Supplementary Materials to
"A Quantitative Theory of the Credit Score"

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A Welfare Metric

Table 7 reports welfare using a wealth equivalent measure whose construction we describe in this section. Given that utility flows in our model are derived not only directly from consumption flows but also from the extreme value shocks attached to consumption choices, we are unable to use a standard Lucas consumption equivalent measure.[61] As an alternative, we construct a measure that answers the question: "how much additional wealth must an agent in state (β, z, ω) be given in the BASE economy in order to be indifferent between being born into the BASE economy or into a given alternative (ALT) economy?" Denoting the value functions in the BASE and ALT economies by $W_{\text{BASE}}(\beta, z, \omega)$ and $W_{\text{ALT}}(\beta, z, \omega)$, respectively, we formally solve for a set of numbers $\phi_{\text{ALT}}(\beta, z, \omega)$ that satisfies

$$W_{\text{BASE}}\left(\beta, z, a(\omega) + \phi_{\text{ALT}}(\beta, z, \omega), s(\omega), e(\omega)\right) = W_{\text{ALT}}\left(\beta, z, a(\omega), s(\omega), e(\omega)\right) \tag{55}$$

Note that if the ALT economy is the FI economy, then s is not a state variable. However, ϕ_{ALT} still depends on s since this shifts the value in the BASE economy. The numbers reported in Table 7 are the ratios of the numbers computed via this expression to mean wealth in the BASE economy, $\overline{a} = \sum_a a \cdot \sum_{\beta,z,s,e} \mu(\beta, z, a, s, e)$.

This measure has several attractive properties. First, by varying wealth in the BASE economy only, we avoid any issues related to the fact that in the NT economy there are no agents with the type score of a newborn and debt, since type scores increase monotonically with age and all newborns are born with zero wealth. Second, the units are directly interpretable as physical quantities of wealth rather than utils. Relatedly, the change in wealth is purely discretionary and not imposed to be translated into consumption immediately. Third, it involves only a trivial calculation given the value functions from each equilibrium; the set of numbers ϕ may be solved via simple bisection.

[61] In particular, the indirect utility function for an agent in state (β, z, ω) may not be written as an appropriately discounted infinite sum of period utility flows.

B Deriving the impact of EV parameters

To ease notation in this section, let an agent's entire state be denoted by $x = (\beta, e, z, a, s)$, and the set of feasible actions for that agent be denoted by $\mathcal{F}(x)$. The goal of this section is to show how the choice probability function σ varies with the extreme value scale parameters α and λ. We first cover the repayment ($d = 0$) actions, and then bankruptcy ($d = 1$). To ease computations in this section rather than compute derivatives with respect to λ or α directly, we will compute them with respect to $1/\lambda$ or $1/\alpha$.[62] Throughout this section, we focus on the first order effects of changes in these parameters by ignoring all derivatives with respect to action-specific value terms, i.e. $\frac{\partial v^{(d,a')}(x)}{\partial \frac{1}{\lambda}}$.

Saving and borrowing actions Equation (9) describes the probability of choosing a feasible action $(0, a')$ conditional on not filing for bankruptcy. Considering first λ,

$$\begin{aligned}
\frac{\partial \tilde{\sigma}^{(0,a')}(x)}{\partial(1/\lambda)} &= \Bigg[\exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\}\frac{v^{(0,a')}(x)}{\lambda}\sum_{(0,\tilde{a})\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\} \\
&\quad -\exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\}\sum_{(0,\tilde{a})\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\}\frac{v^{(0,\tilde{a})}(x)}{\lambda}\Bigg] \Bigg/ \Bigg[\sum_{(0,\tilde{a})\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\}\Bigg]^2 \\
&= \frac{\exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\}}{\sum_{(0,\tilde{a})\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\}}\frac{\sum_{(0,\tilde{a})\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\}\left[v^{(0,a')}(x)-v^{(0,\tilde{a})}(x)\right]}{\sum_{(0,\tilde{a})\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\}} \\
&= \tilde{\sigma}^{(0,a')}(x)\sum_{(0,\tilde{a})\in\mathcal{F}(x)}\tilde{\sigma}^{(0,\tilde{a})}(x)\left[v^{(0,a')}(x)-v^{(0,\tilde{a})}(x)\right].
\end{aligned}$$

