Online Appendix for "Choices and Outcomes in Assignment Mechanisms"

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C Theoretical Appendix

C.1 Notation and Preliminary Results

Let $\Gamma_l(x)$, $l = 0, 1, ...$ be the orthonormal shifted Legendre polynomials on $[0,1]$. The first three polynomials are $\Gamma_0(y) = 1$, $\Gamma_1(y) = \sqrt{3}(2y-1)$, and $\Gamma_2(x) = \sqrt{5}(6x^2 - 6x - 1)$. In general, $\Gamma_l(x) = \sum_{k=0}^{l} \gamma_{l,k} x^k$ for known constants $\gamma_{l,k}$. The Fourier-Legendre approximation of degree m of a function g defined on $[0,1]$ evaluated at x is denoted by $s_m(g;x)$ and is given by:

$$s_m(g;x) = \sum_{l=0}^{m} \Gamma_l(x) \int_0^1 g(y)\,\Gamma_l(y)\,\mathrm{d}y.$$

Because the Legendre polynomials are orthonormal, we can refer to terms $\int_0^1 g(y)\,\Gamma_l(y)\,\mathrm{d}y$ as the l-th coefficient of the Fourier-Legendre approximation of g without explicitly referring to the degree of the approximation. For the rest of this subsection, $F(\cdot)$ is a cumulative distribution function for a random variable with support on $[0,1]$, i.e, $F(1) = 1$.

Lemma 3. *If $g(x)$ a continuous function for all $x \in [a,b] \subset (0,1)$, the partial average $S_n(g;x) = \frac{1}{n}\sum_{m=0}^{n-1} s_m(g;x)$ converges to $g(x)$ uniformly in $[a,b]$.*

Proof. The result is a corollary of Theorem IV.3.2 in Freud (1971). To apply this result, we will use the cumulative distribution function of the uniform distribution on $[0,1]$ as the function $\alpha(x)$ in the statement of the theorem, and the Legendre polynomials $\Gamma_n(x)$ as $p_n(d\alpha;x)$ for $n = 0, 1, 2...$ It is straightforward to check that this sequence of polynomials satisfies the conditions in Theorem I.1.2 of Freud (1971) for the chosen $\alpha(x)$. Moreover, this sequence is unique as noted in the remark below Theorem I.1.2 in Freud (1971).

Therefore, it remains to show that $p_n(d\alpha;x)$ satisfies requirement (3.2) in Chapter IV of Freud (1971). The author notes that Theorem III.3.3 implies that it is sufficient to show

that for every pair x_2 and x_1 in a neighborhood of $x_0 \in [a,b] \subset (0,1)$, $\frac{\alpha(x_2)-\alpha(x_1)}{x_2-x_1}$ is bounded below by some positive constant. This the case because for our chosen $\alpha(x)$, the expression is equal to 1 for every $x_1, x_2 \in (0,1)$.

Finally, $s_m(g;x)$, as defined in equation IV(1.3) of Freud (1971) is the $m-$th order Fourier-Legendre approximation of g. Therefore, by Theorem IV.3.2 in Freud (1971), $S_n(g;x)$ converges to $g(x)$ uniformly in $[a,b] \subset (0,1)$. □

Lemma 4. *The l-th coefficient of the Fourier-Legendre approximation of the function f is a known linear function of $\phi_k = \int_0^1 x^k f(x)\,dx$ for $k = 0,1,...,l$. The partial average $\frac{1}{n}\sum_{m=0}^{n-1} s_m(f,x)$ converges uniformly to $f(\cdot)$ over any interval $[a,b] \subset (0,1)$ on which the function $f(\cdot)$ is continuous.*

Proof. The l-th coefficient of the Fourier-Legendre approximation of degree of f is

$$\int_0^1 \Gamma_l(x) f(x)\,\mathrm{d}x = \int_0^1 \sum_{k=0}^{l} \gamma_{l,k} x^k f(x)\,\mathrm{d}x = \sum_{k=0}^{l} \gamma_{l,k} \int_0^1 x^k f(x)\,\mathrm{d}x = \sum_{k=0}^{l} \gamma_{l,k}\phi_k.$$

The partial average $\frac{1}{n}\sum_{m=0}^{n-1} s_m(f,x)$ converges uniformly to $f(\cdot)$ over any interval $[a,b] \subset (0,1)$ on which the function $f(\cdot)$ is continuous by Lemma 3. □

Lemma 5. *Let $\zeta_k = \int_0^1 x^k \mathrm{d}F(x)$ be the k-th moment of $F(\cdot)$ for $k = 0,1,...$ If $F(\cdot)$ is absolutely continuous, (i) the l-th coefficient of the Fourier-Legendre approximation of F is a linear function of the moments $\zeta_1,...,\zeta_{l+1}$; (ii) the partial average $\frac{1}{n}\sum_{m=0}^{n-1} s_m(F,x)$ converges uniformly to $F(\cdot)$ over any interval $[a,b] \subset (0,1)$.*

Proof. The l-th coefficient of the Fourier-Legendre approximation of $F(x)$ is given by

$$\begin{aligned} c_l &= \int_0^1 \Gamma_l(x) F(x)\,\mathrm{d}x \\ &= \left(\int_0^1 \Gamma_l(x)\,\mathrm{d}x - \int_0^1 \int_0^x \Gamma_l(y)\,\mathrm{d}y \mathrm{d}F(x)\right), \end{aligned}$$

where the second equality follows from integration by parts, which holds by absolute continuity of $F(\cdot)$, and the fact that $F(1) = 1$. For $l = 0$, $\Gamma_0(y) = 1$ and

$$c_0 = 1 - \int_0^1 x \mathrm{d}F(x) = 1 - \zeta_1. \tag{C.1}$$

For $l > 0$, $\int_0^1 \Gamma_l(x)\,\mathrm{d}x = \int_0^1 \Gamma_l(x)\,\Gamma_0(x)\,\mathrm{d}x = 0$. Therefore,

$$\begin{aligned} c_l &= -\int_0^1 \int_0^x \Gamma_l(y)\,\mathrm{d}y\mathrm{d}F(x) = -\int_0^1 \sum_{k=0}^{l} \gamma_{l,k} \frac{1}{k+1} x^{k+1}\mathrm{d}F(x) \\ &= -\sum_{k=0}^{l} \gamma_{l,k} \frac{1}{k+1} \zeta_{k+1}. \end{aligned} \tag{C.2}$$

Equations (C.1) and (C.2) imply that all c_l for $l < n$ can be written in terms of the moments $\zeta_1, \ldots, \zeta_{l+1}$. This proves part (i). Part (ii) follows from Lemma 3 because $[a, b] \subset (0, 1)$ and $F(\cdot)$ is continuous. □

