Replication of 'Identifying the Discount Factor in Dynamic Discrete Choice Models'

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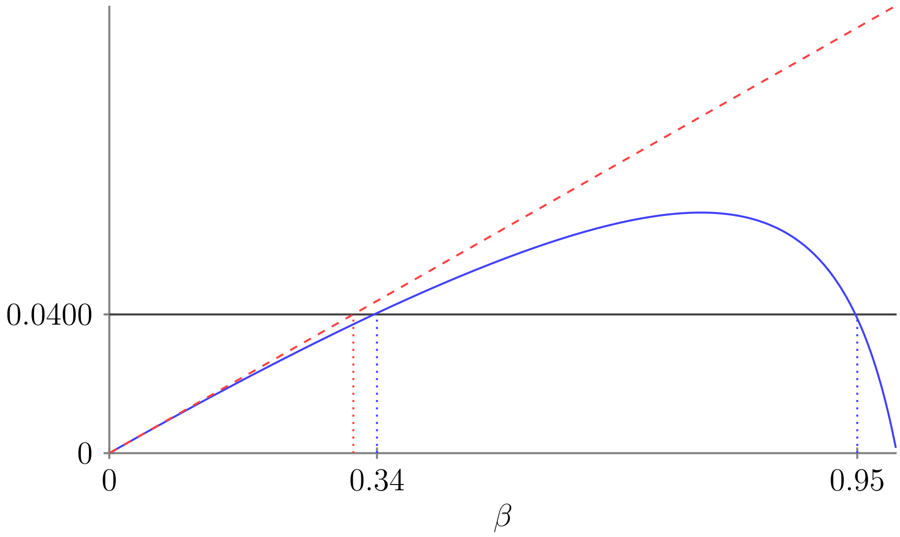
Figure 1: Example in Which the Rank Condition Holds, but Identification Fails

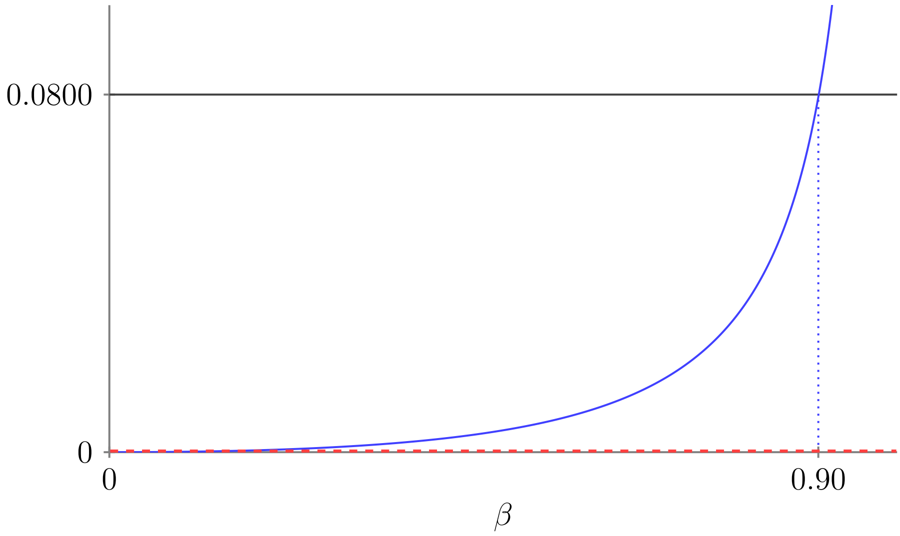


Note: For $J = 3$ states, $K = 2$ choices, $k = l = 1$, $\tilde{x}_1 = x_1$, and $\tilde{x}_2 = x_2$, this graph plots the left hand side of (6) and (12) (solid black horizontal line) and the right hand sides of (6) (dashed red line), and (12) (solid blue curve) as functions of β. The data are $\mathbf{Q}_1(x_1) = \begin{bmatrix} 0.25 & 0.25 & 0.50 \end{bmatrix}$, $\mathbf{Q}_1(x_2) = \begin{bmatrix} 0.00 & 0.25 & 0.75 \end{bmatrix}$,

$$\mathbf{Q}_K = \begin{bmatrix} 0.90 & 0.00 & 0.10 \\ 0.00 & 0.90 & 0.10 \\ 0.00 & 1.00 & 0.00 \end{bmatrix}, \mathbf{p}_1 = \begin{bmatrix} 0.50 \\ 0.49 \\ 0.10 \end{bmatrix}, \text{ and } \mathbf{p}_K = \begin{bmatrix} 0.50 \\ 0.51 \\ 0.90 \end{bmatrix}.$$

