When consensus choice dominates individualism: Jensen's inequality and collective decisions under uncertainty

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Research on collective provision of private goods has focused on distributional considerations. This paper studies a class of problems of decision under uncertainty in which an efficiency argument for collective choice emerges from the mathematics of aggregating individual payoffs. Consider decision making when each member of a population has the same objective function, which depends on an unknown state of nature. If agents knew the state of nature, they would make the same decision. However, they may have different beliefs or may use different decision criteria to cope with their incomplete knowledge. Hence, they may choose different actions even though they share the same objective. Let the set of feasible actions be convex and the objective function be concave in actions, for all states of nature. Then Jensen's inequality implies that consensus choice of the mean privately chosen action yields a larger mean payoff than does individualistic decision making, in all states of nature. If payoffs are transferable, the mean payoff from consensus choice may be allocated to Pareto dominate individualistic decision making. I develop these ideas. I also use Jensen's inequality to show that a planner with the power to assign actions to the members of the population should not diversify. Finally, I give a version of the collective-choice result that holds with consensus choice of the median rather than mean action.

KEYWORDS. Collective choice of private goods, social choice, mechanism design, ambiguity, heterogeneous beliefs.

JEL CLASSIFICATION. D7, D81, H42.

1. INTRODUCTION

Economists working in the utilitarian paradigm have a strong predisposition to think that individualistic decision making regarding private goods is more efficient than collective choice. Research on public provision of private goods, whether normative or positive, has largely focused on distributional considerations. See, for example, Blackorby and Donaldson (1988), Besley and Coate (1991), Bruce and Waldman (1991), Blomquist and Christiansen (1995), and Epple and Romano (1996).

Researchers have occasionally provided efficiency arguments for collective decision making. Garratt and Marshall (1994) argued that public finance of higher education brings into existence a desirable gambling market that would not exist otherwise.

Coate (1995) combined distributional and efficiency concerns by considering an economy with altruistic agents. Fang and Norman (2008) examined a setting where government observation of demand for publicly provided goods reveals private information that is useful in setting tax policy.

This paper develops a distinct efficiency rationale for collective choice of private goods. I study a simple problem of private decision making under uncertainty in which the argument for collective choice emerges directly from the mathematics of aggregating individual payoffs.

I mainly consider decision making when each member of a population faces the same choice problem and wants to optimize the same objective function, which maps actions and an unknown state of nature into outcomes. If agents knew the state of nature, they would make the same decision. However, they may have different beliefs about the state or may cope with their incomplete knowledge by using different decision criteria. Hence, acting independently, they may choose different actions even though they share the same objective.

For example, some agents may believe that they know the state of nature and, hence, choose an action that maximizes the objective function in this state. Others may have probabilistic beliefs and maximize expected utility with heterogeneous subjective probability distributions and risk preferences. Yet others, not having probabilistic beliefs, may use varying criteria for decision making under ambiguity, such as the maximin- or minimax-regret criterion.

Applied economics abounds with research in which agents are assumed to maximize expected utility with heterogeneous beliefs or risk preferences. Economists often suppose that agents have common priors and that heterogeneity in probabilistic beliefs stems from individual observation of different data (private signals). They sometimes argue that agents may hold divergent probabilistic opinions even when all information is public, the reason being that they interpret this information differently. For example, in the field of financial economics, see Miller (1977), Mayshar (1983), Harris and Raviv (1993), and Kandel and Pearson (1995).

Applied study of choice under ambiguity has been much less common, but warrants considerable attention. In research over the past decade, I have studied a variety of settings in which ambiguity arises from partial identification of the relevant state of nature. I have emphasized that heterogeneity in nonprobabilistic beliefs may occur when individuals observe the same data but make different nonrefutable assumptions about what states of nature are feasible. See Manski (2000, 2004, 2005, 2006, 2007, 2009, 2010).

Suppose that the set of feasible actions is convex and the objective function is concave in actions for all states of nature. In this context, Section 2 shows that population coordination on a specific consensus action yields a larger mean payoff than does individualistic decision making in all states of nature. The consensus action with this remarkable property is the mean action that agents would choose independently. This result is an immediate consequence of Jensen's inequality. I use input choice by firms and asset allocation by investors to illustrate the result. I show its robustness to some relaxations of the assumption that each member of the population wants to optimize the same concave objective function.

Section 3 shows that if payoffs are transferable, the mean payoff realized by collective choice of the consensus action may be allocated across the population so that collective choice Pareto dominates individualistic decision making in all states of nature. A Pareto dominating collective-choice mechanism is implementable if agents truthfully reveal the actions they would choose individualistically. I give conditions under which truthful revelation is incentive compatible.