Lemma 6. *Suppose (i) $F(\cdot)$ is absolutely continuous with a density $f(\cdot)$ that is continuous in an interval $[a, b] \subset (0, 1)$, (ii) functions $g(\cdot), \tilde{g}(\cdot)$ are integrable on the unit interval and continuous in the interval $[F(a), F(b)]$, and (iii) $g(y) \neq \tilde{g}(y)$ for some $y \in (F(a), F(b))$, then there exists a finite integer k such that $\int_0^1 g(F(x))\,x^k\mathrm{d}F(x) \neq \int_0^1 \tilde{g}(F(x))\,x^k\mathrm{d}F(x)$. Thus, if $f(\cdot)$ and the scalars $\int_0^1 g(F(x))\,x^k\mathrm{d}F(x)$ for $k = 0, 1, 2, \ldots$ are identified, then $g(\cdot)$ is identified in the interval $(F(a), F(b))$.*

Proof. Consider a pair of real-valued continuous functions $g(\cdot)$ and $\tilde{g}(\cdot)$ defined on the closed unit interval such that $g(y) \neq \tilde{g}(y)$ for some $y \in (F(a), F(b))$. If $F(a) = F(b)$, the conclusion is vacuously true. If $F(a) < F(b)$, continuity of $g(\cdot)$ implies that $g(\tilde{y}) \neq \tilde{g}(\tilde{y})$ for all $\tilde{y}$ in an open neighborhood $B_\delta(y)$ for some $\delta > 0$. Take $\underline{y}, \overline{y} \in B_\delta(y) \cap (F(a), F(b))$ with $\underline{y} < \overline{y}$. Since $F(\cdot)$ is absolutely continuous, $F^{-1}(\underline{y}) < F^{-1}(\overline{y})$, where $F^{-1}(\cdot)$ denotes the quantile function of $F(\cdot)$. By the mean-value theorem, there exist $x^* \in \left(F^{-1}(\underline{y}), F^{-1}(\overline{y})\right)$ such that $f(x^*) > 0$; thus,

$$\Delta \equiv |g(F(x^*))\,f(x^*) - \tilde{g}(F(x^*))\,f(x^*)| > 0.$$

Let $u(x) = g(F(x))\,f(x)$ and $\tilde{u}(x) = \tilde{g}(F(x))\,f(x)$. The function is $u(x)$ is continuous in $[a, b]$ because it is the product of continuous functions. Lemma 3 implies that $\frac{1}{n}\sum_{m=0}^{n-1} s_m(u, x)$ and $\frac{1}{n}\sum_{m=0}^{n-1} s_m(\tilde{u}, x)$ converge respectively to $u(x)$ and $\tilde{u}(x)$ uniformly in $[a, b]$. Thus, there exist an n such that for all $x \in [a, b]$, $\left|\frac{1}{n}\sum_{m=0}^{n-1} s_m(u, x) - u(x)\right| < \frac{\Delta}{3}$ and $\left|\frac{1}{n}\sum_{m=0}^{n-1} s_m(\tilde{u}, x) - \tilde{u}(x)\right| < \frac{\Delta}{3}$. By the triangle inequality, $\left|\frac{1}{n}\sum_{m=0}^{n-1} (s_m(\tilde{u}, x^*) - s_m(u, x^*))\right| >$

$\frac{\Delta}{3}$ which implies that $|s_m(\tilde{u}, x^*) - s_m(u, x^*)| > \frac{\Delta}{3}$ for some $m < n$. Thus, the two Fourier-Legendre approximations $s_m(\tilde{u}, \cdot)$ and $s_m(u, \cdot)$ have a different l-th coefficient for some $l \leq m$. Define $\phi_k = \int_0^1 g(F(x)) x^k \mathrm{d}F(x)$ and $\tilde{\phi}_k = \int_0^1 \tilde{g}(F(x)) x^k \mathrm{d}F(x)$. By Lemma 4, $\phi_k \neq \tilde{\phi}_k$ for some $k \leq l < n$. Thus, the two functions $g(\cdot)$ and $\tilde{g}(\cdot)$ are not observationally equivalent. □

C.2 Proof of Main Results

C.2.1 Proof of Lemma 1

For simplicity of notation, denote $q_n = \left(q_{j(i,1)}, \ldots, q_{j(i,n)}\right)$ and $q_{n-1} = \left(q_{j(i,1)}, \ldots, q_{j(i,n-1)}\right)$, which are truncated from q_i to the first n and $n-1$ offers respectively. For any bounded function $\psi(\cdot)$, $E\left[\psi\left(Y_{i,j(i,n)}\right) \middle| N_i = n, q_i, z, A_i \geq t_{i,j(i,n)}\right]$ is bounded and identified whenever the conditioning event has positive probability. Therefore, it remains to show that

$$E\left[\psi(Y_{i,0}) \middle| N_i = n, q_i, z, A_i \geq t_{i,j(i,n)}\right]$$

is identified. Now, re-write

$$\begin{aligned}
&E\left[\psi(Y_{i,0}) \middle| N_i = n, q_i, z, A_i \geq t_{i,j(i,n)}\right] P\left(N_i = n \middle| q_i, z, A_i \geq t_{i,j(i,n)}\right) \\
=&E\left[\psi(Y_{i,0}) \middle| N_i > n-1, q_i, z, A_i \geq t_{i,j(i,n)}\right] P\left(N_i > n-1 \middle| q_i, z, A_i \geq t_{i,j(i,n)}\right) \\
&- E\left[\psi(Y_{i,0}) \middle| N_i > n, q_i, z, A_i \geq t_{i,j(i,n)}\right] P\left(N_i > n \middle| q_i, z, A_i \geq t_{i,j(i,n)}\right) \\
=&E\left[\psi(Y_{i,0}) \middle| N_i > n-1, q_{n-1}, z, A_i \geq t_{i,j(i,n)}\right] P\left(N_i > n-1 \middle| q_{n-1}, z, A_i \geq t_{i,j(i,n)}\right) \\
&- E\left[\psi(Y_{i,0}) \middle| N_i > n, q_n, z, A_i \geq t_{i,j(i,n)}\right] P\left(N_i > n \middle| q_n, z, A_i \geq t_{i,j(i,n)}\right).
\end{aligned}$$

The first equality follows from set inclusion and the last from Assumption 2. These quantities in the last expression are observed by focussing on the subset of patients that receive the sequence of offer types q_{n-1} and q_n. By assumption, these sequences of offer types is in the support of the sequence of offer types induced by J_i. Since $P\left(N_i = n \middle| q_i, z, A_i \geq t_{i,j(i,n)}\right)$ is identified and strictly positive, $E\left[\psi(Y_{i,0}) \middle| N_i = n, q_n, z, A_i \geq t_{i,j(i,n)}\right]$ is identified. The marginal distributions of $Y_{i,0}$ and $Y_{i,j(i,n)}$ conditional on $N_i = n$, q_i, z and $A_i \geq t_{i,j(i,n)}$ are

identified because the conditional expectations of $\psi(Y_{i,0})$ and $\psi\left(Y_{i,j(i,n)}\right)$ are identified for any bounded function ψ.