Consequently, the left hand side of (6) and (12) equals $\ln(p_1(x_1)/p_K(x_1)) - \ln(p_1(x_2)/p_K(x_2)) = 0.0400$. Moreover, $\mathbf{m}' = \begin{bmatrix} 0.69 & 0.67 & 0.11 \end{bmatrix}$ and $\mathbf{Q}_1(x_1) - \mathbf{Q}_K(x_1) - \mathbf{Q}_1(x_2) + \mathbf{Q}_K(x_2) = \begin{bmatrix} -0.65 & 0.90 & -0.25 \end{bmatrix}$, so that the slope of the dashed red line equals $[\mathbf{Q}_1(x_1) - \mathbf{Q}_K(x_1) - \mathbf{Q}_1(x_2) + \mathbf{Q}_K(x_2)]\, \mathbf{m} = 0.1291$. A unique value of β, 0.31, solves (6), but two values of β solve (12): 0.34 and 0.95.

Figure 2: Example in Which the Rank Condition Fails, but the Discount Factor is Identified

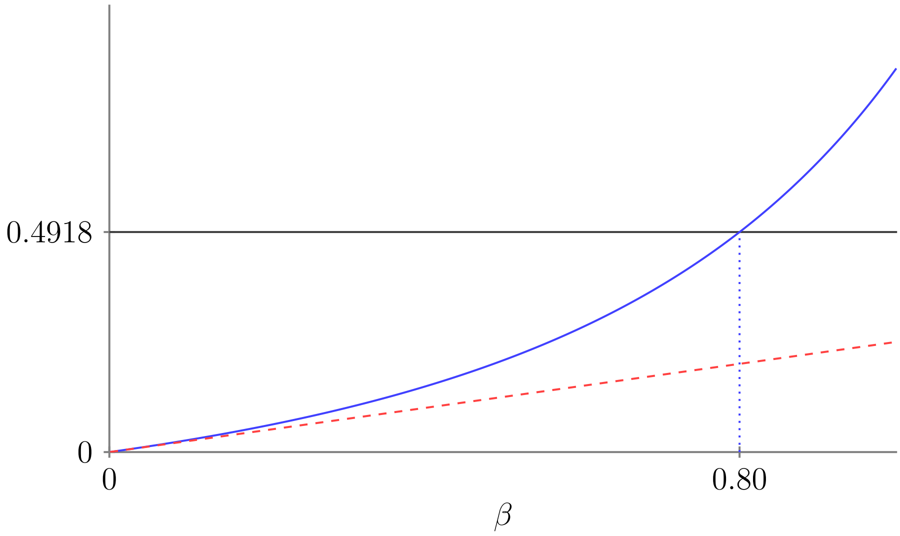


Note: For $J = 3$ states, $K = 2$ choices, $k = l = 1$, $\tilde{x}_1 = x_1$, and $\tilde{x}_2 = x_2$, this graph plots the left hand side of (6) and (12) (solid black horizontal line) and the right hand sides of (6) (dashed red line) and (12) (solid blue curve) as functions of β. The data are $\mathbf{Q}_1(x_1) = \begin{bmatrix} 0.00 & 0.25 & 0.75 \end{bmatrix}$, $\mathbf{Q}_1(x_2) = \begin{bmatrix} 0.25 & 0.25 & 0.50 \end{bmatrix}$,

$$\mathbf{Q}_K = \begin{bmatrix} 0.00 & 1.00 & 0.00 \\ 0.00 & 1.00 & 0.00 \\ 0.00 & 0.00 & 1.00 \end{bmatrix}, \mathbf{p}_1 = \begin{bmatrix} 0.50 \\ 0.48 \\ 0.50 \end{bmatrix}, \text{ and } \mathbf{p}_K = \begin{bmatrix} 0.50 \\ 0.52 \\ 0.50 \end{bmatrix}.$$

