Supplement to “Pensions, household saving, and welfare: A dynamic analysis of crowd out”

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Appendix A: Value function interpolation

The value function in period t can be written as

$$V_t(\mathbf{S}_t) = \max_{c_t, j_t, d_t} \bigl\{u(c_t, j_t | j_{t-1}) + \delta(1-\pi_{\mathrm{t}}) E_{\mathrm{t}}\bigl(V_{t+1}(\mathbf{S}_{t+1} | c_t, j_t, d_t, \mathbf{S}_t)\bigr) + \pi_{\mathrm{t}} B_t(\mathbf{S}_{t+1})\bigr\},$$

where $\mathbf{S}_t$ is the vector of state variables at the beginning of period t, d_t is the DC pension and/or SS claiming choice, if relevant, c is consumption, j is the employment choice, π_{t} is the probability of death at the end of period t, and $B_t(\mathbf{S}_{t+1})$ is the utility derived from leaving a bequest of amount A_{t+1} (an element of $\mathbf{S}_{t+1}$). The expectation of the $t+1$ value function can only be approximated. Let S_{it+1} be the value of the ith continuous state variable ($i = 1, 2, 3$) in period $t+1$. Define k_i as the grid point for state variable i for which $G_{it+1k_i-1} < S_{it+1} \leq G_{it+1k_i}$, where G_{it+1k} is the value of state variable i at grid point k in period $t+1$. For given values of the discrete state variables, the continuation value is approximated by multidimensional local linear interpolation (Judd (1998)),

$$\mathrm{E}_{\mathrm{t}}\bigl(V_{t+1}(S_{1t+1}, S_{2t+1}, S_{3t+1} | c_t, j_t, d_t, \mathbf{S}_t)\bigr) \approx \sum_{b_1=0}^{1}\sum_{b_2=0}^{1}\sum_{b_3=0}^{1} \kappa_{b_1b_2b_3} \bar{V}_{k_1+b_1, k_2+b_2, k_3+b_3t+1},$$

where the last term is the value function evaluated at grid points k_1+b_1, k_2+b_2, k_3+b_3. The weight inside the summation is an inverse function of the Euclidian distance between the point $(S_{1t+1}, S_{2t+1}, S_{3t+1})$ and the cube with vertices $(G_{1k1-1t+1}, G_{1k1t+1})$, $(G_{2k2-1t+1}, G_{2k2t+1})$, $(G_{3k3-1t+1}, G_{3k3t+1})$:

$$\kappa_{b_1b_2b_3} = \frac{\kappa^*_{b_1b_2b_3}}{\displaystyle\sum_{c_1=0}^{1}\sum_{c_2=0}^{1}\sum_{c_3=0}^{1} \kappa^*_{c_1c_2c_3}}$$

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and

$$\kappa^*_{b_1b_2b_3} = \sqrt{3} - \sqrt{\sum_{i=1}^{3}\left(\frac{S_{it+1} - G_{it+1b_i-1}}{G_{it+1b_i} - G_{it+1b_i-1}}\right)^2}.$$

The V_{t+1} terms were computed and stored as part of the solution in period $t+1$. Given the inequalities that define k_i, the maximum possible value of each of the three terms under the square root sign is 1, so the maximum possible value of the square root expression itself is $\sqrt{3}$. Subtracting it from $\sqrt{3}$ ensures that the weight cannot be negative. Scaling by the sum of the κ^* terms ensures that each weight is between 0 and 1 and that the weights sum to 1. In some circumstances, only one or two dimensional interpolation is required.

The grids are chosen so that all possible values of the continuous state variables are interior to the grid, so as to avoid extrapolation. This is straightforward for AIME, which is stationary and has an upper bound determined by SS rules, but the maximum feasible values of assets and the DC balance increase over time. The asset and DC balance grids are set each period to ensure that any feasible value of assets and the DC balance, conditional on the period $t-1$ values, falls within the relevant grid.

The EPDV of the SS and DB benefits are interpolated for periods before the benefit has been claimed, using the same approach. This is necessary because the benefits depend on when they are claimed, which is uncertain and subject to choice. To illustrate in the case of SS, the latest age at which SS can be claimed is 70. At age 70, the SS benefit can be calculated as a function of AIME at each grid point for those points in the age 70 state space at which the SS benefit has not yet been claimed. The EPDV of the benefit for claiming at age 70 is easily calculated since it does not depend on future choices and realizations of random variables, except for the interest rate and mortality. These EPDV values are stored, and used in interpolating the EPDV of SS benefits at age 69 for state points in which the benefit has not been claimed by age 69 and for choices in which it is not claimed at 69. The same approach is used for the EPDV of DB benefits, where the last age at which the benefit can be claimed is, by assumption, 65. The grids used in the solution contain 70 points for assets, 15 for AIME, and 15 for the DC balance.