C.2.2 Proof of Lemma 2

For any $k \leq n$, Assumptions 1 and 2 imply that the observed probability that $D_{i,j(i,1)} = D_{i,j(i,2)} = \ldots = D_{i,j(i,k)} = 0$, i.e., $N_i > k$, can be re-written as equation (5.1). Observe that $\zeta_k = \int_0^1 \varepsilon_D^k \mathrm{d}v(\varepsilon_D; q_j, z)$ is identified for $k \in \{1, \ldots, n\}$ and that $v(\cdot; q_j, z)$ is absolutely continuous by Assumption 3. Thus, the result follows from parts (i) and (ii) of Lemma 5. This concludes the proof of Lemma 2. If Assumption 4(i) holds, Lemma 4 implies that the function $v'(\cdot; q_j, z)$ is identified in $(0,1)$. This result will be used in the proof of Theorem 1.

C.2.3 Proof of Theorem 1

Identification of $E[Y_{i,0}|\nu_{i,D} = \nu]$. For a given $\nu \in (0,1)$, fix z such that there exists $\varepsilon_D \in (0,1)$ with $v(\varepsilon_D; q_j, z) = \nu$. Assumptions 1, 2 and 3 imply that for each $k \leq n$, we can write equation (5.2). $E[Y_{i,0}|\nu_{i,D} = \nu]$ is continuous and integrable by Assumption 4(ii-iii). The hypotheses of Lemma 2 and Assumption 3 imply that the continuous function $v'(\cdot; q_j, z)$ is identified in $(0,1)$ and so is $E[Y_{i,0}|\nu_{i,D} = \nu]$ by Lemma 6 applied to $F(\cdot) = v(\cdot; q_j, z)$.

Identification of $E[Y_{i,j}|\nu_{i,D} = \nu, \varepsilon_{i,j,D} \geq \varepsilon, q_j]$. Assumptions 1, 2 and 3 imply that for each $k \leq n$, we can re-write the observed quantity $E\left[Y_{i,j} \times 1\{N_i = k\} | q_j^k, z\right]$ as

$$\int_0^1 E[Y_{i,j}|\nu_D = v(x; q_j, z), \varepsilon_{i,j,D} \geq x, q_j] \, x^{k-1}(1-x) \, \mathrm{d}v(x; q_j, z)$$

We will invoke Lemma 6 with $F(\cdot) = v(\cdot; q_j, z)$, $f(\cdot) = v'(\cdot; q_j, z)$ and

$$g(\nu_D) = \int_{v^{-1}(\nu_D; q_j, z)}^1 E[Y_{i,j}|\nu_D, \varepsilon_{i,j,D} = \varepsilon, q_j] \, \mathrm{d}\varepsilon.$$

These functions are continuous and integrable by Assumption 4. By the conclusion of Lemma 6, $\int_{v^{-1}(\nu_D; q_j, z)}^1 E[Y_{i,j}|\nu_D, \varepsilon_{i,j,D} = \varepsilon] \, \mathrm{d}\varepsilon$ is identified. Thus, $E[Y_{i,j}|\nu_{i,D} = \nu, \varepsilon_{i,j,D} \geq \varepsilon, q_j]$ is identified for all $\nu_D \in (0,1)$, $\varepsilon_D \in (0,1)$ such that $\nu_D = v(\varepsilon_D; q_j, z)$ for some z in the support of its distribution.

C.3 Donor Unobserved Heterogeneity

We extend our identification results to allow for scalar donor heterogeneity η that is observed by agents but not by the econometrician. For donor j, patients observe both q_j and η_j, whereas the econometrician only observes the former. We modify equations (3.2) and (3.3) to depend on η explicitly. The outcome of patient i assigned organ j is $Y_{i,j} = \hat{g}_1(q_j, x_i, \eta_j, \nu_{i,1}, \varepsilon_{i,j,1})$ and the acceptance rule is $\hat{g}_D(q_j, x_i, \eta_j, z_i, \nu_{i,D}, \varepsilon_{i,j,D}) \in \{0, 1\}$. As in the main text, we assume that $\hat{g}_D$ is non-increasing in $v_{i,D}$ and non-decreasing in $\varepsilon_{i,j,D}$. We also assume that it is non-increasing in η_j and normalize the distribution of η to be uniform on the unit interval. For simplicity, we fix $t_{i,j} = 0$ and we omit x_i from the notation as it will be held fixed.

We are going to exploit the fact that, conditional on observables, organs that are offered in later positions are adversely selected in terms of η_j. We will derive the distribution of η_j conditional having been rejected by a number of observationally equivalent patients. To be able to derive this distribution we will require that the priority of patients that determine the order in which the organ is offered does not depend on donor or patient unobservables. Let $p_{i,j} = \Upsilon(x_i, z_i, q_j, \omega_{i,j})$ be a score that the assignment mechanism assigns to patient i for organ j, where $\omega_{i,j}$ is a scalar tie-breaker. For example, the mechanism may stipulate younger donors to be offered first to young patients, provided that donor age is encoded in the observed type q_j and patient age in x_i. The rule Υ may also require that the donor be offered to patients who have already been added to the waitlist if q_j and x_i include donor arrival and patient registration timestamps. We require that tie-breakers and donor unobserved heterogeneity are drawn independently from all other random variables in our model and from each other:

Assumption 5. *(i) The random variable η_j is drawn i.i.d. for all organs j. (ii) Tie-breakers $\omega_{i,j}$ are drawn i.i.d. for all patients i and organs j.*

The organ offers are generated in the following way. Donors arrive sequentially according to their index j, and each donor will donate one or two kidneys. For each organ and patient, the assignment mechanism generates the score $p_{i,j}$. The patients are then ranked based on their score, and the organ is sequentially offered to the highest-ranked patient first, then the

second highest-ranked, and so on. However, there are two caveats to this process. First, if a patient has already accepted an organ from a previous donor or departed the waitlist for any other reason, they will not receive any subsequent offer. Second, if all of the organs from a given donor have been accepted by patients with higher scores, then there are no more offers for that donor. This process continues until all organs have been offered or until there are no more untransplanted patients remaining.