Consequently, the left hand side of (6) and (12) equals $\ln(p_1(x_1)/p_K(x_1)) - \ln(p_1(x_2)/p_K(x_2)) = 0.0800$. Moreover, $\mathbf{m}' = \begin{bmatrix} 0.69 & 0.65 & 0.69 \end{bmatrix}$ and $\mathbf{Q}_1(x_1) - \mathbf{Q}_K(x_1) - \mathbf{Q}_1(x_2) + \mathbf{Q}_K(x_2) = \begin{bmatrix} -0.25 & 0.00 & 0.25 \end{bmatrix}$, so that the slope of the dashed red line equals $[\mathbf{Q}_1(x_1) - \mathbf{Q}_K(x_1) - \mathbf{Q}_1(x_2) + \mathbf{Q}_K(x_2)]\,\mathbf{m} = 0.0000$. A unique value of β, 0.90, solves (12), but (6) has no solution.

Figure 3: Example of a Dynamic Labor Supply Model that Gives a Monotone Moment Condition

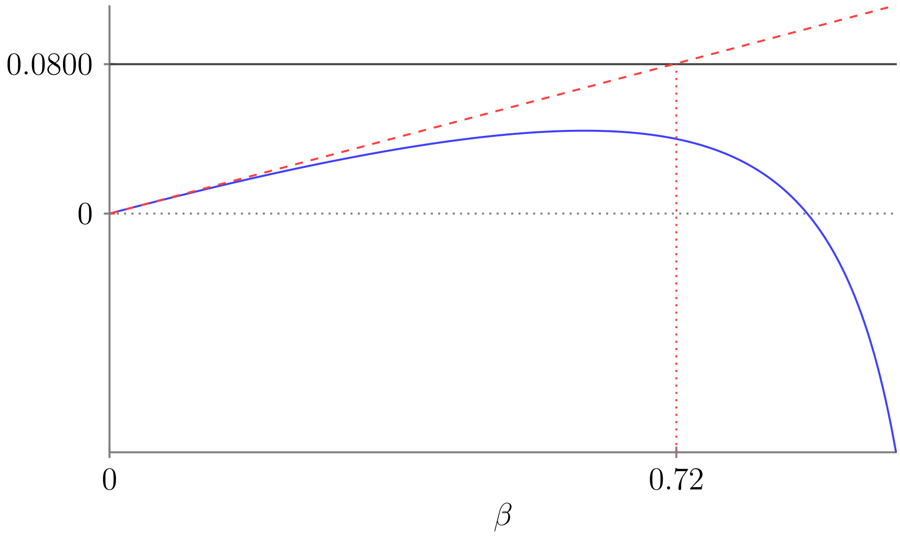


Note: For $J = 3$ states, $K = 2$ choices, $k = l = 1$, $\tilde{x}_1 = x_2$, and $\tilde{x}_2 = x_1$, this graph plots the left hand side of (6) and (12) (solid black horizontal line) and the right hand sides of (6) (dashed red line) and (12) (solid blue curve) as functions of β (we switched the roles of x_1 and x_2 to ensure a positive choice response and visually line up this example with the others). The data are generated from Example 7's stylized dynamic labor supply model, which gives $\mathbf{Q}_1(x_2) = \begin{bmatrix} 0.00 & 0.25 & 0.75 \end{bmatrix}$, $\mathbf{Q}_1(x_1) = \begin{bmatrix} 0.25 & 0.75 & 0.00 \end{bmatrix}$,

$$\mathbf{Q}_K = \begin{bmatrix} 1.00 & 0.00 & 0.00 \\ 0.50 & 0.50 & 0.00 \\ 0.00 & 0.50 & 0.50 \end{bmatrix}, \mathbf{p}_1 = \begin{bmatrix} 0.44 \\ 0.56 \\ 0.71 \end{bmatrix}, \text{and } \mathbf{p}_K = \begin{bmatrix} 0.56 \\ 0.44 \\ 0.29 \end{bmatrix}.$$

