

# Online Appendix to “Optimal Regulation of Noncompete Contracts”

(Not for publication)

Liyan Shi

## B Supplementary Derivations and Proofs

### B.1 Exclusion during Noncompete Period

The calculation below shows that the enforcement of the noncompete clause leading to a production delay of duration  $\pi$  reduces the entrant match value to  $e^{-r\pi}$  fraction.

Consider an entrant match with productivity  $z_t$  at time  $t$ . If it is excluded from production for duration  $\pi$ , production cannot occur during the noncompete period,  $\forall s \in [t, t+\pi)$ . The flow payoff from production starts from time  $t+\pi$ . The joint value is

$$\mathbb{E} \left[ \int_{t+\pi}^T e^{-rs} z_s ds + e^{-r(T-t)} [J^n(z_T, \kappa) + \tau_T(\tilde{\pi}_T(\theta_T))] \right]. \quad (\text{B.1})$$

The productivity does not evolve during the noncompete period:  $z_s = z_t$ , for  $t \leq s \leq t+\pi$ . After performing a change of variable,  $\hat{s} \equiv s - \pi$  and  $\hat{T} \equiv T - \pi$ , equation (B.1) becomes

$$\mathbb{E} \left[ \int_t^{\hat{T}} e^{-r(\hat{s}+\pi)} z_{\hat{s}} d\hat{s} + e^{-r(\hat{T}-t+\pi)} [J^n(z_{\hat{T}}, \kappa) + \tau_{\hat{T}}(\tilde{\pi}_{\hat{T}}(\theta_{\hat{T}}))] \right] = e^{-r\pi} J^n(z_t, \kappa).$$

### B.2 Contracting Cost Specification

This section shows that the model allows for a different interpretation of the contracting cost. Instead of legal costs, I now treat the contracting costs as disutilities workers suffer due to perceived restricted opportunity.

The promise-keeping constraint (1) is modified to

$$\mathbb{E} \left[ \int_0^T e^{-rt} (w_t - \kappa z_t) dt + e^{-rT} J(z_T, \kappa) \right] \geq U_0. \quad (\text{B.2})$$

The firm value in (3) is modified to

$$V^n(z_0, U_0, \kappa) = \max_{\{w_t, \mu_t, \mathcal{M}_t\}_{t \geq 0}} \mathbb{E} \left[ \int_0^T e^{-rt} (z_t - c(\mu_t) z_t - w_t) dt + e^{-rT} \tau_T(\tilde{\pi}_T(\theta_T)) \right]. \quad (\text{B.3})$$

subject to the PK constraint (B.2) and the entrant's IC constraint (4) and IR constraint (5).

Compared to (1) and (3), the firm does not directly bear the contracting costs, but it adjusts for the costs imposed on the worker when making wage payments to deliver the promised utility. Incorporating the PK constraint (B.2) into the firm's objective in (B.3), I obtain the same bilateral efficiency result in Lemma 1. Therefore, it is irrelevant for efficiency implications whether the firm or the worker bears the contracting costs. The only difference is that, in the latter case, the wage adjusts upwards to compensate the worker for the cost.

### B.3 Sequential to Recursive Formulation

This section complements the proof for Lemma 1 in Section A.1 and formally derives the recursive formulation for the joint maximization problem from the sequential one.

I first note that it is without loss of generality to assume that the decision to include a noncompete clause is perfectly persistent over time, i.e., if a firm finds it profitable to include a noncompete clause at the beginning of the match, it will find it profitable to do so in the future. I define the time  $t$  discounting factor for time  $s \geq t$  payoff, adjusting for the job-to-job transition rates in between  $\{\eta_x^i\}_{t \leq x \leq s}$ :

$$R^i(t, s) = \exp\left(-\int_t^s (r + \eta_x^i) dx\right), \quad i \in \{c, n\}.$$

**Absent noncompete clauses.** Consider the case absent noncompete clauses. Using the adjusted discount factors, I rewrite the firm's objective (2) and the PK constraint (1) as:

$$V^c(z_t, U_t) = \max_{\{w_s, \mu_s\}_{s \geq t}} \mathbb{E} \left[ \int_t^\infty R^c(t, s) (z_s - c(\mu_s) z_s - w_s) ds \right]$$

subject to

$$\mathbb{E} \left[ \int_t^\infty R^c(t, s) (w_s + \eta_s^c J^c(z_s)) ds \right] \geq U_t.$$

By the Martingale Representation Theorem, there exists a process  $\{\sigma_t^U\}_{t \geq 0}$  such that  $\{U_t\}_{t \geq 0}$  satisfies the following stochastic differential equations:

$$dU_t = [(r + \eta_t^i) U_t - w_t - \eta_t^i J^c(z_t)] dt + \sigma_t^U dB_t.$$

Thus, the firm's value function follows the HJB equation (dropping the time subscript):

$$(r+\eta^c)V^c(z,U) = \max_{w,\mu,\sigma^U} \left\{ z-c(\mu)z-w+\mu zV_z^c(z,U)+\frac{1}{2}\sigma^2z^2V_{zz}^c(z,U) \right. \\ \left. +V_U^c(z,U)[(r+\eta^c)U-w-\eta^cJ^c(z)]+\frac{1}{2}(\sigma^U)^2V_{UU}^c(z,U)+\sigma z\sigma^U V_{zU}^c(z,U) \right\}. \quad (\text{B.4})$$

Taking the derivative with respect to  $w$ , I obtain

$$V_U^c(z,U) \geq -1 \text{ with “} = \text{” if } w > 0. \quad (\text{B.5})$$

If  $\lambda$  is not too large, the wage non-negativity constraint will never bind. In this case,  $V_U^c(z,U) = -1$ . Further, I obtain that  $V_{UU}^c(z,U) = 0$ ,  $V_{zU}^c(z,U) = 0$ , and  $J^c(z) = V^c(z,U)+U$ . Substituting these equations into the HJB equation (B.4), it becomes equation (7).

**With noncompete clauses.** For the case with noncompete clauses, I follow the same steps as before. I rewrite the firm's objective in (3) and the PK constraint (1) as:

$$V^n(z_t, U_t, \kappa) = \max_{\{w_s, \mu_s, \mathcal{M}_s\}_{s \geq t}} \mathbb{E} \left[ \int_t^\infty R^n(t, s) \left( z_s - c(\mu_s)z_s - \kappa z_s - w_s + \lambda p \int_{\tilde{\theta}_s}^\infty \tau_s(\tilde{\pi}_s(\theta_s)) dF(\theta_s) \right) ds \right]$$

subject to

$$\mathbb{E} \left[ \int_t^\infty R^n(t, s) (w_s + \eta_s^n J^n(z_s, \kappa)) ds \right] \geq U_t,$$

and the entrant firms' IC and IR constraints (4) and (5).

By the same reasoning, there also exists a process  $\{\sigma_t^U\}_{t \geq 0}$  such that  $\{U_t\}_{t \geq 0}$  satisfies

$$dU_t = [(r+\eta_t^i)U_t - w_t - \eta_t^i J^n(z_t, \kappa)] dt + \sigma_t^U dB_t.$$

Thus, the firm's value function follows the HJB equation (dropping the time subscript):

$$(r+\eta^n)V^n(z,U,\kappa) = \max_{w,\mu,\mathcal{M},\sigma^U} \left\{ z-c(\mu)z-\kappa z-w+\lambda p \int_{\tilde{\theta}^n}^\infty \tau(\tilde{\pi}(\theta))dF(\theta)+\mu zV_z^n(z,U) \right. \\ \left. +\frac{1}{2}\sigma^2z^2V_{zz}^n(z,U)+V_U^n(z,U)[(r+\eta^n)U-w-\eta^nJ^n(z,\kappa)]+\frac{1}{2}(\sigma^U)^2V_{UU}^n(z,U)+\sigma z\sigma^U V_{zU}^n(z,U) \right\}. \quad (\text{B.6})$$

If the wage non-negativity constraint does not bind, we also have  $V_U^n(z,U) = -1$ . Further,  $V_{UU}^n(z,U,\kappa) = 0$ ,  $V_{zU}^n(z,U,\kappa) = 0$ , and  $J^n(z,\kappa) = V^n(z,U,\kappa)+U$ . Substituting these equations into the HJB equation (B.6), it becomes equation (8).

## B.4 Proof of Corollary 1

If the hazard rate is constant:

$$\bar{\theta}^{n*}(\kappa) = 1 + \frac{\varepsilon\Delta}{\varepsilon\Delta+1} \frac{1-F(\bar{\theta}^{n*}(\kappa))}{f(\bar{\theta}^{n*}(\kappa))} = 1 + \frac{\varepsilon\Delta}{\varepsilon\Delta+1} \frac{1-F(\bar{\theta}^n)}{f(\bar{\theta}^n)} = 1 + \frac{\varepsilon\Delta}{\varepsilon\Delta+1} (\bar{\theta}^n - 1). \quad (\text{B.7})$$

Mapping to the duration cap and using  $e^{r\pi} - 1 \approx r\pi$  and  $\log\left(1 + \frac{\varepsilon\Delta}{\varepsilon\Delta+1} r\pi\right) \approx \frac{\varepsilon\Delta}{\varepsilon\Delta+1} r\pi$ :

$$\pi(\kappa) = \frac{1}{r} \log(\bar{\theta}^{n*}(\kappa)) = \frac{1}{r} \log\left(1 + \frac{\varepsilon\Delta}{\varepsilon\Delta+1} (e^{r\pi} - 1)\right) \approx \frac{\varepsilon\Delta}{\varepsilon\Delta+1} \pi.$$

The approximation in the last step is a good one when  $r\pi$  is reasonably small.