We can sign this derivative according to

$$\begin{aligned}
\frac{\partial \tilde{\sigma}^{(0,a')}(x)}{\partial(1/\lambda)} > 0 \quad &\iff \quad \sum_{(0,\tilde{a})\in\mathcal{F}(x)}\tilde{\sigma}^{(0,\tilde{a})}(x)\left[v^{(0,a')}(x)-v^{(0,\tilde{a})}(x)\right] > 0 \\
&\iff \quad v^{(0,a')}(x) > \sum_{(0,\tilde{a})\in\mathcal{F}(x)}\tilde{\sigma}^{(0,\tilde{a})}(x)v^{(0,\tilde{a})}(x),
\end{aligned}$$

where the second line uses the fact that $\sum_{(0,\tilde{a})\in F(x)}\tilde{\sigma}^{(0,\tilde{a})}(x) = 1$ by construction. Therefore, the probability of choosing $(0, a')$ conditional on not filing for bankruptcy increases in $1/\lambda$ (decreases in λ) if and only if the conditional value of choosing $(0, a')$, $v^{(0,a')}(x)$, exceeds the expected value of choosing from the set of alternative actions $(0, \tilde{a})$ at the current decision rule.

The inclusive value of repaying, $W_{ND}(x)$, takes the familiar log-sum form of (6). Since it will be

[62] This keeps the analysis clean by avoiding repeated applications of the quotient rule for derivatives to the extent possible.

useful in computing how $\sigma^{(1,0)}(x)$ varies with λ, we compute:

$$\begin{aligned}\frac{\partial W_{ND}(x)}{\partial(1/\lambda)} &= \lambda\left[\frac{\sum_{(0,a')\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\}v^{(0,a')}(x)}{\sum_{(0,a')\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\}} - \lambda\ln\left(\sum_{(0,a')\in\mathcal{F}(x)}\exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\}\right)\right] \\ &= \lambda\left[\sum_{(0,a')\in F(x)}\tilde{\sigma}^{(0,a')}(x)v^{(0,a')}(x) - W_{ND}(x)\right]\end{aligned}$$

which is positive if and only if the average action-value weighted by decision probabilities exceeds the value of filing for bankruptcy.

Bankruptcy Equation (10) defines the probability of filing for bankruptcy as a function of the conditional value of filing for bankruptcy and the inclusive value of repaying. We obtain

$$\begin{aligned}\frac{\partial\sigma^{(1,0)}(x)}{\partial(1/\lambda)} &= -\sigma^{(1,0)}(x)\left(1-\sigma^{(1,0)}(x)\right)\frac{\frac{\partial W_{ND}(x)}{\partial(1/\lambda)}}{\alpha} \\ \frac{\partial\sigma^{(1,0)}(x)}{\partial(1/\alpha)} &= \sigma^{(1,0)}(x)\left(1-\sigma^{(1,0)}(x)\right)\left(v^{(1,0)}(x)-W_{ND}(x)\right)\end{aligned}$$

The first expression above implies that $\frac{\partial\sigma^{(1,0)}(x)}{\partial(1/\lambda)}$ takes the opposite sign of $\frac{\partial W_{ND}(x)}{\partial(1/\lambda)}$. If raising λ raises $W_{ND}(x)$, it makes repaying more attractive and therefore lowers the probability of filing. The second expressions shows that as α decreases, the probability of filing increases if and only if the conditional value of filing exceeds the inclusive value of repaying.

