Relational Contracts: Public versus Private Savings

Online Appendix

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This Online Appendix first provides proofs of the two existence results for "Relational Contracts: Public versus Private Savings". These results establish existence of an optimal relational contract in the unobservable and observable consumption cases. It then discusses anecdotal evidence of the importance of implicit agreements on high consumption levels in employment and other relationships. The findings in the observable consumption case (see Section 5) may shed light on the role of high consumption in these instances.

Proofs of existence results not included in the paper

Proof of Lemma A.2 (Existence of an optimal contract for the unobservable consumption case)

Proof. As we already observed, under the condition "fastest payments" given in ($\mathrm{FP}_t^{\mathrm{un}}$), the relational contract is determined solely by the effort policy $(\tilde{e}_t)_{t\geq 1}$. Hence, the payoff obtained by the principal can be written

$$W\left((\tilde{e}_t)_{t=1}^{\infty}\right) = \sum_{t=1}^{\infty} \delta^{t-1} \tilde{e}_t - \sum_{t=1}^{\infty} \delta^{t-1} \tilde{w}_t$$

where each $\tilde{w}_t$ is recursively obtained from ($\mathrm{FP}_t^{\mathrm{un}}$). Note that, from Lemma A.1, we can restrict attention to effort policies in $[0, z\left(v'\left(b_1\left(1-\delta\right)\right)\right)]^{\infty}$, where z denotes the inverse of ψ'.

Now, let $W^{\sup}$ be the supremum of $W(\cdot)$ over effort policies $(\tilde{e}_t)_{t\geq 1}$ in $[0, z\left(v'\left(b_1\left(1-\delta\right)\right)\right)]^{\infty}$ for which the implied contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ (i.e., the one implied by ($\mathrm{FP}_t^{\mathrm{un}}$)) satisfies the principal's constraints (PC_t) (such contracts are feasible and satisfy all the conditions of Proposition 4.1). Note the set is non-empty; for instance, because effort constant at zero is in the set.

Consider then a sequence of policies $((\tilde{e}_t^n)_{t=1}^{\infty})_{n=1}^{\infty}$ in $[0, z\left(v'\left(b_1\left(1-\delta\right)\right)\right)]^{\infty}$ and with $W\left((\tilde{e}_t^n)_{t=1}^{\infty}\right) > W^{\sup} - 1/n$ for all n, and for which the contract defined by each effort policy (using ($\mathrm{FP}_t^{\mathrm{un}}$)) satisfies the principal's constraints (PC_t). There then exists a sequence $(\tilde{e}_t^{\infty})_{t\geq 1} \in$

$[0, z(v'(b_1(1-\delta)))]^{\infty}$ and a subsequence $\left((\tilde{e}_t^{n_k})_{t\geq 1}\right)_{k\geq 1}$ convergent pointwise to $(\tilde{e}_t^{\infty})_{t\geq 1}$. Let $(\tilde{w}_t^{\infty})_{t\geq 1}$ be the payments corresponding to $(\tilde{e}_t^{\infty})_{t\geq 1}$ determined using (FP_t^{un}).

Note that, for any policy $(\tilde{e}_t)_{t\geq 1}$ in $[0, z(v'(b_1(1-\delta)))]^{\infty}$, using that the payments $(\tilde{w}_t)_{t\geq 1}$ determined by (FP_t^{un}) are bounded, as well as continuity of v,

$$\frac{v\left((1-\delta)\, b_1 + (1-\delta)\sum_{s=1}^{\infty}\delta^{s-1}\tilde{w}_s\right)}{1-\delta} - \sum_{t=1}^{\infty}\delta^{t-1}\psi\left(\tilde{e}_t\right) = \frac{v\left(b_1(1-\delta)\right)}{1-\delta}. \tag{OA.1}$$

Notice also that $\sum_{t=1}^{\infty}\delta^{t-1}\psi\left(\tilde{e}_t^{n_k}\right) \to \sum_{t=1}^{\infty}\delta^{t-1}\psi\left(\tilde{e}_t^{\infty}\right)$ as $k\to\infty$ (by continuity of ψ and discounting). Therefore, we have (by Equation (OA.1), using that v is strictly increasing) that $\sum_{t=1}^{\infty}\delta^{t-1}\tilde{w}_t^{n_k} \to \sum_{t=1}^{\infty}\delta^{t-1}\tilde{w}_t^{\infty}$. Since, also, $\sum_{t=1}^{\infty}\delta^{t-1}\tilde{e}_t^{n_k} \to \sum_{t=1}^{\infty}\delta^{t-1}\tilde{e}_t^{\infty}$, we can conclude that $W\left((\tilde{e}_t^{\infty})_{t\geq 1}\right) = W^{\text{sup}}$.