If the monotone hazard rate is increasing, the second “=” in equation (B.7) becomes a “ $\geq$ ”. Thus, the approximation provides a lower bound for the precise solution. Similarly, if the monotone hazard rate is decreasing, the approximation is an upper bound.

## B.5 Aggregation

This section shows the aggregation of the economy. In steady state, the distribution converges to a stationary one,  $\lim_{t \rightarrow \infty} g(z, \kappa, t) = g(z, \kappa)$ . Building on Proposition 1, I use the conditional distributions in the steady state to compute the aggregate productivity. That is, the density of matches conditional on not being subject to noncompete,  $g^c(z)$ , and the density of matches conditional on being subject to noncompete and of cost type  $\kappa$ ,  $g^n(z; \kappa)$ . These conditional densities follow the stationary version of the KF equations:

$$0 = -\mu^c g_z^c(z) + \frac{1}{2} \sigma^2 g_{zz}^c(z) + \delta[h(z) - g^c(z)] + \lambda \int_1^\infty \left[ g^c\left(\frac{z}{\theta}\right) - g^c(z) \right] dF(\theta) \quad (\text{B.8})$$

$$0 = -\mu^n(\kappa) z g_z^n(z; \kappa) + \frac{1}{2} \sigma^2 z^2 g_{zz}^n(z; \kappa) + \delta[h(z) - g^n(z; \kappa)] \quad (\text{B.9})$$

$$+ \lambda \left\{ p \int_{\bar{\theta}^n}^\infty \left[ g^n\left(\frac{z}{\theta}; \kappa\right) - g^n(z; \kappa) \right] dF(\theta) + (1-p) \int_1^\infty \left[ g^n\left(\frac{z}{\theta}; \kappa\right) - g^n(z; \kappa) \right] dF(\theta) \right\}.$$

**Lemma B.1** (Aggregation). *The steady-state aggregate productivity is  $Z = Z^c(1 - \Phi(\bar{\kappa})) + \int_0^{\bar{\kappa}} Z^n(\kappa) d\Phi(\kappa)$ , where, depending on whether not subject to noncompete clauses, the conditional aggregate productivity is*

$$Z^c = \frac{\delta \int z dH(z)}{\delta - \mu^c - \lambda \int_1^\infty (\theta - 1) dF(\theta)} \quad (\text{B.10})$$

$$Z^n(\kappa) = \frac{\delta \int z dH(z)}{\delta - \mu^n(\kappa) - \lambda \left[ p \int_{\bar{\theta}^n}^\infty (\theta - 1) dF(\theta) + (1-p) \int_1^\infty (\theta - 1) dF(\theta) \right]}. \quad (\text{B.11})$$

*Proof.* First, to calculate  $Z^c = \int_0^\infty z g^c(z) dz$ , multiply the KF equation (B.8) by  $z$  and integrate it from  $z = 0$  to  $z = \infty$ :

$$0 = \int_0^\infty z \left\{ -\mu^c g_z^c(z) + \frac{1}{2} \sigma^2 g_{zz}^c(z) + \delta [h(z) - g^c(z)] + \lambda \int_1^\infty \left[ g^c\left(\frac{z}{\theta}\right) - g^c(z) \right] dF(\theta) \right\} dz$$

Integrating by parts, performing a change of variables, and combining the terms:

$$\delta \int z h(z) dz = \left[ \delta - \mu^c - \frac{1}{2} \sigma^2 - \lambda \int_1^\infty (\theta - 1) dF(\theta) \right] \int_0^\infty z g^c(z) dz,$$

from which I obtain the expression for  $Z^c$  in (B.10).

Similarly, applying the same steps to KF equation (B.9), I obtain the expression for  $Z^n(\kappa)$  in (B.11).  $\square$

The aggregation formulas in (B.10) and (B.11) also illustrates the investment-reallocation trade-off in the intensive margin. Rent extraction leads to higher productivity improvement from investment,  $\mu^n(\kappa) > \mu^c$ , at the expense of lower productivity improvement from worker reallocation,  $\int_{\bar{\theta}^n}^\infty (\theta - 1) dF(\theta) < \int_1^\infty (\theta - 1) dF(\theta)$ . It also shows that, for the functional form  $H(\cdot)$ , the average value of newborn match productivity,  $\int z dH(z)$ , is the relevant statistic.

Given Lemma B.1, I obtain the steady-state *aggregate net output*, or aggregate consumption by accounting for the investment and contracting costs  $Y = Y^c(1 - \Phi(\bar{\kappa})) + \int_0^{\bar{\kappa}} Y^n(\kappa) d\Phi(\kappa)$ , where

$$Y^c = Z^c(1 - c(\mu^c)) \quad \text{and} \quad Y^n(\kappa) = Z^n(\kappa)(1 - c(\mu^n(\kappa)) - \kappa).$$

## B.6 Welfare Measure

Lemma 4 implies that

$$S(z, \kappa) = s(\kappa)z + \frac{\delta}{\rho + \delta} S_0, \quad \text{where } s(\kappa) = s^c \mathbf{1}_{\{i(\kappa)=c\}} + s^n(\kappa) \mathbf{1}_{\{i(\kappa)=n\}}. \quad (\text{B.12})$$

Integrating equation (B.12) across the distribution of the newborn matches, I obtain the average social value of the newborn matches satisfies

$$S_0 = \iint S(z, \kappa) dH(z) d\Phi(\kappa) = \int s(\kappa) d\Phi(\kappa) \int z dH(z) + \frac{\delta}{\rho + \delta} S_0,$$

which implies that

$$S_0 = \frac{\rho + \delta}{\rho} \int s(\kappa) d\Phi(\kappa) \int z dH(z). \quad (\text{B.13})$$

Substituting equation (B.13) into equation (B.12), I obtain that the social value function is

$$S(z, \kappa) = s(\kappa)z + \frac{\delta}{\rho} \int s(\kappa) d\Phi(\kappa) \int z dH(z).$$

**Steady-state welfare.** The steady-state welfare is obtained by aggregating the social values  $S(z, \kappa)$  across the steady-state distribution  $G(z, \kappa)$ :

$$\begin{aligned} \mathcal{W}^{ss} &= \iint S(z, \kappa) dG(z, \kappa) = \iint s(\kappa)z dG(z, \kappa) + \frac{\delta}{\rho} \int s(\kappa) d\Phi(\kappa) \int z dH(z) \\ &= s^c \left( Z^c + \frac{\delta}{\rho} \int z dH(z) \right) (1 - F(\bar{\kappa})) + \int_0^{\bar{\kappa}} s^n(\kappa) \left( Z^n(\kappa) + \frac{\delta}{\rho} \int z dH(z) \right) dF(\kappa). \end{aligned}$$

Recall the expressions for  $s^c$  and  $s^n(\kappa)$  in (35) and (36). The following relations hold:

$$\begin{aligned} s^c \left( Z^c + \frac{\delta}{\rho} \int z dH(z) \right) &= \frac{Y^c}{\rho} \\ s^n(\kappa) \left( Z^n(\kappa) + \frac{\delta}{\rho} \int z dH(z) \right) &= \frac{Y^n(\kappa)}{\rho}. \end{aligned}$$

Thus, the steady state welfare equals the discounted flow of the steady-state net output:

$$\mathcal{W}^{ss} = \frac{1}{\rho} \left[ Y^c (1 - F(\bar{\kappa})) + \int_0^{\bar{\kappa}} Y^n(\kappa) d\Phi(\kappa) \right] = \frac{Y}{\rho}.$$

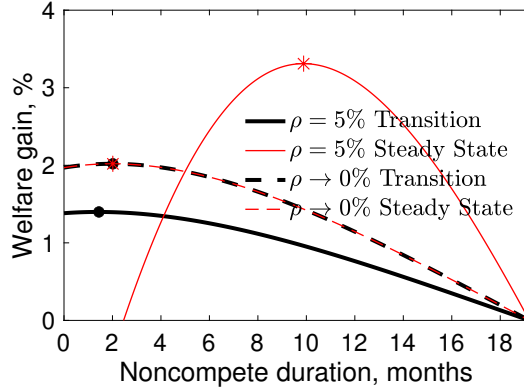
**Time-zero welfare.** The time-zero welfare is obtained by aggregating the social values  $S(z, \kappa)$  across the given initial distribution  $G(z, \kappa, 0)$ :

$$\mathcal{W}^0 = \iint S(z, \kappa) dG(z, \kappa, 0) = \iint s(\kappa)z dG(z, \kappa, 0) + \frac{\delta}{\rho} \int s(\kappa) d\Phi(\kappa) \int z dH(z).$$

**Sensitivity to discount rate.** I carry out an exercise to check how sensitive the optimal noncompete policy and the resulting welfare gain are to the discount rate  $\rho$ . With a lower discount rate, investment has a higher social value: the cost is incurred in the present moment, but the benefit accrues over time. Therefore, the optimal noncompete policy would tilt toward a higher noncompete duration cap to protect more investment.