I also compare collective choice with an alternative in which agents first reveal their choice-relevant attributes to one another and then make individualistic decisions. The outcome of this mechanism depends on what agents learn about each other. Communication-augmented private choice makes collective choice unnecessary if agents learn that heterogeneity stems solely from the existence of private signals. However, collective choice Pareto dominates communication-augmented private choice if agents learn that heterogeneity stems from differing assumptions about the feasible states of nature, differing prior probabilistic beliefs, or differing decision criteria.

Whereas Section 3 concerns decision making by a population whose members must agree on a collective-choice mechanism, Section 4 considers treatment choice by a planner who has the power to assign actions to the members of the population. Here Jensen's inequality implies that any heterogeneous treatment of the population is dominated by assigning the associated mean treatment to all persons. Thus, the planner should not diversify treatment. I explain why this result differs from the positive findings for diversification obtained in Manski (2009, 2010). There I studied different classes of planning problems, where the choice set may not be convex, treatments may be interdependent across persons, and agents may have heterogeneous, nonconcave objective functions.

Except for the discussion of robustness in Section 2, the paper thus far concerns decision making with a convex choice set and a common concave objective function. Section 5 supposes instead that the choice set is ordered and that agents have objective functions that are unimodal in each state of nature. The state-specific mode is invariant across agents, but the objective functions may otherwise vary. In this setting, I show that consensus choice of the median individualistic action makes the majority of agents better off than they would be with private choice in all states of nature. This result is closely related to the median voter theorem.

The analysis of this paper is simple and straightforward. It nevertheless appears to be new to research on collective choice of private goods. The closest connection that I have been able to discover is to the distantly related literature on consensus forecasting. Section 6 makes this connection. Section 7 makes concluding remarks about the broader implications of this work for social choice theory.

2. Analysis with a convex choice set and a concave objective function

2.1 *Notation and concepts*

To begin, let (J, Ω, P) be a probability space of agents, each of whom must choose an action from a set X. Here J lists the agents, the σ-algebra Ω places probability on individual agents, and P is the probability measure. For example, J may be a finite group of

size $|J|$, in which case $P(j) = 1/|J|$ for $j \in J$. Alternatively, J may be a continuum indexed by the unit interval, in which case P is the uniform distribution on $[0, 1]$.

The set S lists states of nature. The objective function $f(\cdot, \cdot) : X \times S \to R$ maps actions and states into the real line. Each agent wants to choose an action that maximizes $f(\cdot, r)$, where $r \in S$ is the actual state of nature. All agents know that $r \in S$, but they may not know the identity of r. Hence, they may not be able to solve the optimization problem $\max_{x \in X} f(x, r)$.

Agents may have different beliefs about r. Some may think that all elements of S are feasible states, while others may believe that only certain elements of S are feasible. Some may place complete subjective probability distributions on the states they consider feasible, while others may have incomplete probabilistic beliefs, or no beliefs at all. Agents may use different decision criteria to make choices with their beliefs. In any case, let x_j be the action that agent $j \in J$ would choose in a regime of individualistic decision making. Then the mean payoff in state s is $\int f(x_j, s) \, dP(j)$.

Consider an alternative regime of collective choice in which all agents choose some consensus action, say $c \in X$. Then the mean payoff in state s is $\int f(c, s) \, dP(j) = f(c, s)$. The question of interest is whether there exists a consensus choice that improves on individualistic decision making. The answer is positive if X is convex and if $f(\cdot, s)$ is concave on X for each $s \in S$.

2.2 *Collective choice of the mean action*

Let $\mu \equiv \int x_j \, dP(j)$ denote the mean individualistic action and assume that μ is finite. Convexity of X implies that $\mu \in X$, so μ is a feasible action. The mean payoff with collective choice of μ is $f(\mu, s)$. Jensen's inequality gives

$$f(\mu, s) \geq \int f(x_j, s) \, dP(j) \quad \text{for all } s \in S. \tag{1}$$

Thus, in all states of nature, the mean payoff with consensus choice of action μ is at least as large as with individualistic decision making.

Let $g(s) \equiv f(\mu, s) - \int f(x_j, s) \, dP(j)$ denote the mean surplus achieved by consensus choice of μ. This surplus is nonnegative by (1). It ordinarily is positive, a familiar sufficient condition being that $f(\cdot, s)$ is strictly concave and $\mathrm{Var}(x_j) > 0$.

It is important to understand that nonnegativity of mean surplus in all states of nature does not imply that agents would choose action μ when making individualistic decisions. To the contrary, x_j is the action that agent j would choose. Thus, each agent j prefers to choose x_j with state-dependent payoff $f(x_j, \cdot)$, rather than choose μ with payoff $f(\mu, \cdot)$.