Consequently, the left hand side of (6) and (12) equals $\ln(p_1(x_2)/p_K(x_2)) - \ln(p_1(x_1)/p_K(x_1)) = 0.4918$. Moreover, $\mathbf{m}' = \begin{bmatrix} 0.57 & 0.82 & 1.23 \end{bmatrix}$ and $\mathbf{Q}_1(x_2) - \mathbf{Q}_K(x_2) - \mathbf{Q}_1(x_1) + \mathbf{Q}_K(x_1) = \begin{bmatrix} 0.25 & -1.00 & 0.75 \end{bmatrix}$, so that the slope of the dashed red line equals $[\mathbf{Q}_1(x_2) - \mathbf{Q}_K(x_2) - \mathbf{Q}_1(x_1) + \mathbf{Q}_K(x_1)]\,\mathbf{m} = 0.2465$. A unique value of β, 0.80, solves (12), but (6) has no solution.

Figure 4: Example of Data that are Consistent with an Exclusion Restriction on Current Values but Not with One on Primitive Utility

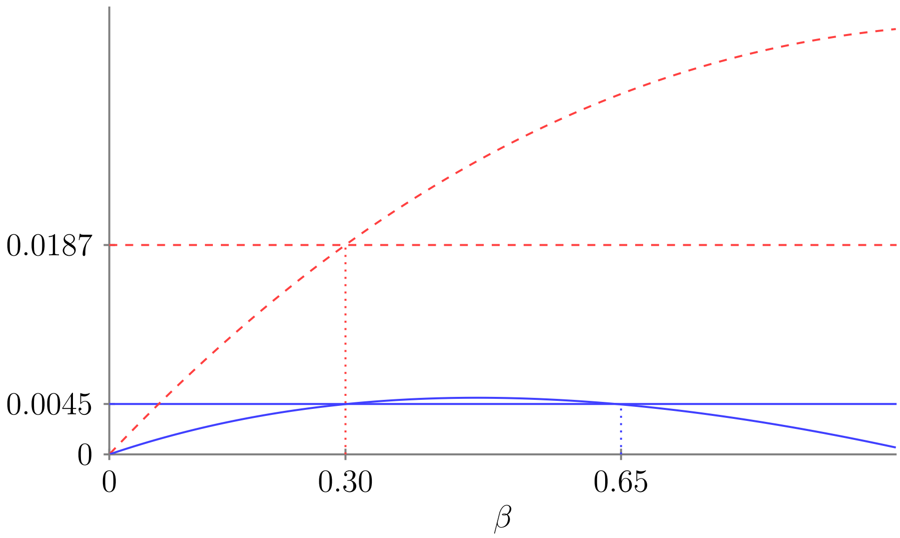


Note: For $J = 3$ states, $K = 2$ choices, $k = l = 1$, $\tilde{x}_1 = x_1$, and $\tilde{x}_2 = x_2$, this graph plots the left hand side of (6) and (12) (solid black horizontal line) and the right hand sides of (6) (dashed red line) and (12) (solid blue curve) as functions of β. The data are $\mathbf{Q}_1(\tilde{x}_1) = \begin{bmatrix} 0.25 & 0.25 & 0.50 \end{bmatrix}$, $\mathbf{Q}_1(\tilde{x}_2) = \begin{bmatrix} 0.00 & 0.25 & 0.75 \end{bmatrix}$,

$$\mathbf{Q}_K = \begin{bmatrix} 0.90 & 0.00 & 0.10 \\ 0.00 & 0.90 & 0.10 \\ 0.00 & 1.00 & 0.00 \end{bmatrix}, \mathbf{p}_1 = \begin{bmatrix} 0.50 \\ 0.48 \\ 0.10 \end{bmatrix}, \text{ and } \mathbf{p}_K = \begin{bmatrix} 0.50 \\ 0.52 \\ 0.90 \end{bmatrix}.$$

Consequently, the left hand side of (6) and (12) equals $\ln(p_1(x_1)/p_K(x_1)) - \ln(p_1(x_2)/p_K(x_2)) = 0.0800$. Moreover, $\mathbf{m}' = \begin{bmatrix} 0.69 & 0.65 & 0.11 \end{bmatrix}$ and $\mathbf{Q}_1(x_1) - \mathbf{Q}_K(x_1) - \mathbf{Q}_1(x_2) + \mathbf{Q}_K(x_2) = \begin{bmatrix} -0.65 & 0.90 & -0.25 \end{bmatrix}$, so that the slope of the dashed red line equals $[\mathbf{Q}_1(x_1) - \mathbf{Q}_K(x_1) - \mathbf{Q}_1(x_2) + \mathbf{Q}_K(x_2)]\,\mathbf{m} = 0.1116$. A unique value of β, 0.72, solves (6), but (12) has no solution.

