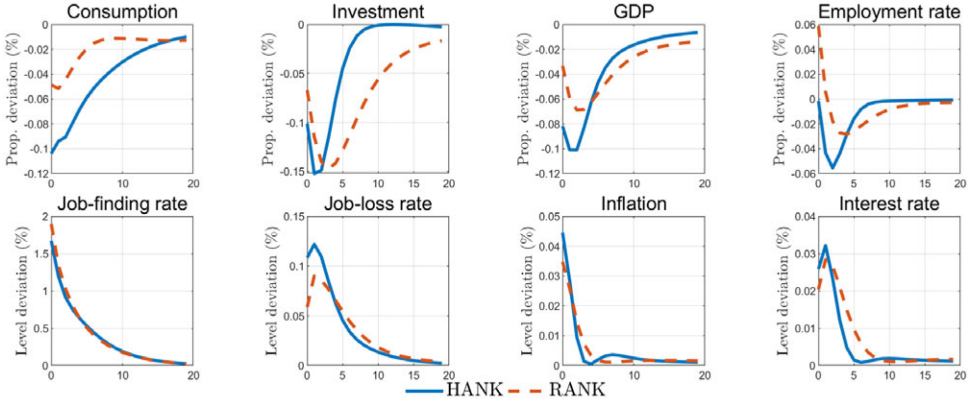


(b) IRFs to a monetary policy shock

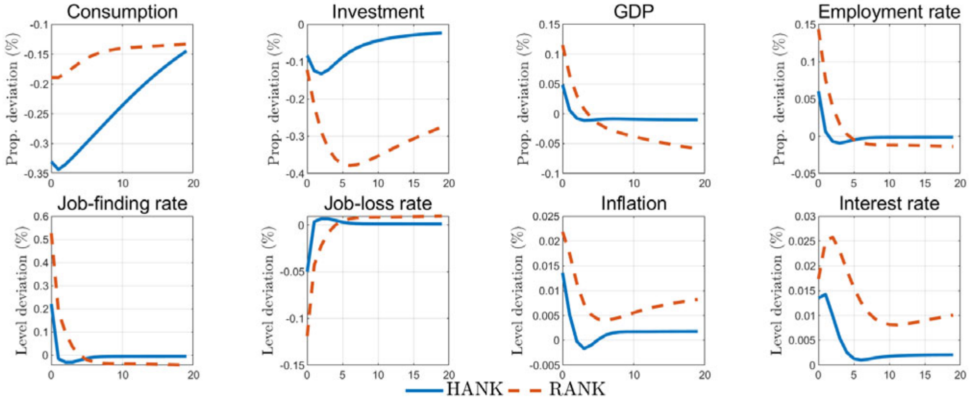


(c) IRFs to a preference shock

FIGURE F.2. IRFs to stationary technology, monetary policy, and preference shocks.



(a) IRFs to a job-separation shock



(b) IRFs to a government purchase shock

FIGURE F.3. IRFs to job-separation and government purchase shocks.

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