The above implies that if payoffs are nontransferable across the population, agents unanimously prefer individualistic choice of $(x_j, j \in J)$ to collective choice of μ, despite the fact that the latter generates a nonnegative mean surplus in all states of nature. Nontransferability of payoffs is critical to this result. Section 3 will show that there exist regimes with collective choice of μ that Pareto dominate individualistic choice of $(x_j, j \in J)$ when payoffs are transferable. This will be accomplished by combining collective choice of μ with a suitable allocation of payoffs after the true state of nature becomes known.

2.3 *Illustrative applications to production decisions*

Result (1) has many applications in which a group of firms or other agents have a common concave production function and want to choose inputs to maximize profit. Here are two examples.

Production by firms when product price is unknown

Let J be a group of price-taking firms that face the same concave production function. In particular, let the set of feasible inputs be $X = [0, \infty)$ and let output be $\log(x+1)$. Let the unit cost of input be 1, and let product price be s. Then the profit function is $f(x, s) = s \cdot \log(x + 1) - x$. Suppose that firms do not know product price when they choose input quantities. For example, the firms may be farms of equal size that must decide how intensively to plant in the spring before knowing the crop price that they will receive at harvest.

Let the set S of feasible prices be a subset of the half-line $[0, \infty)$. Suppose each firm places a subjective distribution on price and maximizes expected profit. Let p_j be the subjective mean price held by firm j. Acting independently, firm j would solve the problem $\max_{x \in X} p_j \cdot \log(x+1) - x$, yielding the input choice $x_j = \max(0, p_j - 1)$. Suppose that $p_j \geq 1$ for all $j \in J$. Then $\mu = E(p) - 1$.

In state s, the mean payoffs with collective choice of input quantity μ and with individualistic decision making are, respectively,

$$f(\mu, s) = s \cdot \log[E(p)] - E(p) + 1,$$

$$\int f(x_j, s)\, dP(j) = sE[\log(p)] - E(p) + 1.$$

Hence, the surplus achieved by consensus choice of μ is $g(s) = s\{\log[E(p)] - E[\log(p)]\}$.

Allocation of an endowment between a safe asset and production with unknown return

Let J be a group of investors, each having an endowment of size 1. Suppose that each investor must allocate his endowment between a safe asset with known return 1 and a productive activity with unknown return. The set of feasible choices is $X = [0, 1]$, where $x \in X$ denotes the fraction of the endowment allocated to the productive activity. Let the return to production be $s \cdot x^{1/2}$, where $S = (0, \infty)$. Then the return to allocation x in state of nature s is $s \cdot x^{1/2} + (1 - x)$.

Suppose that each investor places a subjective distribution on S and maximizes expected return. Let q_j be the subjective mean of s for investor j. Acting independently, j would solve the problem $\max_{x \in X} q_j \cdot x^{1/2} + (1 - x)$, yielding the choice $x_j = \min(1, q_j^2/4)$. Suppose that $q_j \leq 2$ for all $j \in J$. Then $\mu = E(q^2)/4$.

In state s, the mean payoffs with collective choice of input quantity μ and with individualistic decision making are, respectively,

$$f(\mu, s) = (s/2) \cdot [E(q^2)]^{1/2} + 1 - E(q^2)/4,$$

$$\int f(x_j, s)\, dP(j) = (s/2) \cdot E(q) + 1 - E(q^2)/4.$$

Hence, the surplus achieved by consensus choice of μ is $g(s) = (s/2)[E(q^2)^{1/2} - E(q)]$.

2.4 *Relaxing the assumption of a common concave objective function*

Observe that result (1) requires no restrictions on the beliefs that agents hold about the actual state of nature or on the decision criteria they use to cope with incomplete knowledge. The result as stated does presume that each agent has the same concave objective function. Although some semblance of this condition seems essential, some relaxation is possible. I consider two scenarios here.

Collective choice within subpopulations

Let the population be composed of K types. Persons of a given type have the same concave objective function, but this function may vary across types. Let μ_k denote the mean action that would be privately chosen by persons of type k. Consider collective choice in subpopulations, with all persons of type k choosing action μ_k. A possible application to agricultural cooperatives will be discussed in Section 3.4.

Application of (1) to persons of type k gives $f(\mu_k, s) \geq \int f(x_j, s)\, dP(j|k)$ for all $s \in S$. Averaging payoffs across types gives $\sum_k f(\mu_k, s)P(k) \geq \int f(x_j, s)\, dP(j)$, where $P(k)$ is the fraction of type-k persons in the population. Thus, the mean payoff from collective choice within each of the K subpopulations is at least as large as that produced by individualistic decision making.