Our result will then follow if we can show that the contract defined by $(\tilde{e}_t^{\infty})_{t\geq 1}$ satisfies the principal's constraints (PC_t). Suppose with a view to contradiction that there is some t^* at which the principal's constraint does not hold, and so $\tilde{w}_{t^*}^{\infty} > \sum_{s=t^*+1}^{\infty}\delta^{s-t^*}(\tilde{e}_s^{\infty} - \tilde{w}_s^{\infty})$. It is easily verified, from (FP_t^{un}) and the pointwise convergence of $(\tilde{e}_t^{n_k})_{t\geq 1}$ to $(\tilde{e}_t^{\infty})_{t\geq 1}$ and $(\tilde{w}_t^{n_k})_{t\geq 1}$ to $(\tilde{w}_t^{\infty})_{t\geq 1}$, that for large enough k, $\tilde{w}_{t^*}^{n_k} > \sum_{s=t^*+1}^{\infty}\delta^{s-t^{n_k}}(\tilde{e}_s^{n_k} - \tilde{w}_s^{n_k})$, contradicting that the contract determined by $(\tilde{e}_t^{n_k})_{t\geq 1}$ satisfies the principal's constraints (PC_t). □

Proof of Lemma A.17 (Existence of an optimal contract for the observable consumption case)

Proof. If $\delta \geq \frac{c^{FB}(b_1)-(1-\delta)b_1}{e^{FB}(b_1)} \in (0,1)$ then there is a self-enforceable efficient contract (by Proposition 5.2), and so existence is established. The remainder of the proof is needed for the values b_1 such that there is no self-enforceable first-best contract.

Given any sequence $(c_s, b_{s+1})_{s=t}^{\infty}$, and assuming "fastest payments", we can completely define the continuation contract from date t, with effort given at each date $s\geq t$ by $\hat{e}(c_s, b_s, b_{s+1})$ (recall Equation (28) in the paper), and the payment given by $\delta b_{s+1} - b_s + c_s$. We then denote by $\Pi(b_t)$ the sequences $(c_s, b_{s+1})_{s=t}^{\infty}$ which are part of feasible and self-enforceable contracts beginning with balance b_t. Note that these sequences satisfy, for all $s\geq t$,

$$\delta b_{s+1} - b_s + c_s \leq \sum_{\tau=s+1}^{\infty}\delta^{\tau-s}\left(\hat{e}(c_\tau, b_\tau, b_{\tau+1}) - (\delta b_{\tau+1} - b_\tau + c_\tau)\right)$$

as well as

$$v(c_s) + \tfrac{\delta}{1-\delta}v\left((1-\delta)\, b_{s+1}\right) - \tfrac{1}{1-\delta}v((1-\delta)b_s) \geq 0.$$

Note also that $\Pi(b_t)$ is not empty: for instance, it contains the "autarky" continuation contract, where $c_s = (1-\delta)b_t$ and $b_s = b_t$ for all $s\geq t$ (recall that $\hat{e}((1-\delta)b_t, b_t, b_t) = 0$).

Given any $b_t > 0$, let the value of the principal's problem of determining a feasible and self-enforceable contract be given by

$$V(b_t) \equiv \sup_{(c_s,b_{s+1})_{s=t}^{\infty}\in\Pi(b_t)} \sum_{s=t}^{\infty} \delta^{s-t}\left(\hat{e}(c_s,b_s,b_{s+1}) - (\delta b_{s+1} - b_s + c_s)\right).$$

Note that $V(b_t)$ is no greater than the first-best value $V^{FB}(b_t)$. Usual arguments imply that the continuation payoff of the principal in an optimal contract (if it exists) is a fixed point of an operator defined by

$$TW(b_t) \equiv \sup_{c_t>0, b_{t+1}>0} \left(\hat{e}(c_t,b_t,b_{t+1}) - (\delta b_{t+1} - b_t + c_t) + \delta W(b_{t+1})\right) \tag{OA.2}$$

subject to the principal's constraint

$$\delta b_{t+1} - b_t + c_t \leq \delta W(b_{t+1}) \tag{OA.3}$$

and to

$$v(c_t) + \tfrac{\delta}{1-\delta} v\left((1-\delta)\, b_{t+1}\right) - \tfrac{1}{1-\delta} v((1-\delta) b_t) \geq 0. \tag{OA.4}$$

Note that the operator T is monotone: if $W_1 \geq W_2$, then $TW_1 \geq TW_2$. Also, we have $TV^{FB} \leq V^{FB}$. Applying T to both sides, we have that $(T^n V^{FB}(b_t))_{n\geq 1}$ is a decreasing sequence for all $b_t > 0$. Therefore there is some pointwise limit of $T^n V^{FB}$, call it $\bar{V}$. Straightforward continuity arguments show that $\bar{V}$ is a fixed point of T. (See Thomas and Worrall, 1994, for a related argument.)

We now make four observations to be used in the completion of the proof.

