Econometrica Supplementary Material

SUPPLEMENT TO "MUSSA PUZZLE REDUX"

(*Econometrica, forthcoming*)

OLEG ITSKHOKI
Department of Economics, Harvard University

DMITRY MUKHIN
Department of Economics, London School of Economics and Political Science

Online Appendix[1]

A.1 Additional Figures and Tables

Table A1: Macroeconomic volatility across policy regimes in the data

	Δe_t			Δq_t			$\pi_t - \pi_t^*$			$\Delta c_t - \Delta c_t^*$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	0.8	4.5	6.0*	1.5	4.8	3.1*	1.3	1.4	1.1	1.7	2.1	1.3
France	3.5	11.8	3.4*	3.8	11.8	3.1*	1.2	1.3	1.1	2.5	1.8	0.7*
Germany	2.4	12.3	5.1*	2.7	12.4	4.7*	1.4	1.3	0.9	2.6	2.5	1.0
Italy	0.6	10.4	18.8*	1.5	10.3	7.1*	1.4	1.9	1.4*	2.1	2.1	1.0
Japan	0.8	11.6	14.2*	2.7	11.8	4.4*	2.7	2.8	1.0	2.3	2.7	1.2
Spain	4.4	10.8	2.4*	4.8	10.8	2.3*	2.7	2.6	1.0	2.4	2.0	0.8
U.K.	4.2	11.2	2.7*	4.5	11.7	2.6*	1.7	2.5	1.5*	2.7	2.9	1.1
RoW	1.3	9.8	7.8*	1.8	9.9	5.6*	1.2	1.4	1.1	1.7	1.7	1.0

	π_t			Δc_t			Δgdp_t			$i_t - i_t^*$		
	peg	float	ratio	peg	float	ratio	peg	float	ratio	peg	float	ratio
Canada	1.3	1.4	1.1	1.7	1.9	1.1	1.7	1.8	1.0	0.8	1.6	2.1*
France	1.0	1.3	1.3*	1.7	1.5	0.9	1.8	1.1	0.6	0.9	1.8	1.9*
Germany	1.2	1.1	0.9	2.0	2.1	1.0	3.0	2.0	0.7	1.3	1.9	1.5*
Italy	1.0	2.1	2.1*	1.3	1.6	1.2	2.5	1.9	0.8	0.9	3.2	3.6*
Japan	2.6	2.9	1.1	2.0	2.6	1.3	2.3	2.1	0.9	1.4	2.5	1.8*
Spain	2.5	2.5	1.0	1.9	1.4	0.7	2.8	1.4	0.5*	0.7	5.5	7.4*
U.K.	1.6	2.6	1.6*	2.3	2.8	1.2	2.1	2.5	1.2	0.8	2.2	2.9*
U.S.	0.8	1.3	1.5*	1.4	1.6	1.1	1.8	2.0	1.1		—	
RoW	1.6	2.1	1.3	1.9	2.1	1.1	2.4	1.9	0.8	0.6	1.3	2.1*

Note: annualized standard deviations in log points; the peg corresponds to the period from 1960:01 to 1971:07 (except for Canada where it is from 1962:04 to 1970:01); the float is from 1973:08 to 1989:12; * indicates significance of the difference (ratio) of standard deviations under the float and the peg at the 5% level (`robvar` test in Stata). RoW for differences aggregates all non-U.S. countries into RoW and subtracts the U.S. before calculating moments; RoW for levels is a weighted average of the respective moment across non-U.S. countries.

[1] See the published manuscript for the full reference list.

Table A2: Empirical moments: correlations

	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta nx_t$		$\Delta gdp_t, \Delta gdp_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta gdp_t$	
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float
Canada	0.91	0.96	0.05	−0.05	−0.03	0.13	0.32	0.46	0.40	0.25	0.28	0.58
France	0.96	0.99	0.03	−0.03	0.26	0.13	0.09	0.27	−0.24	0.33	0.51	0.57
Germany	0.82	0.99	0.01	−0.18	−0.02	−0.01	−0.01	0.28	−0.11	0.11	0.57	0.58
Italy	0.18	0.98	0.00	−0.11	0.01	0.00	0.04	0.17	−0.18	0.13	0.64	0.45
Japan	0.25	0.98	0.19	0.01	0.01	0.22	−0.08	0.24	0.11	0.23	0.70	0.71
Spain	0.83	0.97	−0.09	−0.18	−0.06	0.17	0.05	0.09	−0.06	0.05	0.56	0.63
U.K.	0.94	0.97	0.11	−0.11	−0.37	−0.16	−0.10	0.30	−0.02	0.22	0.58	0.70
RoW	0.67	0.99	0.02	−0.18	−0.23	0.19	−0.03	0.39	−0.12	0.31	0.63	0.73

Note: see notes to Table A1; cross-country correlation are with the U.S. as the foreign counterpart (indicated w/$*$). Moving average correlations between exchange rates and relative inflation rates are plotted in App. Figure A4.

Table A3: Calibrated parametrers

β	discount factor	0.99
σ	inverse of the IES	2
γ	openness of economy	0.035
φ	inverse Frisch elasticity	1
ϕ	intermediate share in production	0.5
ϑ	capital share	0.3
δ	capital depreciation rate	0.02
θ	elasticity of substitution between H and F goods	1.5
ϵ	elasticity of substitution between different types of labor	4
λ_w	Calvo parameter for wages	0.85
λ_p	Calvo parameter for prices	0.75
ρ	autocorrelation of shocks	0.97
ρ_m	Taylor rule: persistence of interest rates	0.95
ϕ_π	Taylor rule: reaction to inflation	2.15

Table A4: Estimated parameters

	$\sigma_{\hat{\psi}}$	$\sigma_{\tilde{\xi}}$	σ_a	σ_m	ϱ_{a,a^*}	ϱ_{m,m^*}	κ	ϕ_e
No financial shocks, $\hat{\psi}_t \equiv 0$ in (18)								
IRBC	—	12	7.7	—	0.27	—	11	13.5
IRBC$^+$	—	12	6.4	—	0.21	—	7	2.2
NKOE	—	12	—	0.63	—	0.30	22	5
Exogenous financial shocks, $\hat{\psi}_t$ in (18)								
IRBC	0.49	12	1.46	—	0.29	—	13	14
IRBC$^+$	0.48	12	1.24	—	0.39	—	7	3.5
NKOE	0.47	12	—	0.18	—	0.48	20	3.5
Segmented financial markets, $\hat{\psi}_t$ given by (23)†								
IRBC	0.49	12	1.46	—	0.29	—	13	0.85
IRBC$^+$	0.48	12	1.24	—	0.39	—	7	0.18
NKOE	0.47	12	—	0.18	—	0.48	20	0.35
Robustness								
Alt. $\chi_1(\sigma_e^2)$	0.48	12	1.24	—	0.39	—	7	0.38
DCP	0.49	12	1.52	—	0.35	—	9	0.25
UK openness	0.56	12	1.56	—	0.26	—	6	0.23

Note: In all calibrations, shocks are normalized to obtain $\sigma_{e,\text{float}} = \text{std}(\Delta e_t) = 10\%$ under the float; parameter ϕ_e in the Taylor rule is calibrated to generate eightfold reduction in $\text{std}(\Delta e_t)$ under the peg, to $\sigma_{e,\text{peg}} = 1.25\%$. Relative volatility of productivity (monetary) shocks is calibrated to match $\text{corr}(\Delta q_t, \Delta c_t - \Delta c_t^*) = -0.2$ under the float; cross-country correlation ϱ_{a,a^*} (ϱ_{m,m^*}) matches $\text{corr}(\Delta gdp_t, \Delta gdp_t^*) = 0.3$ under the float. Capital adjustment parameter κ ensures that $\frac{\text{std}(\Delta inv_t)}{\text{std}(\Delta gdp_t)} = 2.5$ under the float. The moments are calculated by simulating the model for $T = 100{,}000$ quarters.