Appendix B: Calculating defined benefit pension benefits

HRS respondents who reported any pension coverage at wave 1 were asked for permission to contact their employer to obtain information on the pension plan. For respondents who gave permission and whose employers provided the requested information, the formulas that determine the pension benefit for each plan were coded by the HRS staff and provided to researchers on a restricted access basis, along with pension calculation software. These formulas determine the pension benefit for all possible scenarios involving birth date, age, years in the plan at the time of exit, and salary history. Rather than use the pension calculation software (which is coded in Visual Basic) to directly compute benefits for each individual, I used an approximation approach. This was done so that the benefit calculations could be easily computed in the Fortran program used to solve and simulate the model.

The first step in the approximation uses the pension calculator to compute benefits for each DB plan in which any respondent is enrolled at wave 1, for 5000 artificial individuals, with alternative combinations of birth date, hire date, initial salary, and salary growth rate. For each artificial individual and each plan, I computed the monthly pension benefit and the age at which the individual is first eligible for the benefit for every possible age at which the individual could quit from the year after the hire date through age 75.

I then ran three regressions, separately for each pension plan, using the 5000 observations for each plan. The dependent variables are (1) a binary indicator for whether the individual will ever be eligible for a benefit, for each possible age at exit, (2) the age at which the individual is first eligible for the benefit, conditional on ever being eligible, and (3) the monthly benefit, conditional on eligibility. Each regression is specified with a very flexible functional form, with dummies for age at exit, tenure at exit, and combinations of age and tenure at exit. For the benefit regression, the specification includes average salary in the most recent 5 years, the second most recent 5 years, and so forth, and interactions of the salary averages with age and tenure dummies.

The coefficient estimates from these regressions for each plan are stored and used to compute benefits in the solution and simulation of the model. These regressions are generally very accurate in predicting outcomes. I compared the predictions from the regressions to the values computed directly from the pension calculator. For the "ever eligible" regression, using the rule that the prediction is 0 if the fitted value is less than 0.5 and the prediction is 1 otherwise, the regression predicts every one of the approximately 5000 observations correctly for 78% of the plans, and never predicts more than 13% incorrectly for any plan. Two thirds of the first-age-of-eligibility regressions predict the correct age exactly for every observation, and the 5th and 95th percentiles of the rounded residual distribution are 1 and −1, respectively. Finally, for the annual benefit regressions, the mean prediction error is −2.7 (in thousands of dollars per year), the

TABLE A1. Parameter estimates from log wage, log medical expenditure, layoff, and wife earnings models.

	(1) Log Husband Hourly Wage Rate	(2) Log Wife Earnings $\mid > 0$	(3) Log Family Medical Expenditure	(4) Laid Off
Intercept	0.440 (0.876)	−0.808 (2.64)	−6.621(0.731)	−2.654 (0.090)
Age	0.091 (0.033)	0.128 (0.100)	0.154 (0.022)	
Age squared/100	−0.091 (0.030)	−0.114 (0.091)	−0.078 (0.019)	
Inverse Mills ratio	0.015 (0.062)	0.162 (0.193)		
Dummy if HRS observation				0.401 (0.108)
Mean squared transitory error	0.043	0.340	0.826	
Estimation method	Fixed effect	Fixed effect	Fixed effect	probit
Sample size	3714	3064	11,172	6822

Note: Parameter estimates and variances from these regressions are used in solution and simulation of the model. Standard errors are given in parentheses. Source: Health and Retirement Study and Survey of Income and Program Participation. Samples: (1) Men aged 25–60 with no pension who earned at least $6000 annually, and worked at least 35 hours per week and 40 weeks per year. (2) Women married to men aged 25–65 with no pension. Variances of the transitory errors are net of the individual fixed effects. (3) All households. Total family out-of-pocket medical expenditure. The 0s are replaced with $1. (4) Men aged 25–75 without pensions. HRS dummy is set equal to 0 in solution and simulation.