In addition, the discount rate  $\rho$  also drives the discrepancy between the time-zero welfare and the steady-state welfare. By disregarding the delay in investment payoffs, the steady-state welfare tends to prescribe a less stringent noncompete cap than the time-zero one. When the discount factor  $\rho \rightarrow 0$ , the time-0 welfare coincides with the steady-state welfare, i.e.,  $\mathcal{W}^0 \rightarrow \mathcal{W}^{ss}$ , since the planner values only the streams of steady-state output in the

Figure B.1: Sensitivity of intensive-margin duration cap to the discount rate



Notes: The welfare gains are in the Florida-level enforceability regime, i.e.,  $p = 1$ .

distant future. The discount rate discrepancy between the two measures disappears.

Quantitatively, Figure B.1 illustrates the discount rate effect. It plots the welfare gains *in the intensive margin* from capping noncomplete duration from zero to the private-optimal level at various discount rates. First, with the calibrated model with the discount rate  $\rho = 5\%$  in Section 5.2, the time-zero welfare accounting for the transition path suggests an optimal cap of 1.6 months. In comparison, the steady-state welfare peaks at a duration cap of 10 months. Second, consider an experiment of imposing a zero discount rate, i.e.,  $\rho \rightarrow 0$ , and recalibrate the model. Still, even at the lowest possible discount rate favoring more protection of incumbent investment, the welfare-maximizing duration cap is very stringent at 2 months. Moreover, as the discount rate effect vanishes, the two welfare criteria converge to the same consistent metric, peaking at a two-month cap.

## C Extensions and Discussion

### C.1 Entrant Match Quality Distribution

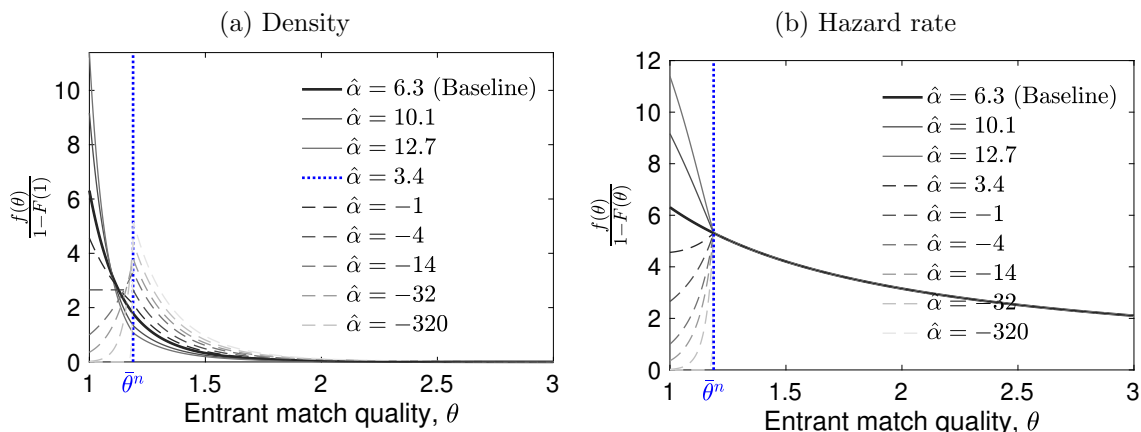
For the entrant match quality, consider a double Pareto distribution centered around  $\theta_c \geq 1$ , with a left tail parameter  $\hat{\alpha}$  and a right tail parameter  $\alpha$ . For  $\theta \geq 1$ , the normalized density is

$$\frac{f(\theta)}{1-F(1)} = \begin{cases} a\theta_c^\alpha\theta^{-(\alpha+1)}, & \theta \geq \theta_c \\ a\theta_c^{\hat{\alpha}}\theta^{-(\hat{\alpha}+1)}, & 1 \leq \theta < \theta_c, \end{cases}$$

where  $a = 1/\left(\frac{1}{\alpha} - \frac{1-\theta_c^{\hat{\alpha}}}{\hat{\alpha}}\right)$ . This distribution nests the baseline one when  $\hat{\alpha} = \alpha$ .

For illustration, I impose that  $\theta_c = \bar{\theta}^n = \frac{\alpha}{\alpha-1}$  such that the private-optimal solution to the

Figure C.1: Entrant match quality distribution



poaching threshold equation (19) is identical to the baseline one. I then vary the shape of the left tail  $\hat{\alpha}$  such that the model generates various magnitudes of mobility decline. A lower  $\hat{\alpha}$  implies that fewer opportunities are blocked and thus a smaller decline in mobility. I adjust the arrival rate  $\lambda$  such that the job-to-job transition rate for workers under noncompete clauses is kept the same as the baseline, which ensures that the remaining model parameters are unchanged when matching the data. Figure C.1 plots the distributions underlying Figure 7 (b) and (d).<sup>1</sup> In extreme cases with thin left tails (very negative  $\hat{\alpha}$ ), the hazard rate can vary a lot along the entrant match distribution and can affect the policy substantially.

## C.2 Selection Effect

This section contains the supplementary details about the two alternative specifications for the selection channel in Section 5.3. The first setting assumes away the contracting cost. This specification shuts down the selection channel in the extensive margin entirely and only allows the intensive margin to operate. I refer to this as a *no-selection* economy.

**Lemma C.1** (No Selection). *Consider an economy with no contracting cost, i.e.,  $\kappa = 0$ .*

- (i) *In the laissez-faire equilibrium, all firm-worker matches include a noncompete clause, the terms of which are characterized in Proposition 1.*
- (ii) *In the social optimum, the planner lets all matches to use a noncompete clause but with a lower duration, which can be implemented by a duration cap as in Corollary 1.*

<sup>1</sup>I rule out pathological cases of entrant match quality distribution. One pathological case is when the entrant types blocked by noncompete clauses are concentrated around  $\theta = 1$ . Since the social gains from reallocation the planner can recuperate are negligible, the unregulated equilibrium is efficient. With the appropriate data, a non-parametric estimation of the distribution  $F(\theta)$  would be valuable.



The economy considered in Lemma C.1 would imply 100% noncompete prevalence, which is in stark contrast with the data. To realign with the overall data, I consider a second setting with a binary cost distribution below. I refer to this as the *exogenous-selection* model.

**Lemma C.2** (Exogenous Selection). *Consider an economy where the contracting cost type follows a two-point distribution:  $\kappa \in \{0, \infty\}$ , with probability  $\bar{\phi}$  the cost is zero.*

- (i) *In the laissez-faire equilibrium, a  $\bar{\phi}$  proportion of the firm-worker matches include a noncompete clause, the terms of which are characterized in Proposition 1.*
- (ii) *In the social optimum, the planner lets  $\bar{\phi}$  proportion of matches to use a noncompete clause but with a lower duration according to formula (28) in Proposition 3.*

The exogenous-selection economy characterized above does not capture the variation of noncompete prevalence across different enforceability regimes. The baseline economy, which I refer here as the *endogenous-selection* version, is parameterized exactly to capture the selection effect that is prominent in the data.

The two alternative specifications are useful for two purposes. First, both economies isolate the selection effect in the extensive margin from the duration setting in the intensive margin, which are otherwise entangled in the benchmark model. Second, they help to illustrate and isolate the role of the contracting costs in shaping the optimal policy and the welfare outcomes. Given that the matches selecting into noncompete contracts do not incur any costs, i.e.,  $\mathbb{E}[\kappa|\kappa \leq \bar{\kappa}] = 0$ , the costs on their own do not affect the welfare number.

I recalibrate the model and report the parameter values in Table C.1. The change in the contracting cost distribution leads to recalibration of the investment cost function in the no-selection and exogenous-selection economies, but the reminders of the parameters from the endogenous-selection economy are unchanged. Recall the investment cost elasticity equation (??). Since the matches do not incur any costs for signing noncompete contracts, i.e.,  $\mathbb{E}[\kappa|\kappa \leq \bar{\kappa}] = 0$ , the payoff for doing so  $\lambda p(\bar{\theta}^n - 1)(1 - F(\bar{\theta}^n)) - \frac{1}{j^c} \mathbb{E}[\kappa|\kappa \leq \bar{\kappa}]$  is higher. Therefore, the investment elasticity that matches the investment response  $(\mathbb{E}[c(\mu^n(\kappa))] - c(\mu^c))/c(\mu^c)$  in the data is lower,  $\varepsilon = 3.2$ .

