Supplemental Appendix for:
Can Deficits Finance Themselves?

This Supplemental Appendix contains further material for the article "Can Deficits Finance Themselves?". We provide: (i) supplementary discussion of our baseline model and results in Sections 2-4; (ii) details for the extensions considered in Section 5; (iii) additional analysis and alternative results for our quantitative investigation in Section 6. The end of this appendix contains all proofs. Additional materials are provided in Angeletos et al. (2024a).

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."—"D." refer to the main article.

A Some further model details

We here provide some additional discussion of our baseline model. Appendix A.1 characterizes labor supply and explains how our model's supply block reduces to the standard NKPC, and Appendix A.2 derives the aggregate demand relation (13). Appendix A.3 then explains how we characterize equilibria under the alternative fiscal policy rule (8).

A.1 The supply block

Recall that the optimal labor supply relation is given as (3), re-stated here for convenience:

$$(1-\tau_y)W_t = \frac{\iota L_t^{\frac{1}{\varphi}}}{\int_0^1 C_{i,t}^{-1/\sigma} di}.$$

Log-linearizing, we obtain (15).

Optimal firm pricing decisions as usual give inflation as a function of real marginal costs. With a standard constant-returns-to-scale, labor-only production function this gives (e.g., see the textbook derivations in Woodford, 2003a; Galí, 2008)

$$\pi_t = \tilde{\kappa} w_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{A.1}$$

where $\tilde{\kappa} = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ is a function of θ (one minus the Calvo reset probability). Combining (A.1) with (15) and imposing that $c_t = \ell_t = y_t$, we obtain

$$\pi_t = \underbrace{\tilde{\kappa}\left(\frac{1}{\varphi} + \frac{1}{\sigma}\right)}_{\equiv\kappa} y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right], \tag{A.2}$$

as required.

A.2 Derivation of the aggregate demand relation (13)

From the fiscal policy in (7), we have that

$$\begin{aligned}
\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k d_{t+k}\right] &= d_t + \mathbb{E}_t\left[\sum_{k=1}^{+\infty}(\beta\omega)^k \frac{1}{\beta}\left((1-\tau_d)\, d_{t+k-1} - \tau_y y_{t+k-1}\right)\right] + (1-\tau_d)\,\omega\varepsilon_t \\
&= d_t + \omega(1-\tau_d)\,\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k d_{t+k}\right] - \omega\tau_y \mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right] + (1-\tau_d)\,\omega\varepsilon_t \\
&= \frac{1}{1-\omega(1-\tau_d)} d_t - \frac{\omega\tau_y}{1-\omega(1-\tau_d)}\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right] + \frac{(1-\tau_d)\,\omega}{1-\omega(1-\tau_d)}\varepsilon_t,
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k t_{t+k}\right] &= \tau_d \mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k d_{t+k}\right] + \tau_y \mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right] - (1-\tau_d)\,\varepsilon_t \\
&= \frac{\tau_d}{1-\omega(1-\tau_d)} d_t + \frac{\tau_y(1-\omega)}{1-\omega(1-\tau_d)}\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right] - \frac{(1-\tau_d)(1-\omega)}{1-\omega(1-\tau_d)}\varepsilon_t
\end{aligned}$$

Together with the aggregate demand relation (12) and market clearing ($c_t = y_t$), we get (13):

$$y_t = \frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{1-\omega(1-\tau_d)}(d_t + \varepsilon_t) + \left(1 - \frac{(1-\omega)\,\tau_y}{1-\omega(1-\tau_d)}\right)\mathbb{E}_t\left[(1-\beta\omega)\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right].$$

A.3 Equilibrium characterization with the alternative fiscal rule

We characterize the equilibrium in our OLG-NK environment with $\omega < 1$, $\tau_y > 0$, and the alternative fiscal rule (8) here. The aggregate demand relation (12) together with $r_t = 0$ and market clearing $y_t = c_t$ lead to the following recursive aggregate demand relation:

$$y_t = \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(d_t - t_t) + \mathbb{E}_t\left[y_{t+1}\right], \tag{A.3}$$

where we use that $\mathbb{E}_t\left[d_{t+1}\right] = \frac{1}{\beta}(d_t - t_t)$ from (5).

We characterize the bounded equilibrium path through backward induction. Given the alternative fiscal rule (8), we know that, for $t \geq H$,

$$d_t - t_t = 0 \Longrightarrow y_t = \mathbb{E}_t\left[y_{t+1}\right].$$

We focus on the equilibrium with $y_t = 0$ for $t \geq H$. As discussed at the end of Section (3), this equilibrium selection can be justified in two ways: considering limits to $\phi = 0$ from above or introducing noise in social memory as in Angeletos and Lian (2023). The sole role of any of these modifications is to remove a class of sunspot equilibria that are inherited from the standard New Keynesian model. Given this selection we use (A.3) to find the equilibrium path of $\{y_t, d_t\}_{t=0}^{H-1}$ through backward induction starting from

$$y_H = \chi_0 d_H \quad \text{with} \quad \chi_0 = 0. \tag{A.4}$$

For $t \leq H-1$, substitute the alternative fiscal rule (8) into (A.3), giving

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\left(d_t+\varepsilon_t\right) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right].$$

As a result, for $t \leq H-1$,

$$y_t = \chi_{H-t}\left(d_t+\varepsilon_t\right) \quad \text{with} \quad \chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y} + \frac{\frac{1}{\beta}\left(1-\tau_y\chi_{H-t}\right)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\chi_{H-t-1}, \tag{A.5}$$

where, according to (8), $\varepsilon_t = 0$ for all $t \neq 0$. Rearranging terms, we find the following recursive formula for the χs:

$$\chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi_{H-t-1}}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi_{H-t-1}}{\beta}\right)\tau_y} = g\left(\chi_{H-t-1}\right), \tag{A.6}$$

where

$$g(\chi) = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi}{\beta}\right)\tau_y} \quad \text{and} \quad g'(\chi) = \frac{1}{\beta}\frac{1}{\left(1+\tau_y\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}+\frac{\chi}{\beta}\right)\right)^2} \geq 0 \quad \forall \chi \geq 0. \tag{A.7}$$

We thus know that

$$\chi_k \in (0, \frac{1}{\tau_y}) \quad \forall k \geq 1 \quad \text{and} \quad \chi_k \text{ increases in } k. \tag{A.8}$$

From (5), $r_t = 0$, (8), and (A.5), we also know that, for $t \leq H$,

$$\mathbb{E}_0\left[d_t\right] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)\left(d_0+\varepsilon_0\right). \tag{A.9}$$

To further characterize the equilibrium it is useful to consider an alternative economy with rigid prices ($\kappa = 0$) but otherwise identical to the baseline economy. Note that this alternative economy shares the same $\{\chi_k\}$ as our main economy, because $\{\chi_k\}$ is independent of κ from (A.6). Let v' denote the self financing share in this alternative economy, i.e.,

$$\nu' \cdot \varepsilon_0 = \nu_y' \cdot \varepsilon_0 \equiv \sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k \right].$$

In this economy, there is no $t = 0$ price level jump and so the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is pre-determined. From (A.5) and (A.9), we have that

$$\nu' = \sum_{t=0}^{H-1} \prod_{j=0}^{t-1} \left(1 - \tau_y \chi_{H-j}\right) \tau_y \chi_{H-t} = 1 - \prod_{j=0}^{H-1} \left(1 - \tau_y \chi_{H-j}\right). \tag{A.10}$$

We can now return to the general case with $\kappa \geq 0$. From the NKPC (16) as well as the definitions in (22) – (23), we have that

$$\nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \nu_y = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu. \tag{A.11}$$

Finally, from the formula of d_0 (18), we know

$$d_0 = -\nu_p \varepsilon_0 \quad \text{and} \quad \nu_y = \left(1 - \nu_p\right) \nu'.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}. \tag{A.12}$$

This completes our characterization of the equilibrium.