Observation 1. First-best value is strictly decreasing. We now show that V^{FB} is a strictly decreasing function. First, notice that, for any $b_t > 0$,

$$V^{FB}(b_t) = \tfrac{1}{1-\delta} \max_w \left\{\psi^{-1}\left(v\left(b_t\left(1-\delta\right)+w\right) - v\left(b_t\left(1-\delta\right)\right)\right) - w\right\}. \tag{OA.5}$$

At the optimal choice of w (which is strictly positive), we have $c^{FB}\left(b_t\right) = b_t\left(1-\delta\right) + w$, and

$$e^{FB}\left(b_t\right) = \psi^{-1}\left(v\left(b_t\left(1-\delta\right)+w\right) - v\left(b_t\left(1-\delta\right)\right)\right).$$

Therefore, by the envelope theorem,

$$\frac{d}{db_t} V^{FB}\left(b_t\right) = \frac{v'(c^{FB}(b_t)) - v'(b_t(1-\delta))}{\psi'(e^{FB}(b_t))} = 1 - \frac{v'(b_t(1-\delta))}{\psi'(e^{FB}(b_t))}.$$

Using the theorem of the maximum and the fact that the objective in Equation (OA.5) is strictly concave, e^{FB} is a continuous function, so V^{FB} is continuously differentiable.

Now note that, for any $b_t > 0$, $\psi'(e^{FB}(b_t)) = v'(c^{FB}(b_t)) < v'(b_t(1-\delta))$, and so $\frac{\mathrm{d}}{\mathrm{d}b_t} V^{FB}(b_t) < 0$, which establishes the result.

Observation 2. Bounded choice variables. Consider the program (OA.2) to (OA.4) given $b_t > 0$ and $W = \bar{V}$. We show that (a) the values of b_{t+1} that satisfy the constraints of this program are contained in a bounded interval $[l^b(b_t), u^b(b_t)]$ with $l^b(b_t) > 0$, and (b) consumption choices c_t are contained in a bounded interval $[l^c(b_t), u^c(b_t)]$ with $l^c(b_t) > 0$.

First, we show that b_{t+1} must be bounded above by some $u^b(b_t)$. Observe that

$$\lim_{b_{t+1}\to\infty} \delta(V^{FB}(b_{t+1}) - b_{t+1}) = -\infty,$$

because V^{FB} is decreasing. Therefore, using $\bar{V} \leq V^{FB}$, we have

$$\lim_{b_{t+1}\to\infty} \delta(\bar{V}(b_{t+1}) - b_{t+1}) = -\infty. \tag{OA.6}$$

Satisfaction of (OA.3) then implies that the choice of b_{t+1} must be bounded above.

We now show that, given b_t, satisfaction of the constraints in Equations (OA.3) and (OA.4) implies that b_{t+1} must be no less than some $l^b(b_t) > 0$. In particular, given any $b_t > 0$, we show that at least one of these constraints is violated whenever b_{t+1} is taken sufficiently close to zero.

The constraints in Equations (OA.3) and (OA.4) are satisfied only if

$$v(c_t) \geq \tfrac{1}{1-\delta}v((1-\delta)b_t) - \tfrac{\delta}{1-\delta}v((1-\delta)b_{t+1}) \quad \text{and} \quad c_t \leq b_t + \delta(V^{FB}(b_{t+1}) - b_{t+1}).$$

Combining these two equations we have

$$V^{FB}(b_{t+1}) \geq \tilde{V}(b_{t+1}) \equiv \frac{v^{-1}\left(\frac{1}{1-\delta}v((1-\delta)b_t) - \frac{\delta}{1-\delta}v((1-\delta)b_{t+1})\right) - b_t}{\delta} + b_{t+1}. \tag{OA.7}$$

Now, notice that the right-hand side of Equation (OA.7) tends to $+\infty$ as $b_{t+1} \to 0$. Hence, joint satisfaction of the constraints requires $\lim_{b_{t+1}\to 0} V^{FB}(b_{t+1}) = +\infty$ and

$$\lim_{b_{t+1}\to 0} \frac{\tilde{V}(b_{t+1})}{V^{FB}(b_{t+1})} \leq 1.$$

We will instead show that this limit is $+\infty$.

From l'Hôpital's rule, we have that

$$\lim_{b_{t+1}\to 0} \frac{\tilde{V}(b_{t+1})}{V^{FB}(b_{t+1})} = \lim_{b_{t+1}\to 0} \frac{\frac{\mathrm{d}}{\mathrm{d}b_{t+1}}\tilde{V}(b_{t+1})}{\frac{\mathrm{d}}{\mathrm{d}b_{t+1}}V^{FB}(b_{t+1})}.$$

Let us first calculate the following limit:

$$\begin{aligned}
\lim_{b_{t+1}\to 0} \frac{\frac{\mathrm{d}}{\mathrm{d}b_{t+1}}\tilde{V}(b_{t+1}) - 1}{\frac{\mathrm{d}}{\mathrm{d}b_{t+1}}V^{FB}(b_{t+1}) - 1} &= \lim_{b_{t+1}\to 0} \frac{-\frac{v'((1-\delta)b_{t+1})}{v'\left(v^{-1}\left(\frac{1}{1-\delta}v((1-\delta)b_t) - \frac{\delta}{1-\delta}v((1-\delta)b_{t+1})\right)\right)}}{-\frac{v'((1-\delta)b_{t+1})}{\psi'(e^{FB}(b_{t+1}))}} \\
&= \lim_{b_{t+1}\to 0} \frac{\psi'(e^{FB}(b_{t+1}))}{v'\left(v^{-1}\left(\frac{1}{1-\delta}v((1-\delta)b_t) - \frac{\delta}{1-\delta}v((1-\delta)b_{t+1})\right)\right)} \\
&= +\infty.
\end{aligned}$$