Table C.2 summarizes the policy outcomes for the two alternative specifications in comparison to the baseline one. The table reports two policies—implementing the social optimum and a complete ban—in a full-enforcement regime  $p = 1$ . In the no-selection and exogenous-selection economies, the social optimum and the duration-cap-only policy achieve the same outcomes, precisely because there is no selection effect. Hence, the analysis for the duration-cap-only policy is omitted.

Table C.1: Recalibrated parameters for the selection channel

Specification	Endo. Selection	No Selection	Exo. Selection
Contracting cost $\kappa$	$\log(\kappa) \sim N(-4.3, 0.98)$	$\kappa \in \{0\}$	$\kappa \in \{0, \infty\}, \bar{\phi} = 0.7$
Investment cost elasticity $\varepsilon$	6.8	3.2	3.2
Investment cost level $\phi$	24.7	50.4	50.4

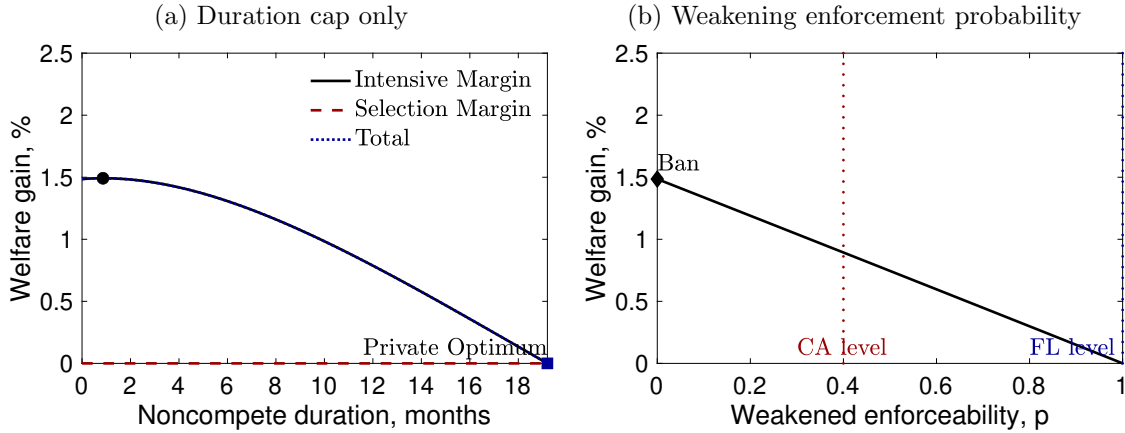
Table C.2: Noncompete policies: alternative specification of the contracting costs

Policy Specification	Social Optimum			Ban		
	Endo. Selection	No Selection	Exo. Selection	Endo. Selection	No Selection	Exo. Selection
<i>Equilibrium</i>						
Duration (months), $\pi$	19.2	19.2	19.2	19.2	19.2	19.2
Prevalence, $F(\bar{\kappa})$	70%	100%	70%	70%	100%	70%
<i>Contract restriction</i>						
Duration cap (months), $\pi^*$	1.6 <sup>a</sup>	0.8 <sup>a</sup>	0.8 <sup>a</sup>	0	0	0
Prevalence, $F(\bar{\kappa}^*)$	$\approx 0\%$	100%	70%	0%	0%	0%
<i>Policy outcome</i>						
$\Delta$ Job-to-job rate	1.26%	1.67%	1.17%	1.26%	1.80%	1.26%
$\Delta$ Investment rate	-1.19%	-1.47%	-1.03%	-1.19%	-1.65%	-1.16%
Welfare gain (transition)	2.25%	2.15%	1.49%	2.25%	2.14%	1.48%
<i>Decomposition: Total</i>						
= Reallocation	1.53%	2.27%	1.58%	1.55%	2.28%	1.58%
+ Investment	-0.13%	-0.12%	-0.09%	-0.16%	-0.14%	-0.10%
+ Selection	0.85%	0%	0%	0.87%	0%	0%

*Notes:* The superscript *a* indicates that, while the optimal duration cap  $\pi^*(\kappa)$  depends on the cost type  $\kappa$ , the variation is negligible quantitatively. The welfare gain (transition) computes the gain along the transition path after imposing the policy in the laissez-faire steady state in the Florida-level enforceability regime, i.e.,  $p = 1$ . The decomposition follows the order of imposing the reallocation, investment, and selection outcomes resulting from the policy prescribed.

To implement the social optimum, the no-selection and exogenous-selection economies prescribe a lower duration cap than the baseline endogenous-selection model. The cap reduces from 1.6 months  $\pi$  to 0.8 months. The reason is that, with a lower investment elasticity, the social-optimal point along the investment-reallocation trade-off in equation (28) favors capturing more reallocational gain. With the extensive margin shut down, this shift is driven entirely by the intensive margin consideration. Figure C.2(a) further illustrates this compositional change. The total welfare gain from a duration-cap-only policy peaks at 0.8 months. This is in contrast with the endogenous-selection economy, where the selection margin pushes the duration cap to zero as shown in Figure 6(a).

Figure C.2: Welfare gain: exogenous-selection economy



Notes: The welfare gains are in the Florida-level enforceability regime, i.e.,  $p = 1$ .

Overall, regardless of whether the selection effect is operative, all specifications of the model imply that the optimal duration cap is very low, to the extent that a complete ban achieves roughly the same welfare outcome. However, the two alternative specifications imply a welfare gain of a lower magnitude than the endogenous-selection model. For example, the welfare gain from implementing the social optimum reduces from 2.25% to 2.15% in the no-selection model and 1.49% in the exogenous-selection model. Unsurprisingly, the welfare decomposition shows that the gap between the exogenous-selection economy and the baseline one comes from the selection margin.

### C.3 Free Entry

In this section, I discuss the free-entry extension in Section 5.2. To account for potential entry channel in general equilibrium, I endogenize the arrival rate of outside opportunities by introducing a random labor search market and costly free entry of new firms.

Consider the following extension to the model in Section 2. A measure-one of employed workers search on-the-job, and a measure-one of ex-ante identical entrants post vacancies. Each entrant decides to post  $v$  vacancies by incurring a cost  $K(v)Z$ , which is proportional to the aggregate productivity  $Z$ . The job-posting cost function  $K(v)$  is increasing, differential, and convex in  $v$ . Suppose that the workers and the vacancies are matched with a Cobb-Douglas meeting technology  $\lambda(v) = \lambda_0 1^{1-\omega} v^\omega$ , where the entry congestion parameter  $\omega \in [0,1]$ . Thus, the arrival rate of outside matches for employed workers is  $\lambda(v)$ , and the vacancy filling rate for a posted job is  $\lambda(v)/v$ . Upon being matched to a worker of productivity  $z$ , the entrant match draws quality  $\theta$  from a distribution with cumulative density function  $F(\theta)$ .

The free entry condition is such that the marginal value of posting one additional vacancy

equals the marginal cost. In steady state, this condition is

$$\begin{aligned} \frac{\lambda(v)}{v} \iint \left\{ \int_{\bar{\theta}^c}^{\infty} [J^c(z\theta) - J^c(z)] dF(\theta) \mathbf{1}_{\{i(\kappa)=c\}} + \left[ p \int_{\bar{\theta}^n}^{\infty} [J^n(z\theta, \kappa) - J^n(z\bar{\theta}^n, \kappa)] dF(\theta) \right. \right. \\ \left. \left. + (1-p) \int_{\bar{\theta}^c}^{\infty} [J^n(z\theta, \kappa) - J^n(z, \kappa)] dF(\theta) \right] \mathbf{1}_{\{i(\kappa)=n\}} \right\} dG(z, \kappa) \leq K'(v)Z, \end{aligned} \quad (\text{C.1})$$

which holds with strict equality if the vacancy posting is positive,  $v > 0$ .

**Transition path.** Relative to the fixed entry economy, the free-entry economy here features an aggregate state, the amount of vacancies  $v$  posted. This state affects the decisions of incumbent matches through the arrival rate of outside offers workers receive. In the expression of the joint value functions, one modification is made: the arrival rate  $\lambda$  is replaced by  $\lambda(v)$ . Before studying the policy outcomes, I first show that the economy exhibits simple transition dynamics after a policy change.