†In segmented market models: $\sigma_{\hat{\psi}} = \chi_1 \sigma_\psi = \omega \sigma_e^2 \sigma_\psi$ under the float. Note that ω and σ_ψ are not separately identified, and $\omega\sigma_\psi = \sigma_{\hat{\psi}}/\sigma^2_{e,\text{float}}$, where $\sigma_{e,\text{float}} = 0.1$. Parameter $\chi_2 = 0.001$ under the float. Both $\sigma_{\hat{\psi}}$ and χ_2 are reduced $\sigma^2_{e,\text{float}}/\sigma^2_{e,\text{peg}} = 8^2$ times under the peg. See footnote 32 in the paper.

Table A5: Quantitative results: correlations

	$\Delta q_t, \Delta e_t$		$\Delta q_t, \Delta c_t - \Delta c_t^*$		$\Delta q_t, \Delta nx_t$		$\Delta gdp_t, \Delta gdp_t^*$		$\Delta c_t, \Delta c_t^*$		$\Delta c_t, \Delta gdp_t$		$\Delta gdp_t, \Delta nx_t$		β_F	
	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float	peg	float
DATA	0.67	0.99	0.02	−0.18	−0.23	0.19	−0.03	0.39	−0.12	0.31	0.63	0.73	−0.15	−0.38	0.0	−0.5
NO FINANCIAL SHOCKS, $\hat{\psi}_t \equiv 0$ in (18)																
IRBC	0.86	0.99	0.98	0.98	0.46	0.46	0.30	0.30	0.34	0.34	1.00	1.00	0.40	0.40	0.8	0.9
IRBC$^+$	0.65	0.98	0.86	0.97	0.53	0.52	0.84	0.30	0.77	0.33	0.89	0.99	−0.24	0.45	0.6	0.9
NKOE	0.90	0.99	0.21	0.96	−0.20	−0.14	0.87	0.30	0.94	0.32	1.00	1.00	0.38	0.16	0.9	1.0
EXOGENOUS FINANCIAL SHOCKS, $\hat{\psi}_t$ in (18)																
IRBC	0.86	0.99	−0.19	−0.19	0.73	0.73	0.30	0.30	0.21	0.21	0.91	0.91	0.13	0.13	0.0	−0.8
IRBC$^+$	0.78	1.00	−0.80	−0.19	0.75	0.68	0.43	0.29	0.09	0.42	0.99	0.89	−0.77	0.32	−0.1	−0.8
NKOE	0.79	1.00	−0.84	−0.20	0.59	0.63	0.34	0.31	0.07	0.41	0.99	0.87	−0.50	0.32	−0.1	−1.4
SEGMENTED FINANCIAL MARKETS, ψ_t in (23)																
IRBC	0.96	0.99	0.83	−0.19	−0.52	0.73	0.30	0.30	0.36	0.21	0.97	0.91	0.24	0.13	1.0	−0.8
IRBC$^+$	0.93	1.00	0.42	−0.19	−0.39	0.68	0.30	0.29	0.68	0.42	0.89	0.89	0.71	0.32	1.0	−0.8
NKOE	0.95	1.00	0.08	−0.20	−0.55	0.63	0.38	0.31	0.77	0.41	0.97	0.87	0.72	0.32	1.0	−1.4
ROBUSTNESS																
Alt. $\chi_1(\sigma_e^2)$	0.91	1.00	−0.17	−0.19	−0.01	0.68	0.28	0.29	0.58	0.42	0.92	0.89	0.39	0.32	0.1	−0.8
DCP	0.91	1.00	0.54	−0.20	−0.18	0.78	0.39	0.31	0.69	0.39	0.91	0.92	0.64	0.21	1.0	−0.5
UK openness	0.91	1.00	0.63	−0.19	−0.09	0.81	0.37	0.30	0.72	0.38	0.84	0.65	0.63	0.51	1.0	−0.6

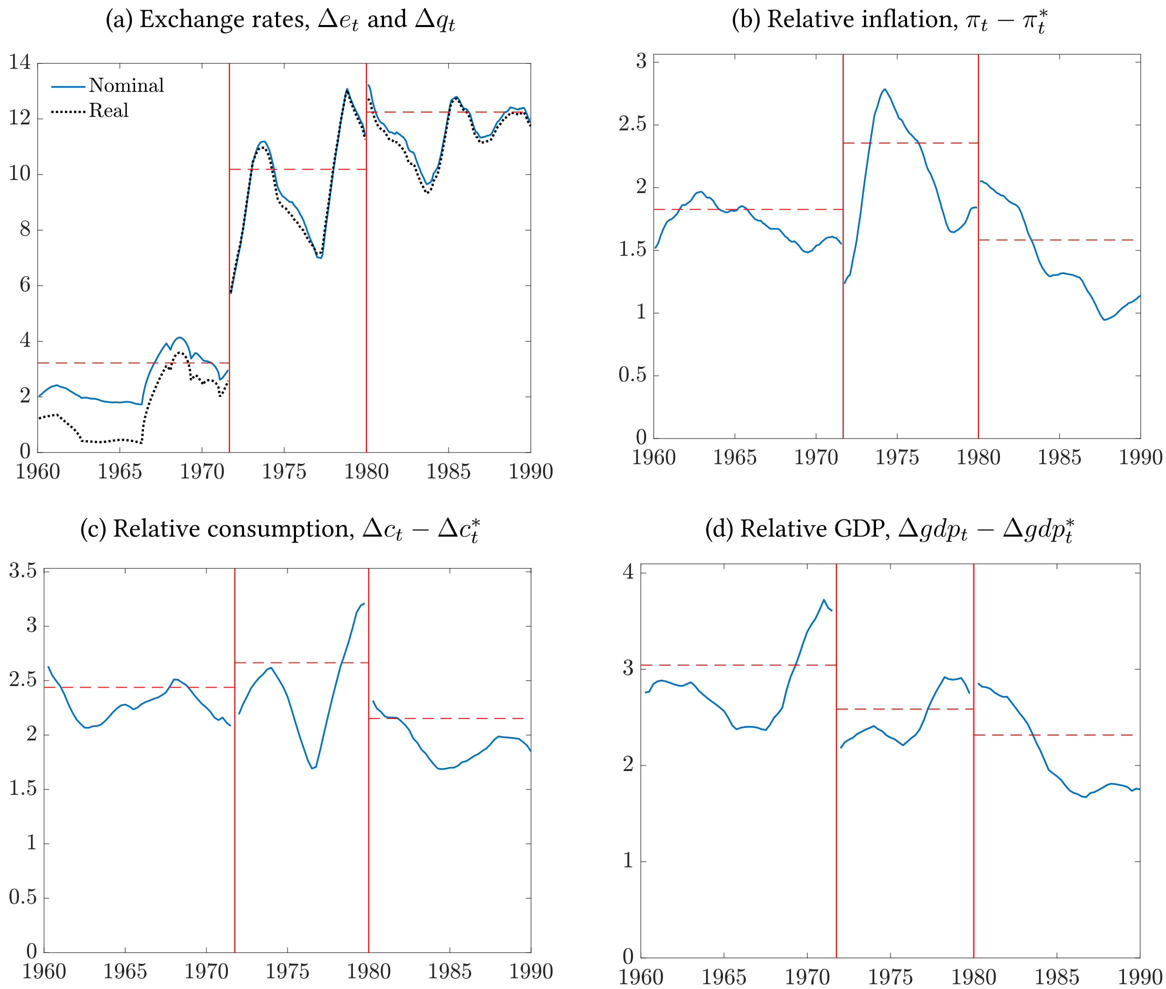
Note: see text of the paper and notes to Table 1.

