C. Supplementary Appendix (For Online Publication)

This supplementary appendix provides proofs for the lemmas stated in Appendix B.2. To reduce notation, we denote $S(v) := \overline{P}(t(v))$.

Lemma 8. *For any v and $z < y \in T(v)$, we have $\overline{P}(z) < \overline{P}(y)$.*

Proof. Suppose that there are v and $z < y$ such that $\overline{P}(z) \geq \overline{P}(y)$. We prove that $y \notin T(v)$. Since $\overline{P}$ is increasing, it is constant on $[z, y]$; call that value $\bar{p}$. It follows that

$$\begin{aligned}
&u(\bar{p})[F(v) - F(y)] + \delta R(y) \\
\leq &u(\bar{p})[F(v) - F(y)] + \delta \left\{u(\bar{p})[F(y) - F(z)] + R(z)\right\} \\
< &u(\bar{p})[F(v) - F(z)] + \delta R(z),
\end{aligned}$$

where the first inequality is because the payoff from any type in $[z, y]$ is at most $u(\bar{p})$ (and hence $R(y) - R(z) \leq u(\bar{p})[F(y) - F(z)]$). Thus, $y \notin T(v)$. Q.E.D.

Below, we will use the fact that T is upper hemicontinuous. This follows from the generalized theorem of the maximum in Ausubel and Deneckere (1989b, p. 527). The theorem is applicable because: (i) the maximand function $u(\overline{P}(y))[F(v) - F(y)] + \delta R(y)$ is upper semicontinuous as a function of y for every v, which in turn is because $\overline{P}$ is upper semicontinuous, and u and F are continuous and increasing on the relevant range $\{y : y \leq v \text{ and } \overline{P}(y) \leq 1\}$;[45] and (ii) for any sequence $v_n \to v$, the maximand function converges uniformly.

Proof of Lemma 5. Step 1: We begin by specifying beliefs and strategies:

- μ is derived from Bayes' rule whenever possible; if at history $h = (h', a)$ a probability 0 rejection occurs, $\mu(h)$ puts probability 1 on $\overline{v}$ if $\overline{v} \leq 1/2$ and probability 1 on 0 if $\overline{v} > 1/2$ (in the latter case, $\underline{v} \leq 0$ by assumption);
- At any history $h = (h', a)$, any Vetoer type not in the support of Proposer's current belief plays an arbitrary best response; type $v \geq 0$ in the support accepts

[45] There is no loss in restricting attention to this range by a similar argument to that in the proof of Lemma 8.

a if and only if $a \in [0, \overline{P}(v)]$; type $v < 0$ in the support accepts if and only if $u_V(a,v) \geq u_V(0,v)$;

- Proposer's first offer is $S(\overline{v})$. To describe the rest of Proposer's strategy, consider any history $h = (h', a)$. Given Vetoer's strategy and the belief updating specified above, if Proposer holds a non-degenerate belief upon rejection of a then this belief equals $F_{[\underline{v},d]}$ for some d. We stipulate that if $a = \overline{P}(d) = P(d)$, then Proposer offers $S(d)$; if $a = \overline{P}(d) > P(d)$, then Proposer offers $\lim_{d'\uparrow d} S(d')$; if $a \in [\lim_{d'\uparrow d} \overline{P}(d'), \overline{P}(d))$, then Proposer randomizes between $\lim_{d'\uparrow d} S(d')$ and $S(d)$ so that type d is indifferent between a in the current period and the lottery in the next period; and for any $a \notin [\overline{P}(\underline{v}), \overline{P}(\overline{v})]$, Proposer offers $S(d)$. Finally, whenever Proposer's belief is degenerate on $x \geq 0$ ($x \in \{0, \overline{v}\}$), Proposer offers $\min\{2x, 1\}$ in all future periods.

Observe that at any history, Proposer's subsequent on path offers are decreasing, either trivially if the current belief is degenerate, or for any non-degenerate belief because the belief cutoffs are decreasing by definition and $\overline{P}$ and t are increasing.

Step 2: We verify that Proposer is playing a best response to Vetoer's strategy given beliefs μ. As this is obvious whenever he has a degenerate belief, assume he has a non-degenerate belief. As noted above, any such belief is of the form $F_{[\underline{v},d]}$ for some d. Proposer's strategy prescribes some randomization (possibly degenerate) between $S(d)$ and $\lim_{d'\uparrow d} S(d')$.