In the main text, we defined J_i to be the ordered set of organs offered to patient i if they refused all offers made to them and they were registered indefinitely. To determine whether $j \in J_i$, one can run the allocation algorithm for all donors $j' < j$ and all patients except i and verify whether there was a patient with a score lower than $p_{i,j}$ that was offered the organ. It is clear that the event $j \in J_i$ depends on all the other patients observables and unobservables, but given our assumptions, does not depend on patient i's unobservables (ν_i, ε_i) given patients observables x_i and z_i. Thus, Assumptions 1, 2 and 5 are mutually compatible.

The acceptance and rejection sets, which now depend on η_j are separated by the function

$$\epsilon\left(\nu_{i,D}, \eta_j, q_j, z\right) = \sup\left\{\varepsilon_D \in \left[0,1\right] : \hat{g}_D\left(q_j, \eta_j, z, \nu_{i,D}, \varepsilon_D\right) = 0\right\},$$

where we follow the convention that the supremum of the empty set is 0. This function is non-decreasing in its first two arguments.

Throughout the argument, condition on observed donor type q_j and scarcity z. Given the unobservable η, consider the conditional probability that the donor will be rejected by a patient drawn from the (unconditional) population of patients. This probability is $\pi\left(\eta; q_j, z\right) = \int_0^1 \epsilon\left(\nu, \eta, q_j, z\right) \mathrm{d}\nu$. Since η is a uniformly distributed random variable, $\pi\left(\eta; q_j, z\right)$ is a random variable with cdf denoted by $F_\pi\left(\cdot | q_j, z\right)$.

Let R_k denote the event that a randomly drawn donor is consecutively rejected by the first k patients drawn from the (unselected) population of patients.

$$P\left(R_k | q_j, z\right) = \int \pi^k \mathrm{d}F_\pi\left(\pi | q_j, z\right). \tag{C.3}$$

Under Assumption 5, the probabilities $P\left(R_k|q_j,z\right)$ are data. We will use this equation to identify $F_\pi\left(\cdot|q_j,z\right)$.

Organs that are offered in later positions are adversely selected in terms of η_j. One complication is that both the number and type of patients who have previously rejected the organ induce selection on the unobserved donor type of the set of rejected organs that are offered to patients down the list. To simplify the argument, focus attention on offers for organs that were offered to and rejected by k observationally equivalent patients who have not received any other offer in the past. Formally, we will define K_{ij} as a random variable that is equal to minus one if some patient $i' \neq i$ that received an offer for organ j before i, either accepted it—i.e., $D_{i'j}=1$—or had received a previous offer—i.e., $j \neq j\left(i',1\right)$. Otherwise, K_{ij} is the number of patients i' who received an offer for organ j before i. Let the type of an offer be summarized by the pair $\{q_j,k_{ij}\}$.

Given the unobservable selectivity ν, consider the conditional probability that the patient rejects a donor drawn from the (unconditional) population of donors with observable characteristic q_j and history k. This probability is

$$\rho\left(\nu;q_j,k,z\right)=\int_0^1 \epsilon\left(\nu,F_\pi\left(\pi|q_j,z\right),q_j,z\right)\frac{\pi^k}{\int_0^1 \tilde{\pi}^k \mathrm{d}F_\pi\left(\tilde{\pi}|q_j,z\right)}\mathrm{d}F_\pi\left(\pi|q_j,z\right) \tag{C.4}$$

with cdf $F_\rho\left(\cdot,q_j,k,z\right)$. When $k=0$, donors are not selected based on their unobserved η, so after the change of variables $\eta=F_\pi\left(\pi|q_j,z\right)$, $\rho\left(\nu;q_j,0,z\right)=\int_0^1 \epsilon\left(\nu,\eta,q_j,z\right)d\eta$. For positive k, the distribution of η is selected. Unobservably worse organs, i.e., those with low π due to low η, become relatively scarce. Because $\rho\left(\nu;q_j,k,z\right)$ depends on the random draw ν, $\rho\left(\nu;q_j,k,z\right)$ is a random variable. In the absence of donor unobserved heterogeneity, $F_\rho\left(\cdot,q_j,k,z\right)$ is equal to $v\left(\cdot,q_j,z\right)$ for all k. Let $\{q_j,k\}^n$ be the set of offers consisting of n consecutive offers of type $\{q_j,k\}$. We can write an expression analogous to (5.1) for the probability of a randomly selected patient rejecting l consecutive offers from $\{q_j,k\}^n$ as:

$$P\left(N_i>l|\left\{q_j,k\right\}^n,z\right)=\int_0^1 \rho^l \mathrm{d}F_\rho\left(\rho,q_j,k,z\right). \tag{C.5}$$

Because the probabilities $P\left(N_i>l|\left\{q_j,k\right\}^n,z\right)$ are directly identified, this equation will be

used to show identification of $F_\rho(\rho, q_j, k, z)$.

We are now ready to derive identification results analogous to those of the main text in a model with unobserved heterogeneity η.

C.3.1 Identifying Conditional Expected Outcomes

Lemma 1 yields that the marginal distributions of $Y_{i,j(i,n)}$ and $Y_{i,0}$ conditional on $N_i = n$, $z_i = z$, $K_{ij} = k$ and q_i are identified for all $n \leq |q_i|$ such that $P\left(N_i = n | q_i, k, z, A_{i,0} \geq t_{j(i,n)}\right) > 0$,

$$\left(\left\{q_{j(i,1)}, k_{ij(i,1)}\right\}, \left\{q_{j(i,2)}, k_{ij(i,2)}\right\}, ..., \left\{\left\{q_{j(i,n)}, k\right\}\right\}\right)$$

and

$$\left(\left\{q_{j(i,1)}, k_{ij(i,1)}\right\}, \left\{q_{j(i,2)}, k_{ij(i,2)}\right\}, ..., \left\{\left\{q_{j(i,n-1)}, k_{ij(i,n-1)}\right\}\right\}\right)$$

belong to the support of the distribution of offer-types induced by the distribution of J_i.

C.3.2 Identifying the Choice Model

We follow a similar argument to that of lemma 2 to identify F_π and F_ρ as an intermediate step to show identification of $\epsilon(\cdot, \cdot, q_j, z)$ under a stronger version of Assumption 3:

Assumption 6. *For each q_j and z, (i) $\epsilon(\cdot, \cdot, q_j, z)$ has continuous positive derivatives with respect to its first two arguments in $(0,1)^2$, (ii) for every $(\nu, \eta) \in (0,1)^2$, there exists a pair of dominating functions $\bar{\epsilon}_\eta(\cdot)$ and $\bar{\epsilon}_\nu(\cdot)$, integrable in the unit interval, such that $\frac{\partial}{\partial \eta}\epsilon(\cdot, \eta', q_j, z) < \bar{\epsilon}_\eta(\cdot)$ on $(0,1)$ for every η' in a neighborhood of η and $\frac{\partial}{\partial \nu}\epsilon(\nu', \cdot, q_j, z) < \bar{\epsilon}_\nu(\cdot)$ on $(0,1)$ for every ν' in a neighborhood of ν.*

Similar to Assumption 3, Assumption 6 requires that there are no (interior) values of ν_D for which the patient either accepts or rejects all organs of type $q_j, \eta_j \in (0,1)$ when faced with scarcity z. Moreover, it also requires that there are no (interior) values of η for which patient of unobserved type $\nu_D \in (0,1)$ either accepts or rejects all organs of type q_j, η when faced with scarcity z. Part (ii) allows us to obtain derivatives of $\pi(\eta; q_j, z)$ and $\rho(\nu; q_j, k, z)$ by differentiating under the integral sign.