B Details on model extensions

We elaborate on the various extensions considered in Section 5: equilibria under more general monetary rules and the richer aggregate demand block are discussed here, while the remaining extensions are presented in the Additional Materials.

B.1 More general monetary policy

Here we supplement the discussion of monetary policy in Subsection 5.1. First, we explain how we measure the degree of self-financing when real rates are variable. Second, we investigate the model's determinacy regions, extending Leeper (1991).

Measuring the degree of self-financing when $\phi \neq 0$. With variable expected real rates, the government's intertemporal budget constraint in (21) has to be re-written as follows:

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} \mathbb{E}_0 \left[r_k \right] = \tau_d \left(\varepsilon_0 + \sum_{k=0}^{+\infty} \beta^k \mathbb{E}_0 \left[d_k \right] \right) + \sum_{k=0}^{+\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k \right] + \frac{D^{ss}}{Y^{ss}} \left(\pi_0 - \mathbb{E}_{-1} \left[\pi_0 \right] \right).$$

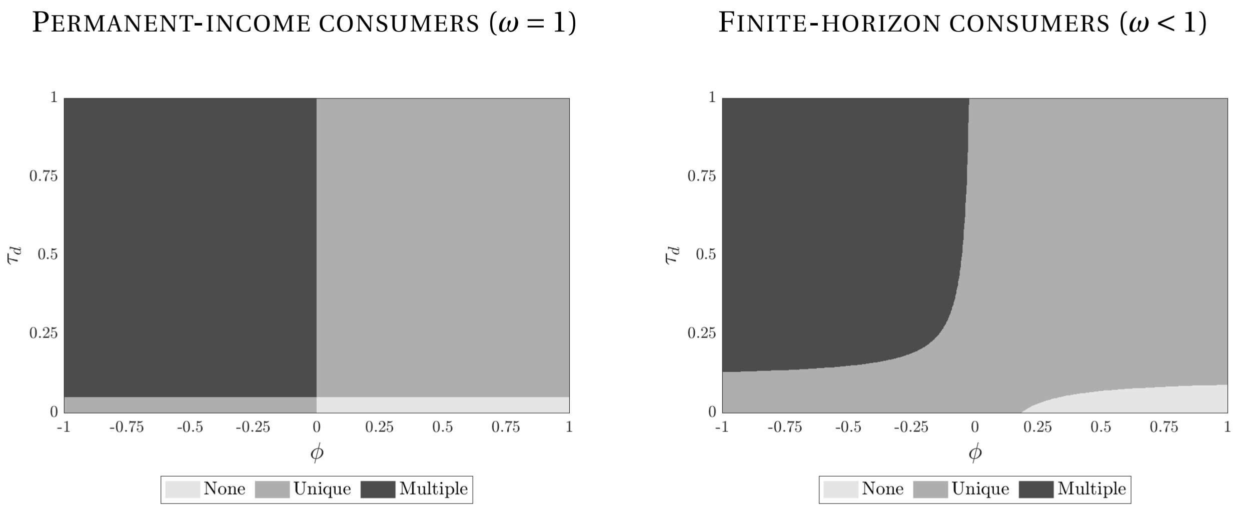

Figure B.1: Determinacy regions for $\omega = 1$ (left panel) as well as $\omega \ll 1$ (right panel) as a function of the fiscal and monetary policy rule coefficients τ_d (y-axis) and ϕ (x-axis), with $\tau_y \gg 0$.

The right hand side is the same as before, while the new term on the left-hand side, $\frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} \mathbb{E}_0 [r_k]$, captures how the time-varying expected real interest rate in (10) changes the costs of servicing the outstanding public debt. We interpret this new term as analogous to the deficit shock itself and accordingly define the share of self-financing as

$$\nu \equiv \frac{\sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k\right] + \frac{D^{ss}}{Y^{ss}} \pi_0}{\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{\infty} \beta^{k+1} \mathbb{E}_0 \left[r_k\right]} \tag{B.1}$$

This reduces to the original definition when $\phi = 0$.

Determinacy regions. We begin by providing a visual illustration of equilibrium determinacy—i.e., the famous Leeper (1991) regions—in our OLG model. The two panels in Figure B.1 show whether a bounded equilibrium (i.e., the standard solution concept of Blanchard and Kahn (1980)) exists and is unique under different assumptions on monetary policy (ϕ) and fiscal policy (τ_d in (7)), for a standard permanent-income model (left panel) and our OLG economy (right panel); throughout we set $\tau_y \gg 0$, allowing for the feedback from economic activity to fiscal surpluses at the heart of our results.

The figure reveals that the determinacy properties of the two economies are materially different. Results for the permanent-income model are well-known and require little explanation: equilibrium uniqueness requires that fiscal policy is passive ($\tau_d > 1 - \beta$) and monetary policy is active ($\phi > 0$), or vice-versa; if both rules are active then no bounded equilibrium exists, and if both are passive then there are multiple bounded equilibria. With discounting on the household side (i.e., $\omega < 1$), the regions of equilibrium determinacy look rather different. Perhaps most importantly, the benchmark monetary rule of $\phi = 0$ now induces unique bounded equilibrium for any τ_d, consistent with Proposition 1. Intuitively, with $\omega < 1$, determinacy comes from the fact that public debt directly enters the aggregate demand relation. Moreover, existence of a bounded equilibrium is related to $\tau_y > 0$, which implies that output directly affects the government budget. As a result, self-financing is now strong enough to pull debt as well as spending towards zero, even if interest rates do not provide any further Euler equation tilting. This automatic stabilization of government debt also shrinks the equilibrium non-existence region in the bottom right corner of the figure.

B.2 A more general aggregate demand relation

In Section 5.2 we showed explicitly how several popular models of the household consumption-savings problem can be written in our general form (30). We here elaborate further on: (i) our discussion of the well-known spender-saver model; (ii) what happens in general with a margin of permanent-income households; and (iii) the model of cognitive discounting of Gabaix (2020).

Self-financing in the spender-saver model. We provide a visual illustration of self-financing—or the lack thereof—in Figure B.2. The top panel shows impulse responses and the self-financing share in the spender-saver model, while the bottom panel does the same for a spender-OLG hybrid model.

The top panel reveals that, in the spender-saver model, the self-financing share ν is always 0. In particular, we see that the date-0 boom is always exactly offset (in present-value terms) by a bust at date H—the date of the delayed fiscal adjustment. The intuition is that the presence of permanent-income households breaks our discounting and front-loading properties: the date-H tax hike invariably affects date-0 demand, and part of the deficit-financed boom is delayed to the infinite future. The empirically testable flipside of this "connection at infinity" is an infinitely large elasticity of household asset demand to interest rates (e.g., see the discussion in Kaplan and Violante, 2018). With the spender-OLG model ($\omega < 1$), we break this unrealistic feature of the model, return to our discounting and front-loading properties, and thus see that the date-H bust endogenously gets smaller and smaller as we again converge to full self-financing (right panel).

Adding a margin of permanent-income consumers. We now elaborate further on what happens to our self-financing results in the presence of a margin of (at least near-)permanent-income consumers. We proceed in two steps. First, we elaborate on the discontinuity of ν in the presence of such a permanent-income margin, connecting with our discussion in Section 4.4 on the "order of limits." Second, we investigate what happens under alternative configurations of policy.