Table A2. Selection equations for husband's wage and wife's employment models.

	Pr(Husband's Wage Observed)	Pr(Wife Earnings > 0)
Intercept	0.088 (0.074)	−0.842 (0.535)
Age	0.038 (0.003)	0.097 (0.024)
Age squared/100	−0.044 (0.004)	−0.119 (0.027)
HS grad	0.001 (0.001)	0.135 (0.090)
Some college	0.038 (0.009)	0.262 (0.075)
College grad	0.050 (0.009)	0.618 (0.086)
>College	0.070 (0.010)	0.947 (0.161)
Black	−0.038 (0.009)	0.611 (0.350)
Hispanic	0.035 (0.010)	−0.131 (0.165)
Other race	−0.004 (0.013)	0.004 (0.188)
Poor health	−0.082 (0.003)	−0.236 (0.021)
Number of kids at home	0.010 (0.001)	−0.050 (0.012)
Asset income/1,000,000	−0.025 (0.023)	−2.39 (1.39)
Estimation method	Probit	Probit
Sample size	26,977	4298

Note: The selection equations are used to generate the inverse Mills ratios used in Table A1. The selection equation for women is used in the solution and simulation of the model as well. All variables other than age and age squared are held constant at their means in solution and simulation (the resulting equation is $-1.184 + 0.097 \times \text{age} - 0.119 \times \text{agesq}/100$). Standard errors are in parentheses. Source: Health and Retirement Study and Survey of Income and Program Participation. Samples: (1) Men aged 25–65. (2) Women married to men aged 25–65 without pensions.

Table A3. Simulated earnings summary statistics by age.

Age:	25	35	45	55	65	75
Mean hourly wage offer	8.34	12.38	14.98	14.77	11.85	7.75
Probability wife earnings > 0	0.69	0.78	0.79	0.72	0.55	0.29
Mean annual wife earnings offer	6.34	11.60	16.77	19.29	17.62	12.83

Note: The wife's earnings are in thousands of 1992 dollars, and the husband's hourly wage rate is in 1992 dollars per hour.

Table A4. Simulated out-of-pocket family medical expenditure summary statistics by age.

Age:	25	35	45	55	65	75	85	95
Mean	0.06	0.17	0.43	0.93	1.70	2.66	3.39	3.93
Median	0.04	0.11	0.28	0.62	1.11	1.82	2.23	2.60
10th percentile	0.01	0.04	0.09	0.19	0.34	0.55	0.69	0.85
90th percentile	0.13	0.38	0.91	1.93	3.57	5.65	7.34	8.14

Note: Medical expenditure is measured in units of thousands of 1992 dollars.

median error is −0.6, the 75th percentile of the prediction error is 0.6, and the 25th percentile is −8.5. Comparing the benefits predicted from this approach with the actual benefit reported by HRS respondents who retired during the panel, given actual quit dates, yields a mean prediction error of 3.0 and a median of 2.7.

Figures A1–A3 report data on employment dynamics used in calibration, along with the simulated outcomes. Figures A4–A9 and A10–A15 show data on the employment level, employment dynamics, and assets for DC and DB pension holders, respectively. These data were not used in calibration. Simulated data from the model are shown as well for comparison.

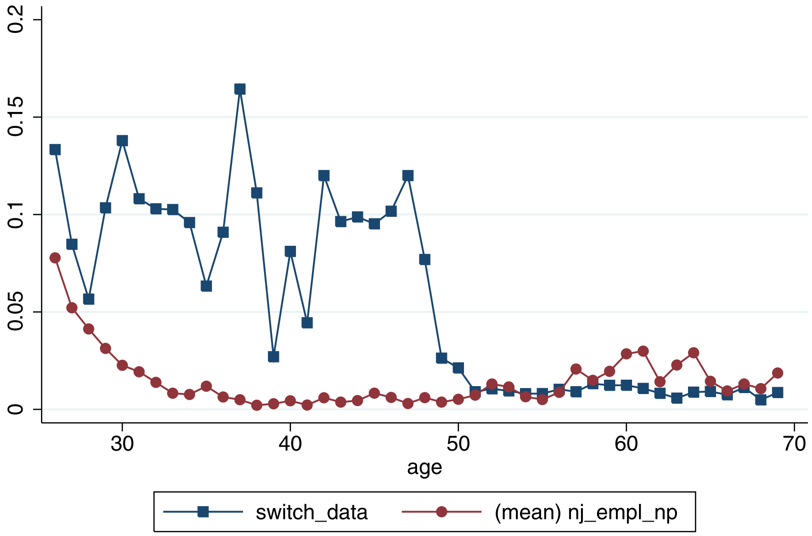

FIGURE A1. Actual and simulated rate of job-to-job change: NP.