Lemma 7. *If Assumption 6 holds, then (i)* $F_\pi(\cdot|q_j,z)$ *is absolutely continuous on* $[0,1]$, $F_\pi(0|q_j,z)=0$, $F_\pi(1|q_j,z)=1$. *(ii)* $F_\pi(\cdot|q_j,z)$ *has a strictly positive and continuous derivative on* $(\pi(0;q_j,z),\pi(1;q_j,z))$, *(iii)* $F_\rho(\cdot|q_j,k,z)$ *is absolutely continuous on* $[0,1]$, $F_\rho(0|q_j,k,z)=0$, $F_\rho(1|q_j,k,z)=1$ *and (iv)* $F_\rho(\cdot|q_j,z)$ *has a strictly positive and continuous derivative on* $(\rho(0;q_j,k,z),\rho(1;q_j,k,z))$.

Proof. Assumption 6(i) implies that $\pi(\cdot;q_j,z)$ is strictly increasing. By assumption 6(ii) and the dominated convergence theorem, $\pi(\cdot;q_j,z)$ has a strictly positive and continuous derivative on $(0,1)$. Theorem 2 in Villani (1984) implies that the inverse of $\pi(\cdot;q_j,z)$ exists and is absolutely continuous. The inverse of $\pi(\cdot;q_j,z)$ equals $F_\pi(\cdot|q_j,z)$ since η is uniformly distributed. Because the domain of $\pi(\cdot;q_j,z)$ is $[0,1]$, the range of $F_\pi(\cdot|q_j,z)$ is also $[0,1]$. Monotonicity and absolute continuity imply $F_\pi(0|q_j,z)=0$ and $F_\pi(1|q_j,z)=1$. The derivative of $F_\pi(\cdot|q_j,z)$ at $\pi\in(\pi(0;q_j,z),\pi(1;q_j,z))$ is the reciprocal of the derivative of $\pi(\cdot;q_j,z)$ at $F_\pi(\pi|q_j,z)\in(0,1)$; thus, the derivative is positive and continuous. This concludes the proof of parts (i) and (ii). Parts (iii) and (iv) follow by the exact same arguments replacing $\pi(\cdot;q_j,z)$ by $\rho(\cdot;q_j,k,z)$. □

Lemma 8. *If Assumptions 1, 2, 5 and 6 are satisfied, and* $\{q_j,k\}^n$ *is in the support of the distribution of offer-types induced by* J_i, *then the Fourier-Legendre approximations* $s_{n-1}(F_\pi(\cdot|q_j,z),x)$ *and* $s_n(F'_\pi(\cdot|q_j,z),x)$ *are identified for each* $z\in(0,1)$ *and* q_j. *Similarly,* $s_{n-1}(F_\rho(\cdot|q_j,k,z),x)$ *and* $s_n\left(F'_\rho(\cdot|q_j,k,z),x\right)$ *are identified. In particular, if the hypotheses hold for all* n, *then* $F_\pi(\cdot|q_j,z)$, $F'_\pi(\cdot|q_j,z)$, $F_\rho(\cdot|q_j,k,z)$ *and* $F'_\rho(\cdot|q_j,k,z)$ *are identified.*

Proof. Assumptions 1, 2 and 5 imply that the observed probability that the first k offers made to observationally identical patients who have not received any previous offer can be written as in (C.3). Note that $a_k=\int_0^1\pi^k\mathrm{d}F_\pi(\pi|\,q_j,z_i)$ is observed for $k\in\{1,\ldots,n\}$ and that $F_\pi(\cdot|q_j,z)$ is absolutely continuous by Lemma 7 and has derivative $F'_\pi(\cdot|q_j,z)$. Similarly, the observed probability that an individual rejects l offers of type $\{q_j,k\}$ can be written as equation (C.5). Observe that $\zeta_l=\int_0^1\rho^l\mathrm{d}F_\rho(\rho|\,q_j,k,z_i)$ is identified for $l\in\{1,\ldots,n\}$ and that $F_\rho(\cdot|q_j,k,z)$ is absolutely continuous by Lemma 7 and has derivative $F'_\rho(\cdot|q_j,k,z)$. Thus, the results follow by Lemmas 4 and 5. □

Lemma 9. *If Assumptions 1, 2, 5 and 6 are satisfied and* $\{q_j, k\}^n$ *is in the support of the distribution of offer-types induced by* J_i *for all integers* $k = 0, 1, 2, ..., n$ *and* $n = 1, 2, ...,$ *then* $\epsilon(\cdot, \cdot, q_j, z)$ *is identified in* $(0,1)^2$ *for each* z *and* q_j *in the support of the data. Therefore,* $P(D_{i,j} = 1 | \nu_{i,D} = \nu_D, \eta_j, q_j, z)$ *is identified.*

Proof. Consider any closed interval $I \subset (\rho(0; q_j, k, z), \rho(1; q_j, k, z))$. Let $S_n(F_\rho(\cdot|q_j, k, z), x) = \frac{1}{n}\sum_{m=0}^{n-1} s_m(F_\rho(\cdot|q_j, k, z), x)$. For each $n = 0, 1, 2...$ define f_n as the solution to the problem $\min_{g \in \mathcal{N}_n} \left\| S_{n|I} - g \right\|_\infty$, where $S_{n|I}$ is the restriction of S_n to I and $\mathcal{N}_n$ is the set of non-decreasing $n-$Lipschitz functions $I \to [0,1]$. The set is compact, so f_n exists. Let $\tilde{f}_n(x) = n^{-1}((n-1) f_n(x) + x)$. The strictly increasing function $\tilde{f}_n(x)$ is $I \to [0,1]$.