Neighborhoods of a common concave function

Suppose that each agent $j \in J$ has a person-specific, not necessarily concave, objective function $f_j(\cdot, \cdot) : X \times S \to R$. Then the mean payoffs in state of nature s with individualistic decision making and collective choice of μ are $\int f_j(x_j, s)\, dP(j)$ and $\int f_j(\mu, s)\, dP(j)$, respectively. Collective choice achieves a nonnegative surplus if $\int f_j(\mu, s)\, dP(j) \geq f_j(x_j, s)\, dP(j)$. This inequality can hold if agents do not share the same concave objective function. In particular, it holds if all agents have objective functions that are sufficiently close to a common concave function.

For each state s, let there exist a $\lambda(s) > 0$ and a concave function $f(\cdot, s) : X \to R$ such that

$$\sup_{w \in X, j \in J} |f_j(w, s) - f(w, s)| < \lambda(s).$$

As earlier, let $g(s) \equiv f(\mu, s) - \int f(x_j, s)\, dP(j)$ denote the mean surplus that occurs with function f. Mean surplus with the actual objective functions $f_j(\cdot, s)$, $j \in J$, satisfies the inequality

$$\int f_j(\mu, s)\, dP(j) - \int f_j(x_j, s)\, dP(j) \geq g(s) - 2\lambda(s).$$

Hence, the condition $\lambda(s) \leq g(s)/2$ suffices to ensure that actual surplus is nonnegative.

3. Pareto dominant collective-choice mechanisms

3.1 *Allocation of transferable payoffs*

I observed in Section 2.2 that if payoffs are nontransferable across the population, agents unanimously prefer individualistic choice of $(x_j, j \in J)$ to collective choice of μ, despite the fact that the latter generates nonnegative mean surplus in all states of nature. This section shows that if payoffs are transferable, there exist collective-choice mechanisms that Pareto dominate private choice.

Let $\gamma \equiv [\gamma_j(s), (j, s) \in J \times S]$ be any set of positive real numbers such that $\int \gamma_j(s) \, dP(j) = 1$ for all $s \in S$. Consider a collective choice mechanism in which agents take the consensus action μ and, if the state of nature turns out to be s, agent j receives the payoff

$$h_j(\gamma, s) \equiv f(x_j, s) + \gamma_j(s)g(s). \tag{2}$$

These collective-choice payoffs weakly dominate payoffs with private choice; that is, $h_j(\gamma, s) \geq f(x_j, s)$ for all $s \in S$ and $h_j(\gamma, s) > f(x_j, s)$ for some s. Payoffs (2) are feasible because $\int h_j(\gamma, s) \, dP(j) = f(\mu, s)$. Hence, the collective-choice mechanism is feasible and Pareto dominates individualistic decision making. That is, each agent j prefers collective choice of μ with state-dependent payoff $f(x_j, \cdot) + \gamma_j(\cdot)g(\cdot)$ to individualistic choice of x_j with payoff $f(x_j, \cdot)$.

Observe that a collective-choice mechanism of this type benefits even an agent who knows the actual state of nature with certainty. The consensus action μ may differ from what such an agent knows to be the optimal action. Nonetheless, he receives at least the payoff that he would have achieved by private choice.

Our finding that consensus choice of a private good Pareto dominates individualistic decision making goes against conventional economic wisdom. It has been common to think that, absent distributional considerations, private choice must be Pareto superior to collective choice. The standard utilitarian argument is that when agents want to make different choices, collective choice of a single good creates deadweight loss.

The present analysis shows that it is important to ask why agents want to make different choices. The standard utilitarian argument is correct in deterministic settings, where variation in private choices stems from heterogeneity in choice sets or objective functions. In decisions under uncertainty, variation in private choices may stem from heterogeneity in agents' beliefs about the state of nature or in their decision criteria. The latter sources of heterogeneity are the driving force behind result (1). When agents have the same convex choice set and concave objective function, (1) shows that consensus choice creates mean surplus. This enables the Pareto improvement achieved by mechanism (2).

Contrast with collective insurance

The collective choice mechanism introduced here differs from collective insurance. In insurance systems, agents make individualistic decisions and then smooth payoffs across states of nature. Insurance increases payoffs in states with relatively bad private outcomes and decreases payoffs in states with relatively good private outcomes.

Formally, agent j privately chooses action x_j and receives payoff $f(x_j, s) + \varphi_j(s) - c_j$ should state s occur. Here $c_j > 0$ is a state-invariant insurance premium. The term $\varphi_j(s)$ is a state-dependent insurance payment, with $\varphi_j(s) = 0$ in states where $f(x_j, s)$ is relatively large and $\varphi_j(s) > c_j$ in states where $f(x_j, s)$ is relatively small.