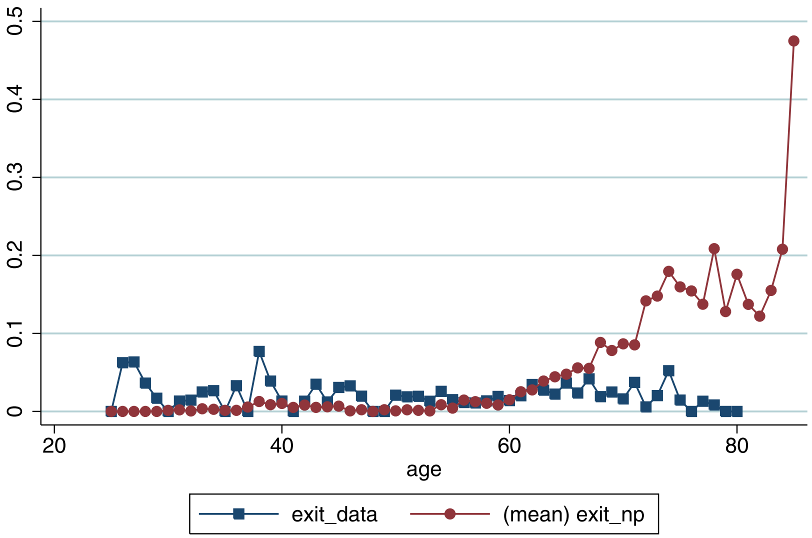

FIGURE A2. Actual and simulated exit rate from employment: NP.

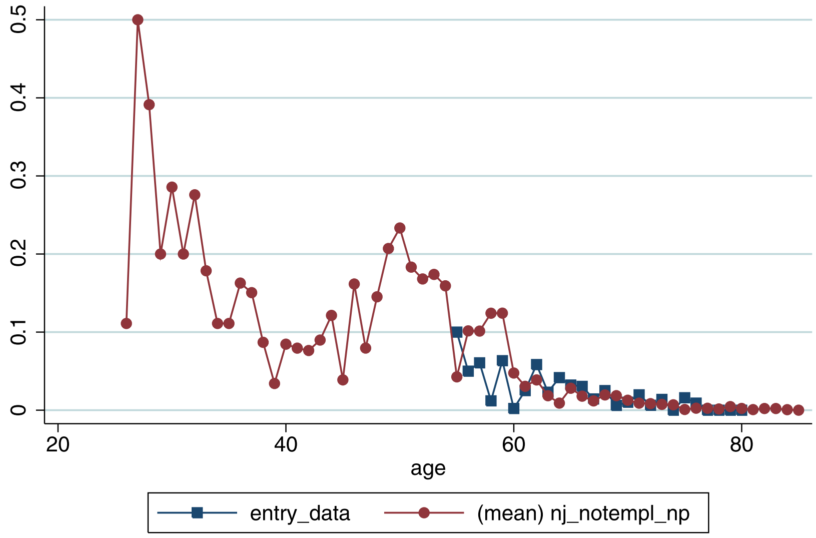

FIGURE A3. Actual and simulated rate of entry to employment: NP.

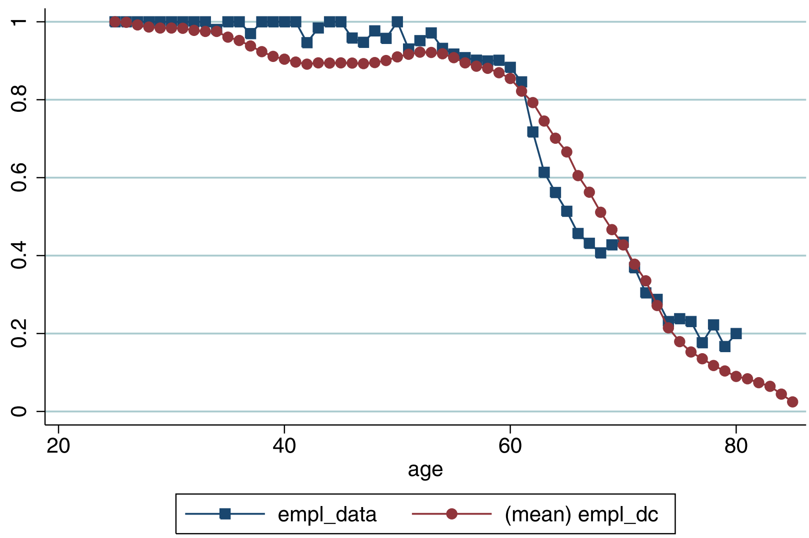

FIGURE A4. Actual and simulated employment rate: DC.