Figure 5: Example with Two Moment Conditions of Which One Identifies the Discount Factor



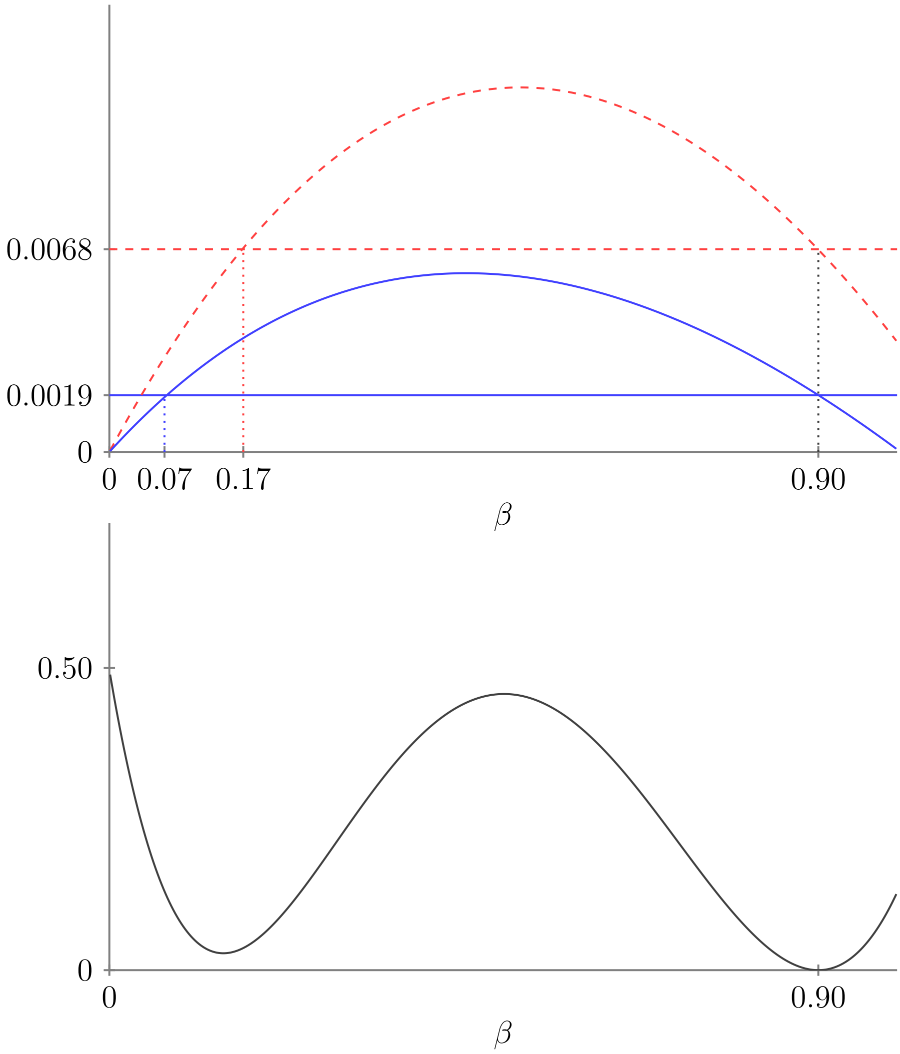
Note: For $J = 4$ states, $K = 2$ choices, and $k = l = 1$, this graph plots the left (horizontal lines) and right hand sides (curves) of (12) as functions of β, for $\tilde{x}_1 = x_1$ and $\tilde{x}_2 = x_2$ (corresponding to $u_1(x_1) = u_1(x_2)$; dashed red line and curve) and $\tilde{x}_1 = x_3$ and $\tilde{x}_2 = x_4$ (corresponding to $u_1(x_3) = u_1(x_4)$; solid blue line and curve). The data are