We first claim that $S(d)$ is an optimal proposal. Given Vetoer's strategy, $R(d)$ is an upper bound on Proposer's payoff. Furthermore, it follows from Lemma 8 that Vetoer's strategy has all types above $t(d)$ accepting $S(d)$ and all types strictly below rejecting. The claim follows.

We next claim that $\lim_{d'\uparrow d} S(d')$ is also an optimal proposal. Since T is upper hemicontinuous, $\lim_{d'\uparrow d} t(d') \in T(d)$. Hence, given Vetoer's strategy, $\overline{P}(\lim_{d'\uparrow d} t(d'))$ is an optimal proposal. It therefore suffices to show that $\lim_{d'\uparrow d} S(d') = \overline{P}(\lim_{d'\uparrow d} t(d'))$, or equivalently, $\lim_{d'\uparrow d} \overline{P}(t(d')) = \overline{P}(\lim_{d'\uparrow d} t(d'))$. Note that $\lim_{d'\uparrow d} \overline{P}(t(d')) \leq \overline{P}(\lim_{d'\uparrow d} t(d'))$ because t and $\overline{P}$ are increasing. But if $\lim_{d'\uparrow d} \overline{P}(t(d')) < \overline{P}(\lim_{d'\uparrow d} t(d'))$ then continuity of R and u and strict monotonicity of u in the relevant range imply the contradiction

$$R(d) = u(\lim_{d'\uparrow d} \overline{P}(t(d')))[F(d) - F(\lim_{d'\uparrow d} t(d'))] + \delta R(\lim_{d'\uparrow d} t(d'))$$
$$< u(\overline{P}(\lim_{d'\uparrow d} t(d')))[F(d) - F(\lim_{d'\uparrow d} t(d'))] + \delta R(\lim_{d'\uparrow d} t(d')) = R(d).$$

All that remains is to verify that at a history $h = (h', a)$ with $a \in [\lim_{d'\uparrow d} \overline{P}(d'), \overline{P}(d))$, there is a randomization between $S(d)$ and $\lim_{d'\uparrow d} S(d')$ that makes type d indifferent between a in the current period and the lottery in the next period. To confirm this, note that since P is right-continuous and $P(v) \geq v$ for any v, we have

$$u_V(\lim_{d'\uparrow d} P(d'), d) \geq u_V(a, d) \geq u_V(P(d), d).$$

The existence of a suitable randomization now follows from continuity of $u_V(\cdot, d)$ and Equation (7).

Step 3: We verify that Vetoer is playing a best response at each history. Consider any history (h, a) with $\mu(h) = F_{[\underline{v},q]}$. Since types outside of the support of Proposer's belief play a best response by assumption, we only consider types in $[\underline{v}, q]$.

- If $a > \overline{P}(q)$, Vetoer's strategy prescribes that no type below q accepts, and Proposer will propose $S(q)$ next period. Since type q is indifferent between $P(q)$ in the current period and $S(q)$ next period, and $S(q) \leq P(q) \leq \overline{P}(q) < a$, type q prefers $S(q)$ next period to a in the current period. The same holds for all lower types, and hence Vetoer is playing a best response.
- If $a < 0$, then: (i) it is clearly a best response for all types $v \geq 0$ to reject; and (ii) types $v < 0$ accept if and only if they prefer a to 0, which is a best response because Proposer will never make a strictly negative offer in the continuation equilibrium.
- If a is positive but below the range of $\overline{P}$, all types $v \geq 0$ accept. After a rejection, Proposer will either perpetually offer 0 or $2\overline{v}$, yielding a continuation payoff of 0 to all types, and so it is a best response for any type $v \geq 0$ to accept a.
- Otherwise, a is between $\overline{P}(\underline{v})$ and $\overline{P}(q)$.

  If $a = \overline{P}(d) = P(d)$ for some $d \leq q$, Vetoer's strategy prescribes that all and only those types above d accept.[46] On path, Proposer will propose $S(d)$ next period followed by lower offers; since type d is indifferent between a in the current period and $S(d)$ next period, and all future offers are below a, SCED implies that it is a best response for all higher types to accept and for all lower types to reject. Hence, Vetoer is playing a best response.