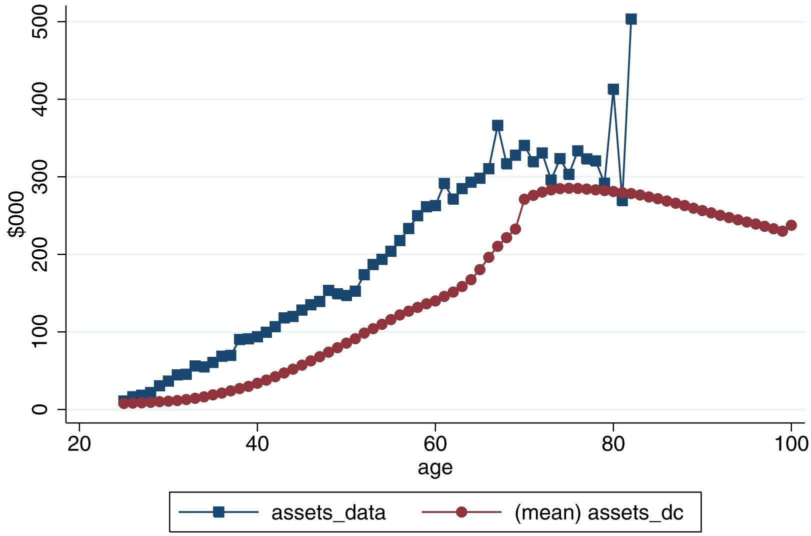

Figure A5. Actual and simulated assets: DC.

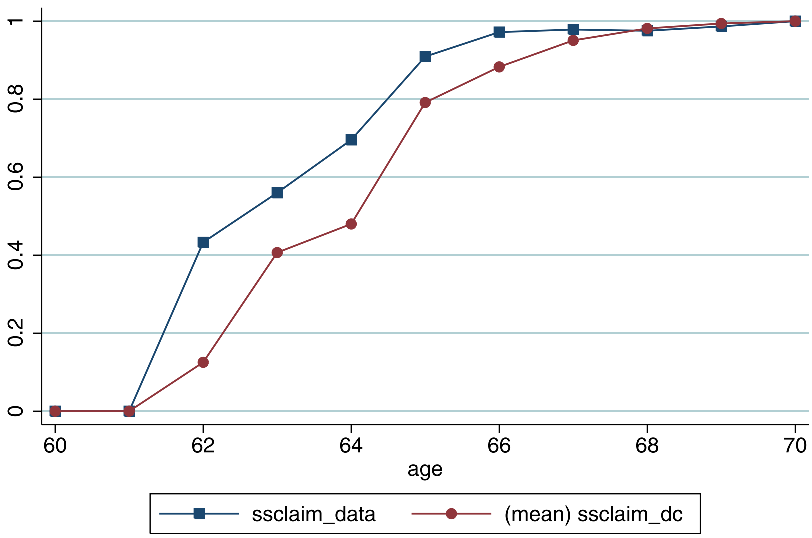

Figure A6. Actual and simulated social security claiming age: DC.