To see the last equality, note that $\lim_{b_{t+1}\to 0} V^{FB}(b_{t+1}) = +\infty$ implies $\lim_{b_{t+1}\to 0} e^{FB}(b_{t+1}) = +\infty$. Also, the denominator takes positive values and has a finite limit as $b_{t+1} \to 0$. Using that $\frac{\mathrm{d}}{\mathrm{d}b_{t+1}} V^{FB}(b_{t+1}) < 0$, it is then readily seen that in fact

$$\lim_{b_{t+1}\to 0} \frac{\frac{\mathrm{d}}{\mathrm{d}b_{t+1}}\tilde{V}(b_{t+1})}{\frac{\mathrm{d}}{\mathrm{d}b_{t+1}}V^{FB}(b_{t+1})} = +\infty.$$

We have therefore shown that, given a date-t balance $b_t > 0$, the choices of b_{t+1} that are available in the program (OA.2) to (OA.4) come from some bounded set $\left[l^b(b_t), u^b(b_t)\right]$ with $l^b(b_t) > 0$. It is then immediate (using Equations (OA.3) and (OA.4)) that consumption c_t must be chosen from some bounded interval $[l^c(b_t), u^c(b_t)]$ with $l^c(b_t) > 0$ as well.

Observation 3. The function $\bar{V}$ is continuous. We now show that $\bar{V}$ is continuous. We will use that there is a decreasing and strictly positive function $\kappa(b)$ such that $\bar{V}(b) \geq \kappa(b)$ for all $b > 0$. This can be seen by recalling that, for all $b > 0$, $\bar{V}(b)$ is the limit of $T^n V^{FB}(b)$, and by verifying that the latter is, for all n, at least a positive payoff obtainable from constant consumption and balances.

Suppose for a contradiction that there is a point of discontinuity in $\bar{V}$, call it $\check{b} > 0$. Then there is $\varepsilon > 0$ and a sequence $(\check{b}_n)_{n=1}^{\infty}$ convergent to $\check{b}$ with $|\bar{V}(\check{b}_n) - \bar{V}(\check{b})| \geq \varepsilon$ for all n. We will suppose first there is a subsequence $(\check{b}_{n_k})_{k=1}^{\infty}$ along which $\bar{V}(\check{b}_{n_k}) \leq \bar{V}(\check{b}) - \varepsilon$ for all k.

Denote $\check{c}$ and $\check{b}'$ consumption and next-period balance that achieve within $\varepsilon/2$ of the supremum in the program (OA.2) to (OA.4) when the initial balance is $b_t = \check{b}$ and $W = \bar{V}$. We may assume that $\hat{e}(\check{c}, \check{b}, \check{b}')$ is strictly positive, as otherwise $\delta\bar{V}(\check{b}') - (\delta\check{b}' - \check{b} + \check{c}) > 0$ and $\check{c}$ can be increased, so the payment $\delta\check{b}' - \check{b} + \check{c}$ increases, and the implied effort $\hat{e}(\check{c}, \check{b}, \check{b}')$ increases, yielding an increase in profit. To obtain a contradiction we then note that, for k sufficiently large, it is possible in the program given $b_t = \check{b}_{n_k}$ and $W = \bar{V}$ to choose subsequent balance $b_{t+1} = \check{b}'$ and consumption c_t equal to $\check{c} + \check{b}_{n_k} - \check{b}$. Note that the payment $\delta b_{t+1} - b_t + c_t$ is the same as when the initial balance is $\check{b}$ (hence equal to $\delta\check{b}' - \check{b} + \check{c}$) while, as k becomes large, effort $\hat{e}(c_t, b_t, b_{t+1})$ is arbitrarily close to $\hat{e}(\check{c}, \check{b}, \check{b}')$. This shows that $\bar{V}(\check{b}_{n_k}) = T\bar{V}(\check{b}_{n_k}) > \bar{V}(\check{b}) - \varepsilon$, a contradiction.