If there is $d \leq q$ such that $a = \overline{P}(d) > P(d)$, Vetoer's strategy prescribes that all and only those types above d accept. Proposer will propose $\lim_{d' \uparrow d} S(d')$ next period, followed by lower offers. Since type d' is indifferent between $P(d')$ in the current period and $S(d')$ next period, continuity of u implies that type d is indifferent between $\lim_{d' \uparrow d} P(d') = \overline{P}(d) = a$ in the current period and $\lim_{d' \uparrow d} S(d')$ next period. Hence, Vetoer is playing a best response.

If there is $d \leq q$ such that $a \in [\lim_{d' \uparrow d} \overline{P}(d'), \overline{P}(d))$, Vetoer's strategy again prescribes that all and only those types above d accept. Proposer will randomize next period between $\lim_{d' \uparrow d} S(d')$ and $S(d)$ to make type d indifferent between accepting a or getting the lottery next period. Therefore, Vetoer is playing a best response. Q.E.D.

Proof of Lemma 6. Step 1: Suppose $\underline{v} > 0$. We claim that there is $\varepsilon > 0$ such that (R, P) given by

$$R(v) := u(2\underline{v})F(v)$$
$$P(v) := v + \sqrt{v^2 - 4\delta\underline{v}(v - \underline{v})}$$

supports a skimming equilibrium on $[\underline{v}, \underline{v} + \varepsilon]$. Plainly, R and P are continuous, given that F is continuous. Also, P is increasing and hence $\overline{P} = P$. Some algebra confirms that $R(v)$ is the value from securing acceptance from all types below v on action $2\underline{v}$, while $P(v)$ is the action that makes type v indifferent between accepting that action now and getting action $2\underline{v}$ in the next period. Therefore, it is sufficient for us to show that there is $\varepsilon > 0$ such that for all $v \in [\underline{v}, \underline{v} + \varepsilon]$ the unique maximizer of the RHS of Equation (6) is $\underline{v}$, which implies $t(v) = \underline{v}$.

To that end, observe that the derivative of the objective function in Equation (6)

[46] If there are multiple values of d satisfying $a = \overline{P}(d)$, all types above the lowest one accept.

with respect to y is

$$u'(\overline{P}(y))\overline{P}'(y)[F(v)-F(y)]-u(\overline{P}(y))f(y)+\delta u(2\underline{v})f(y). \tag{11}$$

Since $0<u(2\underline{v})\leq u(\overline{P}(y))$ and f is bounded away from 0, the sum of the last two terms in expression (11) is strictly negative and bounded away from 0. Since $u'(\overline{P}(y))$ is bounded (by concavity), $\overline{P}'(y)$ is bounded (as $v^2-4\delta\underline{v}(v-\underline{v})>0$ for all v), F is continuous, and $v,y\in[\underline{v},\underline{v}+\varepsilon]$, the first term in expression (11) goes to 0 as $\varepsilon\to 0$. It follows that there is $\varepsilon>0$ such that expression (11) is strictly negative for all $y\in[\underline{v},\underline{v}+\varepsilon]$, and hence the maximum of the RHS of Equation (6) is attained uniquely at $t(v)=\underline{v}$ whenever $v\leq\underline{v}+\varepsilon$.

Step 2: Suppose (R_{v^*},P_{v^*}) supports a skimming equilibrium on $[\underline{v},v^*]$, where $0<\underline{v}<v^*<\overline{v}$. We will show that there is (R,P) that supports a skimming equilibrium on $[\underline{v},\overline{v}]$ with the property that $P(v)=P_{v^*}(v)$ and $R(v)=R_{v^*}(v)$ for all $v\in[\underline{v},v^*]$.