We begin by considering what happens to the self-financing share ν in our baseline model studied in Sections 3 - 4, but when a margin—here one percent—of near-PIH consumers is added, with some $\bar{\omega} > \omega$. Throughout we focus on our "H-policy" (8). Results are displayed in Figure B.3, which shows

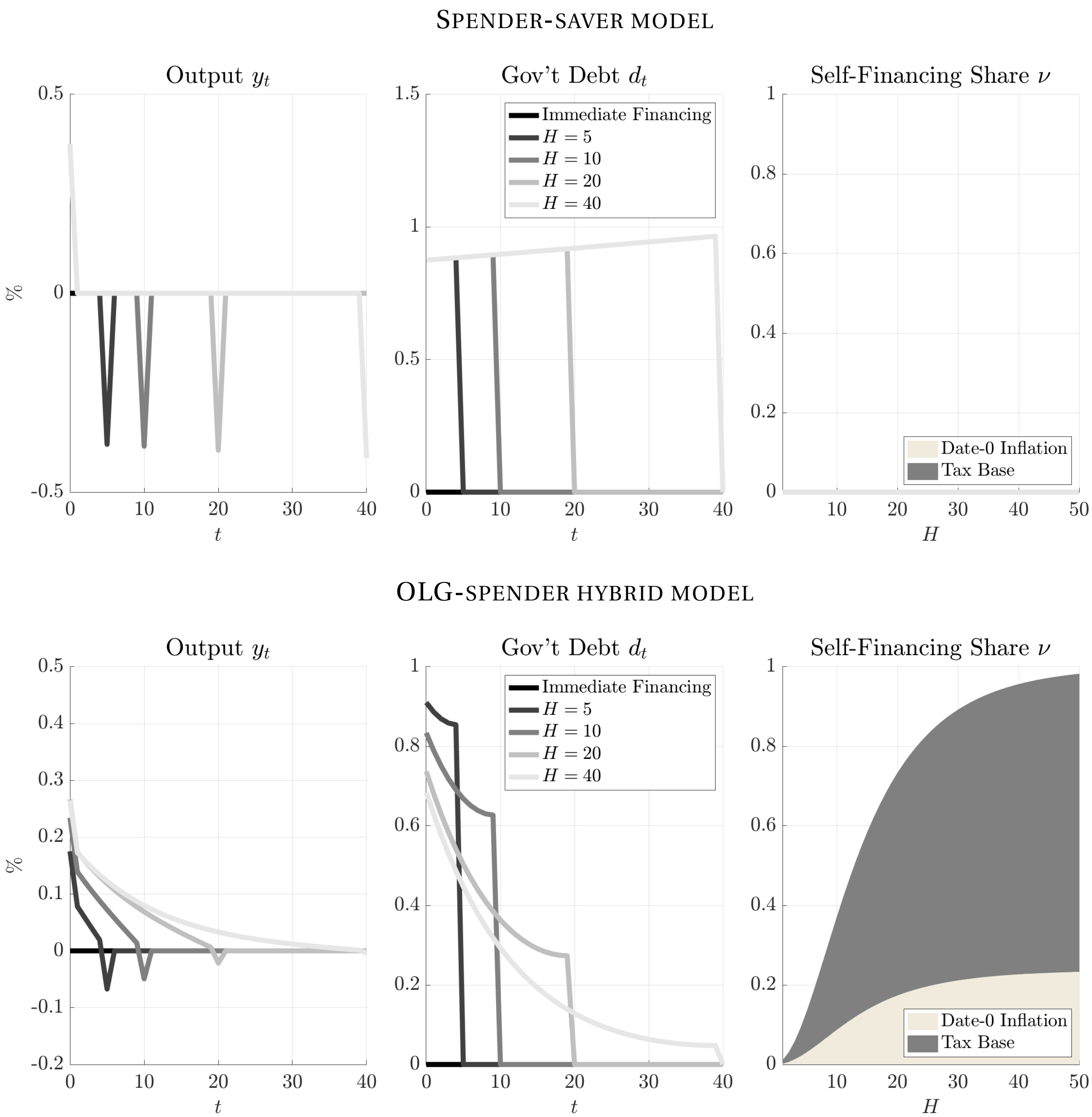

Figure B.2: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to a shock ε_0 equal to one per cent of steady-state output, as a function of H in a spender-saver model. Bottom panel: same as above, but in an OLG-spender hybrid economy.

the self-financing share ν for different H (shaded lines), as a function of $\bar{\omega}$ (x-axis). Consistent with the discussion in Section 5.2, we see that, as $\bar{\omega} \to 1$, the self-financing share ν converges to zero. However, for every $\bar{\omega} < 1$, as H is increased, the self-financing share increases, eventually converging to one—our "order of limits" discussion. Ultimately, the practical relevance of our results is thus a quantitative question, addressed by the analysis in Section 6.

We next show that alternative policy mixes deliver a self-financing share ν that is continuous in the

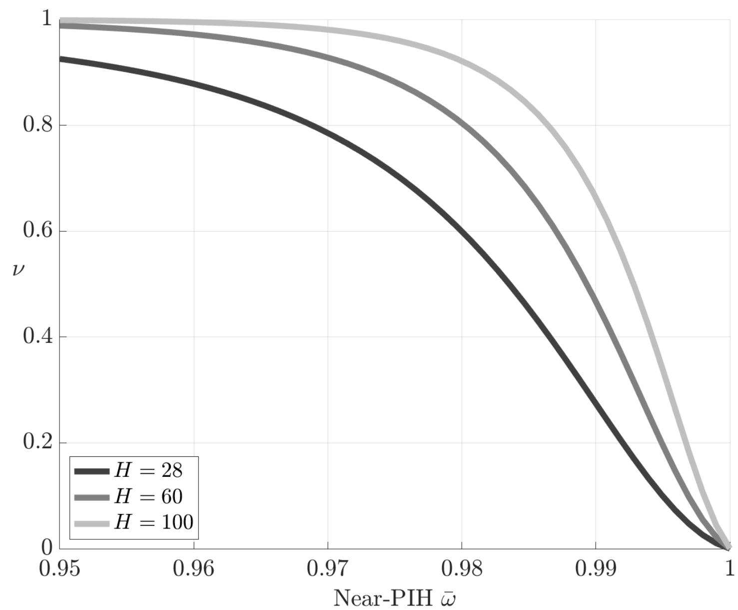

Figure B.3: Self-financing share in an augmented version of the model of Sections 3 - 4, featuring a margin (1 percent) of households with higher OLG survival coefficient $\bar{\omega}$. ν is plotted as a function of $\bar{\omega}$ (x-axis) and H (shaded lines).

margin of exact permanent-income consumers. Specifically, we consider a PIH-OLG hybrid economy with a share μ of OLG consumers (with a survival rate ω) and a residual share $1-\mu$ of PIH consumers. Note that this environment nests the classical spender-saver model with $\omega = 0$. We consider a variant of fiscal policy (8), where the tax burden of fiscal adjustment at date H is imposed in a group-specific manner, reducing the post-tax financial wealth of both permanent-income consumers and spenders to its steady-state value 0. That is, for a PIH consumer i, $t_{i,t} = t_t^{\text{PIH}} = \tilde{a}_t^{\text{PIH}}$ for $t \geq H$, while for an OLG consumer i, $t_{i,t} = t_t^{\text{OLG}} = \tilde{a}_t^{\text{OLG}}$, where superscripts PIH and OLG capture group-specific averages and $\tilde{a}_t$ captures real financial wealth. For $t < H$, the fiscal policy is exactly the same as (8)

$$t_{i,t} = t_t = \begin{cases} \tau_y y_{i,t} - \varepsilon_0 & t = 0 \\ \tau_y y_{i,t} & t \in \{1, \cdots, H-1\} \end{cases}.$$

Given the fiscal policy for $t \geq H$ and similar to the main analysis, for $t \geq H$,

$$c_t^{\text{PIH}} = c_t^{\text{OLG}} = y_t = 0.$$

Since expected real interest rates are fixed, consumption of PIH consumers for $t \leq H-1$ is given by:

$$c_t^{\text{PIH}} = 0.$$

As a result, for $t \leq H-1$,

$$y_t = \mu c_t^{\text{OLG}} \quad \text{and} \quad t_t^{\text{OLG}} = \tau_y y_t = \mu \tau_y c_t^{\text{OLG}}. \tag{B.2}$$