Table A6: Variance decomposition (%)

	peg			float		
	ψ	$\tilde{\xi}$	a or m	ψ	$\tilde{\xi}$	a or m
Real exchange rate, $\mathrm{var}(\Delta q_t)$						
IRBC	0	48	52	82	11	7
IRBC$^+$	0	40	60	90	7	4
NKOE	0	40	60	88	7	5
Consumption, $\mathrm{var}(\Delta c_t)$						
IRBC	0	1	99	10	1	89
IRBC$^+$	0	16	84	4	0	95
NKOE	0	30	70	6	0	94

Note: This table shows a variance decomposition of the real exchange rate and consumption into contribution shares (in %) of various shocks in three model specifications with endogenous financial shocks under the two exchange rate regimes (see Table 1 in the text). IRBC and IRBC$^+$ specifications feature productivity shocks (a_t, a_t^*) and NKOE specification features monetary shocks $(\varepsilon_t^m, \varepsilon_t^{m*})$.

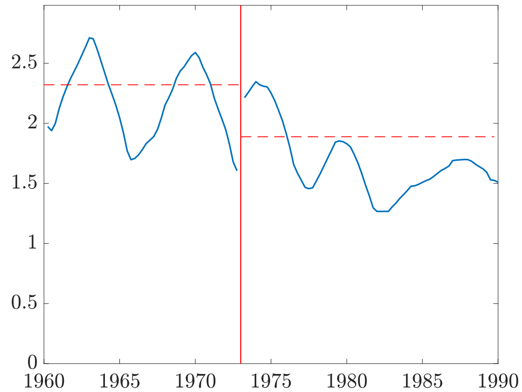
Figure A1: Macroeconomic volatility: alternative breakpoints at 1971:08 and 1980:01



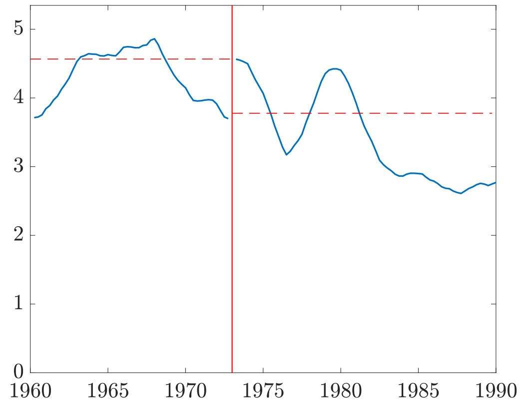
Note: as in Figure 3, annualized standard deviations for the U.S. against the RoW, estimated as triangular moving averages with a window over 18 months (panels a, b) or 10 quarters (panels c, d) before and after, treating 1971:08 and 1980:01 as the end points for the three regimes; the dashed lines correspond to average standard deviations within each interval.

Figure A2: Macroeconomic volatility over time: GDP and net exports

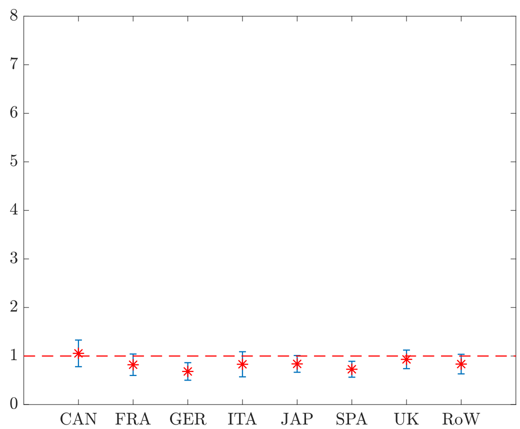
(a) GDP, Δgdp_t



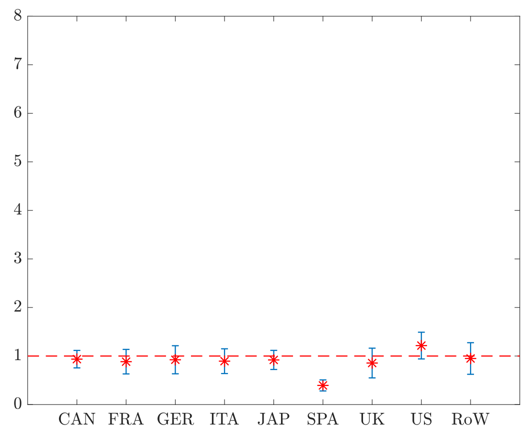
(b) Net exports, Δnx_t



(c) Relative GDP, $\Delta gdp_t - \Delta gdp_t^*$



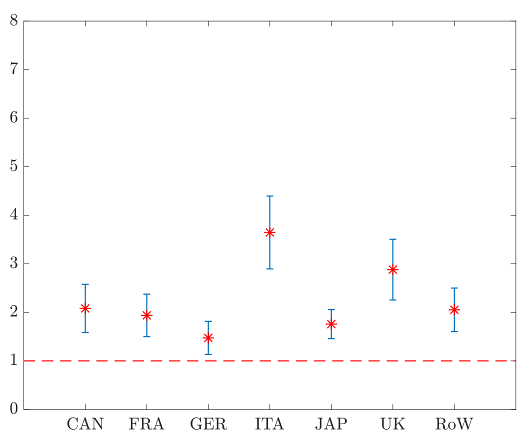
(d) Net exports, Δnx_t



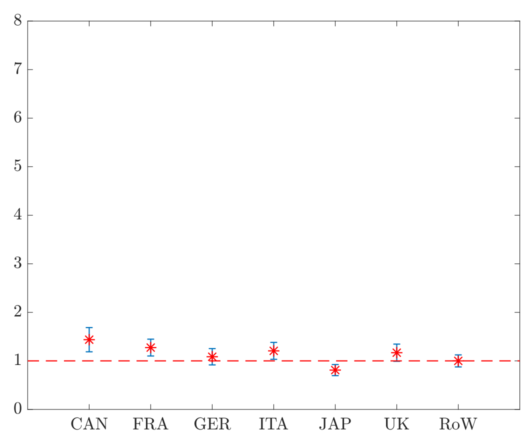
Note: see notes to Figure 3 for upper panels (moving average standard deviations, in log points) and notes to Figure 4 for lower panels (ratio of standard deviations float/peg).

Figure A3: Volatility ratio float/peg for financial variables

(a) Relative interest rates, $i_t - i_t^*$



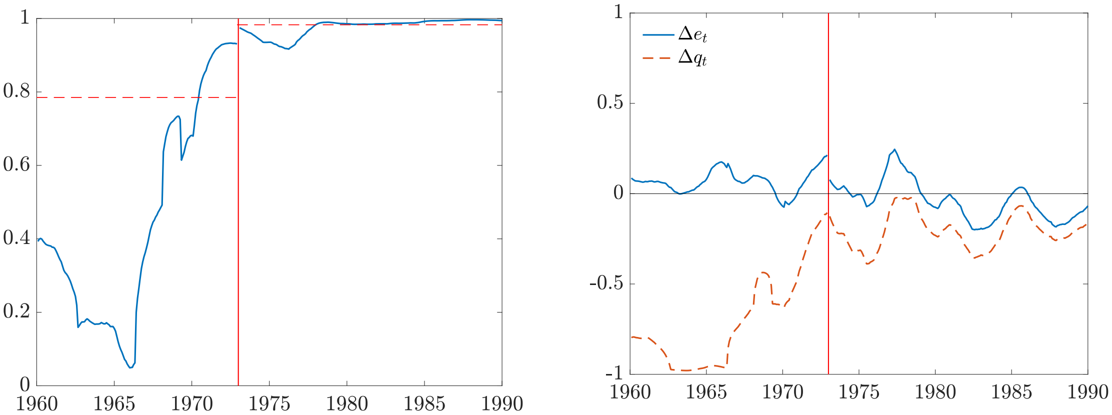
(b) Relative stock market returns, $r_t^s - r_t^{s*}$



Note: see notes to Figure 4.