C Extreme value shocks in the alternative economies

In this section, we explore our alternative economies under different extreme value parameterizations in order to measure how these parameters affect our counterfactuals. We present two robustness exercises relative to our results in Table 7 of Section 6. Specifically, the "benchmark" column in Table 10 simply provides our model moments and welfare measures in the FI and NT economies relative to the BASE economy using our estimated parameters (i.e. replicates Table 7). We compare these same measures under two alternative parameterizations: one in which we raise α by 10% in *both* the base and alternative economies and one in which we similarly raise λ by 10%. The results of this analysis are presented in the remaining columns of Table 10. It should be noted that these 10% increases in α and λ can have large effects on outcomes in the BASE economy (to which we are comparing the alternative outcomes) since the estimated values were chosen to match the data in the benchmark. Furthermore, 10% parametric decreases in α and λ can yield even bigger deviations between model and data moments, since this correspond to significant increases in the informational content of actions, especially for α.

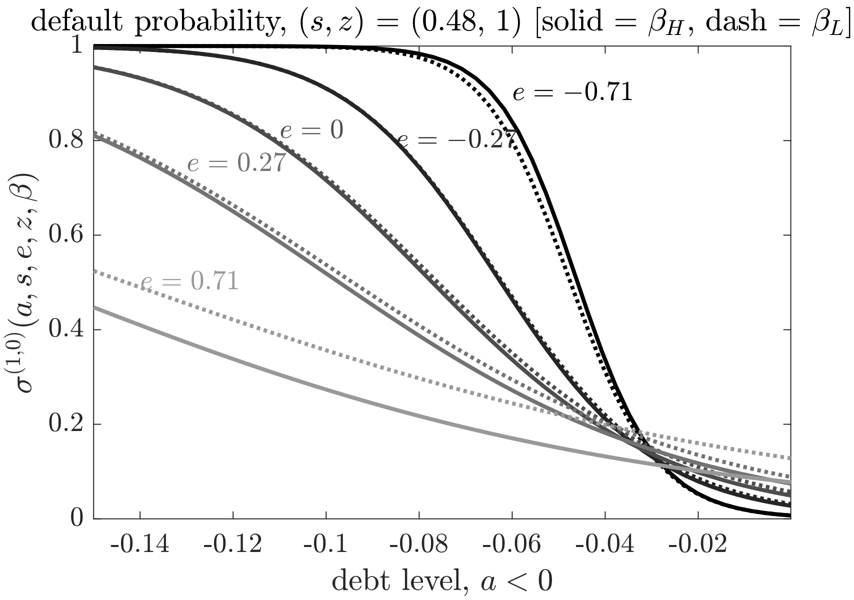
Starting with Panel A for the NT economy in Table 10, we note that the differences across columns are not large relative to the BASE values. This continues to be the case for the FI economy. Panel B shows that the positive welfare mean gain in the NT economy is eliminated as we raise α and λ indicating that the negative effects on dynamic incentives outweigh the static insurance benefits at already high levels of pooling. The welfare numbers in Panel B for the FI economy indicate that there can be larger gains in the presence of higher α and λ since inference is harder in the BASE economy, so that FI revelation of type benefits welfare.

Table 10: Alternative Economies and Extreme Value Parameters

model	**No Tracking (NT)**			**Full Information (FI)**		
parameterization	benchmark	high α	high λ	benchmark	high α	high λ
α value	0.0290	**0.0319**	0.0290	0.0290	**0.0319**	0.0290
λ value	0.0015	0.0015	**0.0017**	0.0015	0.0015	**0.0017**
Panel A: % difference from BASE model with same parameters						
bankruptcy rate	1.12	1.02	1.07	-0.13	0.27	-0.09
average int. rate	1.44	1.49	1.45	-2.52	-2.57	-2.48
int. rate dispersion	7.57	5.28	5.80	-2.15	-1.40	-1.58
fraction in debt	-0.15	-0.23	-0.17	0.00	0.05	0.01
debt to income ratio	0.39	0.35	0.35	-0.04	0.17	-0.02
Panel B: wealth equivalent welfare measure, newborns						
low z	0.060	-0.003	-0.001	0.121	0.547	0.139
median z	-0.000	-0.001	0.000	0.058	0.188	0.064
high z	-0.000	-0.001	0.000	0.104	0.163	0.103
mean	0.020	-0.002	0.000	0.094	0.299	0.102

Notes: Each entry in Panel A is the difference, in percentage points of the BASE moment, of the moment in the indicated alternative economy (FI or NT) relative to the BASE economy for the same parameterization. The high α and high λ parameterizations raise the value of these parameters by 10% in each case. Panel B reports the amount of additional wealth an agent would have to be given in the baseline economy in order to be indifferent between being born into the indicated alternative economy in the indicated state and being born in the baseline economy. The units for Panel B are percentages of mean wealth. The "base" columns for each economy match the "all" columns for each economy from Table 7.