The remaining case is where there is a subsequence $(\check{b}_{n_k})_{k=1}^{\infty}$ for which $\bar{V}(\check{b}) \leq \bar{V}(\check{b}_{n_k}) - \varepsilon$. For each k, consider the program (OA.2) to (OA.4) when $b_t = \check{b}_{n_k}$ and $W = \bar{V}$, and pick $c_t = \check{c}_{n_k}$ and $b_{t+1} = \check{b}'_{n_k}$ satisfying the constraints and such that

$$\hat{e}(\check{c}_{n_k}, \check{b}_{n_k}, \check{b}'_{n_k}) - (\delta\check{b}'_{n_k} - \check{b}_{n_k} + \check{c}_{n_k}) + \delta\bar{V}(\check{b}'_{n_k}) > \bar{V}(\check{b}_{n_k}) - \left|\check{b} - \check{b}_{n_k}\right|. \quad \text{(OA.8)}$$

Using Equation (OA.6), there exists $u^b > 0$ such that, necessarily, $\check{b}'_{n_k} \leq u^b$ for all k. Using (OA.7) and the arguments from Observation 2, there exists $l^b > 0$ such that $\check{b}'_{n_k} \geq l^b$ for all k. From this, and using constraints (OA.3) and (OA.4), we conclude also that there are $l^c, u^c > 0$ with $\check{c}_{n_k} \in [l^c, u^c]$ for all k.

Note we may assume that $\hat{e}(\check{c}_{n_k}, \check{b}_{n_k}, \check{b}'_{n_k})$ remains bounded below by some $\bar{e} > 0$ for all k sufficiently large. This follows from examining the program (OA.2) to (OA.4), and by the existence of a strictly positive function $\kappa(b)$ such that $\bar{V}(b) \geq \kappa(b)$ for all $b > 0$, as mentioned above. In particular, if there is no such lower bound, we can find an $\bar{e} > 0$ small enough that the following is true. For all large enough k, if $\hat{e}(\check{c}_{n_k}, \check{b}_{n_k}, \check{b}'_{n_k}) < \bar{e}$, $\check{c}_{n_k}$ can be increased to yield effort $\hat{e}(\check{c}_{n_k}, \check{b}_{n_k}, \check{b}'_{n_k}) \geq \bar{e}$ (preserving, in particular, the constraint (OA.3), as this must initially have sufficient slack). These changes can be made so as to increase the payoffs in the program (OA.2) to (OA.4), so the inequality (OA.8) continues to hold for all k.

Finally, for any large enough k, in the problem (OA.2) to (OA.4) with $b_t = \check{b}$ and $W = \bar{V}$, we may specify $c_t = \check{c}_{n_k} + \check{b} - \check{b}_{n_k}$ and $b_{t+1} = \check{b}'_{n_k}$ (thus specifying the same payment $\delta\check{b}'_{n_k} - \check{b}_{n_k} + \check{c}_{n_k}$ and next-period balance $\check{b}'_{n_k}$ as when the initial balance is $\check{b}_{n_k}$). As $k \to \infty$, we have

$$\hat{e}(\check{c}_{n_k} + \check{b} - \check{b}_{n_k}, \check{b}, \check{b}'_{n_k}) - (\delta\check{b}'_{n_k} - \check{b}_{n_k} + \check{c}_{n_k}) + \delta\bar{V}(\check{b}'_{n_k}) - \bar{V}(\check{b}_{n_k}) \to 0.$$

But, for all large enough k,

$$\bar{V}(\check{b}) \geq \hat{e}(\check{c}_{n_k} + \check{b} - \check{b}_{n_k}, \check{b}, \check{b}'_{n_k}) - (\delta\check{b}'_{n_k} - \check{b}_{n_k} + \check{c}_{n_k}) + \delta\bar{V}(\check{b}'_{n_k})$$

and it cannot be that $\bar{V}\left(\check{b}\right) \leq \bar{V}\left(\check{b}_{n_k}\right) - \varepsilon$. This establishes the contradiction.

Observation 4. The function $\bar{V}$ is strictly decreasing. To see that $\bar{V}$ is decreasing, we show that if W is a strictly positive and nonincreasing function with $W \leq V^{FB}$, then TW is strictly decreasing. That $\bar{V}$ is nonincreasing then follows because $\bar{V}$ is the pointwise limit of $T^n V^{FB}$ as $n \to \infty$. That it is strictly decreasing follows because $\bar{V} = T\bar{V}$.

The claim follows by a similar argument to Lemma A.13. Consider any strictly positive and nonincreasing function W, with $W \leq V^{FB}$. Note that TW is continuous (the argument is the same as for the continuity of $\bar{V}$ above and so omitted). Also, $TW \leq V^{FB}$. Similar to the proof of Lemma A.13, if TW fails to be strictly decreasing, we can find $b^* > 0$ such that, for all $\nu > 0$ sufficiently small, $TW(b^* - \nu) \leq TW(b^*)$. It is therefore enough to reach a contradiction by showing that, for any $\nu > 0$ sufficiently small, in fact $TW(b^* - \nu) > TW(b^*)$.

To establish the claim, note that given $b_t = b^*$ in the optimization of Equations (OA.2) to (OA.4), the value $TW(b^*)$ can be derived by considering choices of c_t and b_{t+1} such that, for some fixed $\eta > 0$, either $c_t > b_t(1-\delta) + \eta$ or $b_{t+1} > b_t + \eta$. Otherwise, the supremum would be approached by choices such that $\hat{e}(c_t, b_t, b_{t+1})$ and $\delta b_{t+1} - b_t + c_t$ approach zero, which is not the case as c_t (and hence the payment $c_t + \delta b_{t+1} - b_t$) could be increased achieving a higher payoff than the claimed supremum (that this would be possible follows by the assumption that W is strictly positive).