$$\mathbf{Q}_1 = \begin{bmatrix} 0.40 & 0.26 & 0.18 & 0.18 \\ 0.33 & 0.29 & 0.36 & 0.27 \\ 0.19 & 0.26 & 0.18 & 0.45 \\ 0.08 & 0.18 & 0.29 & 0.09 \end{bmatrix}, \mathbf{Q}_K = \begin{bmatrix} 0.17 & 0.26 & 0.13 & 0.43 \\ 0.13 & 0.07 & 0.20 & 0.60 \\ 0.20 & 0.30 & 0.10 & 0.40 \\ 0.25 & 0.15 & 0.50 & 0.10 \end{bmatrix},$$

$$\mathbf{p}_1' = \begin{bmatrix} 0.60 & 0.59 & 0.88 & 0.88 \end{bmatrix}, \text{ and } \mathbf{p}_K' = \begin{bmatrix} 0.40 & 0.41 & 0.12 & 0.12 \end{bmatrix}.$$

Consequently, the left hand sides of (12) equal $\ln(p_1(x_1)/p_K(x_1)) - \ln(p_1(x_2)/p_K(x_2)) = 0.0187$ and $\ln(p_1(x_3)/p_K(x_3)) - \ln(p_1(x_4)/p_K(x_4)) = 0.0045$. A unique value of β, 0.30, solves (12) for $\tilde{x}_1 = x_1$ and $\tilde{x}_2 = x_2$ (dashed red line and curve). Two values of β solve (12) for $\tilde{x}_1 = x_3$ and $\tilde{x}_2 = x_4$ (solid blue line and curve), of which one coincides with the solution to the first moment condition.

Figure 6: Example with Two Moment Conditions that Jointly Identify the Discount Factor but Individually Do Not



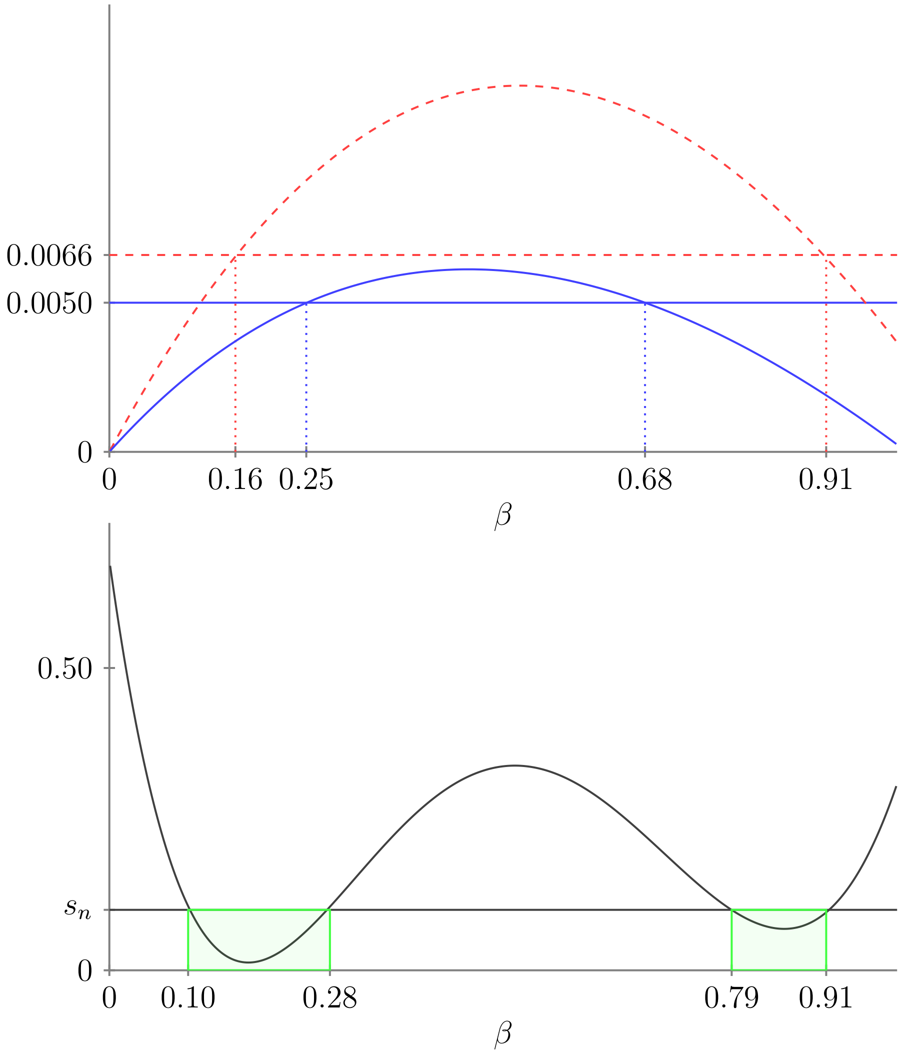
Note: For $J = 4$ states, $K = 2$ choices, and $k = l = 1$, the graph in the top panel plots the left (horizontal lines) and right hand sides (curves) of (12) as functions of β, for $\tilde{x}_1 = x_1$ and $\tilde{x}_2 = x_2$ (corresponding to $u_1(x_1) = u_1(x_2)$; dashed red line and curve) and $\tilde{x}_1 = x_3$ and $\tilde{x}_2 = x_4$ (corresponding to $u_1(x_3) = u_1(x_4)$; solid blue line and curve). The graph in the bottom panel plots the corresponding squared Euclidian distance between the left and right hand sides of (12) as a function of β (in multiples of 10^{-4}). The data are