Figure 16: Default Probability by Earnings and Type



D Other Results

Default probabilities by earnings level Figure 16 illustrates that, as is a feature of many default models, the probability of default is increasing in debt. It is also evident that default probabilities are decreasing in earnings for those with sufficiently large debt, another standard feature of default models. For very small debt, however, the lowest earners (who have the highest marginal utility of consumption) are least likely to default in order to avoid bearing the costs (κ and $\kappa_1 \times \exp(e)$) of default.

Credit Access Following Default For all $a' < 0$, define the two price schedules

$$\begin{aligned} q_D^{a'}(e, a, s) &\equiv q^{a'}\left(e, 0, \psi^{(1,0)}(e, a, s)\right), \\ q_N^{a'}(e, a, s) &\equiv q^{a'}\left(e, 0, \psi^{(0,0)}(e, a, s)\right), \end{aligned}$$

where the former corresponds to default (D) and the latter corresponds to no default (N). In order to compute an "average" effect of defaulting, we can weight the price differences for each action by the stationary distribution of agents who have the option to default. Specifically, define

$$\overline{\mu}(e, a, s) = \frac{\sum_{\beta,z} \mu(\beta, e, z, a, s)}{\sum_{\beta,z,\tilde{a}<0} \mu(\beta, e, z, \tilde{a}, s)} \text{ for all } a < 0.$$

Table 11: Age Profile of Credit Rankings

Age Bins	Mean, Score Pctl	SD, Score Pctl	Corr($\Delta \text{Pctl}_{03}^{04}$, $\Delta \text{Pctl}_{04}^{05}$)
21-25 years	0.32	0.20	-0.22
26-30 years	0.35	0.23	-0.18
31-35 years	0.40	0.26	-0.20
36-40 years	0.44	0.28	-0.21
41-45 years	0.47	0.28	-0.21
46-50 years	0.50	0.28	-0.21
51-55 years	0.54	0.28	-0.19
56-60 years	0.58	0.28	-0.20

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. All entries are averages over the four quarters of 2004.

Then, we can compute the aggregate metrics for each debt choice $a' < 0$

$$\Delta^q(a') = \sum_{e,a<0,s} \overline{\mu}(e,a,s) \left[\frac{q_N^{a'}(e,a,s)}{q_D^{a'}(e,a,s)} - 1 \right].$$

E Numbers for Figures 1, 2, and 3

Table 11 reports the mean and standard deviations of the credit rankings in each bin, averaged over the four quarters of 2004. These moments were used in the regressions that determine the coefficients in the first 4 rows of the middle panel of Table 2. The correlations reported in the final column of Table 11 are the averages of these correlations over the four quarters of 2004. The final row of the middle panel of Table 2 reports the average over age bins of the correlations in the final column Table 11. The average credit ranking by age bin of people in the base sample of bankrupts reported in Table 12.

Table 12: Default Event Study Data

Years	26-30 years	31-35 years	36-40 years	41-45 years
-4	0.25	0.28	0.27	0.29
-3	0.22	0.26	0.26	0.27
-2	0.19	0.23	0.22	0.24
-1	0.14	0.18	0.17	0.20
0	0.11	0.13	0.12	0.13
1	0.17	0.19	0.20	0.21
2	0.19	0.21	0.22	0.24
3	0.20	0.22	0.23	0.25
4	0.20	0.23	0.22	0.25

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. The data presented in this table corresponds to the black lines in Figure 3. Since calculations are performed quarterly, the indicated year is the start of the year; that is, year -4 is the observation preceeding the bankruptcy by 16 quarters.