Consumption for OLG consumers for $t \leq H-1$ is given by:

$$\begin{aligned} c_t^{\text{OLG}} &= \left(1-\beta\omega\right)\left(\tilde{a}_t^{\text{OLG}} + \mathbb{E}_t\left[\sum_{k=0}^{+\infty}\left(\beta\omega\right)^k\left(y_{t+k} - t_{t+k}^{\text{OLG}}\right)\right]\right) \\ &= \left(1-\beta\omega\right)\left(\tilde{a}_t^{\text{OLG}} + \left(1-\tau_y\right)\mu\mathbb{E}_t\left[\sum_{k=0}^{H-t-1}\left(\beta\omega\right)^k c_{t+k}^{\text{OLG}}\right] - \left(\beta\omega\right)^{H-t}\tilde{a}_H^{\text{OLG}}\right). \end{aligned} \tag{B.3}$$

Together with OLG consumers' budget,

$$\begin{aligned} \tilde{a}_{t+1}^{\text{OLG}} &= \frac{1}{\beta}\left(\tilde{a}_t^{\text{OLG}} + y_t - t_t^{\text{OLG}} - c_t^{\text{OLG}}\right) \\ &= \frac{1}{\beta}\left(\tilde{a}_t^{\text{OLG}} - \left(1-\left(1-\tau_y\right)\mu\right)c_t^{\text{OLG}} + \varepsilon_0 \mathbb{1}_{\{t=0\}}\right), \end{aligned} \tag{B.4}$$

(B.3) and (B.4) fully characterize $\{c_t^{\text{OLG}}\}_{t=0}^{H-1}$ and hence $\{y_t^{\text{OLG}}\}_{t=0}^{H-1}$ given $\tilde{a}_0^{\text{OLG}} = d_0$.

Note that when $\mu = 1$ (all consumers are OLG consumers), (B.3) and (B.4) fully characterize the equilibrium in our baseline economy studied in Sections (3) and (4). That is, for $t \leq H-1$,

$$c_t = \left(1-\beta\omega\right)\left(d_t + \left(1-\tau_y\right)\mathbb{E}_t\left[\sum_{k=0}^{H-t-1}\left(\beta\omega\right)^k c_{t+k}\right] - \left(\beta\omega\right)^{H-t} d_H\right), \tag{B.5}$$

with

$$d_{t+1} = \frac{1}{\beta}\left(d_t - \tau_y c_t + \varepsilon_0 \mathbb{1}_{\{t=0\}}\right). \tag{B.6}$$

Now, let $\{c_t^{\text{Baseline}}\}_{t=0}^{H-1}$ denote the path characterized by (B.5) and (B.6) given d_0, with $\tau_y' = 1-\left(1-\tau_y\right)\mu$. From (B.3) and (B.4), we know that, for all $t \leq H-1$,

$$c_t^{\text{OLG}} = c_t^{\text{Baseline}} \quad \text{and} \quad y_t = \mu c_t^{\text{Baseline}}.$$

Now first of all suppose that prices are fully rigid (i.e., we have that $\kappa = 0$). Then $d_0 = b_0 = 0$, and so the self-financing share of the OLG-PIH economy is given by

$$\nu = \frac{\mu\tau_y}{1-\left(1-\tau_y\right)\mu}\nu^{Baseline},$$

where

$$\nu \equiv \frac{\tau_y \sum_{t=0}^{H-1}\beta^t y_t}{\varepsilon_0} \quad \text{and} \quad \nu^{Baseline} \equiv \frac{\tau_y' \sum_{t=0}^{H-1}\beta^t c_t^{Baseline}}{\varepsilon_0}.$$

As $H \to \infty$, $\nu^{Baseline} \to 1$, and so we find that

$$\nu \to \frac{\mu\tau_y}{1-\left(1-\tau_y\right)\mu}$$

Note that $\frac{\mu\tau_y}{1-(1-\tau_y)\mu}$ is continuous in μ and limits to 1 as the margin of PIH consumers vanishes. Away from the rigid-price case, from (D.10), the limiting self-financing share is given by

$$\nu \to \frac{\frac{\mu\tau_y}{1-(1-\tau_y)\mu}}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\frac{\mu\tau_y}{1-(1-\tau_y)\mu}},$$

again continuous in the margin of exact PIH consumers.

Cognitive discounting. Under cognitive discounting, a shock h periods in the future is additionally discounted by a factor of θ, with $\theta = 1$ corresponding to the standard full-information, rational-expectations model and $\theta = 0$ corresponding to myopic households. It is immediate that cognitive discounting added to our baseline OLG model gives the adjusted aggregate demand relation

$$c_t = (1-\beta\tilde{\omega})\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\tilde{\omega}\theta)^k(y_{t+k} - t_{t+k})\right]\right), \tag{B.7}$$

where $\tilde{\omega}$ is the survival rate. This fits into our demand structure (30) with $M_y = M_d = 1-\beta\tilde{\omega}$, $\delta = 1$ and $\omega = \tilde{\omega}\theta$. It is immediate that, for $\tilde{\omega} < 1$ and $\theta < 1$, Assumption 1 holds. Differently from the baseline OLG case, however, Assumption 2 does not hold automatically; plugging into (31) and re-arranging we find that we need

$$\tau_y > \frac{\tilde{\omega}(1-\theta)}{1-\tilde{\omega}\theta}\frac{1-\beta}{1-\beta\tilde{\omega}} \tag{B.8}$$

This relation holds automatically for $\theta = 1$, but need not hold for $\theta < 1$; intuitively, as already discussed in the main text, $\theta < 1$ dampens demand spillovers from the future to the present and thus slows down the Keynesian boom. (B.8) is, however, a very mild condition: even for $\theta = 0$, as long as β is close to one and for values of $\tilde{\omega}$ as considered in Section 6, Assumption 2 holds even for small τ_y.

C Quantitative analysis

This section supplements our quantitative analysis in Section 6. We first provide some missing details on the specification of our spender-OLG hybrid model in Appendix C.1, and then consider model variants with more flexible prices and more aggressive monetary reactions in Appendices C.2 and C.3. Several further related results are presented in the Additional Materials.

C.1 Further details on the hybrid model

We first elaborate on the model environment and discuss in greater detail the model's implications for household consumption behavior, contrasting it in particular with the predictions of quantitative HANK-type models.

Model. The only change relative to our baseline model of Section 2 is that we generalize the household block to also feature a margin μ of spenders—that is, households who do not hold any assets and immediately spend any income they receive. The remaining fraction $1-\mu$ of households are exactly as described in Section 2.1. Both groups of households receive labor income as well as dividends and pay taxes, but only the OLG block holds government bonds.

We will make assumptions ensuring that both groups of households receive the same labor and dividend income, pay the same taxes (up to a between-group steady-state transfer), and have identical steady-state consumption. First, we assume that unions assign identical hours worked to both groups, and that dividends also accrue equally to both. Second, we assume that the government in lump-sum fashion redistributes between the two groups to ensure identical steady-state consumption; given that government bonds are held by the OLG block, this will generally require lump-sum transfers to spenders. Under those assumptions, it is first of all immediate that the supply block of the economy—notably (16)—is unchanged. Next, the demand block of the economy generalizes (29) as follows:

$$c_t = (1-\beta\omega)\cdot d_t + \left[\mu + (1-\mu)(1-\beta\omega)\right]\cdot\left((y_t - t_t) + \frac{(1-\mu)(1-\beta\omega)}{\mu+(1-\mu)(1-\beta\omega)}\mathbb{E}_t\left[\sum_{k=1}^{\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\right). \tag{C.1}$$

Replacing (29) by (C.1) is the only difference between our baseline OLG economy and the generalized hybrid model. Relative to (29), the most important change in (C.1) is that we allow the MPC out of income to be larger than that out of wealth. As we discuss next, this minimal departure from our baseline OLG model is all that is needed to ensure (approximate) consistency with consumption-savings behavior even in quantitative HANK-type models.