A collective insurance system is feasible if mean payments do not exceed mean premiums in every state of nature; that is, if $\int \varphi_j(s) \, dP(j) \le \int c_j \, dP(j)$ for all $s \in S$. This set of inequalities formalizes the idea that it is infeasible to insure against systemic risks. Insurance is feasible only if, in every state of nature, the subpopulation of agents who experience bad private outcomes and receive payments is sufficiently small that mean premiums cover their payments.

In contrast to insurance, our agents all choose the consensus action μ. In every state of nature, they receive payoffs (2) that are at least as large as their private outcomes under individualistic decision making. These payoffs are feasible because collective choice of μ generates a nonnegative mean surplus in every state of nature.

3.2 *Truthful revelation of private choices*

Implementation of the collective-choice mechanism requires knowledge of the actions that agents would choose privately. Research on mechanism design has long sought to determine when it is incentive compatible for agents to reveal private information truthfully. Here we would like each agent to announce his private choice truthfully.

In a regime of private choice, agent j receives payoff $f(x_j, s)$ in state s. Let x_j^a denote the action that j announces under the collective-choice mechanism and let $\mu^a \equiv \int x_j^a \, dP(j)$ denote the mean of the announced actions. Then

$$h_j^a(\gamma, s) \equiv f(x_j^a, s) + \gamma_j(s) g^a(s) \tag{2$'$}$$

is the collective-choice payoff based on announcements. Here $g^a(s) \equiv f(\mu^a, s) - \int f(x_j^a, s) \, dP(j)$ is the mean surplus in state s.

Private- and collective-choice payoffs differ, so agent j would not necessarily announce $x_j^a = x_j$. However, truthful revelation is incentive compatible in some settings. I show here that it is if the population is a continuum and agents either maximize expected payoff or minimize maximum regret.

Suppose that the population is a continuum. Then the action announced by agent j does not affect $g^a(s)$. Here, as in analysis of other collective decision problems, the idea of a continuum of agents is a simplifying idealization, meant to approximate a large finite population. In a large finite population, an agent's announced action negligibly affects mean surplus.

Suppose that j maximizes his subjective expected payoff. Then $x_j^a = \arg\max_{x \in X} \int f(x, s) \, d\pi_j + \int \gamma_j(s) g^a(s) \, d\pi_j$, where π_j is the agent's subjective distribution on S. In the private-choice regime, this agent would choose $x_j = \arg\max_{x \in X} \int f(x, s) \, d\pi_j$. The expression $\int \gamma_j(s) g^a(s) \, d\pi_j$ does not vary with x. Hence, $x_j^a = x_j$.

Suppose that j minimizes maximum regret. In this case, $x_j^a = x_j$ because, for each $x \in X$ and $s \in S$,

$$\max_{w \in X}[f(w, s) + \gamma_j(s) g^a(s)] - [f(x, s) + \gamma_j(s) g^a(s)] = \max_{w \in X} f(w, s) - f(x, s).$$

The left-hand side of the equation is regret in state s for announcing action x. The right-hand side is regret in state s for choosing x in a regime of private choice. The equation holds because the population is a continuum, so $g^a(s)$ does not vary with w or x.

3.3 *Communication-augmented private choice*

An alternative to the collective-choice mechanism proposed here is private choice after communication between agents. For example, economists have studied the operation of trade associations, in which firms reveal private signals to one another and then make individualistic decisions. See, for example, Gal-Or (1985), Shapiro (1986), and Vives (1990).

Under our maintained assumptions, each agent's payoff in a regime of individualistic decision making is independent of the actions chosen by other agents. Hence, agents should be indifferent regarding communication of private information to other agents. To make the best case for private choice, suppose that agents truthfully reveal all of their choice-relevant attributes to one another and that this is common knowledge. Thus, they announce their assumptions about the feasible states of nature, their prior probabilistic beliefs if such exist, the private signals they receive, and the decision criteria they use. They then make communication-augmented private choices.

The outcome of this mechanism depends on what agents learn about each other. Two polar scenarios illustrates the possibilities. First suppose agents learn that heterogeneity stems solely from the existence of private signals about the state of nature. Then agents are identical after they reveal their private signals. Hence, they make identical communication-augmented private choices. There is, therefore, no need for a collective-choice mechanism.

Now suppose agents learn that there are no private signals. Instead, heterogeneity stems from differing assumptions about the feasible states of nature, differing probabilistic beliefs, or differing decision criteria. For example, they may learn that they all maximize expected utility but have different risk preferences. Then, absent social influences such as a preference for conformity, communication reveals no information relevant to individual decision making. Hence, the collective choice mechanism of Section 3.1 Pareto dominates communication-augmented private choice.