Figure A4: Correlations of exchange rates and prices over time

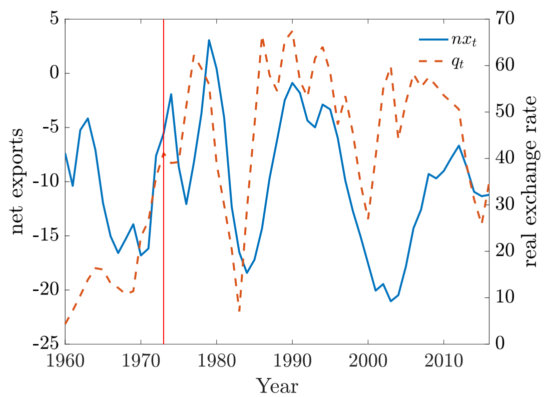
(a) $\text{corr}(\Delta q_t, \Delta e_t)$ (b) $\text{corr}(\bullet, \Delta\pi_t - \Delta\pi_t^*)$ for Δe_t and Δq_t



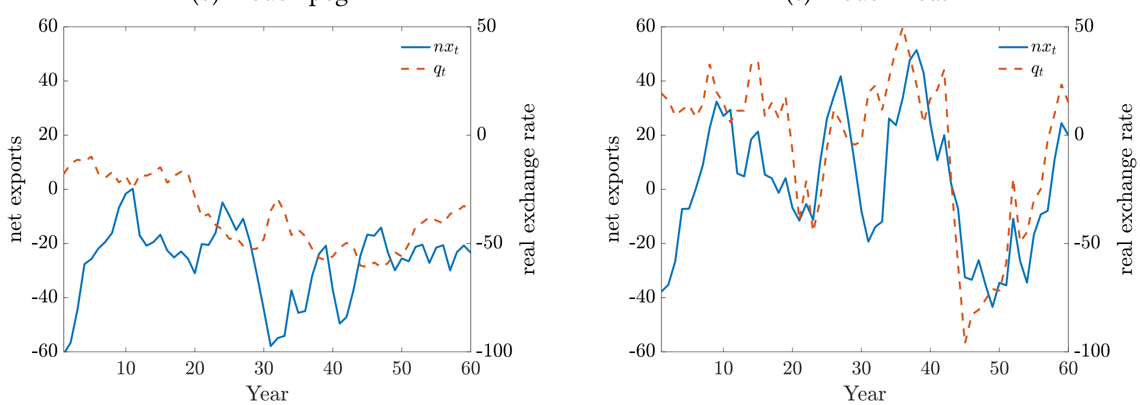
Note: triangular moving average correlations estimated with a window over 18 months before and after, treating 1973:01 as the end point for the two regimes; the dashed lines in the left panel correspond to average values under the two regimes.

Figure A5: The real exchange rate and the trade balance

(a) Data



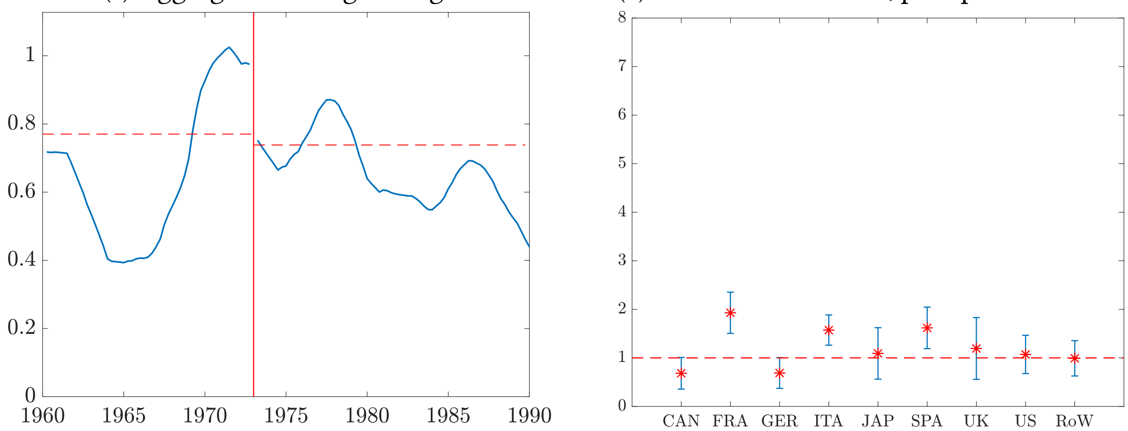
(b) Model: peg (c) Model: float



Note: panel (a) extends the figure from Alessandria and Choi (2021) using annual data for the U.S., while panels (b) and (c) show the series simulated from the IRBC$^+$ version of the model with endogenous financial shocks under the two exchange rate regimes. See Table 1 and Appendix Table A5 for the moments of nx_t and q_t.

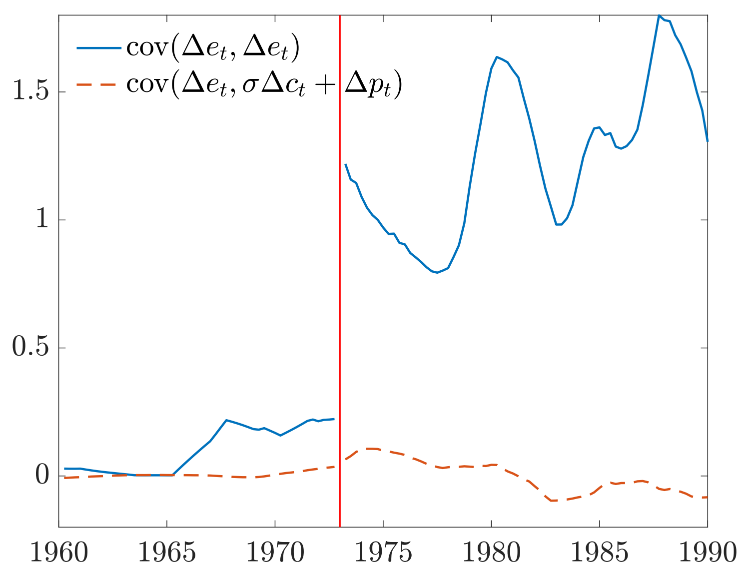
Figure A6: Volatility of official foreign reserves-to-GDP ratio