Household consumption-savings behavior. By our discussion in Additional Materials E.1 we know that the role of the household consumption-savings decision in driving our self-financing result is fully governed by the matrix of intertemporal marginal propensities to consume. The left top panel of Figure 4 provides a visual illustration of this matrix in our spender-OLG hybrid model, as implied by the generalized demand block (C.1).

The figure plots the spending response over time to (anticipated) income gains at different dates. We emphasize two takeaways. First, the response to a date-0 income gain—that is, the first column of $\mathscr{M}$—agrees with prior empirical evidence (Fagereng et al., 2021), as discussed in Section 6.1. Second, the higher-order columns are qualitatively and quantitatively similar to those implied by HANK-type models. This observation has been made previously in Auclert et al. (2023) and Wolf (2021). For our purposes, the important takeaway is that our analysis is indeed quantitatively relevant—as far as our question of self-financing is concerned, our model will have very similar predictions as richer quantitative HANK-type models. We further illustrate this observation in Additional Materials E.6.1.

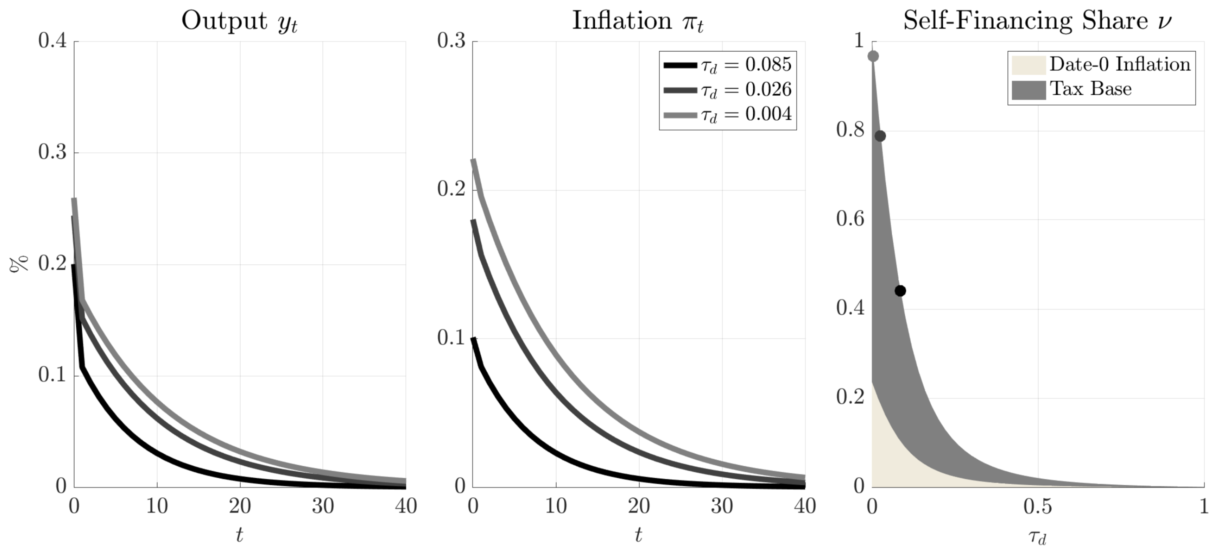

Figure C.1: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to a shock ε_0 equal to one per cent of steady-state output, as a function of τ_d, with more flexible prices. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

C.2 Self-financing with more flexible prices

Figure C.1 repeats our analysis of Section 6.2 in a variant of the baseline model with more flexible prices, setting $\kappa = 0.1$—a value at the large end of recent empirical evidence and arguably more relevant for the inflationary post-covid environment. With more flexible prices, a non-trivial share—here around 20 percent—of self-financing comes from adjustments in prices rather than quantities. We note that alternative assumptions on the shape of the NKPC (e.g., a hybrid NKPC) or on government debt maturity could further impact that split; we leave this investigation to Angeletos et al. (2024b).

C.3 Active monetary reaction

Figure C.2 shows the degree of self-financing ν as a function of τ_d under the active monetary (Taylor-type) rule (34), for $\psi \in \{1, 1.25, 1.5\}$, with the three panels corresponding to the three displayed values of ψ, and throughout assuming our baseline flat NKPC (i.e., $\kappa = 0.0062$). The figure provides a visual illustration that complements Table 2. In all three cases, full self-financing is possible; in particular, all panels are qualitatively and quantitatively similar to our headline results in Figure 3. Even more aggressive monetary reactions, either through higher ψ or with higher κ, would be necessary to prevent full self-financing from being feasible.

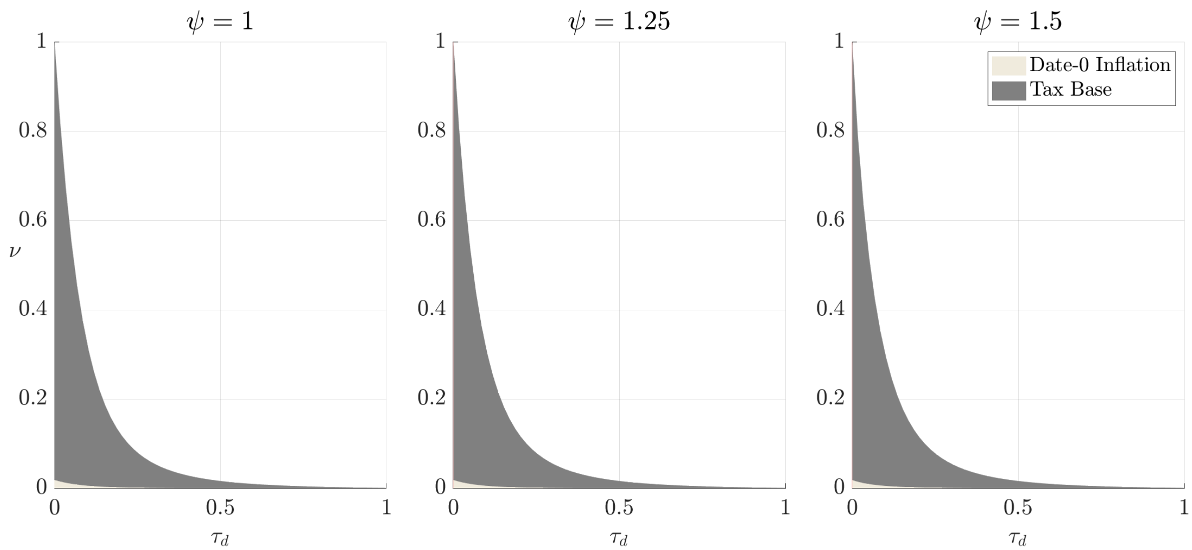

Figure C.2: Total self-financing share ν in response to a shock ε_0 equal to one per cent of steady-state output, as a function of τ_d, for the Taylor-type monetary policy rule $i_t = \psi\pi_t$, with $\psi \in \{1, 1.25, 1.5\}$.