**Lemma C.3.** *Consider the steady state of the laissez-faire equilibrium in the free-entry economy. After imposing a noncompete regulation, the vacancy posting  $v$  jumps immediately to the new steady-state level. Correspondingly, the contract choices  $\{I(\kappa), \pi\}$  and on-the-job investments  $\{\mu^c, \mu^n(\kappa)\}$  also jump immediately to their new steady-state levels. However, the distribution of matches  $G(z, \kappa, t)$  converges slowly to the new steady-state one.*

*Proof.* The proof follows a guess-and-verify method. I first guess that the entry decision  $v$  immediately adjusts to the new steady-state level. Given this guess, Lemma 2 holds. That is, the contract choices  $\{I(\kappa), \pi\}$  and on-the-job investments  $\{\mu^c, \mu^n(\kappa)\}$  also jump to their new steady-state levels. At time  $t$ , the value of making a successful match for an entrant is

$$\begin{aligned} \iint \left\{ \int_{\bar{\theta}^c}^{\infty} [J^c(z\theta) - J^c(z)] dF(\theta) \mathbf{1}_{\{i(\kappa)=c\}} + \left[ p \int_{\bar{\theta}^n}^{\infty} [J^n(z\theta, \kappa) - J^n(z\bar{\theta}^n, \kappa)] dF(\theta) \right. \right. \\ \left. \left. + (1-p) \int_{\bar{\theta}^c}^{\infty} [J^n(z\theta, \kappa) - J^n(z, \kappa)] dF(\theta) \right] \mathbf{1}_{\{i(\kappa)=n\}} \right\} dG(z, \kappa, t) \end{aligned}$$

can be calculated according to

$$\left\{ \int_1^{\infty} (\theta - 1) dF(\theta) j^c (1 - \Phi(\bar{\kappa})) + \left[ p \int_{\bar{\theta}^n}^{\infty} (\theta - \bar{\theta}^n) dF(\theta) + (1-p) \int_1^{\infty} (\theta - 1) dF(\theta) \right] \int_0^{\bar{\kappa}} j^n(\kappa) \phi(\kappa) \right\} Z_t,$$

which is proportional to the aggregate productivity  $Z_t$ . The vacancy posting cost in (C.1) is also proportional to the aggregate productivity  $Z_t$ . This verifies that indeed the vacancy

Table C.3: Recalibrated parameters for the entry channel

Specification	Fixed Entry	Free Entry
Matching level $\lambda_0$	0.14	0.14
Entrant congestion $\omega$	0	1
Vacancy posting cost $k$	0	0.038

posting reaches its new steady state level immediately. □

**Entry channel.** This extended model nests the baseline model in Section 2 when entry is fully congested and job posting is costless, i.e.,  $\omega = 0$  and  $K(v) = 0$ . I label the baseline model as the “*fixed-entry*” economy here.

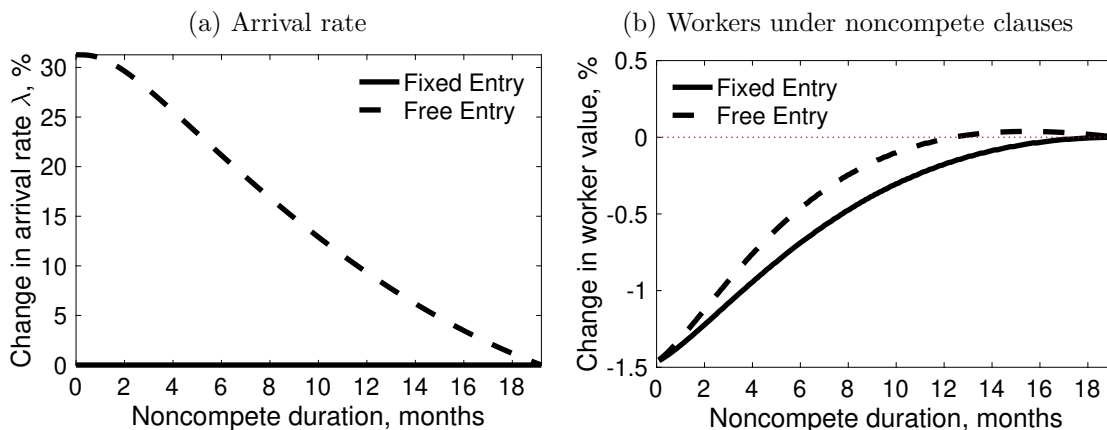
For the remainder of the analysis, I consider the other extreme case absent any entry congestion, i.e.,  $\omega = 1$ . In this case, the arrival rate of outside opportunity is  $\lambda(v) = \lambda_0 v$ , and the vacancy filling rate is constant  $\lambda_0$ . This economy is a suitable basis for introducing the trade-off associated with noncompete contracts. This economy starts from the *efficient turnover premise* as our baseline model: absent the interference of noncompete clauses, the economy features efficient job-to-job reallocation. Moreover, this economy also starts from the *efficient entry premise*: it features efficient entry when the entrants are competing for the workers on level playing field with the incumbents. To see why, note that the Hosios condition holds: absent noncompete clauses, the entrants capture the entire surplus from new matches, aligned with their share of contribution to the creation of new matches.<sup>2</sup> However, when we introduce endogenous investment, there is underinvestment given the holdup problem. Noncompete clauses protect and encourage investments at the expenses of distorting both the job-to-job reallocation and the entry.

For illustration, I specify a posting cost function  $K(v) = \frac{k}{3}v^3$  and pick the value of  $k$  such that the equilibrium level of vacancy posting is normalized to  $v = 1$  in the full-enforcement regime. The parameters for this free-entry model are reported in Table C.3 alongside the baseline fixed-entry model. The remaining parameters reported in Table 4 are unchanged.

Figure 8(b) shows the welfare gains from policies capping the noncompete duration in the fixed-entry and free-entry models. In the free-entry model, while the intensive and extensive margins are roughly equal to the magnitudes in the fixed-entry model, a new entry margin arises and increases the total welfare gain. To supplement the results presented in Section 5.3, I plot the changes in the entry rate in Figure C.3(a). Compared to the fixed arrival rate in the baseline model, the vacancy posted and the arrival rate increase as we impose a

<sup>2</sup>This corresponds to a special case of the [Postel-Vinay and Robin \(2002\)](#) model with endogenous entry studied by [Gautier et al. \(2010\)](#).

Figure C.3: Entry channel: duration-cap-only policies



Notes: The policy counterfactual is in the Florida-level enforceability regime, i.e.,  $p = 1$ .

cap on the noncompete duration. Therefore, the job-to-job transition rate increases for two reasons: workers are more likely to find a new opportunity, and they are more likely to take on the new opportunity.

**Implications for worker welfare.** As mentioned in Section 3.3, the individual agents take the arrival rate as given and do not internalize the effects of their contracts on the aggregate vacancy posting. As noncompete contracts make entry less profitable, entry firms post fewer vacancies, and workers are less likely to get an outside offer. This entry margin can potentially offset the extra compensation individual workers get from signing noncompete contracts in partial equilibrium. One could think of this force as noncompete contracts reducing aggregate entry and worsening overall labor market monopsony.

I use the illustrative example above to show this effect. To measure worker welfare, I use the initial value obtained by newborn workers, i.e., the discounted stream of their lifetime wage. Figure C.3(b) shows the change in the value for workers under noncompete clauses. In the fixed-entry model, the worker value decreases as the noncompete duration decreases. In the free-entry model, the worker value increases initially as the noncompete duration increases, when the increase in the arrival rate of outside matches dominates the decrease in rent per match.

The example above is for illustration and probably a little stark numerical. I leave a more careful quantitative assessment for future investigation. Moreover, to analyze the distributional effects on workers properly, the generalized three-party bargaining by Cahuc et al. (2006) would allow more flexibility than the Postel-Vinay and Robin (2002) framework. Instead of the entrant capturing the full surplus from job-to-job reallocation, the entrant and the worker split that surplus. Therefore, the generalized three-party bargaining allows the

possibility that noncompete clauses can hurt the workers in the same way they hurt entrants.

## C.4 Bargaining Games

Bargaining takes place in several instances in the model: (1) the bilateral bargaining between a newborn firm and a newborn worker when forming a match, (2) the three-party bargaining when an entrant arrives to poach an employed worker, and (3) negotiating for a buyout payment after the entrant poaches the worker. Furthermore, bargaining under asymmetric information tends to introduce additional complexities compared to the standard perfect-information settings. So additional clarifications of the information structure and the bargaining games are in order.

**Bilateral firm-worker bargaining.** The two parties involved in a match are *perfectly informed* about the characteristics of their match.<sup>3</sup> Thus, for a newborn match entering into the economy, the negotiation process follows the alternating-offer bargaining game by [Rubinstein \(1982\)](#), which delivers the Nash-bargaining solution. The bargaining powers are determined by the relative impatience of the two bargaining parties, with the worker's bargaining weight denoted by  $\beta$ .

In addition, to focus on the contracting problem concerning job-to-job mobility, the possibilities of worker unemployment and firm replacement of workers are eliminated. If failing to reach an agreement, both the firm and the worker exit the economy. Thus, their outside options are normalized to zero.

Summarizing, at the formation of the match, the worker receives  $\beta$  share of the maximized joint value  $J(z_0, \kappa)$  obtained, as captured by equation (15). This value is delivered by all future wage payments anticipated in all jobs promised in the long-term contract.

**Three-party bargaining.** When an outside offer arrives for an employed worker, the two competing firms are *asymmetrically informed* about their respective match productivity. While the worker is perfectly informed about both the incumbent match and the new match, she stands in as the object for the bidding and does not play an active role.

When bidding for the worker, given the information asymmetry between the two competing firms, the bargaining protocol departs from the standard ones in the models by [Postel-Vinay and Robin \(2002\)](#) and [Cahuc et al. \(2006\)](#). Instead of the two firms making alternating offers to the worker, the firms bid in an ascending (English) auction.