It would be useful to consider scenarios where communication reveals heterogeneity in private signals as well as in assumptions or decision criteria. Then communication and collective choice may both be of value. Study of such scenarios is beyond the scope of this paper.

3.4 *Application to agricultural cooperatives*

Is the collective choice mechanism introduced here only of theoretical interest or might it have useful applications? The mechanism may prove helpful to stimulate the formation and guide the operation of agricultural cooperatives. I use crop production to illustrate.

Consider a geographically concentrated group of price-taking farmers. Crop production approximates the conditions assumed in this paper reasonably closely. Farmers

share the common objective of profit maximization. They typically have concave production functions and convex choice sets with diminishing returns to inputs of seed, fertilizer, labor, and irrigation. The farmers in a given locale face approximately homogeneous climate, soil conditions, input costs, and crop prices. Hence, their profit functions are approximately homogeneous. Finally, profits are a transferable payoff. Thus, the conditions for application of result (1) and mechanism (2) approximately hold.

The surplus generated by consensus choice is greatest in settings with considerable uncertainty, where agents with disperse beliefs and decision criteria make heterogeneous private choices. Farmers making planting decisions in the spring face considerable uncertainty about the crop prices they will receive in the fall, as well as uncertainty about crop yield arising from the difficulty of predicting weather during the planting season. In a private-choice regime, farmers may cope with this uncertainty in different ways, varying the allocation of fields to alternative crops, the timing of planting, and so on. Hence, consensus choice may generate meaningful surplus.

A cooperative could determine the consensus input bundle in late winter, prior to initiation of the annual production cycle in the spring. Each farmer in the cooperative would report the input bundle he would choose if he made production decisions individualistically. The mean announced bundle would become the consensus choice.

To the best of my knowledge, agricultural cooperatives do not currently function in this manner. They may communicate information, share farming machinery, market crops, operate credit unions, and perform other services for members. However, I am not aware of cooperatives that make consensus production decisions and allocate payoffs as proposed here.

4. Implications for planning: Uniform treatment dominates diversification

Sections 2 and 3 considered decision making when the members of a population collectively agree on a consensus choice. This section concerns a planner who has the power to assign actions. The planner's objective is to maximize the mean payoff.

When studying planning problems, actions are often called treatments. The planner's problem is to choose treatments. A planner with incomplete knowledge of the state of nature has partial knowledge of treatment response. I have previously studied various planning problems that do not have a convex choice set and homogeneous concave objective function. I show here that this structure qualitatively affects findings.

Suppose that a planner can assign each agent any feasible action. Thus, the planner can choose any element of the Cartesian product set $X^{|J|}$. Let $w \equiv (w_j, j \in J)$ be any set of assigned actions with finite mean $\mu_w \equiv \int w_j \, dP(j)$. Jensen's inequality gives

$$f(\mu_w, s) \geq \int f(w_j, s) \, dP(j) \quad \text{for all } s \in S. \tag{3}$$

Result (3) shows that, in each state, the mean payoff when the planner assigns every agent action μ_w is at least as large as the payoff with the possibly heterogeneous assignments $(w_j, j \in J)$. In other words, any diversified treatment of the population is dominated by assigning the associated mean treatment to all persons.

This finding differs sharply from one that I reported previously (Manski (2007, 2009)). There I studied a class of planning problems in which diversified treatment of observationally identical persons is always undominated. In particular, I studied problems where the choice set X contains only two elements, say a and b. I also permitted heterogeneous objective functions ($f_j, j \in J$).

Let $\alpha(s) \equiv \int f_j(a, s) \, dP(j)$ denote the mean payoff in state s when all agents receive treatment a and, similarly, let $\beta(s) \equiv \int f_j(b, s) \, dP(j)$. Consider a treatment allocation that randomly assigns a fraction $\delta \in [0, 1]$ of the population to treatment b and $1 - \delta$ to treatment a. Let the population be a continuum. Then the payoff to allocation δ in state s is $(1 - \delta)\alpha(s) + \delta\beta(s)$. Suppose that the planner faces ambiguity; that is, there exists a state s with $\alpha(s) > \beta(s)$ and another state s' with $\alpha(s') < \beta(s')$. Then it is immediate that all $\delta \in [0, 1]$ are undominated. Moreover, it can be shown that the minimax-regret allocation always diversifies treatment.