$$\mathbf{Q}_1 = \begin{bmatrix} 0.43 & 0.26 & 0.18 & 0.18 \\ 0.33 & 0.29 & 0.36 & 0.27 \\ 0.19 & 0.26 & 0.18 & 0.45 \\ 0.05 & 0.18 & 0.29 & 0.09 \end{bmatrix}, \ \mathbf{Q}_K = \begin{bmatrix} 0.17 & 0.26 & 0.13 & 0.43 \\ 0.13 & 0.07 & 0.20 & 0.60 \\ 0.20 & 0.30 & 0.10 & 0.40 \\ 0.25 & 0.15 & 0.50 & 0.10 \end{bmatrix},$$

$$\mathbf{p}_1' = \begin{bmatrix} 0.92 & 0.92 & 0.63 & 0.63 \end{bmatrix}, \text{ and } \mathbf{p}_K' = \begin{bmatrix} 0.08 & 0.08 & 0.37 & 0.37 \end{bmatrix}.$$

Consequently, the left hand sides of (12) equal $\ln\left(p_1(x_1)/p_K(x_1)\right) - \ln\left(p_1(x_2)/p_K(x_2)\right) = 0.0068$ and $\ln\left(p_1(x_3)/p_K(x_3)\right) - \ln\left(p_1(x_4)/p_K(x_4)\right) = 0.0019$. A unique value of β, 0.90, solves (12) for both $\tilde{x}_1 = x_1$ and $\tilde{x}_2 = x_2$ (dashed red line and curve) and $\tilde{x}_1 = x_3$ and $\tilde{x}_2 = x_4$ (solid blue line and curve). In addition, each of these two moment conditions has one other solution.

Figure 7: Example with Two Moment Conditions that Jointly Identify the Discount Factor but Individually Do Not, Using Noisy Estimates of the Choice Probabilities



Note: This figure redraws Figure 6 for the same values of $\mathbf{Q}_1$ and $\mathbf{Q}_K$, but randomly perturbed values of its choice probabilities $\mathbf{p}_1$ and $\mathbf{p}_K$. Rounded to two digits, the perturbed choice probabilities equal those reported below Figure 6. Consequently, the perturbation to $\mathbf{m} = -\ln \mathbf{p}_K$ is very small too, so that the right hand sides of (12) are very close to those plotted in Figure 6. The left hand sides of (12), however, now equal $\ln(p_1(x_1)/p_K(x_1)) - \ln(p_1(x_2)/p_K(x_2)) = 0.0066$ (instead of 0.0068) and $\ln(p_1(x_3)/p_K(x_3)) - \ln(p_1(x_4)/p_K(x_4)) = 0.0050$ (instead of 0.0019). The resulting moment conditions again have two solutions. However, they no longer share a common solution and the squared Euclidian distance in the bottom panel never attains zero. The green shaded areas highlight the intervals $[0.10, 0.28]$ and $[0.79, 0.91]$ of values of β at which the distance is below some critical level s_n (which is taken to be 0.10×10^{-4} in this example).