D Proofs

D.1 Proof of Proposition 1

Note that we restrict that $\omega \in (0,1)$, $\tau_y \in (0,1)$, and $\tau_d \in (0,1)$. We first write (13) recursively:

$$\begin{aligned} y_t - \mathscr{F}_1 \cdot (d_t + \varepsilon_t) &= (1-\beta\omega)\mathscr{F}_2 \cdot y_t + \beta\omega\mathbb{E}_t\left[y_{t+1} - \mathscr{F}_1 \cdot (d_{t+1} + \varepsilon_{t+1})\right] \\ &= (1-\beta\omega)\mathscr{F}_2 \cdot y_t + \beta\omega\mathbb{E}_t\left[y_{t+1} - \mathscr{F}_1 \cdot \frac{1}{\beta}\left[(1-\tau_d)(d_t+\varepsilon_t) - \tau_y y_t\right]\right]. \end{aligned}$$

After rearranging terms and using the formula of $\mathscr{F}_1$ and $\mathscr{F}_2$ (as stated after (13)), we have

$$\begin{aligned} y_t &= \frac{(1-\omega(1-\tau_d))\mathscr{F}_1}{1-\omega\tau_y\mathscr{F}_1 - (1-\beta\omega)\mathscr{F}_2}(d_t+\varepsilon_t) + \frac{\beta\omega}{1-\omega\tau_y\mathscr{F}_1 - (1-\beta\omega)\mathscr{F}_2}\mathbb{E}_t\left[y_{t+1}\right] \\ &= \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}(d_t+\varepsilon_t) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right]. \end{aligned}$$

Applying period-t expectations $\mathbb{E}_t[\cdot]$ to (17), we have

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t[y_{t+1}] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y \end{pmatrix} \begin{pmatrix} d_t+\varepsilon_t \\ y_t \end{pmatrix} \qquad \text{(D.1)}$$

The two eigenvalues of the system are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \frac{1}{\beta}(1-\tau_d) = 0,$$

with

$$\begin{aligned}
\lambda_1 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\sqrt{\left(1+\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2-4\frac{1}{\beta}(1-\tau_d)}}{2} \\
&= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\
&> \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\left|1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \geq 1
\end{aligned} \tag{D.2}$$

and

$$\begin{aligned}
\lambda_2 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1+\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2-4\frac{1}{\beta}(1-\tau_d)}}{2} \\
&= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\
&< \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\left|\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)-1\right|}{2} \leq 1,
\end{aligned} \tag{D.3}$$

with $\lambda_2 > 0$ too since $\lambda_1\lambda_2 = \frac{1}{\beta}(1-\tau_d) > 0$. Let $\left(1, \chi_2\right)'$ denote the eigenvector associated with λ_2, where

$$\lambda_2 = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_2\right) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\lambda_2} > 0. \tag{D.4}$$

This means that any bounded path of $\{d_t, y_t\}_{t=0}^{+\infty}$ that satisfies (D.1) takes the form of

$$y_t = \chi(d_t+\varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t+\varepsilon_t),$$

where χ and ρ_d are uniquely given by

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1). \tag{D.5}$$

In other words, the equilibrium takes the form of (19) and satisfies (20).[36] From (16), inflation satisfies $\pi_t = \frac{\kappa}{1-\beta\rho_d}y_t$.

Note that the total amount of nominal public debt outstanding at the start of $t=0$, $B_0 = B^{ss}$, is given. From (16) and (18), we know d_0 is uniquely pinned down by

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0[y_k] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\varepsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_0. \tag{D.6}$$

[36]To see the first part of (20), combine (13) with (19).

Similarly, for $t \geq 1$,

$$d_t - \mathbb{E}_{t-1}[d_t] = -\frac{D^{ss}}{Y^{ss}}(\pi_t - \mathbb{E}_{t-1}[\pi_t]) = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\left(\mathbb{E}_t\left[y_{t+k}\right] - \mathbb{E}_{t-1}\left[y_{t+k}\right]\right)$$
$$= -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_t - \mathbb{E}_{t-1}[d_t] + \varepsilon_t) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_t. \tag{D.7}$$

Together with (16) and (19), this pins down the unique bounded equilibrium path of $\{\pi_t, d_t, y_t\}_{t=0}^{+\infty}$.

D.2 Proof of Theorem 1

We start with the case based on the baseline fiscal policy (7). We begin by considering an alternative economy with rigid prices ($\kappa = 0$) but otherwise identical to the baseline economy. Let ν' denote the self financing share in this alternative economy, which similarly to (23) is given as

$$\nu' \cdot \varepsilon_0 = \nu'_y \cdot \varepsilon_0 \equiv \sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0\left[y_k\right].$$

Note that this alternative economy shares the same χ and ρ_d as our main economy, because χ and ρ_d are independent of κ from (20). Moreover, all self-financing in this alternative economy comes from tax base changes. In particular, there is no $t = 0$ price level jump and so the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is pre-determined. From (19) and (23) we know that

$$\nu' = \tau_y\frac{\chi}{1-\beta\rho_d}. \tag{D.8}$$

Now, consider the general case with $\kappa \geq 0$. From NKPC (16) and the definitions in (22) – (23), we have

$$\nu_p = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y}\nu_y = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa\frac{D^{ss}}{Y^{ss}}}\nu \tag{D.9}$$

From the formula of d_0 in (18), we know

$$d_0 = -\nu_p\varepsilon_0 \quad \text{and} \quad \nu_y = \left(1-\nu_p\right)\nu'.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}. \tag{D.10}$$

From the second part of (20), we know that

$$\frac{\chi}{1-\beta\rho_d} = \frac{\chi}{\tau_d + \tau_y \chi}. \tag{D.11}$$

From (D.3) and (D.5), we know

$$\rho_d = \lambda_2 = f(a,b) \equiv \frac{a+b+1-\sqrt{(a+b-1)^2+4b}}{2} \tag{D.12}$$

where $a = \frac{1}{\beta}(1-\tau_d) > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{(a+b-1)}{2\sqrt{(a+b-1)^2+4b}} > 0$, we know that ρ_d decreases with τ_d. From (D.4) and (D.5), we then know $\chi = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y - \rho_d}$ also decreases in τ_d. From (D.11), we know $\frac{\chi}{1-\beta\rho_d}$ decreases in τ_d. Finally, from (D.8) and (D.10), we know ν decreases in τ_d. This finishes the proof of Part 1.

For Part 2, from (D.3) and (20), we know that ρ_d and χ are continuous in $\tau_d \in [0,1)$, and

$$\begin{aligned} \rho_d^{\text{full}} \equiv \lim_{\tau_d \to 0^+} \rho_d &= \frac{\left(\frac{1}{\beta}+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) - \sqrt{\left(1-\frac{1}{\beta}-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2 + 4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \\ &< \frac{\frac{1}{\beta}+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) - \left|\frac{1}{\beta}+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)-1\right|}{2} \le 1 \end{aligned} \tag{D.13}$$

$$\chi^{\text{full}} \equiv \lim_{\tau_d \to 0^+} \chi = \frac{1-\beta\rho_d^{\text{full}}}{\tau_y} > 0 \tag{D.14}$$

From (D.11), we know $\lim_{\tau_d \to 0^+} \frac{\chi}{1-\beta\rho_d} = \frac{1}{\tau_y}$. From (D.8) and (D.10), we know $\lim_{\tau_d \to 0^+} \nu = 1$. Next, the fact that $\lim_{k\to\infty} \mathbb{E}_t[d_{t+k}] \to 0$ follows directly from $\rho_d^{\text{full}} < 1$. Moreover, the limiting equilibrium path as $\tau_d \to 0^+$ is the unique bounded equilibrium when $\tau_d = 0$, characterized by (19), (D.6), and (D.7) with $\rho_d = \rho_d^{\text{full}}$ and $\chi = \chi^{\text{full}}$. In other words, there is no discontinuity at $\tau_d = 0$.