Result (3) shows that the situation differs dramatically if all convex combinations of a and b are feasible treatments and all agents have the same concave objective function f. Then, for all $\delta \in (0, 1)$ and $s \in S$, assigning every agent to treatment $(1 - \delta)a + \delta b$ yields at least as large a payoff as does the diversified allocation assigning fraction δ to b and $1 - \delta$ to a. Formally,

$$f[(1 - \delta)a + \delta b, s] \geq (1 - \delta)f(a, s) + \delta f(b, s)$$
$$= (1 - \delta)\alpha(s) + \delta\beta(s) \quad \text{for all } s \in S.$$

Application to medical treatment

Medical treatment with partial knowledge of treatment response illustrates when diversification is and is not a reasonable strategy. Consider first an organ disease with two alternative treatments. One is surgery to repair the organ and the other is replacement of the organ with a transplant. Convex combinations of these treatments are not feasible—one can only repair or replace. In a setting of this sort, diversification warrants consideration when it is not clear which treatment is better. Some fraction of patients would have the organ repaired and the remaining fraction would receive transplants. The minimax-regret criterion provides a coherent method to choose the fractions.

Now consider exercise as a treatment intended to increase life-span. Here convex combinations of treatments are feasible: one can exercise in low, high, or intermediate intensities. Suppose that the objective function is concave and homogeneous across the relevant patient population, with diminishing marginal returns to higher intensity of exercise. Then a planner should not vary intensity across patients. Any diversified treatment strategy is dominated by one in which all patients exercise at the mean of the diversified intensities.

5. Analysis with an ordered choice set and unimodal objective functions

Section 2.4 showed that collective choice of the mean action may dominate individualistic decision making in circumstances where agents do not have the same concave

objective function. However, this goes only so far. Result (1) certainly does not hold universally.

This section presents a weaker result that holds when X is an ordered set of actions and each agent j has an objective function $f_j(\cdot, \cdot): X \times S \to R$ that is unimodal on X for each $s \in S$. Set X need not be convex; it may, for example, be a discrete subset of the real line. Objective functions may vary across agents, except that they share the same mode. Formally, I assume that, for each $s \in S$, $x(s) \equiv \arg\max_{x \in X} f_j(x, s)$ is invariant across $j \in J$.

Let m denote the median individualistic action; that is, $m \equiv \inf\{t : P(x_j \le t) \ge 1/2\}$. I will show that, in every state of nature, a majority of agents receive at least as high a payoff with consensus choice of action m as with individualistic decision making. That is,

$$P[f_j(m, s) \ge f_j(x_j, s)] \ge 1/2 \quad \text{for all } s \in S. \tag{4}$$

PROOF OF EQUATION (4). Result (4) is a special case of a result that holds for any quantile of the distribution of individualistic actions. For $\kappa \in (0, 1)$, let q_κ denote the κ-quantile individualistic action; that is, $q_\kappa \equiv \inf\{t : P(x_j \le t) \ge \kappa\}$. The general result is

$$P[f_j(q_\kappa, s) \ge f_j(x_j, s)] \ge \min(\kappa, 1 - \kappa) \quad \text{for all } s \in S. \tag{5}$$

To show (5), consider separately the cases in which $q_\kappa = x(s)$, $q_\kappa < x(s)$, and $q_\kappa > x(s)$.

First let $q_\kappa = x(s)$. Then $P[f_j(q_\kappa, s) \ge f_j(x_j, s)] = 1$. Next let $q_\kappa < x(s)$. Then $P[f_j(q_\kappa, s) \ge f_j(x_j, s)] \ge P(x_j \le q_\kappa) \ge \kappa$. Now let $q_\kappa > x(s)$. Then $P[f_j(q_\kappa, s) \ge f_j(x_j, s)] \ge P(x_j \ge q_\kappa) \ge 1 - \kappa$. Combining these cases yields (5). Result (4) is the special case with $\kappa = 1/2$. $\square$

Result (4) brings to mind the median voter theorem of Black (1948), which also considered an ordered set of actions and unimodal objective functions. Indeed (4) shows that, given knowledge of the state of nature, a majority of agents prefer consensus choice of action m to the set $(x_j, j \in J)$ of individualistic actions. However, (4) does not imply that a majority of agents prefer m ex ante, before the actual state is known. Recall that x_j is the action that agent j chooses in a regime with private choice. Thus, the population unanimously prefers private choice of $(x_j, j \in J)$ to consensus choice of m.

While result (4) is interesting, it is less powerful than (1), in the sense that it does not provide the foundation for a Pareto dominant collective-choice mechanism. The mean surplus in state s with collective choice of m is $\int f_j(m, s) \, dP(j) - \int f_j(x_j, s) \, dP(j)$. Result (4) does not determine the sign of this quantity. If there exists a state of nature where mean surplus is negative, it is not possible in this state to allocate payoffs so that all agents are better off than they would be with private choice.