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<sup>3</sup>The information friction within a firm-worker match in labor search models studied by, for example, [Guerrieri \(2008\)](#) are absent in this regard.

If the worker is not bound by noncompete restriction, the bidding process generates the Bertrand competition outcome identical to the one in [Postel-Vinay and Robin \(2002\)](#), with the winning firm paying a price equal to the reservation value of the losing firm. In this case, the worker gets zero share of the rent from reallocation, while the entrant firm retains the full share, implicitly setting the worker’s bargaining power to zero. When a noncompete clause exists and is enforceable, it reduces the outside match value and impairs the poaching firm’s ability to bid. Thus, a better outside match is needed to win the bid.

**Firm-firm bargaining.** If a buyout stage ensues, an additional bargaining process takes place between the incumbent and entrant firms to determine the buyout payment. In this stage, the incumbent firm still has imperfect information about the entrant match value. It has learned that the entrant match is above the poaching threshold but not its precise value. I assume that the incumbent makes a take-it-or-leave-it offer to the entrant match, implicitly assigning all the bargaining power to the incumbent firm.

## D A One-Period Model

This section provides a simple one-period model. This model encapsulates the essential features of the dynamic model in Section 2 but strips away the legal enforcement and contracting cost, hence shutting down the selection channel into noncompete arrangements. I use this simple model to discuss the role of model assumptions, show a simplified version of the results from the dynamic model, and illustrate some additional insights.

### D.1 Environment

Consider an economy in which production lasts for one period. Noncompete exclusion leads to delays in production. If production is delayed by  $\pi \in [0, \infty)$  duration, the agents discount the production value by a factor of  $e^{-r\pi}$ .

The events occur in the following sequence. There is a worker matched to an incumbent firm. They can undertake investment in their productivity  $z \in \mathcal{Z}$  by incurring cost  $c(z)$ . Employment opportunity with an entrant firm arrives with probability  $\lambda$ . The outside match productivity  $z' = z\theta \in \mathcal{Z}'$ , where the entrant match quality  $\theta \in \Theta = [\theta_m, \infty)$  is drawn according to the cumulative distribution function  $F(\cdot)$ . Employment and production take place. The usual assumptions in Section 2 are retained.

**Definition D.1** (Allocation). An allocation  $(\bar{\theta}, z)$  consists of (i) the poaching threshold  $\bar{\theta}$  such that entrants with  $\theta > \bar{\theta}$  poaches the worker, and (ii) the level of investment  $z$ .



**Information.** Information is asymmetric: the firms do not observe each other's productivity. Given the information constraint, the maximum payoff the incumbent and the worker can jointly achieve is to charge the entrant a monopoly price to poach away the worker. The contract below implements the monopoly price.

**Contract.** The incumbent firm and the worker enter a contract ex ante. The firm can commit to the contract, which delivers a promised level of utility  $U_0$  to the worker. The contract includes (i) the initial wage payment from the firm to the worker  $w_0$ , (ii) the firm's wage bidding strategy against the entrant,  $w : \mathcal{Z} \rightarrow \mathbb{R}_+$ , and (iii) the noncompete clause excluding the worker from working at the entrant firm for  $\pi$  duration. The firm has limited liability in delivering wage payments. Hence, the maximum wage bidding that it can commit is the entire production output  $z$ . To summarize, the contract is denoted by  $\mathcal{C} = \{w_0, w, \pi\}$ .

When an entrant arrives, the firms and the worker play a two-stage game:

- (i) *Wage Bidding.* The incumbent and the entrant bid for the worker in an ascending (English) auction: the wage is raised continuously from the initial level  $w_0$  until one firm drops out. Denote the entrant's bidding strategy by  $w^e : \mathcal{Z}' \rightarrow \mathbb{R}_+$ . If the entrant wins, i.e.,  $w^e(z') > w(z)$ , the worker moves to the entrant, and a second stage ensues.
- (ii) *Buyout.* The incumbent chooses a buyout price  $\tau : \mathcal{Z} \rightarrow \mathbb{R}_+$ .<sup>4</sup> The entrant decides whether to buy out the noncompete clause.

Note that the buyout payment  $\tau$  is chosen ex post in the buyout stage. I will show later that, even if the contract does not specify the buyout payment  $\tau$  ex ante as in Section 2, the incumbent firm would choose the same amount ex post. Therefore, it does not matter whether the buyout price is ex-ante stipulated or ex-post bargained.

**Belief.** The incumbent and entrant firm's prior beliefs about each other's productivity are denoted by  $G(z'|z)$  and  $G^e(z|z')$ . If the worker is poached by the entrant, the incumbent updates its posterior belief of the entrant's productivity, denoted by  $P(z'|w^e(z') > w(z))$ .

## D.2 Equilibrium

In the buyout stage, the entrant decides to buy out if and only if  $\tau(z) \leq (1 - e^{-r\pi})z'$ . The incumbent's expected payoff from buyout payment is

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<sup>4</sup>Without loss of generality, I restrict the buyout price as a function of only the incumbent productivity. It is impossible to set the price contingent on the entrant type, because no information about it is revealed other than it is above some threshold.

$$\chi(\mathcal{C}, w^e, \tau | z) = \int \tau(z) \mathbb{1}_{\{\tau(z) \leq (1-e^{-r\pi})z'\}} dP(z' | w^e(z') \geq w(z)).$$

In the wage bidding stage, the expected payoffs for the incumbent, the worker, and the entrant are, respectively,

$$\begin{aligned} V(\mathcal{C}, w^e, \tau | z) &= \int [(z-w_0) \mathbb{1}_{\{w^e(z') < w_0\}} + (z-w^e(z')) \mathbb{1}_{\{w_0 < w^e(z') < w(z)\}} \\ &\quad + \chi(\mathcal{C}, w^e, \tau | z) \mathbb{1}_{\{w^e(z') \geq w(z)\}}] dG(z' | z) \\ U(\mathcal{C}, w^e, \tau | z) &= \int [w_0 \mathbb{1}_{\{w^e(z') < w_0\}} + w^e(z') \mathbb{1}_{\{w_0 < w^e(z') < w(z)\}} + w(z) \mathbb{1}_{\{w^e(z') \geq w(z)\}}] dG(z' | z) \\ V'(\mathcal{C}, w^e, \tau | z') &= \int [\max\{e^{-r\pi} z', z' - \tau(z)\} - w(z)] \mathbb{1}_{\{w^e(z') \geq w(z)\}} dG^e(z | z'). \end{aligned}$$

The equilibrium notation is Perfect Bayesian Equilibrium.

**Definition D.2** (Equilibrium). An equilibrium consists of strategies  $\{\mathcal{C}, w^e, \tau, z\}$  and beliefs  $\{G, G^e, P\}$  such that,

- (i) The incumbent's ex-ante contract and the investment are optimal:

$$\max_{\mathcal{C}, z} V(\mathcal{C}, w^e, \tau | z) - c(z), \quad (\text{D.1})$$

subject to the PK constraint

$$U(\mathcal{C}, w^e, \tau | z) = U_0; \quad (\text{D.2})$$

- (ii) The entrant's bidding strategy is optimal in the bidding stage:

$$\max_{w^e} V'(\mathcal{C}, w^e, \tau | z'), \quad \forall z'; \quad (\text{D.3})$$

- (iii) The incumbent's buyout price is optimal in the buyout stage:

$$\max_{\tau} \chi(\mathcal{C}, w^e, \tau | z). \quad (\text{D.4})$$

- (iv) the incumbent's posterior belief is updated according to

$$P(z' | w^e(z') \geq w(z)) = \frac{\int_{z\theta_m}^{z'} \mathbb{1}_{\{w^e(z') \geq w(z)\}} dG(z' | z)}{\int_{z\theta_m}^{\infty} \mathbb{1}_{\{w^e(z') \geq w(z)\}} dG(z' | z)}. \quad (\text{D.5})$$

**Bilateral Efficiency.** Given the firm's commitment and risk-neutral preferences, the bilateral efficiency result applies here. Incorporating the PK constraint (D.2) into the incumbent's

objective (D.1), I obtain the bilateral joint payoff

$$\begin{aligned} J(\mathcal{C}, w^e, \tau | z) &\equiv V(\mathcal{C}, w^e, \tau | z) + U(\mathcal{C}, w^e, \tau | z) \\ &= \int [z \mathbb{1}_{w^e(z') < w(z)} + (w(z) + \chi(\mathcal{C}, w^e, \tau | z)) \mathbb{1}_{\{w^e(z') \geq w(z)\}}] dG(z' | z). \end{aligned} \quad (\text{D.6})$$