Now, considering the aforementioned values of (c_t, b_{t+1}) chosen when $b_t = b^*$, there is a strictly positive function $\kappa(\nu)$ such that the following is true. For all ν sufficiently small, we have that if $c_t > b_t(1-\delta) + \eta$,

$$\hat{e}(c_t - \nu, b_t - \nu, b_{t+1}) > \hat{e}(c_t, b_t, b_{t+1}) + \kappa(\nu).$$

Also, if $b_{t+1} > b_t + \eta$, then

$$\hat{e}\big(c_t, b_t - \nu, b_{t+1} - \tfrac{\nu}{\delta}\big) + \delta W\big(b_{t+1} - \tfrac{\nu}{\delta}\big) > \hat{e}(c_t, b_t, b_{t+1}) + \delta W(b_{t+1}) + \kappa(\nu).$$

In either case, this shows that given a choice (c_t, b_{t+1}) when the balance is $b_t = b^*$, a payoff that is greater by at least $\kappa(\nu)$ can be obtained when the balance is reduced by ν. This can be achieved by keeping the payment $\delta b_{t+1} - b_t + c_t$ unchanged and either decreasing consumption by ν (keeping the next-period balance the same) or decreasing the next-period balance by $\frac{\nu}{\delta}$ (keeping consumption the same). That the latter is possible given the constraints follows because W is nonincreasing. This shows that indeed $TW(b^* - \nu) \geq TW(b^*) + \kappa(\nu)$ whenever ν is sufficiently small, which establishes the result.

Completion of the proof. We now show that, for any $b_1 > 0$, $\bar{V}(b_1) = V(b_1)$. Also, this payoff is attained by a feasible self-enforceable contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$, with $\tilde{b}_1 = b_1$. The latter is sufficient for $\bar{V}(b_1) = V(b_1)$ as $\bar{V}(b_1) \geq V(b_1)$ follows straightforwardly from the definition of $\bar{V}$.

Given any date-1 balance $b_1 > 0$, a sequence $(c_t, b_{t+1})_{t=1}^{\infty}$ can be determined by iteratively solving the program given by Equations (OA.2) to (OA.4) for $W = \bar{V}$. That a maximum exists, given each balance b_t, follows by Observations 2 and 3 above. We can then define a contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$. For all t, we have $\tilde{c}_t = c_t$, $\tilde{b}_t = b_t$, $\tilde{w}_t = \delta \tilde{b}_{t+1} - \tilde{b}_t + \tilde{c}_t$, and $\tilde{e}_t = \hat{e}(\tilde{c}_t, \tilde{b}_t, \tilde{b}_{t+1})$. We now argue that this contract gives the principal a payoff $\bar{V}(b_1)$ and that it is feasible and self-enforceable.

First, we argue that $\bar{V}(\tilde{b}_t)$ is bounded along the sequence of balances $(\tilde{b}_t)_{t=1}^{\infty}$. To see this, recall Observation 4 that $\bar{V}$ is a decreasing function. Also, balances remain positive and bounded away from zero. This follows because there is a $\bar{b} > 0$ such that, if the balance $\tilde{b}_t$ is less than $\bar{b}$, $\bar{V}(\tilde{b}_t)$ equals $V^{FB}(\tilde{b}_t)$, and the balance necessarily remains constant from then on (and effort and consumption from then on are equal to first-best values). (That the first-best payoff is achievable when $b_t \leq \bar{b}$, for some $\bar{b} > 0$, follows by the argument in Step 3 of the proof of Lemma A.12.) We can then conclude that, for all t,

$$\bar{V}(\tilde{b}_t) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left(\tilde{e}_\tau - \tilde{w}_\tau\right).$$

In particular, the contract $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ attains the payoff $\bar{V}(\tilde{b}_1)$ and also the principal's constraints (PC_t) are satisfied (using Equation (OA.3)).

Now let's show that the agent's constraints (AC_t^{ob}) are satisfied. It will be enough to show that, for all t,

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t}(v(\tilde{c}_\tau) - \psi(\tilde{e}_\tau)) = v(\tilde{b}_t(1-\delta)). \tag{OA.9}$$

From the perspective of date t, the agent's payoff from obediently working and consuming until date $s > t$ and then "quitting" in the subsequent period and smoothing the available balance is

$$\sum_{\tau=t}^{s} \delta^{\tau-t}(v(\tilde{c}_\tau) - \psi(\tilde{e}_\tau)) + \delta^{s+1-t} v(\tilde{b}_{s+1}(1-\delta)).$$

By construction of the proposed contract, this is constant in s for any fixed t, and equal to $v(\tilde{b}_t(1-\delta))$. Equation (OA.9) will follow if we can show that $\delta^{s+1-t}v(\tilde{b}_{s+1}(1-\delta)) \to 0$ as $s \to \infty$.