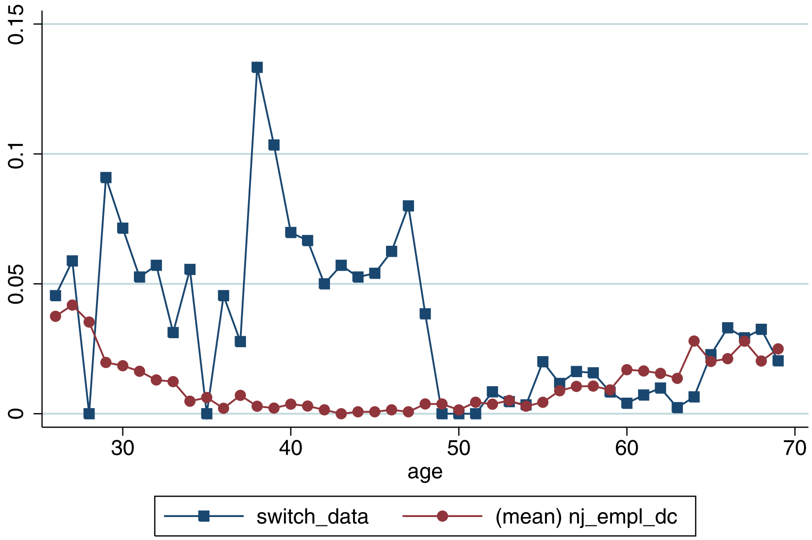

FIGURE A7. Actual and simulated rate of job-to-job change: DC.

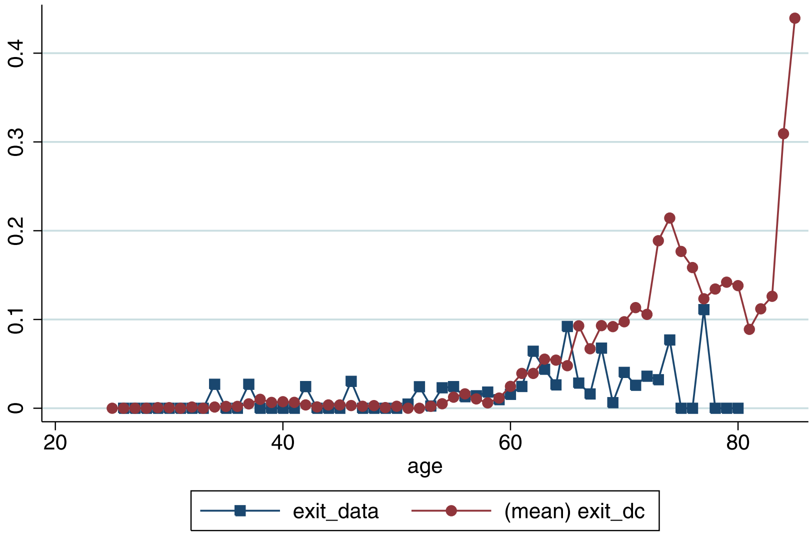

FIGURE A8. Actual and simulated exit rate from employment: DC.

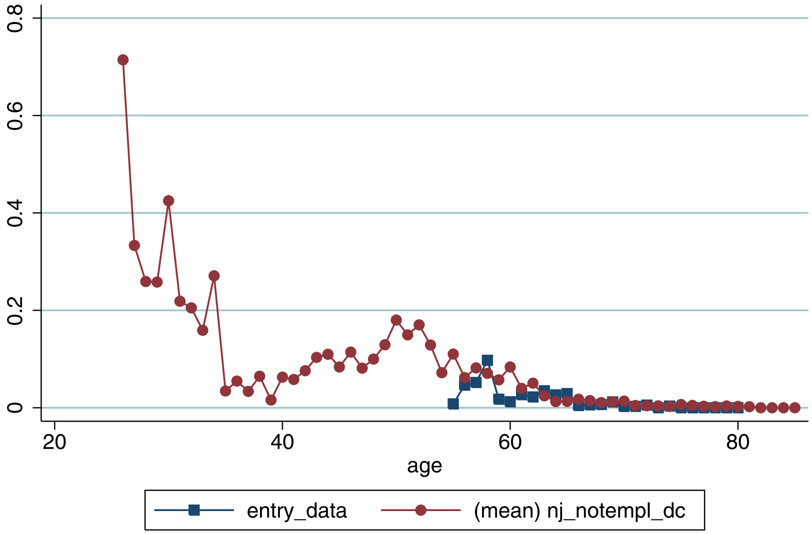

FIGURE A9. Actual and simulated rate of entry to employment: DC.