Now we turn to the alternative fiscal policy rule in (8), for which we use the equilibrium characterization in Appendix A.3. For the case with rigid prices ($\kappa = 0$), one can see from (A.10) that ν' increases in H, which proves Part I. For Part II and to find $\lim_{H\to\infty} \nu'$, first note that, from (A.8), $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, increasing sequence. As a result, there exists $\chi^{\text{full,NM}}$ such that $\lim_{k\to\infty} \chi_k = \chi^{\text{full,NM}}$ and $\chi^{\text{full,NM}} = g\left(\chi^{\text{full,NM}}\right) \in \left(0, \frac{1}{\tau_y}\right)$, where $g(\cdot)$ is defined in (A.7). From (A.10), we know that $\lim_{H\to\infty} \nu' = 1$. From (D.4) and (D.5), we also know that $g\left(\chi^{\text{full}}\right) = \chi^{\text{full}}$ where χ^{full} defined in (D.14) parametrizes the output response in the full self-financing limit ($\tau_d \to 0$) with the baseline fiscal rule (7). From the definition of $g(\cdot)$ in (A.7), we know that there is a unique $\chi > 0$ such that $g(\chi) = \chi$ when $\omega < 1$ and $\tau_y \in (0,1)$. As a result, $\chi^{\text{full,NM}} = \chi^{\text{full}} < \frac{1}{\tau_y}$ and $\lim_{k\to+\infty} \chi_k = \chi^{\text{full}}$. That is, these two limits ($\tau_d \to 0$ and $H \to \infty$) share the same equilibrium path. Moreover, $\lim_{k\to\infty} \frac{1-\tau_y\chi_k}{\beta} = \frac{1-\tau_y\chi^{\text{full}}}{\beta} = \rho_d^{\text{full}} < 1$. From (A.9), we know that $\lim_{H\to\infty} \mathbb{E}_0[d_H] = 0$.

Finally, for the general case with $\kappa \ge 0$, the desired result follows directly from the rigid price case together with (A.12) and (D.10).

D.3 Proof of Proposition 2

Since $\pi_0 = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_0[y_k]$ (by the NKPC), it follows that the debt erosion effect is proportional to the tax base effect:

$$\nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \nu_y. \tag{D.15}$$

(24) follows directly from (D.15). The rest of the Proposition 2 follows directly from Theorem 1 and (24).

D.4 Proof of Proposition 3

Consider the baseline fiscal policy (7). From (D.12), we know

$$\rho_d = \frac{a+b+1-\sqrt{(a+b+1)^2-4a}}{2} = \frac{2a}{a+b+1+\sqrt{(a+b+1)^2-4a}},$$

where $a = \frac{1}{\beta}(1-\tau_d) > 0$ and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. From the second part of the equation, we know that ρ_d decreases in $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)$ and increases in ω. From the second half of (20), we then know that χ decreases in ω. From (D.8), (D.10) and (D.11), we know that ν decreases in ω.

Now we turn to the alternative fiscal policy in (8), which we use the equilibrium characterization in Appendix A.3. First note that, from (A.4) and (A.6), each χ_k $(k \geq 1)$ decreases in ω. From (A.10), we know that the self-financing share in the rigid-price $\kappa = 0$ decreases in ω. From (A.12), we know the self-financing share in the general $\kappa \geq 0$ case decreases in ω.

D.5 Proof of Theorem 2

As a preparation, for any bounded equilibrium in the form of (19), from (16) and (18) ,

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k \mathbb{E}_0[y_k] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\varepsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_0.$$

From the definition (B.1), we know

$$\nu = \frac{\frac{\left(\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}\right)\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}}{1+\frac{\beta\frac{D^{ss}}{Y^{ss}}\phi\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}} = \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{\chi}{1-\beta\rho_d}}{1+\left(\kappa+\beta\phi\right)\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}} = \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\chi}{\tau_d+\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\chi}, \tag{D.16}$$

where we use $\rho_d = \frac{1-\tau_d}{\beta} - \frac{\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta}\chi$ from (5).

As mentioned in the main text, we restrict $\phi \in [-1/\sigma, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}})$. Aggregating individual demand relation (11), together with monetary policy (28), goods and asset market clearing, and the government budget (5) lead to the following aggregate demand relation:

$$y_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\right) - \left(\sigma\omega - (1-\beta\omega)\frac{D^{ss}}{Y^{ss}}\right)\beta\phi\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k y_{t+k}\right]$$
$$= \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\sigma\phi-\frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}(d_t - t_t) + \frac{1}{1+\sigma\phi-\frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}\mathbb{E}_t\left[y_{t+1}\right].$$

Together with the baseline fiscal policy (7) we arrive at the following equation:

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}(d_t+\varepsilon_t) + \frac{1}{1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\mathbb{E}_t\left[y_{t+1}\right].$$

Applying the period-t expectation operator $\mathbb{E}_t[\cdot]$ to (5), we have

$$\begin{pmatrix}\mathbb{E}_t[d_{t+1}]\\ \mathbb{E}_t\left[y_{t+1}\right]\end{pmatrix} = \begin{pmatrix}\frac{1-\tau_d}{\beta} & -\frac{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta}\\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}{\beta\omega}\end{pmatrix}\begin{pmatrix}d_t+\varepsilon_t\\ y_t\end{pmatrix} \tag{D.17}$$

The two eigenvalues are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1-\tau_d}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+(1+\sigma\phi)\frac{1-\tau_d}{\beta}=0. \tag{D.18}$$

From $\phi\in[-1/\sigma,\frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}})$ and $\tau_d\in[0,1]$, we know that $\lambda_1+\lambda_2\geq 0$ and $\lambda_1\lambda_2\geq 0$, so $\lambda_1\geq 0$ and $\lambda_2\geq 0$.

We first prove Part 1 of Theorem 2. That is, if

$$\phi < \bar{\phi} \equiv \frac{\frac{(1-\beta\omega)(1-\omega)}{\omega}\tau_y}{\sigma(1-\beta)+\frac{(1-\beta\omega)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}}} < \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}, \tag{D.19}$$

as $\tau_d\to 0$ (from above), there exists a unique bounded equilibrium, and it is such that $\nu\to 1$ (from below) and $\lim_{k\to\infty}\mathbb{E}_t[d_{t+k}]\to 0$. That is, full self-financing obtains as fiscal adjustment is indefinitely delayed.

Since the eigenvalue of (D.18) is continuous in τ_d at 0, we have

$$\lim_{\tau_d\to 0^+}\lambda_1 = \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}{2}$$
$$= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4(1+\sigma\phi)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2} \tag{D.20}$$

and

$$\lim_{\tau_d\to 0^+}\lambda_2 = \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}{2}$$
$$= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}. \tag{D.21}$$

When $\phi \in [-1/\sigma, 0)$, from (D.20) and (D.21),

$$\lim_{\tau_d\to 0^+}\lambda_1 \geq \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\left|\frac{1}{\beta}+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)-1-\sigma\phi\right|}{2} \geq \frac{1}{\beta}+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right) > 1$$
$$\lim_{\tau_d\to 0^+}\lambda_2 \leq \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\left|1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} \leq 1+\sigma\phi < 1$$

When $\phi \in [0, \bar{\phi})$, from (D.19), we have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right) > \sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence

$$\lim_{\tau_d\to 0^+}\lambda_1 > \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\left|1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} \geq 1+\sigma\phi \geq 1$$
$$\lim_{\tau_d\to 0^+}\lambda_2 = \frac{2\frac{1+\sigma\phi}{\beta}}{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}} \tag{D.22}$$
$$< \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2-4\frac{1+\sigma\phi}{\beta}}} = \frac{2\frac{1+\sigma\phi}{\beta}}{1+\frac{1+\sigma\phi}{\beta}+\left|\frac{1+\sigma\phi}{\beta}-1\right|} = 1.$$

Thus, as long as (D.19) holds, and as $\tau_d \to 0^+$, there exists a unique a bounded equilibrium in the form of (19), with $\rho_d = \lambda_2$ (where $\lim_{\tau_d\to 0^+}\rho_d < 1$) and $\chi = \frac{1-\tau_d-\beta\rho_d}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}} > 0$, and $\{\pi_t, y_t, d_t\}$ given by (16), (19), (D.6), and (D.7). The fact that $\lim_{\tau_d\to 0^+}\nu \to 1$ follows directly from (D.16). The fact that and $\lim_{k\to\infty}\mathbb{E}_t[d_{t+k}] \to 0$ follows directly from $\lim_{\tau_d\to 0^+}\rho_d \in [0,1)$.