$$\left\| \tilde{f}_n - F_{\rho|I} \right\|_\infty \leq \left\| \tilde{f}_n - f_n \right\|_\infty + \left\| f_n - S_{n|I} \right\|_\infty + \left\| S_{n|I} - F_{\rho|I} \right\|_\infty,$$

where $F_{\rho|I}$ is the restriction of F_ρ to I. By Lemma 7, $F'_\rho(\cdot|q_j, z)$ is continuous; thus, it has a finite supremum norm. For all $n > \left\| F'_{\rho|I}(\cdot|q_j, k, z) \right\|_\infty$, $\left\| f_n - S_{n|I} \right\|_\infty \leq \left\| S_{n|I} - F_{\rho|I} \right\|_\infty$ because $F_{\rho|I} \in \mathcal{N}_n$. Thus, $\left\| \tilde{f}_n - F_{\rho|I} \right\|_\infty \leq \left\| \tilde{f}_n - f_n \right\|_\infty + 2\left\| S_{n|I} - F_{\rho|I} \right\|_\infty$. The first term is bounded by n^{-1} and, by lemma 3, $\left\| S_{n|I} - F_{\rho|I} \right\|_\infty \to 0$. Thus, $\tilde{f}_n$ converges uniformly to $F_{\rho|I}$. By Theorem 2 in Barvinek et al. (1991), $\tilde{f}_n^{-1}$ converges locally uniformly to $\rho(\cdot; q_j, k, z)$, the inverse of $F_{\rho|I}(\cdot, q_j, k, z)$, in the interior of the image of $F_{\rho|I}(\cdot|q_j, k, z)$. $\tilde{f}_n^{-1}$ is identified from $\frac{1}{n}\sum_{m=0}^{n-1} s_m(F_\rho(\cdot|q_j, k, z), x)$ which is identified by Lemma 8. Thus, $\rho(\nu; q_j, k, z)$ is identified for all $\nu \in (0,1)$.

Rearranging equation (C.4),

$$P(R_k | q_j, z)\, \rho(\nu; q_j, k, z) = \int \epsilon(\nu, F_\pi(\pi|q_j, z), q_j, z)\, \pi^k F_\pi(\mathrm{d}\pi|q_j, z). \tag{C.6}$$

We will apply lemma 6 with $\phi_k = P(R_k | q_j, z)\, \rho(\nu; q_j, k, z)$ for varying values of k, $g(\cdot) = \epsilon(\nu, \cdot, q_j, z)$, and $F(\cdot) = F_\pi(\cdot|q_j, z)$. Lemma 8 states that $F'_\pi(\cdot|q_j, z)$ is identified. Therefore, for every $\nu \in (0,1)$, $\epsilon(\nu, \cdot, q_j, z)$ is identified in the open unit interval. □

C.3.3 Identifying Selection on Unobservables

To obtain an identification result for expected outcomes analogous to Theorem 1, we need to strengthen Assumption 4:

Assumption 7. *For each q_j, the function $E\left[Y_{i,j}\middle|\,\nu_D,\eta,\varepsilon_{i,j,D}\geq\varepsilon_D,q_j\right]$ is continuous in ν_D, η and ε_D for $(\nu_D,\eta,\varepsilon_D)\in(0,1)^3$.*

Theorem 2. *Suppose that Assumptions 4, 7 and the hypotheses for Lemma 9 hold. Then, the quantities $E\left[Y_{i,0}\middle|\,\nu_{i,D}=\nu_D\right]$ and $E\left[Y_{i,j}\middle|\,\nu_{i,D}=\nu_D,\eta,\varepsilon_{i,j,D}\geq\varepsilon_D,q_j\right]$ are identified for all $\varepsilon_D\in(0,1)$, $\eta\in(0,1)$ and $\nu_D\in(0,1)$ such that there exists z in the support of its distribution with $\epsilon\left(\nu_D,\eta,q_j,z\right)=\varepsilon_D$.*

Proof. Identification of $E\left[Y_{i,0}\middle|\,\nu_{i,D}=\nu_D\right]$ follows from Theorem 1 for a sequence of offers $\{q_j,0\}^n$ for any z. Now, consider the sequence of offers $\{q_j,k\}^n$ for $k\geq 0$. The expression for the expected survival conditional on a transplant can be rearranged to yield:

$$\begin{aligned}&E\left[Y_{i,j}\times 1\left\{N_i=n\right\}\middle|\,\{q_j,k\}^n,z\right]\\&=\int_0^1 E\left[Y_{ij}\middle|\,\nu_{i,D}=F_\rho\left(\rho;q_j,k,z\right),\{q_j,k\},z\right]\rho^{n-1}\left(1-\rho\right)\mathrm{d}F_\rho\left(\rho;q_j,k,z\right).\end{aligned}$$

As discussed in subsection C.3.1, the left-hand side of this equation is identified by Lemma 1. Let $g\left(\nu\right)=E\left[Y_{ij}\middle|\,\nu_{i,D}=\nu,\{q_j,k\},z\right]$. This function is continuous and integrable over ν by Assumptions 4(iii) and 7 and it is therefore identified by Lemma 6 with $F\left(x\right)=F_\rho\left(x;q_j,k,z\right)$. By the law of iterated expectations:

$$\begin{aligned}&E\left[Y_{ij}\middle|\,\nu_{i,D}=\nu_D,\{q_j,k\},z\right]P\left(R_k\middle|\,q_j,z\right)\\&=\int E\left[Y_{i,j}|\nu_{i,D}=\nu_D,\eta=F_\pi\left(\pi|q_j,z\right),\varepsilon_{i,j,D}\geq\epsilon\left(\nu_D,F_\pi\left(\pi|q_j,z\right),q_j,z\right),q_j\right]\pi^k\mathrm{d}F_\pi\left(\pi|q_j,z\right),\end{aligned}$$

where $\epsilon\left(\cdot,\cdot,q_j,z\right)$ is identified by Lemma 9. Let

$$g\left(\eta\right)=E\left[Y_{i,j}|\nu_{i,D}=\nu_D,\eta,\varepsilon_{i,j,D}\geq\epsilon\left(\nu_D,\eta,q_j,z\right),q_j\right]$$

This function is continuous by Assumption 7, integrable over η by Assumption 6. Thus, it

is identified by Lemma 6 with $F(x) = F_\pi(x|q_j, z)$. Therefore, the conditional expectation $E[Y_{i,j}|\nu_{i,D} = \nu_D, \eta, \varepsilon_{i,j,D} \geq \varepsilon_D, q_j]$ is identified for all $\varepsilon_D \in (0,1)$, $\eta \in (0,1)$ and $\nu_D \in (0,1)$ such that there exists z in the support of its distribution with $\epsilon(\nu_D, \eta, q_j, z) = \varepsilon_D$. □

This result also implies identification of the analogous quantities for any bounded transformation $\psi(\cdot)$ of $Y_{i,0}$ and $Y_{i,j}$.

C.4 Dynamic Selection

The results in this subsection extend the results of Theorem 2 for the selected set of patients who remain in the list until time t. This group of patients is selected in terms of their potential outcomes and unobserved selectivity ν_D. To be on the list at time t, a patient has to have survived to time t, i.e., $A_i > t$, and rejected all previous offers.