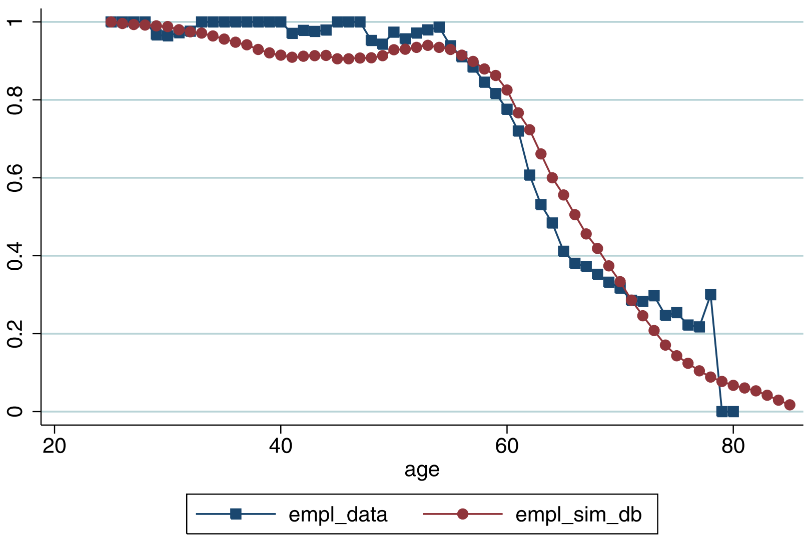

FIGURE A10. Actual and simulated employment rate: DB.

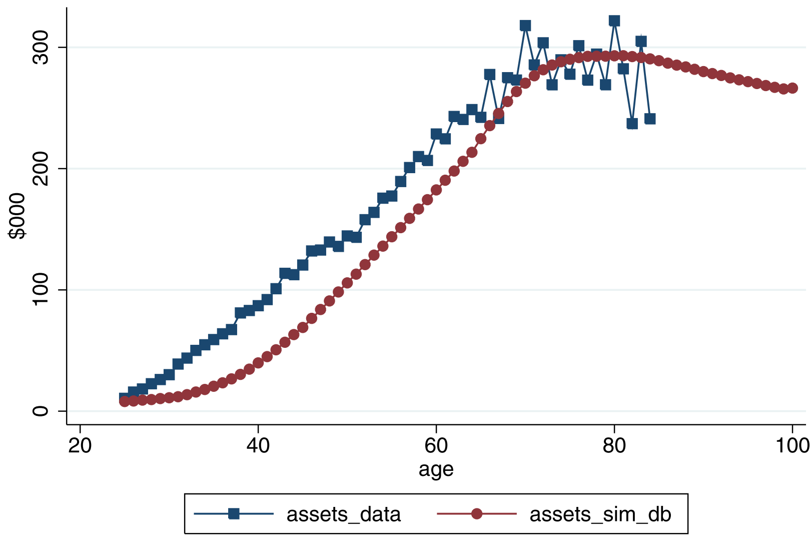

Figure A11. Actual and simulated assets: DB.

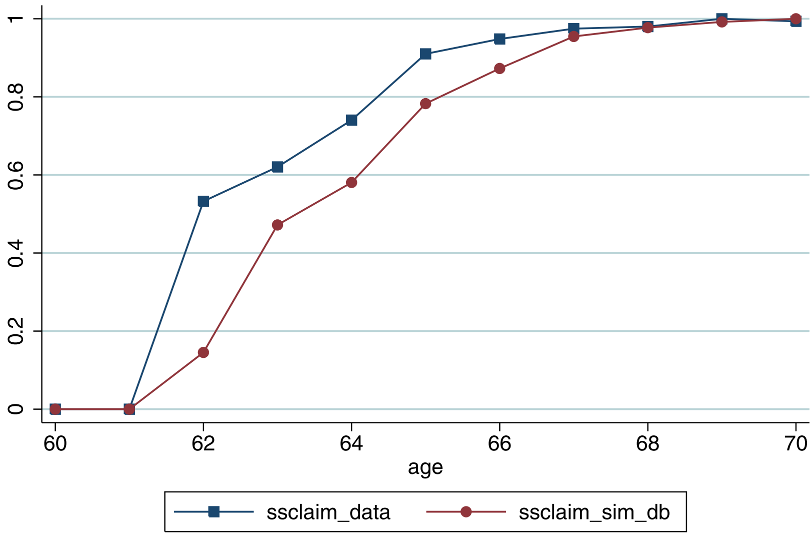

Figure A12. Actual and simulated social security claiming age: DB.