(a) Aggregate moving average (b) Individual countries, post/pre-1973 ratio



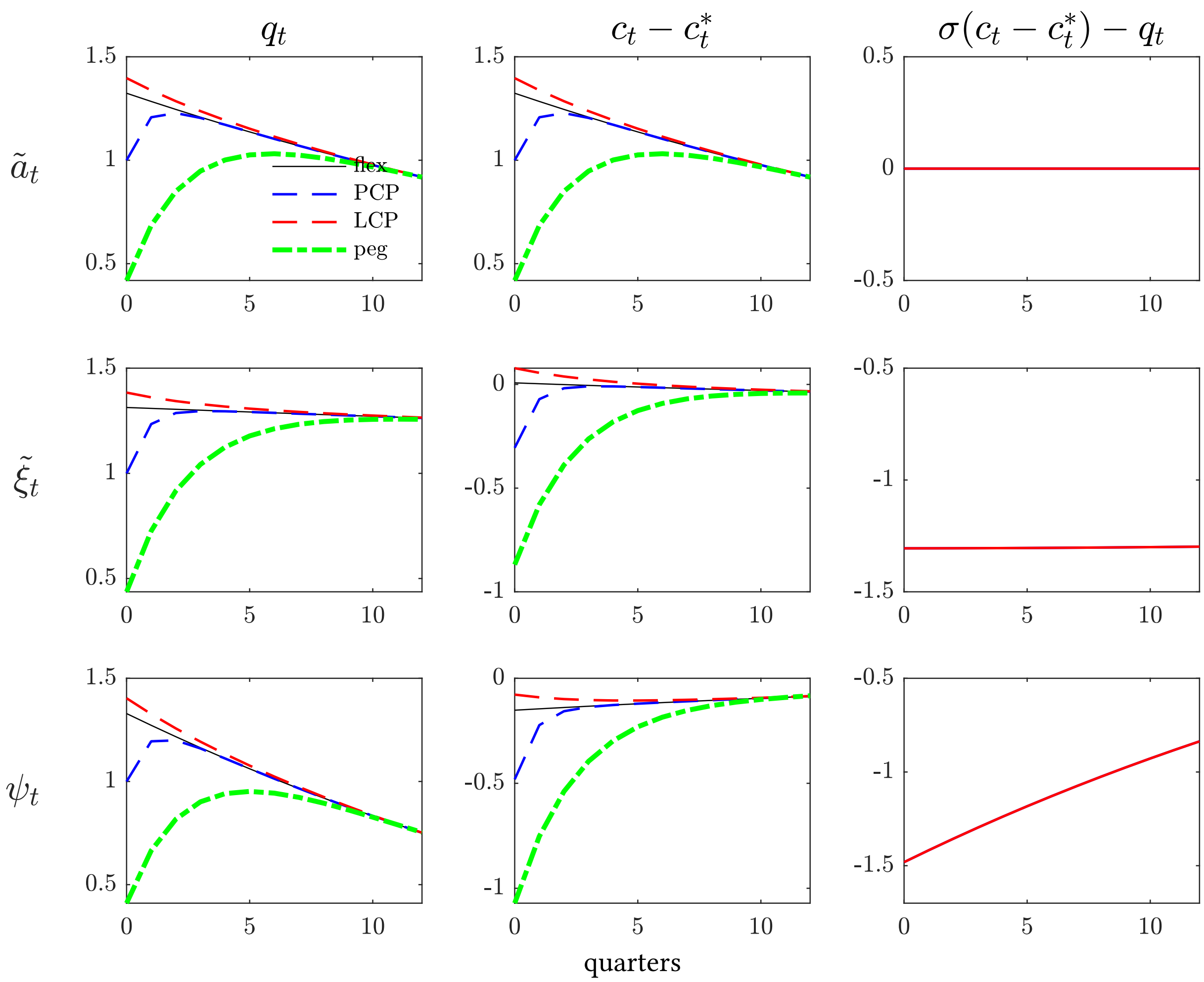
Note: std $\left(\Delta \frac{\text{FX}_t}{\text{GDP}_t}\right)$, using quarterly data on official foreign reserves from IMF IFS database, constructed as in Figures 3 and 4.

Figure A7: Covariance of the nominal exchange rate



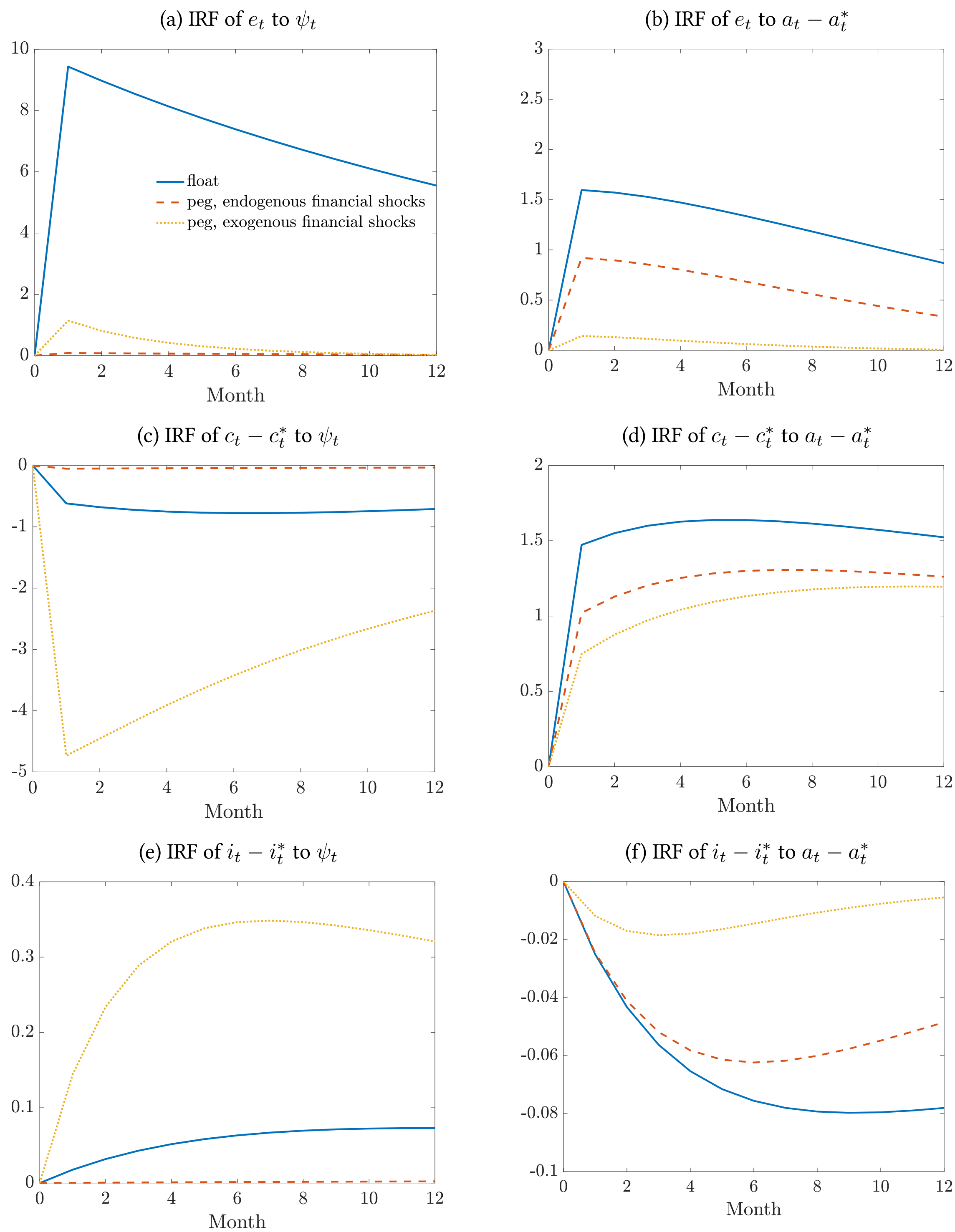
Note: Triangular moving average covariances of the nominal exchange rate changes with itself (i.e., the variance) and with the representative-agent stochastic discount factor ($\sigma \Delta c_t + \Delta p_t$ for $\sigma = 2$), treating 1973:01 as the end point for the two regimes; quarterly data.

Figure A8: Impulse response functions



Note: Impulse responses of q_t, $c_t - c_t^*$ and $z_t = \sigma(c_t - c_t^*) - q_t$ (columns) to shocks $\tilde{a}_t = a_t - a_t^*$, $\tilde{\xi}_t = \xi_t - \xi_t^*$ and exogenous $\hat{\psi}_t$ (rows) under (1) flexible prices (independent of monetary regime), (2) peg (independently of PCP or LCP), (3) PCP-float and (4) LPC-float, in 'conventional' models under the Cole-Obstfeld parameter restriction. Note that the impulse responses of q_t and $c_t - c_t^*$ change with both the supply side (flex prices vs PCP vs LCP) and the monetary policy regime (peg vs float), however, the IRF of z_t does not depend on these details of equilibrium environment, and hence the unconditional statistical properties of z_t also do not depend on the monetary regime, illustrating Proposition 1. We use a simplified version of the calibrated model (as described in Tables A3 and A4 for the IRBC$^+$ model with exogenous financial shocks), with $\sigma = \theta = 1$ and $\phi = \vartheta = \lambda_w = 0$.

Figure A9: Impulse responses: float vs. peg



Note: The figure shows the impulse responses of nominal exchange rate, relative consumption, and relative nominal interest rates to a one standard deviation noise-trader currency demand shock ψ_t and relative productivity shock $a_t - a_t^*$ under the two exchange rate regimes in the IRBC$^+$ quantitative model with segmented asset markets and with exogenous financial shocks (see Table 1). The impulse responses are the same in the two models under the floating regime (see Section 6), but differ markedly under the peg: financial shocks ψ_t are transmitted into the interest rate and consumption by monetary rule that stabilizes the nominal exchange rate in the conventional model with exogenous UIP shocks, which in turn are endogenously muted in the model with segmented markets.