**Lemma D.1** (Bilateral Efficiency). *The contract and the investment maximize the bilateral joint value between the firm and the worker*

$$\max_{\mathcal{C}, z} J(\mathcal{C}, w^e | z). \quad (\text{D.7})$$

### D.3 Private Optimum

Building on Lemma D.1, I now solve for the equilibrium.<sup>5</sup>

**Proposition D.1** (Private-Optimal Contract). *The private-optimal allocation is characterized by*

$$\bar{\theta} = 1 + \frac{1 - F(\bar{\theta})}{f(\bar{\theta})} \quad (\text{D.8})$$

$$c'(z) = 1 + (\bar{\theta} - 1)(1 - F(\bar{\theta})). \quad (\text{D.9})$$

*It is implemented by a contract which embeds wage bidding,  $w(z) = z$ , and a noncompete clause subject to be bought out:*

$$\pi = \frac{1}{r} \log(\bar{\theta}) \quad (\text{D.10})$$

$$\tau(z) = z(\bar{\theta} - 1). \quad (\text{D.11})$$

*Proof.* Since the bidding strategy  $w^e(z')$  is strictly increasing in  $z'$ , there exists a unique poaching threshold  $\bar{z} = z\bar{\theta}$  such that  $w^e(z\bar{\theta}) = w(z)$ ,  $\forall z$ . Performing a change of variable from  $z'$  to  $\theta$ , the Bayes rule for the posterior belief (D.5) simplifies to:

$$P(\theta | \theta \geq \bar{\theta}) = \frac{F(\theta) - F(\bar{\theta})}{1 - F(\bar{\theta})}, \quad \forall \theta \geq \bar{\theta}.$$

The bidding process reveals the entrant match quality  $\theta$  only up to the poaching threshold. Given that no information revelation occurs to allow screening in the buyout stage, the

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<sup>5</sup>Given the information asymmetry, the incumbents can obtain the maximum payoff by charging entrants a monopoly price to poach the workers. Wage bidding and noncompete buyouts implement this outcome.

buyout menu bunches to a single price.<sup>6</sup> The incumbent's problem (D.4) of choosing the buyout price  $\tau(z)$  is equivalent to choosing a buyout threshold denoted by  $\theta_0$ , which satisfies  $\tau(z) = (1 - e^{-r\pi})z\theta_0$ . Since the incumbent can at least charge a buyout price such that the entrant at the poaching threshold would be indifferent, without loss of generality, I restrict the buyout threshold to be above the poaching threshold,

$$\theta_0 = \operatorname{argmax}_{\theta \geq \bar{\theta}} (1 - F(\theta))\theta.$$

At the poaching threshold, the wage bidding satisfies  $w(z) = w^e(z\bar{\theta}) = e^{-r\pi}z\bar{\theta}$ . The latter equality is obtained from (D.3). Substituting into the bilateral joint value in (D.6):

$$F(\bar{\theta})z + (1 - F(\bar{\theta}))e^{-r\pi}z\bar{\theta} + (1 - F(\theta_0))(1 - e^{-r\pi})z\theta_0 \leq [F(\theta_0) + (1 - F(\theta_0))\theta_0]z.$$

This inequality follows from two relations:  $(1 - F(\bar{\theta}))\bar{\theta} \leq (1 - F(\theta_0))\theta_0$  and  $F(\bar{\theta}) \leq F(\theta_0)$ . The maximum of the left-hand side is obtained when  $\theta_0 = \bar{\theta}$ , which can be ensured by contracting the incumbent firm's bidding strategy as  $w(z) = e^{-r\pi}z\bar{\theta}$ . Since the incumbent firm has limited liability, i.e.,  $e^{-r\pi}\bar{\theta} \leq 1$ , it needs at least a noncompete duration of  $\log(\bar{\theta})/r$ . There exists potentially a continuum of payoff-equivalent Perfect Bayesian Equilibria, which all achieve the same allocation and payoff. These equilibria are indexed by the level of noncompete duration,  $\pi \in [\log(\bar{\theta})/r, 1]$ . The corresponding wage bidding strategies are  $w(z) = e^{-r\pi}z\bar{\theta}$  and  $w^e(z') = e^{-r\pi}z'$ ,  $\forall z' \leq z\bar{\theta}$ , and buyout price is  $\tau(z) = (1 - e^{-r\pi})z\bar{\theta}$ . In reality, firms can have problems enforcing the clauses if the duration is excessively long. Therefore, I select the contract with the minimum duration and the maximum wage bidding.

Finally, I take the first-order condition with respect to  $z$  in (D.7) and obtain (D.9).  $\square$

Proposition D.1 is a one-period version of Proposition 1 in Section 3.1 and captures the key intuitions behind the results. The poaching threshold equation (D.8) is identical to equation (19). These equations are driven by the exact same consideration. In turn, the noncompete duration equation (D.10) is identical to one in Proposition 1. Since the one-shot economy removes the dynamics, the investment equation (D.9) and the buyout price in (D.11) are simplified versions of the dynamic ones.

**Discussion: firm commitment.** The discussion above illustrates that it is not essential to contract on the amount of buyout payment ex ante nor to have the incumbent firm commit

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<sup>6</sup>As in Aghion and Bolton (1987), information friction is crucial here to prevent ex-post efficient renegotiation and generate mobility distortion. This is in contrast with Spier and Whinston (1995) (with unverifiable information) and Segal and Whinston (2000) where ex-post renegotiation can take place.

to it. The same result holds when the payment is determined through ex-post bargaining. However, it is essential that the firm commits to the long-term wage contract, which ensures bilateral efficiency. If, to the contrary, the incumbent cannot commit at all, it maximizes its own payoff when bidding for the worker

$$\max_{\bar{\theta}} \int_0^{\bar{\theta}} [1 - e^{-r\pi\theta}] dF(\theta) + (1 - F(\bar{\theta})) e^{-r\pi\bar{\theta}}.$$

The optimality condition with respect to the poaching threshold here is  $\bar{\theta} - \pi(1 - F(\bar{\theta})) / f(\bar{\theta}) = 1$ . Comparing to equation (D.8), it implies that, as long as  $\pi < 1$ , the poaching threshold is below the level with commitment. Since the contract is no longer bilateral efficient, disagreement occurs: the firm prefers a longer duration while the worker prefers a shorter one.

## D.4 Social Optimum

As in Section 3.4, to characterize the social optimum, I consider a planner who designs the noncompete contract but leaves the investment decisions in the hands of the firm. This problem is equivalent to one where the planner chooses the allocation subject to the constraint that incentivizing firm investment inevitably generates distortions in reallocation. Formally, the planner's problem can be stated as

$$\max_{\bar{\theta}, z} z \left[ 1 + \int_{\bar{\theta}}^{\infty} (\theta - 1) dF(\theta) \right] - c(z) \quad (\text{D.12})$$

subject to the investment incentive in (D.9).

**Proposition D.2** (Social-Optimal Contract). *In the social-optimal allocation, the poaching threshold is characterized by*

$$\bar{\theta}^* = 1 + \frac{\varepsilon \Delta(\bar{\theta}^*)}{\varepsilon \Delta(\bar{\theta}^*) + 1} \frac{1 - F(\bar{\theta}^*)}{f(\bar{\theta}^*)}, \quad (\text{D.13})$$

where the investment elasticity  $\varepsilon \equiv \frac{c'(z)}{c'(z)z}$  and  $\Delta(\bar{\theta}) \equiv \frac{\int_{\bar{\theta}}^{\infty} (\theta - \bar{\theta}) dF(\theta)}{1 + (\bar{\theta} - 1)(1 - F(\bar{\theta}))}$ .<sup>7</sup> The corresponding investment  $z^*$  satisfies equation (D.9).

*Proof.* Differentiating the planner's objective in with respect to  $\bar{\theta}$  and accounting for how the investment  $z$  responds to  $\bar{\theta}$ :

$$\left[ 1 + \int_{\bar{\theta}}^{\infty} (\theta - 1) dF(\theta) - c'(z) \right] \frac{\partial z}{\partial \bar{\theta}} = z(\bar{\theta} - 1) f(\bar{\theta}).$$

<sup>7</sup>For illustration, I specify the same investment cost function as in Section 5.1,  $c(z) = \frac{\varphi}{1 + 1/\varepsilon} z^{1 + \frac{1}{\varepsilon}}$ .

Substituting equation (D.9) into the equation above:

$$\int_{\bar{\theta}}^{\infty} (\theta - \bar{\theta}) dF(\theta) \frac{\partial z}{\partial \bar{\theta}} = z(\bar{\theta} - 1) f(\bar{\theta}). \quad (\text{D.14})$$

Differentiating the investment incentive condition (D.9) with respect to  $\bar{\theta}$ :

$$\frac{\partial z}{\partial \bar{\theta}} = \frac{1}{c''(z)} [1 - F(\bar{\theta}) - (\bar{\theta} - 1) f(\bar{\theta})] = \varepsilon \frac{z}{1 + (\bar{\theta} - 1)(1 - F(\bar{\theta}))} [1 - F(\bar{\theta}) - (\bar{\theta} - 1) f(\bar{\theta})]. \quad (\text{D.15})$$

Combining equations (D.14) and (D.15), I obtain

$$\varepsilon \frac{\int_{\bar{\theta}}^{\infty} (\theta - \bar{\theta}) dF(\theta)}{1 + (\bar{\theta} - 1)(1 - F(\bar{\theta}))} [1 - F(\bar{\theta}) - (\bar{\theta} - 1) f(\bar{\theta})] = (\bar{\theta} - 1) f(\bar{\theta}),$$

from which I obtain the optimality condition (D.13).  $\square$

Similarly, Proposition D.2 is aligned with Proposition 3 in Section 3.4. The social-optimal poaching threshold characterized in equation (D.13) is identical to one in equation (28), except the wedge term  $\Delta(\bar{\theta})$  is a simplified one-period expression of the dynamic one. The social optimum can be implemented by a duration cap  $\pi^* = \frac{1}{r} \log(\bar{\theta}^*)$ .