Pick $v'\in(v^*,\overline{v}]$ as large as possible such that

$$u(1)[F(v')-F(v^*)]\leq(1/2)(1-\delta)R_{v^*}(v^*). \tag{12}$$

Note that v' is well-defined because F is continuous and $R_{v^*}(v^*)>0$ (this inequality holds because of $v^*>\underline{v}$ and the property noted at the end of the paragraph following Definition 1). Moreover, letting $\overline{f}$ denote an upper bound for f, it holds that

$$v'-v^*\geq\frac{(1/2)(1-\delta)R_{v^*}(v^*)}{u(1)\overline{f}}>0. \tag{13}$$

We extend R_{v^*} to $R_{v'}$ defined on $[\underline{v},v']$ by setting $R_{v'}(v):=R_{v^*}(v)$ for $v\in[\underline{v},v^*]$, and for $v\in(v^*,v']$,

$$R_{v'}(v):=\max_{y\in[\underline{v},v^*]}\left\{u(\overline{P}_{v^*}(y))[F(v)-F(y)]+\delta R_{v^*}(y)\right\}$$

and define $t_{v'}(v)$ to be the largest value in the argmax correspondence. Observe that $\overline{P}_{v^*}$ is upper semicontinuous (since P_{v^*} is right-continuous by assumption, and hence $\overline{P}_{v^*}$ is right-continuous) and R_{v^*} is continuous; hence, $R_{v'}(v)$ and $t_{v'}(v)$ are well-defined. We extend P_{v^*} to $P_{v'}$ defined on $[\underline{v},v']$ by setting $P_{v'}(v):=P_{v^*}(v)$ for

$v \in [\underline{v}, v^*]$, and for $v \in (v^*, v']$ by letting $P_{v'}(v)$ be the largest value satisfying

$$u_V(P_{v'}(v), v) = \delta u_V(\overline{P}_{v^*}(t_{v'}(v)), v).$$

So $(R_{v'}, P_{v'})$ satisfies Equation (7). We can apply the generalized theorem of the maximum in Ausubel and Deneckere (1989b, p. 527) analogously to the discussion after Lemma 8 and conclude that $R_{v'}$ is continuous and $T_{v'}$ is non-empty and upper hemicontinuous. Therefore, $t_{v'}$ is upper semicontinuous and, since it is increasing, right-continuous. These properties of $t_{v'}$ and the hypothesis that P_{v^*} is right-continuous imply that $P_{v'}$ is right-continuous. $(R_{v'}, P_{v'})$ also satisfies Equation (6), i.e.,

$$R_{v'}(v) = \max_{y \in [\underline{v}, v]} \left\{ u(\overline{P}_{v'}(y))[F(v) - F(y)] + \delta R_{v'}(y) \right\}$$

for all $v \in [\underline{v}, v']$, because for all $y \in [v^*, v]$,

$$\begin{aligned} & u(\overline{P}_{v'}(y))[F(v) - F(y)] + \delta R_{v'}(y) \\ \leq & u(1)[F(v) - F(y)] + \delta R_{v'}(y) \\ \leq & (1/2)(1-\delta) R_{v^*}(v^*) + \delta R_{v'}(y) \\ \leq & (1/2)(1-\delta) R_{v'}(y) + \delta R_{v'}(y) \\ < & R_{v'}(v). \end{aligned}$$

Here the second inequality is because the choice of v' satisfies inequality (12) and the second inequality is because $R_{v^*}(v^*) = R_{v'}(v^*)$ and $R_{v'}$ is increasing. Therefore, the maximum is attained for $y \in [\underline{v}, v^*)$ and the claim follows since $R_{v'}(y) = R_{v^*}(y)$ for any such y.

We have established that $(R_{v'}, P_{v'})$ supports a skimming equilibrium on $[\underline{v}, v']$. Since $R_{v'}$ is increasing, it follows from inequality (13) that a finite number of repetitions of this argument extends (R_{v^*}, P_{v^*}) to the entire $[\underline{v}, \overline{v}]$ interval.

Step 3: By an approximation argument analogous to that in Ausubel and Deneckere (1989b, Theorem 4.2), there exists (R, P) that supports a skimming equilibrium on $[\underline{v}, \overline{v}]$ if $\underline{v} = 0$; we omit details. The case of $\underline{v} < 0$ is handled by setting $R(v) = 0$ and $P(v) = 0$ for all $v < 0$, and pasting that to a solution when we take $\underline{v} = 0$ and set the distribution on $[0, \overline{v}]$ to be the conditional distribution $F_{[0,\overline{v}]}$. Q.E.D.