For Part 2 of Theorem 2, note that, when $\phi > \bar{\phi}$, we from (D.19) have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right) < \sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence, from (D.22),

$$\lim_{\tau_d \to 0^+} \lambda_2 > \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta} + 1 + \frac{\sigma\phi}{\beta} + \sqrt{\left(1 + \frac{1+\sigma\phi}{\beta}\right)^2 - 4\frac{1+\sigma\phi}{\beta}}} = 1.$$

As a result, there exists no bounded equilibrium if the fiscal adjustment is infinitely delayed (i.e., if $\tau_d \to 0$ from above).

For $\tau_d > 0$, we have[37]

$$\lambda_2 = f(a,b) \equiv \frac{a + b + 1 + \sigma\phi - \sqrt{(a + b - (1 + \sigma\phi))^2 + 4(1 + \sigma\phi)\, b}}{2}$$

where $a = \frac{1}{\beta}(1 - \tau_d) > 0$ and $b = \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{a+b-(1+\sigma\phi)}{2\sqrt{(a+b-(1+\sigma\phi))^2+4(1+\sigma\phi)b}} > 0$, we know that λ_2 decreases with τ_d and $\lim_{\tau_d \to 1^-} \lambda_2 = 0$. As a result, for $\phi > \bar{\phi}$, there exists an $\underline{\tau}_d(\phi) \in (0,1)$ such that $\lambda_2 \le 1$ if and only if $\tau_d \ge \underline{\tau}_d(\phi)$. As a result, for $\phi > \bar{\phi}$, any bounded equilibrium exists if and only if $\tau_d \ge \underline{\tau}_d(\phi)$. For any $\phi \in \left(\bar{\phi}, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}\right)$, $\chi = \frac{1-\tau_d-\beta\rho_d}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}} \le \frac{1-\underline{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}$. From (D.16), $\nu \le \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{1-\underline{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}}{\bar{\tau}_d(\phi)+\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{1-\underline{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}} \equiv \bar{\nu}(\phi) < 1$. This proves Part 2 of Theorem 2.

D.6 Proof of Theorem 3

Given the baseline fiscal policy (7), we focus on a bounded equilibrium similar to (19), taking the form of

$$y_t = \chi_d d_t + \chi_\varepsilon \varepsilon_t, \quad \mathbb{E}_t[d_{t+1}] = \rho_d d_t + \rho_\varepsilon \varepsilon_t \quad \text{with} \quad \chi_d, \chi_\varepsilon > 0, \;\; \rho_d \in (0,1). \tag{D.23}$$

For (D.23) to be an equilibrium, it needs to satisfy (17) and (30). For (D.23) to satisfy to the government budget (17), we need

$$\rho_d = \frac{1}{\beta}\left(1 - \tau_d - \tau_y\chi_d\right) \quad \text{and} \quad \rho_\varepsilon = \frac{1}{\beta}\left(1 - \tau_d - \tau_y\chi_\varepsilon\right). \tag{D.24}$$

For (D.23) to satisfy aggregate demand (30) (together with market clearing $c_t = y_t$), we need

$$\chi_d = M_d + M_y\left[\left(1-\tau_y\right)\chi_d - \tau_d\right]\left(1 + \delta\sum_{k=1}^{+\infty}\left(\beta\omega\rho_d\right)^k\right) = \frac{M_d - \tau_d M_y\left(1 + \frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1 - M_y\left(1-\tau_y\right)\left(1 + \frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}. \tag{D.25}$$

and

$$\chi_\varepsilon = M_y\left(1 + \left(\left(1-\tau_y\right)\chi_\varepsilon - \tau_d + \delta\left(\left(1-\tau_y\right)\chi_d - \tau_d\right)\sum_{k=1}^{+\infty}(\beta\omega)^k\rho_d^{k-1}\rho_\varepsilon\right)\right). \tag{D.26}$$

[37] The formula for the function f is slightly adjusted compared to (D.12) in the baseline analysis, to accommodate $\phi \neq 0$.

(D.24) and (D.25) together mean that ρ_d needs to be the root of the following equation:

$$h(\rho_d, \tau_d) \equiv \frac{1-\tau_d-\beta\rho_d}{\tau_y} - \frac{M_d - \tau_d M_y\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1-M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)} = 0.$$

When Assumption 2 holds, we first show that there exists a unique $\rho_d^{\text{full}} \in (0,1)$ such that

$$h\left(\rho_d^{\text{full}}, 0\right) = \frac{1-\beta\rho_d^{\text{full}}}{\tau_y} - \frac{M_d}{1-M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega\rho_d^{\text{full}}}{1-\beta\omega\rho_d^{\text{full}}}\right)} = 0.$$

Note that that $h(0,0) = \frac{1}{\tau_y} - \frac{M_d}{1-M_y(1-\tau_y)} > 0$ because $\tau_y > 0$, $M_y \in (0,1)$ and $M_y \geq M_d$. Then, there are two cases. First, $M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right) > 1$. In this case, there exists $\bar{\rho} \in (0,1)$ such that

$$M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega\bar{\rho}}{1-\beta\omega\bar{\rho}}\right) = 1.$$

For $\rho_d \in [0,\bar{\rho})$, $h(\rho_d, 0)$ monotonically decreases in ρ_d and $\lim_{\rho_d \to (\bar{\rho})^-} h(\rho_d, 0) = -\infty$. As a result, there exists a unique $\rho_d^{\text{full}} \in (0,\bar{\rho})$ such that we have $h\left(\rho_d^{\text{full}}, 0\right) = 0$. For $\rho_d \in (\bar{\rho}, 1)$, $\frac{M_d}{1-M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)} < 0$. As a result, $h(\rho_d, 0) > 0$.

Second, $M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right) < 1$. From Assumption 2, in this case,

$$h(1,0) = \frac{1-\beta}{\tau_y} - \frac{M_d}{1-M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right)} < 0,$$

and $h(\rho_d, 0)$ monotonically decreases in $\rho_d \in [0,1]$. As a result, there exists a unique $\rho_d^{\text{full}} \in (0,1)$ such that $h\left(\rho_d^{\text{full}}, 0\right) = 0$.

Note that $h(\rho_d, \tau_d)$ is continuously differentiable in the neighborhood of $\left(\rho_d^{\text{full}}, 0\right)$. Because $\frac{\partial h\left(\rho_d^{\text{full}}, 0\right)}{\partial \rho_d} < 0$, we can use the implicit function theorem to show that for each τ_d in a right neighborhood of 0, there exists a unique $\rho_d(\tau_d) \in (0,1)$ such that $h(\rho_d(\tau_d), \tau_d) = 0$ and $\lim_{\tau_d \to 0^+} \rho_d(\tau_d) = \rho_d^{\text{full}} \in (0,1)$. For $\tau_d \to 0^+$, given $\rho_d(\tau_d)$, one can find $\rho_\varepsilon(\tau_d)$ and $\chi_d(\tau_d), \chi_\varepsilon(\tau_d) > 0$ from (D.24) – (D.26), and constitute a bounded equilibrium based on (16)–(18). The fact that $\lim_{\tau_d \to 0^+} \nu = 1$ follows directly from boundedness and $\lim_{\tau_d \to 0^+} \frac{\tau_d\left(\varepsilon_0 + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_0[d_k]\right)}{\varepsilon_0} = 0$, using (21) and (22). The fact that $\lim_{k\to\infty} \mathbb{E}_t[d_{t+k}] \to 0$ follows directly from $\rho_d^{\text{full}} \in (0,1)$. This finishes the proof.