A.2 Data

Quarterly data for FX reserves and monthly data for nominal exchange rates, consumer prices, discount interest rates and stock market returns come from the IFM IFS database (IFS 2024), while monthly data for stock market prices and quarterly data for GDP, consumption, imports and exports are from the OECD database (OECD 2024). Additional data on interest rates is from GFD (2024) and CEIC (2024). See further details about the data in the README supplement. Our analysis focuses on the "convertible phase" of the Bretton Woods period from 1960 to 1973 and the period of floating from 1973 to 1990, where the end date is chosen to keep the length of the two periods comparable and to exclude the Great Moderation of the 1990s. Before estimating empirical moments, we use extrapolation to replace missing data in the raw series and the following two outliers: (1) civil unrests in France in May-June 1968, which led to over a 20% fall in production and (2) missing values of GDP, imports and exports for Canada in 1960. The outliers in stock returns and changes in interest rates are eliminated using winsorization. We compute first differences of net exports normalized by total trade and log first differences of all other variables, and annualize the log changes by multiplying the quarterly series by $\sqrt{4}$ and the monthly series by $\sqrt{12}$. The series for France, Germany, Italy, Japan, Spain and the U.K. are aggregated into the RoW variables using the average PPP-adjusted GDP shares in 1960–1990 as weights.

A.3 Full Quantitative Model

This section provides a complete description of the general modeling framework. For simplicity, we focus on home households and firms with the understanding that the problems of foreign agents are symmetric.

Households A representative home household maximizes the expected utility:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+1/\nu} L_t^{1+1/\nu} \right), \tag{A1}$$

where $\nu \equiv 1/\varphi$ is the Frisch elasticity, subject to the flow budget constraint:

$$P_t C_t + P_t I_t + \frac{B_{t+1}}{R_t} \leq W_t L_t + R_t^K K_t + B_t + \Pi_t, \tag{A2}$$

where R_t^K is the nominal rental rate of capital and I_t is the gross investment into the domestic capital stock K_t, which accumulates according to a standard rule with depreciation δ and quadratic capital adjustment costs:

$$K_{t+1} = (1-\delta)K_t + \big[I_t - \frac{\kappa}{2} \frac{(\Delta K_{t+1})^2}{K_t}\big]. \tag{A3}$$

The domestic households allocate their within-period consumption expenditure $P_t C_t$ between home and foreign varieties of the goods

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft} = \int_0^1 \Big[P_{Ht}(i) C_{Ht}(i) + P_{Ft}(i) C_{Ft}(i) \Big] \mathrm{d}i \tag{A4}$$

to minimize expenditure on aggregate consumption, defined implicitly by a Kimball (1995) aggregator:

$$\int_0^1 \left[(1-\gamma) e^{-\gamma \xi_t} g\bigg(\frac{C_{Ht}(i)}{(1-\gamma) e^{-\gamma \xi_t} C_t} \bigg) + \gamma e^{(1-\gamma)\xi_t} g\bigg(\frac{C_{Ft}(i)}{\gamma e^{(1-\gamma)\xi_t} C_t} \bigg) \right] \mathrm{d}i = 1, \tag{A5}$$

where the aggregator function $g(\cdot)$ in (A5) satisfies: $g'(\cdot) > 0$, $g''(\cdot) < 0$ and $-g''(1) \in (0,1)$, and two normalizations: $g(1) = g'(1) = 1$. The solution to the optimal expenditure allocation results in the following homothetic demand schedules:

$$C_{Ht}(i) = (1-\gamma)e^{-\gamma\xi_t} h\left(\frac{P_{Ht}(i)}{\mathcal{P}_t}\right) C_t \qquad \text{and} \qquad C_{Ft}(j) = \gamma e^{(1-\gamma)\xi_t} h\left(\frac{P_{Ft}(j)}{\mathcal{P}_t}\right) C_t, \tag{A6}$$

where $h(\cdot) = g'^{-1}(\cdot) > 0$ and satisfies $h(1) = 1$ and $h'(\cdot) < 0$. The function $h(\cdot)$ controls the curvature of the demand schedule, and we denote its point elasticity with $\theta \equiv -\frac{\partial \log h(x)}{\partial \log x}\big|_{x=1} = -h'(1) > 1$. The consumer price level P_t and the auxiliary variable $\mathcal{P}_t$ in (A6) are two alternative measures of average prices in the home market (different by a second-order term in cross-sectional price dispersion), which are defined implicitly by (A4) and (A5) after substituting in the demand schedules (A6). The taste shock ξ_t in (A5) is defined such that it has no first-order effects on the consumer prices level P_t.

The CES demand is nested as a special case of the Kimball aggregator (A5) with $g(z) = 1 + \frac{\theta}{\theta-1}\big(z^{1-1/\theta} - 1\big)$, resulting in the demand schedule $h(x) = x^{-\theta}$ and price index:

$$P_t = \mathcal{P}_t = \left[\int_0^1 \Big((1-\gamma)e^{-\gamma\xi_t} P_{Ht}(i)^{1-\theta} + \gamma e^{(1-\gamma)\xi_t} P_{Ft}(i)^{1-\theta}\Big)\,\mathrm{d}i\right]^{1/(1-\theta)}.$$

The import price index is defined conventionally as $P_{Ft} = \big(\int_0^1 P_{Ft}(i)^{1-\theta}\mathrm{d}i\big)^{1/(1-\theta)}$, and aggregate imports are given by $P_{Ft}C_{Ft} = \int_0^1 P_{Ft}(i)C_{Ft}(i)\mathrm{d}i = \gamma e^{(1-\gamma)\xi_t}(P_{Ft}/P_t)^{1-\theta}P_tC_t$, with a corresponding generalization under the Kimball aggregate. With a symmetric expression for aggregate exports, $P^*_{Ht}C^*_{Ht} = \gamma e^{(1-\gamma)\xi^*_t}(P^*_{Ht}/P^*_t)^{1-\theta}P^*_tC^*_t$, we can express net exports as:

$$NX_t = \mathcal{E}_tP^*_{Ht}C^*_{Ht} - P_{Ft}C_{Ft} = P_{Ft}C_{Ft}\left[e^{-(1-\gamma)\tilde{\xi}_t}\frac{\mathcal{E}_t(P^*_{Ht})^{1-\theta}(P^*_t)^{\theta}}{P_{Ft}^{1-\theta}P_t^{\theta}}\frac{C^*_t}{C_t} - 1\right],$$

where $\tilde{\xi}_t \equiv \xi_t - \xi^*_t$. Using the definition of $\mathcal{Q}_t$ and $\mathcal{S}_t$, we obtain (15) in the text.

International risk-sharing condition Home Euler equations for (A1)–(A3) are given by:

$$\beta R_t \mathbb{E}_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right\} = 1,$$

$$1 + \kappa\frac{\Delta K_{t+1}}{K_t} = \beta\mathbb{E}_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left[R^K_{t+1} + (1-\delta) + \kappa\frac{\Delta K_{t+2}}{K_{t+1}} + \frac{\kappa}{2}\left(\frac{\Delta K_{t+2}}{K_{t+1}}\right)^2\right]\right\}.$$

In the more general setup of (3)–(4) in the text, the Euler equations are given by:

$$\beta\mathbb{E}_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\frac{e^{-\zeta_{t+1}}\mathcal{D}^j_{t+1}}{\Theta^j_t}\right\} = 1 \qquad \forall j \in J_t,$$

where ζ^j_{t+1} is the tax, and we denote by $\mathcal{R}^j_{t+1} = \frac{\mathcal{D}^j_{t+1}/\Theta^j_t}{P_{t+1}/P_t}$ the pre-tax real return on asset j. Then the foreign Euler equations can be written as:

$$\beta\mathbb{E}_t\left\{\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\sigma}\frac{P^*_t}{P^*_{t+1}}\frac{e^{-\zeta^*_{t+1}}\mathcal{D}^j_{t+1}/\mathcal{E}_{t+1}}{\Theta^j_t/\mathcal{E}_t}\right\} = 1 \qquad \forall j \in J^*_t,$$

and we denote with $\mathcal{R}^{j*}_{t+1} = \mathcal{R}^{j}_{t+1}\frac{\mathcal{Q}_t}{\mathcal{Q}_{t+1}} = \frac{\mathcal{D}^j_{t+1}/\mathcal{E}_{t+1}}{\Theta^j_t/\mathcal{E}_t}\frac{1}{P^*_{t+1}/P^*_t}$ the pre-tax foreign real return on asset j. Using the expression for $\mathcal{R}^j_{t+1}$ and subtracting the home Euler equation from the foreign one for assets $j \in J_t \cap J^*_t$ results in the risk-sharing condition (16).