Our main result requires an additional mild restriction on the conditional distribution of A_i:

Assumption 8. *For any $t > 0$, $\log P(A_i \geq t|\nu_D)$ is a continuous function of ν_D on the closed unit interval.*

One implication of this assumption is that if $P(A_i \geq t) > 0$ implies $P(A_i \geq t|\nu_D) > 0$ for all $\nu_D \in [0,1]$. With this assumption, we show identification in the presence of dynamic selection:

Theorem 3. *Suppose that Assumption 8 and the hypothesis of Theorem 2 hold, allowing for $t_{i,j} > 0$. Then, the probability $P(D_{ij} = 1|\nu_{i,D} = \nu_D, \eta, A_i \geq t_{i,j})$ and the expectation $E[\psi(Y_{ij})|\nu_{i,D} = \nu_D, \eta, \varepsilon_{ij,D} \geq \varepsilon_D, A_i \geq t_{i,j}]$ are identified for any bounded function $\psi(\cdot)$, and all $\varepsilon_D \in (0,1)$, $\eta \in (0,1)$ and $\nu_D \in (0,1)$ such that there exist z in the support of its distribution with $\epsilon(\nu_D, \eta, q_j, z) = \varepsilon_D$ and $P(A_i \geq t_{i,j})$.*

The argument is developed in two steps. In the first step, we identify the conditional distribution of ν_D for agents that remain on the list until time t (Lemma 11). The second step takes this conditional distribution and combines it with the arguments that parallel those in Theorem 2.

Let $h_t(v)$ be the cdf of ν_D conditional on remaining on the list until t: $h_t(v) = \int_0^v \frac{P(A \geq t|\nu_D)}{P(A_i \geq t)} d\nu_D$.

Lemma 10. *If Assumption 8 holds, then for every t such that $P(A_i \geq t) > 0$, $h_t(\cdot)$ is a strictly increasing function with a strictly positive and continuous derivative that maps the closed unit interval to itself.*

Proof. Note that $h_t(0) = 0$ and $h_t(1) = 1$. Moreover, Assumption 8 implies that $h_t'(v) = \frac{P(A_i \geq t|v)}{P(A_i \geq t)}$ is strictly positive and continuous. □

Lemma 11. *Suppose that the hypothesis of Theorem 2 hold. The function $h_t(v)$ is identified for every t such that $P(A_i \geq t) > 0$.*

Proof. Let $\{q_j, k\}$ be a donor-type that arrives at the same time as patient i. Because the image of $F_\rho(\cdot, q_j, k, z)$ is the unit interval (Lemma 7), for any $\nu_D \in (0,1)$ and z, there exists $\varepsilon_D \in (0,1)$ such that $\nu_D = F_\rho(\varepsilon_D; q_j, k, z)$. Theorem 2 implies that for every $t \geq 0$, $P(A_i \geq t|\nu_D) = E[1\{A_i \geq t\}|\nu_D]$ is identified. Thus, for all t such that $P(A_i \geq t) > 0$, the function $h_t(v)$ is identified. □

Proof of Theorem 3:

Proof. Take any $\varepsilon_D \in (0,1)$, $\eta \in (0,1)$ and $\nu_D \in (0,1)$ satisfying the stated hypotheses. Conditional on $A_i \geq t$, the random variable $h_{i,D,t} = h_t(\nu_{i,D})$ is uniformly distributed and, by the properties of $h(\cdot)$ stated in Lemma 10, the function $\kappa_t(h_{i,D,t}, \eta_j, q_j, z) = \epsilon\left(h_t^{-1}(h_{i,D,t}), \eta_j, q_j, z\right)$ inherits the same properties of $\epsilon(\cdot, \cdot, q_j, z)$. If $\epsilon(\cdot, \cdot, q_j, z)$ satisfies Assumption 6, then $\kappa_t(\cdot, \cdot, q_j, z)$ has continuous and positive derivatives with respect of its first two arguments, $\bar{\epsilon}_\eta(\cdot)\|h_t'(\cdot)\|_\infty$ is a dominating function for $\frac{\partial}{\partial \eta}\kappa_t(\cdot, \eta', q_j, z)$ for η' in a neighborhood of η_j, and $\epsilon_\nu(\cdot)$ is a dominating function for $\frac{\partial}{\partial h}\kappa_t(h', \cdot, q_j, z)$ for h' in a neigborhood of $h_{i,D,t}$. Lemma 9 implies that, $P(D_{i,j} = 1|h_{i,D,t} = h_{D,t}, \eta_j, q_j, z, A_i \geq t)$ is identified and Theorem 2 implies that $E[\psi(Y_{i,j})|h_{i,D,t} = h_{D,t}, \eta, \varepsilon_{i,j,D} \geq \varepsilon_D, q_j, A_i \geq t]$ is identified. The conclusion follows because $h_t(\cdot)$ is invertible (Lemma 10) and identified (Lemma 11). □

D Additional Figures and Tables

Table D.1: Top 10 offers: Balance

	Age (1)	Diabetes (2)	Female (3)	Weight (4)	Height (5)
log(1 + # Top 10 Offers in 2 Years)					
KDPI <= 50%	-0.0235	-0.00363	0.00291	-0.217*	-0.0586
	(0.0707)	(0.00287)	(0.00281)	(0.106)	(0.0730)
KDPI > 50% or Missing	0.0776	0.00357	-0.00557*	0.242*	0.208*
	(0.0722)	(0.00295)	(0.00281)	(0.110)	(0.0841)
DSA FE, Year FE, and Blood Type FE	x	x	x	x	x
Control for Pediatric at Listing	x	x	x	x	x
CPRA Category Controls	x	x	x	x	x
F-test p-Value					
Number of Observations	128753	127244	128753	127221	126477
R-Squared	0.027	0.024	0.074	0.039	0.035
Distribution of # Top 10 Offers in 2 Years					
Mean					
Std. Dev.					

Notes: * p<0.05, ** p<0.01, *** p<0.001
The sample for all regressions is non-pediatric patients who registered between 2000 and 2008. Dependent variables are as indicated in the column headers. All regressions control for DSA fixed effect, registration year fixed effect, blood type fixed effect, time on dialysis, and indicators for CPRA = 0, 20 <= CPRA < 80, CPRA >= 80, and CPRA missing at registration. Standard errors, clustered by DSA, registration year, and blood type, are in parentheses. F-test tests against the null hypothesis that the coefficients on the instruments are zero.