To establish our claim note that, for each date τ, $\tilde{c}_\tau \geq (1-\delta)\tilde{b}_{\tau+1}$. This follows from the optimality of $c_\tau = \tilde{c}_\tau$ and $b_{\tau+1} = \tilde{b}_{\tau+1}$ in the maximization (OA.2) to (OA.4), with $W = \bar{V}$ and initial balance $b_\tau = \tilde{b}_\tau$. If instead $\tilde{c}_\tau < (1-\delta)\tilde{b}_{\tau+1}$, c_τ can be increased and $b_{\tau+1}$ decreased, holding the payment $\delta b_{\tau+1} - b_\tau + c_\tau$ unchanged, increasing $v(c_\tau) + \frac{\delta}{1-\delta}v((1-\delta)b_{\tau+1})$ (by strict concavity of v) and hence increasing effort $\hat{e}(c_\tau, b_\tau, b_{\tau+1})$. Recalling again that $\bar{V}$ is a decreasing function (Observation 4), the objective in Equation (OA.2) is increased, and both constraints (OA.3) and (OA.4) remain intact.

Using Equation (1) and the previous observation, we have that, for any date s, $\tilde{b}_{s+1} \leq \tilde{b}_s + \tilde{w}_s$. Iterating, for any $t \leq s$, we have $\tilde{b}_{s+1} \leq \tilde{b}_t + \sum_{\tau=t}^{s} \tilde{w}_\tau$. Recalling that $\bar{V}(\tilde{b}_{\tau+1})$ remains bounded across dates τ, payments $\tilde{w}_\tau$ must remain bounded (due to (OA.3)), say by a value $\bar{w} > 0$. Let also $\check{b} = v^{-1}(0)$. We then have, for any dates t and s, $t \leq s$,

$$\begin{aligned}
\delta^{s+1-t} v(\tilde{b}_{s+1}(1-\delta)) &\leq \delta^{s+1-t} v\left((1-\delta)\left(\tilde{b}_t + \sum_{\tau=t}^{s} \tilde{w}_\tau\right)\right) \\
&\leq \delta^{s+1-t} v\big((1-\delta)(\tilde{b}_t + \bar{w}(s+1-t))\big) \\
&\leq \delta^{s+1-t} v'(\check{b})(1-\delta)(\tilde{b}_t + \bar{w}(s+1-t)). \qquad \text{(OA.10)}
\end{aligned}$$

The right-hand side approaches zero as $s \to \infty$ for fixed t. This, together with the fact that balances are positive and bounded away from zero as noted above, establishes $\delta^{s+1-t}v(\tilde{b}_{s+1}(1-\delta)) \to 0$. This proves the equality in Equation (OA.9).

Finally, we check feasibility of $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$. Similar to the observations from Equation (OA.10), we have for fixed t that $\delta^{s+1-t}\tilde{b}_{s+1} \to 0$ as $s \to \infty$. This implies that the agent's intertemporal budget constraint (2) is satisfied as an equality. Equation (1) is satisfied by choice of each $\tilde{w}_t$ given the sequence of balances and consumption. We have already argued that these payments remain bounded. Because balances $\tilde{b}_t$ are bounded away from zero, bounded payments in turn imply bounded efforts $\tilde{e}_t$.

It remains to check that consumption $\tilde{c}_t$ is bounded. Note that all of the arguments in the proof of this lemma are unaffected if the feasibility requirement of bounded consumption is dropped, so we have established that $(\tilde{e}_t, \tilde{c}_t, \tilde{w}_t, \tilde{b}_t)_{t\geq 1}$ is optimal with the relaxed feasibility condition. In addition, because the agent's intertemporal budget constraint (2) is satisfied as an equality, Equation (35) in Lemma A.11 holds for all t. Given this, the argument in Lemma A.11 continues to apply, implying that consumption $\tilde{c}_t$ is weakly decreasing. Hence consumption is in fact bounded, and all the original feasibility conditions are satisfied. □

Anecdotal evidence of the importance of high consumption in relationships

The results of the paper establish that the principal benefits from a relationship with high consumption, as low savings prevent it from deteriorating in the future. As mentioned in the paper, Henderson and Spindler (2004) have provided various examples of how high consumption seems to be induced in practice, permitting more effective incentive provision. We review here their examples and suggest some of our own.

Henderson and Spindler (2004) argue that firms seek to reduce the savings especially of top employees through "payment-in-kind (perks), deferred compensation (corporate loans), and the encouragement of employees' conspicuous consumption" (p. 1835). They argue that these tools are being used precisely to resolve agency problems: "Employees who reduce savings are more reliable over the long term than employees who do not, since reduced savings makes employees more dependent on remaining employed into the future" (p. 1835). Therefore high consumption can serve a useful purpose in the agency relationship with senior management and it is not necessarily the case that high perks are simply a sign of corporate excess and poor governance.