## D.5 Noncompete Exclusion and “Damaged Goods”

In the model in Section 2, entrant firms always fully buy out noncompete, and, therefore, exclusion never takes place in equilibrium. This section provides an extension by introducing two features: (1) additional business stealing by entrants; and (2) knowledge depreciation during the noncompete period. I show that in this extension the equilibrium contract features a continuum buyout menu to screen the entrant type. I use the simple one-period model to build the extension, but the insights would also apply to the dynamic model.

Suppose that, when the worker departs to join an entrant, there is an additional business stealing inflicted upon the incumbent employer. Specifically, the incumbent firm not only loses worker production  $z$  but also suffers (additional) stolen business  $\nu z$ , where  $\nu \geq 0$ . During the noncompete exclusion, the worker’s knowledge about the incumbent firm depreciates at rate  $\gamma \geq 0$  over time, and the amount of stolen business declines. By enforcing the noncompete clause for  $\tilde{\pi} \leq \pi$  duration, the amount of stolen business reduces to  $e^{-\gamma \tilde{\pi}} \nu z$ . Adjusting for the time delay, the present value of this stolen business is  $e^{-(r+\gamma)\tilde{\pi}} \nu z$ . Taken together, the surplus of reallocating the worker from the incumbent to the entrant is

$$R(\theta, \tilde{\pi}) \equiv (e^{-r\tilde{\pi}} \theta - 1 - e^{-(r+\gamma)\tilde{\pi}} \nu) z.$$

In the absence of business stealing  $\nu = 0$  and the knowledge depreciation  $\gamma = 0$ , the extended model nests the baseline one in Section D.1.

**Assumption D.1** (Monotone Hazard Rate). *The hazard rate  $\frac{f(\theta)}{1-F(\theta)}$  is increasing in  $\theta$ .*

**Assumption D.2** (Log-Submodularity). *The rent of worker reallocation  $R(\theta, \tilde{\pi})$  is log-submodular in the entrant match quality  $\theta$  and the noncompete duration enforced  $\tilde{\pi}$ :*

$$\frac{\partial^2 \log(R(\theta, \tilde{\pi}))}{\partial \theta \partial \tilde{\pi}} < 0, \forall \theta, \tilde{\pi} \geq 0.$$

Assumption D.1 is a stronger assumption than the regulatory condition (34) in the benchmark model. Assumptions D.1 and D.2 are necessary for price discrimination to be profitable for the incumbent employers (see Anderson and Dana (2009)).<sup>8</sup> It breaks the bunching result in Proposition 1 and leads to separation. This assumption holds with sufficiently large  $\nu$  and  $\gamma$ . In fact, it requires that  $\gamma\nu > r$ . Proposition D.1 is modified to:

**Proposition D.3.** *Under Assumptions D.1 and D.2, in the private-optimal allocation, the poaching threshold  $\bar{\theta}$  is characterized by*

$$\bar{\theta} = \frac{r+\gamma}{\gamma} \left( \frac{\gamma}{r} \nu \right)^{\frac{r}{r+\gamma}} + \frac{1-F(\bar{\theta})}{f(\bar{\theta})}. \quad (\text{D.16})$$

*The corresponding noncompete duration  $\pi$  still specifies equation (D.10). The buyout menu features a continuum price-quantity options, depending on the duration the entrant wants to buy out. The entrant firms that poach the worker buy out to reduce the exclusion period to*

$$\tilde{\pi}(\theta) = \max \left\{ \frac{1}{\gamma} \left[ \log \left( \frac{r+\gamma}{r} \nu \right) - \log \left( \theta - \frac{1-F(\theta)}{f(\theta)} \right) \right], 0 \right\}, \forall \theta \geq \bar{\theta}. \quad (\text{D.17})$$

*Thus, threshold entrants are subject to some exclusion,  $\tilde{\pi}(\bar{\theta}) = \frac{1}{r+\gamma} \log \left( \frac{\gamma}{r} \nu \right) > 0$ .*

*Proof.* The steps follow the proof for Proposition 1 in Section A.2. The binding IR constraint at the poaching threshold  $\bar{\theta}$  implies that the corresponding buyout payment is

$$\tau(\tilde{\pi}(\bar{\theta})) = R(\bar{\theta}, \tilde{\pi}(\bar{\theta})).$$

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<sup>8</sup>Note that the incumbent has “ownership” in the worker’s future employment during the stipulated noncompete period of length  $\pi$ , and the “quantity” purchased by the entrant is  $\pi - \tilde{\pi}$ . Assumption D.2 can be equivalently stated as the rent being log-supermodular in the entrant quality  $\theta$  and the “quality” purchased  $\pi - \tilde{\pi}$ , as formulated by Anderson and Dana (2009).

Using the Envelope condition for the IC constraint, I obtain the buyout payment for  $\theta \geq \bar{\theta}$ :

$$\tau(\tilde{\pi}(\theta)) = R(\theta, \tilde{\pi}(\theta)) - \int_{\bar{\theta}}^{\theta} R_{\theta}(\tilde{\theta}, \tilde{\pi}(\tilde{\theta})) d\tilde{\theta}. \quad (\text{D.18})$$

The problem of maximizing expected buyout revenue becomes

$$\begin{aligned} & \max_{\tilde{\pi}(\theta), \bar{\theta}} \int_{\bar{\theta}}^{\infty} \left[ R(\theta, \tilde{\pi}(\theta)) - \int_{\bar{\theta}}^{\theta} R_{\theta}(\tilde{\theta}, \tilde{\pi}(\tilde{\theta})) d\tilde{\theta} \right] dF(\theta) \\ &= \max_{\tilde{\pi}(\theta), \bar{\theta}} \int_{\bar{\theta}}^{\infty} \left[ R(\theta, \tilde{\pi}(\theta)) - \frac{1-F(\theta)}{f(\theta)} R_{\theta}(\theta, \tilde{\pi}(\theta)) \right] dF(\theta). \end{aligned}$$

The first-order conditions with respect to  $\tilde{\pi}(\theta)$  and  $\bar{\theta}$  are, respectively,

$$R_{\tilde{\pi}}(\theta, \tilde{\pi}(\theta)) - \frac{1-F(\theta)}{f(\theta)} R_{\theta\tilde{\pi}}(\theta, \tilde{\pi}(\theta)) \geq 0 \text{ with “} = \text{” if } \tilde{\pi}(\theta) > 0, \forall \theta \geq \bar{\theta} \quad (\text{D.19})$$

$$R(\bar{\theta}, \tilde{\pi}(\bar{\theta})) - \frac{1-F(\bar{\theta})}{f(\bar{\theta})} R_{\theta}(\bar{\theta}, \tilde{\pi}(\bar{\theta})) = 0. \quad (\text{D.20})$$

Given Assumption D.2,

$$\frac{\partial^2 \log(R(\theta, \tilde{\pi}))}{\partial \theta \partial \tilde{\pi}} = \frac{R_{\theta\tilde{\pi}}(\theta, \tilde{\pi}) R(\theta, \tilde{\pi}) - R_{\theta}(\theta, \tilde{\pi}) R_{\tilde{\pi}}(\theta, \tilde{\pi})}{R(\theta, \tilde{\pi})} < 0,$$

which implies  $\gamma \nu e^{-(r+\gamma)\pi} > r, \forall \pi > 0$ . Thus, this assumption requires that  $\gamma \nu > r$ .

Further, equation (D.19) holds with equality, and I obtain the entrant's buyout decision in equation (D.17). Given Assumptions D.1,  $\theta - \frac{1-F(\theta)}{f(\theta)}$  is strictly increasing in  $\theta$ ; therefore,  $\tilde{\pi}(\theta)$  is decreasing in  $\theta$ . Higher type entrants buy out more noncompete duration. Combining equation (D.19) with equation (D.20), I obtain the poaching threshold equation (D.16).  $\square$

The contract in Proposition D.3 features a continuum buyout menu, in contrast to the single buyout price in the baseline model. Noncompete is enforced in equilibrium to price discriminate against less productive entrant firms, whereas in the baseline model noncompete is always fully bought out and never enforced.

This extension reconciles the model with selective noncompete enforcement observed in actual practices. It also rationalizes instances of more complex arrangements such as the one shown in Figure F.3 which specifies a two-part buyout menu. More broadly, noncompete enforcement can be likened to the “damaged goods” phenomenon in industrial organization where a monopolist intentionally damages goods to achieve price discrimination (Deneckere and McAfee (1996)). In this setting, the incumbent firm as the monopolist selectively and



partially enforces the noncompete clause to create a damaged version of worker human capital to achieve price discrimination against entrant firms.

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