Production Home output is produced according to a Cobb-Douglas technology in labor L_t, capital K_t and intermediate inputs X_t:

$$Y_t = \left(e^{a_t}K_t^{\vartheta}L_t^{1-\vartheta}\right)^{1-\phi}X_t^{\phi}, \tag{A7}$$

where ϑ is the elasticity of the value added with respect to capital and ϕ is the elasticity of output with respect to intermediates. Intermediates (as well as investment goods) are the same bundle of home and foreign varieties as the final consumption bundle (A5). The marginal cost of production is thus:

$$MC_t = \tfrac{1}{\varpi}\left[e^{-a_t}(R^K_t)^{\vartheta}W_t^{1-\vartheta}\right]^{1-\phi}P_t^{\phi}, \quad \text{where } \varpi \equiv \phi^{\phi}\left[(1-\phi)\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}\right]^{1-\phi}, \tag{A8}$$

where $R^K_t = (1-\phi)\vartheta Y_t/K_t$ is the marginal product of capital and W_t is the wage rate that clears the labor market (see below). The aggregate *value-added productivity* follows an AR(1) process in logs:

$$a_t = \rho_a a_{t-1} + \sigma_a\varepsilon^a_t, \qquad \varepsilon^a_t \sim iid(0,1), \tag{A9}$$

where $\rho_a \in [0,1]$ is the persistence parameter and $\sigma_a \geq 0$ is the volatility of the innovation.

Profits and price setting The firm maximizes profits from serving the home and foreign markets:

$$\mathbb{E}_0\sum\nolimits_{t=0}^{\infty}\mathbf{M}_t\Pi_t(i), \quad \text{where} \quad \Pi_t(i) = \left(P_{Ht}(i) - MC_t\right)Y_{Ht}(i) + \left(P^*_{Ht}(i)\mathcal{E}_t - MC_t\right)Y^*_{Ht}(i), \tag{A10}$$

where $\mathbf{M}_t \equiv \beta^t\frac{C_t^{-\sigma}}{P_t}$ is the nominal present-value stochastic discount factor. In the absence of nominal frictions, profit maximization results in the markup pricing rules, with a common price across all domestic firms $i \in [0,1]$ in a given destination market and expressed in the destination currency:

$$P_{Ht}(i) = P_{Ht} = \mu\left(\frac{P_{Ht}}{\mathcal{P}_t}\right)\cdot MC_t \qquad \text{and} \qquad P^*_{Ht}(i) = P^*_{Ht} = \mu\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right)\cdot\frac{MC_t}{\mathcal{E}_t}, \tag{A11}$$

where $\mu(x) \equiv \frac{\tilde\theta(x)}{\tilde\theta(x)-1}$ is the markup function (with $\mu'(\cdot) \leq 0$) and $\tilde\theta(x) = -\frac{\partial\log h(x)}{\partial\log x}$ is the elasticity schedule for the demand curve in (A6).

Nominal rigidities We introduce Calvo sticky prices and wages in a conventional way (see e.g. Galí 2008). Denote with ϵ the elasticity of substitution between varieties of labor, and let λ_p and λ_w be the Calvo probability of price and wage non-adjustment. Then the resulting New Keynesian Phillips Curves (NKPC) for nominal-wage and domestic-prices inflation can be written respectively as:

$$\pi^w_t = k_w\left[\sigma c_t + \frac{1}{\nu}\ell_t + p_t - w_t\right] + \beta\mathbb{E}_t\pi^w_{t+1}, \quad \text{where} \quad k_w = \frac{(1-\beta\lambda_w)(1-\lambda_w)}{\lambda_w(1+\epsilon/\nu)},$$

$$\pi_{Ht} = k_p\Big[(1-\alpha)mc_t + \alpha p_t - p_{Ht}\Big] + \beta\mathbb{E}_t\pi_{Ht+1}, \quad \text{where} \quad k_p = \frac{(1-\beta\lambda_p)(1-\lambda_p)}{\lambda_p},$$

where $\alpha \in [0,1)$ is the strategic complementarity elasticity defined by $\alpha = \frac{-\mu'(x)}{1-\mu'(x)}\Big|_{x=1}$, and $(1-\alpha) = \frac{1}{1-\mu'(x)}\Big|_{x=1}$ is the cost pass-through elasticity (under flexible prices), and $\mu'(\cdot)$ is the elasticity of the markup function in (A11). The NKPC for export prices depends on the currency of invoicing and is given by:

$$\pi^*_{Ht} = k_p\Big[(1-\alpha)(mc_t - e_t) + \alpha p^*_t - p^*_{Ht}\Big] + \beta \mathbb{E}_t \pi^*_{Ht+1}, \qquad \text{under LCP,}$$

$$(\pi^*_{Ht} + \Delta e_t) = k_p\Big[(1-\alpha)mc_t + \alpha(p^*_t + e_t) - (p^*_{Ht} + e_t)\Big] + \beta \mathbb{E}_t\left(\pi^*_{Ht+1} + \Delta e_{t+1}\right), \quad \text{under PCP.}$$

Note that the DCP case with all international trade invoiced in foreign currency can be expressed as a mix of the two other regimes — home exporters use LCP and foreign exporters use PCP.

Good and factor market clearing The labor market clearing requires that L_t equals simultaneously the labor supply of the households and the labor demand of the firms, and equivalently for L^*_t in foreign. Similarly, equilibrium in the capital market requires that K_t (and K^*_t) equals simultaneously the capital supply of the households and the capital demand of the local firms. Goods market clearing requires that the total production by the home firms is split between supply to the home and foreign markets respectively, $Y_t = Y_{Ht} + Y^*_{Ht}$, and satisfies the local demand in each market for the final, intermediate and capital goods:

$$Y_{Ht} = C_{Ht} + X_{Ht} + I_{Ht} = (1-\gamma)h\left(\frac{P_{Ht}}{\mathcal{P}_t}\right)\big[C_t + X_t + I_t\big], \tag{A12}$$

$$Y^*_{Ht} = C^*_{Ht} + X^*_{Ht} + I^*_{Ht} = \gamma h\left(\frac{P^*_{Ht}}{\mathcal{P}^*_t}\right)\big[C^*_t + X^*_t + I^*_t\big]. \tag{A13}$$

Lastly, we combine the household budget constraint (A2) with profits (A10), aggregated across all home firms, as well as the market clearing conditions above to obtain the home country budget constraint:

$$\frac{B_{t+1}}{R_t} - B_t = NX_t \qquad \text{with} \qquad NX_t = \mathcal{E}_t P^*_{Ht} Y^*_{Ht} - P_{Ft} Y_{Ft}, \tag{A14}$$

where NX_t denotes net exports expressed in units of the home currency.