While our theory can also explain the use of perks or corporate loans to encourage dependency, the model provided in our paper most directly captures the encouragement of conspicuous consumption.[1] Henderson and Spindler (2004) argue precisely that high consumption expenditures may be an implicit requirement of an employer. They argue: "Contrary to the conventional wisdom that agents wear expensive clothes and drive fancy cars in order to impress principals, it may well be that principals *require* their agents to engage in such consumption, because spending money on these items increases an agent's reliance upon the future relationship with her principal" (p. 1869).

To develop their argument, Henderson and Spindler (2004) note ideas such as in Fournier (1991) that "products can help in the creation and management of identities at the group and society levels [...] by serving as unambiguous announcements of role and position" (p. 14). This suggests that the acquisition of certain types of goods can seem close to being necessary for maintaining a certain role or position in a firm;[2] Henderson and Spindler argue that such a perception can benefit the firm by preventing top employees from accumulating wealth. Among their examples are the cars driven by corporate employees: "BMW makes a line of automobiles of gradated expense that are meant to be marketed to those at various stages of the corporate ladder; entry-level employees in the "executive segment" are meant to purchase, of course, "entry level" BMWs. Or there may be certain posh suburbs, expensive restaurants, or fashion designers that an employee is expected to spend her money on" (p. 1869). Another of their examples is

[1] Our findings suggest that, if consumption is otherwise hidden, the principal could benefit by paying the agent partly in kind (perks), effectively forcing the agent to consume. Also, loans to the employee could relax borrowing constraints to allow the employee to consume at a higher level.

[2] The view that established norms may require high spending is similar to an observation of Postlewaite (1998). He argues that excessive consumption might be sustained due to a need to meet cultural norms rather than necessarily being a result of signaling.

historical, coming from Louis XIV of France: "Louis adopted extravagantly expensive fashions, which his courtiers were expected to emulate. The courtiers thus spent all of their money on these fashions and became entirely dependent upon Louis' allowance to them. In that case, as in the above examples, the employee destroys value through extravagant and wasteful consumption, which serves to bind herself to the firm (or sovereign, as the case may be)" (p. 1870). The case of Enron is then compared to Louis XIV in providing an example of corporate culture of high spending developed through leadership: Chairman Ken Lay and CEO Jeff Skilling "created a "culture of excess" that, according to one executive, "could spoil you pretty well." Lay and Skilling drove fancy cars and built mansions in Aspen, Colorado and tiny Houston neighborhoods. Their minions followed suit.... According to the special report prepared by the board of directors after Enron was wiped out, Enron's senior leadership created a culture of spending to excess that permeated the ranks of top executives..." (p. 1870). Henderson and Spindler thus argue that even the infamous case of Enron can prove their point.

To add our own examples, similar ideas may apply to supply relationships between small firms and large procurers. A possible case in point relates to the US poultry industry, where chicken farmers supply to a few large firms that dominate the industry. An article at *The Guardian* illustrates what it sees as a common situation through the example of a farmer that contracted with chicken producer Tyson Foods.[3] The farmer in question entered an exclusive agreement with Tyson. After some time, Tyson began demanding additional expenditures on equipment such as extra feed bins and chicken houses the farmer believed unnecessary. The farmer commented: "If we are independent contractors, then why does the company have the right to tell us what equipment to use?" After the farmer failed to comply with the demands, the relationship deteriorated and, in the end, it was terminated. The connection to our theory is that Tyson asked for expenditures by the farmer that (according to the farmer) were to a degree superfluous, but as in our theory could have served the purpose of making him financially dependent. In fact, another farmer in the article commented precisely that such financial dependence is the producers' objective: "As long as they keep us in debt we have to keep raising their chickens. They don't want farmers to pay off their farms."

Still a further example of an institution encouraging high employee consumption and low savings is the "trucking system" that operated for instance in eighteenth and nineteenth century Britain. According to Hilton (1957), "in the nineteenth century the truck system consisted mainly of compulsion to deal with the employer's grocery store at risk of reprimand or discharge" (p. 237). The requirement to shop at the company store was typically only backed by an implicit threat. As Bailey (1859) writes about the typical worker under the system: "He is not obliged to go to the tommy-shop or the butty collier's drinking-shop,– of course not, – none of the workmen ever were; it is of their own choice to go there, – choice between that and having no work to do" (p. 17). As for the principal's optimal contract in our model, failure to consume at the specified level is associated with an implicit threat of termination. Bailey also describes

[3] See "Fowl play: The chicken farmers being bullied by big poultry," by Alison Moodie, published at *The Guardian* on April 22nd, 2017. https://www.theguardian.com/sustainable-business/2017/apr/22/chicken-farmers-big-poultry-rules

the impoverishment of workers under this system. While the conventional explanation of the system is the expropriation of profits through elevated prices of goods, our theory suggests the impoverishment of the worker through the demand of a certain level of consumption could be a benefit in itself. The idea of impoverishment and dependence is perhaps best captured by the song Sixteen Tons by Merle Travis, first recorded in 1946, about a coal miner:

You load sixteen tons, what do you get?
Another day older and deeper in debt
St. Peter, don't you call me 'cause I can't go
I owe my soul to the company store

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