6. JENSEN'S INEQUALITY AND RESEARCH ON CONSENSUS FORECASTING

The analysis in this paper appears to be entirely new to research on collective choice of private goods. However, a version of result (1) has received sporadic recognition in research on the distantly related subject of consensus forecasts. I discuss some of the history here.

For over a century, beginning at least as early as Galton (1907), researchers studying the accuracy of forecasts have studied settings in which multiple agents are asked to give point forecasts of a quantity and their forecasts are combined to create a *consensus forecast*. It is particularly common to define the consensus forecast as the cross-sectional mean of the individual forecasts.

Empirical studies have regularly found that the cross-sectional mean forecast is more accurate than the individual forecasts used to form the mean. Clemen (1989, p. 559) put it this way in a review article:

> The results have been virtually unanimous: combining multiple forecasts leads to increased forecast accuracy. This has been the result whether the forecasts are judgmental or statistical, econometric or extrapolation. Furthermore, in many cases one can make dramatic performance improvements by simply averaging the forecasts.

Researchers have been intrigued by this empirical regularity, which has recently become known popularly as the wisdom of crowds (Surowiecki (2004)). Various reasons have been suggested. In fact, the empirical regularity follows from Jensen's inequality.

Let $y_n, n = 1, \ldots, N$, be a set of individual point forecasts of an unknown real quantity θ, let P_N denote the cross-sectional distribution of the forecasts, and let $\mu_N \equiv \int y \, dP_N$ denote the cross-sectional mean forecast. Let $L(\cdot, \cdot) : R \times R \to [0, \infty)$ be a loss function used to measure the consequence of prediction error. Research on forecasting has typically used absolute loss $L(y, \theta) = |y - \theta|$ or square loss $L(y, \theta) = (y - \theta)^2$. When these or any other convex loss function are used, Jensen's inequality gives $L(\mu_N, \theta) \leq \int L(y, \theta) \, dP_N$ for all $\theta \in R$. Thus, whatever the actual value of the quantity being forecast, the loss associated with the mean forecast is no larger than the mean loss of the individual forecasts.

This simple result has long been known in statistical decision theory. There θ is a parameter to be estimated and $(y_n, n = 1, \ldots, N)$ is a *randomized estimate*, meaning that the statistician draws an integer i at random from the set $(1, \ldots, N)$ and uses y_i to estimate θ. Suppose that a convex loss function is used to measure precision of estimation. Then Jensen's inequality implies that loss using the *nonrandomized estimate* μ_N is smaller than expected loss using the randomized estimate. See Hodges and Lehmann (1950).

Research on consensus forecasts has largely disregarded the result as it has sought to explain why mean forecasts perform better than individual forecasts. A notable exception is McNees (1992), who exposited the matter clearly in the context of absolute and square loss. He also recognized the consensus-forecasting version of result (4); that is, the median forecast of any event must be at least as close to the truth as at least half of the individual forecasts, whatever the truth may be. McNees (1992, p. 705) observed that much research on forecasting did not acknowledge "these simple, well-known, yet often ignored arithmetic principles."

More recently, Larrick and Soll (2006) referred to the application of Jensen's inequality to consensus forecasting as the "averaging principle" and reported experimental research showing that a majority of their student subjects did not understand the principle. Thus, the power of Jensen's inequality may be plain in abstraction but not as evident in application.

7. Conclusion

The simple analysis of this paper adds to our understanding of how human heterogeneity may affect social choices. In the introduction to his Nobel Prize lecture, Sen (1999, p. 349) pointed to heterogeneity as the core problem of social choice theory, writing:

> If there is a central question that can be seen as the motivating issue that inspires social choice theory, it is this: how can it be possible to arrive at cogent aggregative judgments about the society..., given the diversity of preferences, concerns, and predicaments of the different individuals *within* the society?

A substantial part of research in social choice theory, including much of Sen's work, focuses on distributional issues stemming from heterogeneity in "predicaments." The vexing problem driving the modern study of voting mechanisms initiated by Arrow (1951) is the generic absence of a coherent collective preference ordering over actions when individual preferences over actions are heterogeneous.

The broad message of this paper is that analysis of social choice under uncertainty requires specific attention to heterogeneity in individual beliefs and decision criteria. A classical utilitarian argument holds that when agents have heterogeneous preferences over private actions, collective choice of a single action prevents persons from exercising their preferences and, hence, creates deadweight loss. However, this argument was developed in the study of deterministic settings where heterogeneity in preferences over actions can stem only from heterogeneity in choice sets or objective functions. The present analysis has shown that the classical argument does not hold in certain settings of choice under uncertainty with heterogeneous beliefs or decision criteria. The extent to which this conclusion extends to other uncertainty settings is an open question.

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