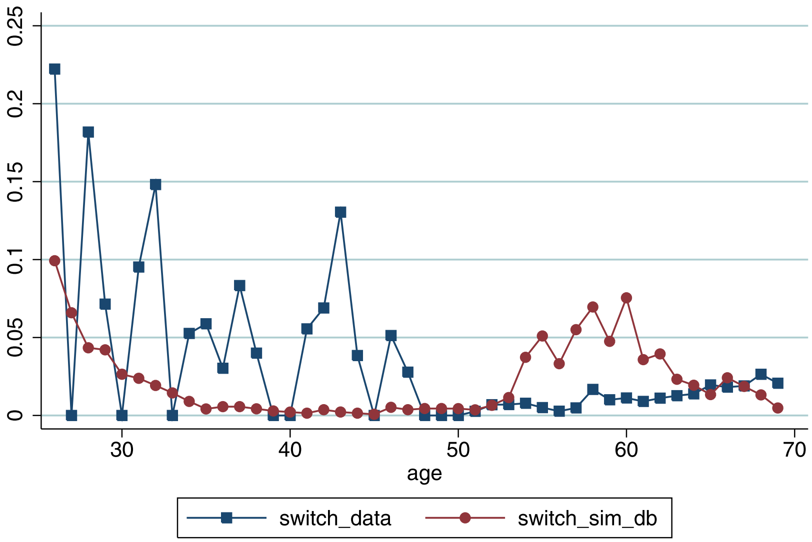

FIGURE A13. Actual and simulated rate of job-to-job change: DB.

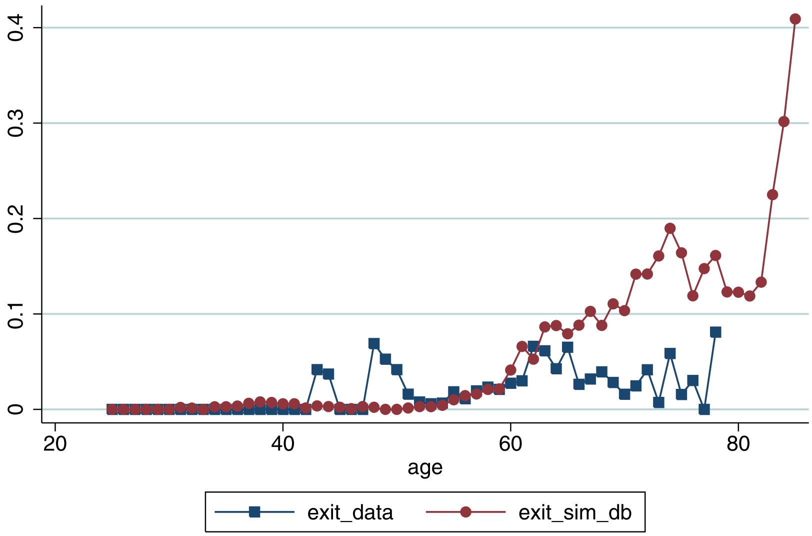

FIGURE A14. Actual and simulated exit rate from employment: DB.

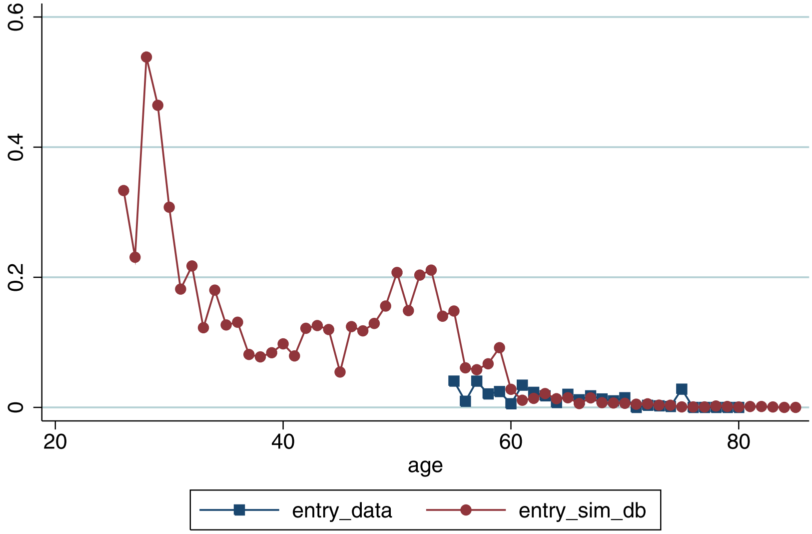

Figure A15. Actual and simulated rate of entry to employment: DB.

Reference

Judd, K. L. (1998), *Numerical Methods in Economics*. MIT Press, Cambridge. [1]

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