Table D.2: Scarcity Instruments: Balance

	Age	Diabetes	Female	Weight	Height
	(1)	(2)	(3)	(4)	(5)
Log(1 + No. Donors)					
Patients Waited 0-1 years	-0.303	0.00190	0.00920	-0.0573	-0.398
	(0.331)	(0.0125)	(0.0114)	(0.512)	(0.325)
Patients Waited 1-2 years	0.147	-0.0130	0.00229	0.341	0.0570
	(0.299)	(0.0117)	(0.0111)	(0.461)	(0.313)
Patients Waited 2-3 years	-0.243	0.000383	0.0119	-0.325	-0.00656
	(0.271)	(0.0104)	(0.00918)	(0.400)	(0.271)
Patients Waited 3-4 years	0.263	0.0153	-0.0270**	0.164	0.126
	(0.223)	(0.00906)	(0.00819)	(0.347)	(0.226)
Patients Waited 4-5 years	-0.0120	-0.0114	0.0139*	-0.444*	-0.247
	(0.153)	(0.00601)	(0.00550)	(0.219)	(0.153)
Log(1 + No. Offers)					
Patients Waited 0-1 years	0.370	0.0165*	-0.00680	0.382	0.384
	(0.195)	(0.00822)	(0.00744)	(0.318)	(0.217)
Patients Waited 1-2 years	-0.0250	0.0000367	0.000268	-0.293	-0.198
	(0.215)	(0.00846)	(0.00780)	(0.330)	(0.231)
Patients Waited 2-3 years	0.0945	0.000665	-0.00727	0.361	0.0513
	(0.213)	(0.00819)	(0.00713)	(0.318)	(0.223)
Patients Waited 3-4 years	-0.112	-0.0123	0.0212**	-0.178	-0.145
	(0.195)	(0.00766)	(0.00692)	(0.299)	(0.199)
Patients Waited 4-5 years	0.0654	0.0125*	-0.0159**	0.312	0.167
	(0.132)	(0.00526)	(0.00494)	(0.197)	(0.134)
Year FE, DSA FE, and blood type FE	x	x	x	x	x
F-test p-Value					
Number of Observations	87303	87299	87303	86175	85598
R-Squared	0.023	0.020	0.003	0.025	0.017

Notes: * p<0.05, ** p<0.01, *** p<0.001
The sample for all regressions is adult patients who registered on the waitlist between 1999Q4 and 2005Q4. Each regression is on patient level, where the dependent variable is the patient characteristics in the column header at registration. Each regression has five regressors indexed by $k = 0, 1, 2, 3, 4$, where the kth regressor for patient i is computed as the number of unique donors (offers) such that: the offer is made to patients who are in the same DSA as i, have the same blood type as i, and have waited the same number of years as i; the offer is made between $4k+1$ and $4k+4$ quarters, inclusive, from the quarter when i registers (e.g. if i registers in 2002Q1, then the offer must be made between 2003Q2 and 2004Q1 for $k = 1$. All regressions control for DSA fixed effect, registration year fixed effect, blood type fixed effect, an indicator for pediatric at registration, and indicators for CPRA = 0, 20 <= CPRA < 80, CPRA >= 80, and CPRA missing at registration. Robust standard errors, clustered by DSA, registration year, and blood type, are in parentheses. F-test tests against the null hypothesis that the coefficients on the five regressors are zero.

Table D.3: Robustness

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Realized Assignment	8.55	9.29	9.22	8.67	10.02	9.18	8.78	9.18	9.38	9.29
Random Assignment among										
All Patients	7.31	7.54	7.40	6.95	8.20	7.71	7.05	7.33	7.54	7.49
Transplanted Patients	7.78	8.51	8.38	7.75	9.13	8.41	8.11	8.40	8.54	8.50
No Choice	8.42	8.05	7.96	7.65	8.66	8.32	7.57	7.79	8.10	8.05
Optimal Assignment among										
Transplanted Patients	11.26	11.04	10.91	10.55	12.06	11.10	10.32	11.00	11.88	11.04
All Patients Based on Only Observables	9.08	9.93	9.85	9.29	10.79	9.83	9.39	9.86	10.75	9.93
All Patients	11.26	14.08	13.97	13.33	15.79	14.62	13.28	13.44	14.94	14.02
Box-Cox ρ										
Survival without Transplant	0.5	0.5	0.5	0.5	0.5	0.4	0.5	0.6	0.5	0.5
Survival with Transplant	0.6	0.6	0.6	0.6	0.5	0.6	0.7	0.6	0.6	0.6
Instruments										
# Past Donors		x		x	x	x	x	x	x	x
# Past Offers			x							
# Future Donors										
Donor Unobservables		x	x		x	x	x	x	x	x
Other Unobservables		x	x	x	x	x	x	x	x	x
DSA X Time Trend									x	
Region X Donor Characteristics										x

Notes: Robustness of the results presented in Figure 4. The baseline specification is presented in column (2). The remaining specifications vary the instruments, the presence of η_j, or the Box-Cox shape parameters as indicated in the table.

Table D.4: Characteristics of Transplanted Patients (Observational Model)

		Random Assignment		No Choice		Realized Assignment		Optimal Assignment	
	All Patients	Transplanted Patients	LYFT	Transplanted Patients	LYFT	Transplanted Patients	LYFT	Transplanted Patients	LYFT
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Age < 18	3.1%	3.1%	14.54	6.3%	15.74	5.7%	15.33	5.9%	16.52
Age 18 - 35	11.6%	12.2%	11.40	12.0%	12.62	13.1%	12.83	18.3%	13.06
Age 36 - 59	54.8%	55.5%	7.61	52.6%	8.49	54.3%	8.70	59.1%	9.63
Age >= 60	30.4%	29.2%	4.29	29.1%	4.93	26.9%	4.73	16.8%	5.50
White	42.0%	42.5%	6.96	47.8%	8.11	46.4%	8.21	39.1%	9.82
Black	32.7%	31.8%	7.30	30.0%	8.30	30.7%	8.53	33.0%	9.63
Hispanic	16.7%	16.7%	8.05	15.2%	9.46	14.8%	9.51	18.4%	10.80
Other	8.6%	9.0%	7.69	7.1%	8.66	8.2%	8.85	9.5%	10.16
Diabetic	41.4%	40.0%	5.10	37.0%	5.64	33.3%	5.63	27.7%	6.66
On Dialysis at Registration	83.0%	82.4%	7.10	81.8%	8.10	80.1%	8.27	79.6%	9.68
0 HLA Mismatches	-	0.1%	9.20	16.6%	8.33	12.9%	8.37	7.4%	12.01
0 DR Mismatches	-	4.0%	7.51	36.1%	8.42	22.3%	8.60	12.2%	11.11
HLA Mismatches	-	4.77	-	3.60	-	3.90	-	3.81	-
Untransplanted Survival	6.90	6.95	-	6.97	-	7.11	-	7.61	-

Notes: Optimal assignment is computed using the observational model with no patient or kidney unobservables and no scarcity instrument. LYFT reported in this table is computed using the baseline specification.

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