A.4 Proof of Proposition 1

This proposition follows from the dynamic system (18)–(19), which transforms the risk-sharing condition (16) and the flow budget constraint (14)–(15) by defining the residual terms $\hat{\psi}_t$ and $\hat{\xi}_t$. Define $z_t \equiv \sigma(c_t - c_t) - q_t$. The Cole-Obstfeld parameter restriction $\sigma = \theta = 1$ implies $\hat{\theta} = \theta = 1$.[2] In this case, (14)–(15) result in $\beta b_{t+1} - b_t = \gamma\big[-z_t - (1-\gamma)\hat{\xi}_t\big]$ with $\hat{\xi}_t = \tilde{\xi}_t$ up to higher order terms, which is a special case of (19). Iterating this condition forward and using the no-bubble condition $\lim_{j\to\infty}\beta^j b_{t+j} = 0$, we obtain

$$\gamma\sum_{j=0}^{\infty}\beta^j z_{t+j} = b_t - \gamma(1-\gamma)\sum_{j=0}^{\infty}\beta^j\hat{\xi}_{t+j}.$$

Condition (18), in turn, results in a martingale property $\mathbb{E}_t\Delta z_{t+1} = \hat{\psi}_t$, or equivalently

$$\mathbb{E}_t z_{t+j} = z_t + \sum_{\ell=0}^{j-1}\mathbb{E}_t\hat{\psi}_{t+\ell} \qquad \text{for any } j > 0.$$

[2]Note that a weaker parameter restriction $\sigma\hat{\theta} = 1$ is a sufficient requirement for Proposition 1.

Combining the two expressions, we obtain:

$$z_t = \frac{1-\beta}{\gamma} b_t - \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \big\{ \beta \hat{\psi}_{t+j} + (1-\beta)(1-\gamma)\hat{\xi}_{t+j} \big\}.$$

Substituting into (19), we solve for Δb_{t+1} and Δz_{t+1}, yielding:

$$\frac{\beta}{\gamma} \Delta b_{t+1} = (1-\gamma)\hat{\xi}_t + \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \big\{ \beta \hat{\psi}_{t+j} + (1-\beta)(1-\gamma)\hat{\xi}_{t+j} \big\}, \tag{A15}$$

$$\Delta z_{t+1} = \frac{1-\beta}{\beta} \Big[(1-\gamma)\hat{\xi}_t + \sum_{j=0}^{\infty} \beta^j \mathbb{E}_t \big\{ \beta \hat{\psi}_{t+j} + (1-\beta)(1-\gamma)\hat{\xi}_{t+j} \big\} \Big] \tag{A16}$$
$$- \sum_{j=0}^{\infty} \beta^j (\mathbb{E}_{t+1} - \mathbb{E}_t) \big\{ \beta \hat{\psi}_{t+1+j} + (1-\beta)(1-\gamma)\hat{\xi}_{t+1+j} \big\}$$

which only depends on the path of $\{\hat{\psi}_t, \hat{\xi}_t\}_t$. Therefore, in conventional models according to Definition 1, the properties of Δz_t do not depend on the monetary policy or exchange rate regime.

The cases of complete markets and financial autarky need to be considered separately. In the case of complete markets, we have from (17) that $\Delta z_t = -\tilde{\zeta}_{t+1}$, where $\tilde{\zeta}_{t+1}$ is a component of $\hat{\psi}_t$ corresponding to the relative exogenous risk-sharing wedges. In the case of financial autarky, we have $nx_t = 0$, which from derivation above implies $z_t = -(1-\gamma)\hat{\xi}_t$. Therefore, the result of Proposition 1 applies as well in these two limiting cases. ■

A.5 Segmented Financial Market: Proof of Lemma 1

The structure of the financial markets is as described in Section 5, and we generalize it to allow for mass m of intermediaries and mass n of noise traders, instead of unit masses. Specifically, we now have $N^*_{t+1} = n\psi_t$ in (21) and $D^*_{t+1} = m d^*_{t+1}$, where d^*_{t+1} denotes the position of a representative arbitrageur which solves (22).

The proof of Lemma 1 follows two steps. First, it characterizes the solution to the portfolio problem (22) of the arbitrageurs to derive their policy function (24). Second, it combines this solution with the financial market clearing (20) to derive the equilibrium condition (25).

(a) Portfolio choice: *The solution to the portfolio choice problem* (22) *when the time periods are short is given by:*

$$\frac{d^*_{t+1}}{P^*_t} = -\frac{i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega \sigma_e^2}, \tag{A17}$$

where $i_t - i^*_t \equiv \log(R_t/R^*_t)$, $\sigma_e^2 \equiv \mathrm{var}_t(\Delta e_{t+1})$ *and* $\sigma_{e\pi^*} = \mathrm{cov}_t(\Delta e_{t+1}, \Delta p^*_{t+1})$.

Proof: The proof follows Campbell and Viceira (2002, Chapter 3 and Appendix 2.1.1). Consider the objective of the arbitrageur's problem (22) and rewrite it as:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \big(1 - e^{x^*_{t+1}}\big) e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t} \right) \right\}, \tag{A18}$$

where we used the definition of $\tilde{R}^*_{t+1} = R^*_t - R_t \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}$ and the following algebraic manipulation:

$$\frac{\tilde{R}^*_{t+1}}{P^*_{t+1}} \frac{d^*_{t+1}}{R^*_t} = \frac{\tilde{R}^*_{t+1}/R^*_t}{P^*_{t+1}/P^*_t} \frac{d^*_{t+1}}{P^*_t} = \frac{1 - \frac{R_t}{R^*_t}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}}{e^{\pi^*_{t+1}}} \frac{d^*_{t+1}}{P^*_t} = \left(1 - e^{x^*_{t+1}}\right) e^{-\pi^*_{t+1}} \frac{d^*_{t+1}}{P^*_t}$$

and defined the log Carry trade return and foreign inflation rate as

$$x^*_{t+1} \equiv i_t - i^*_t - \Delta e_{t+1} = \log(R_t/R^*_t) - \Delta \log \mathcal{E}_{t+1} \qquad \text{and} \qquad \pi^*_{t+1} \equiv \Delta \log P^*_{t+1}.$$

When time periods are short, (x^*_{t+1}, π^*_{t+1}) correspond to the increments of a vector normal diffusion process $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ with time-varying drift $\boldsymbol{\mu}_t$ and time-invariant conditional variance matrix $\boldsymbol{\sigma}$:

$$\begin{pmatrix} \mathrm{d}\mathcal{X}^*_t \\ \mathrm{d}\mathcal{P}^*_t \end{pmatrix} = \boldsymbol{\mu}_t \mathrm{d}t + \boldsymbol{\sigma} \mathrm{d}\mathcal{W}_t, \tag{A19}$$

where $\mathcal{W}_t$ is a standard two-dimensional Brownian motion. Indeed, as we show below, in equilibrium x^*_{t+1} and π^*_{t+1} follow stationary linear stochastic processes (ARMAs) with correlated innovations, and therefore

$$(x^*_{t+1}, \pi^*_{t+1}) \,\big|\, \mathcal{I}_t \sim \mathcal{N}(\boldsymbol{\mu}_t, \boldsymbol{\sigma}^2),$$

where $\mathcal{I}_t$ is the information set at time t, and the drift and variance matrixes are given by:

$$\boldsymbol{\mu}_t = \mathbb{E}_t \begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} \\ \mathbb{E}_t \pi^*_{t+1} \end{pmatrix} \quad \text{and} \quad \boldsymbol{\sigma}^2 = \mathrm{var}_t \begin{pmatrix} x^*_{t+1} \\ \pi^*_{t+1} \end{pmatrix} = \begin{pmatrix} \sigma^2_e & -\sigma_{e\pi^*} \\ -\sigma_{e\pi^*} & \sigma^2_{\pi^*} \end{pmatrix},$$

where $\sigma^2_e \equiv \mathrm{var}_t(\Delta e_{t+1})$, $\sigma^2_{\pi^*} \equiv \mathrm{var}_t(\Delta p^*_{t+1})$ and $\sigma_{e\pi^*} \equiv \mathrm{cov}_t(\Delta e_{t+1}, \Delta p^*_{t+1})$ are time-invariant (annualized) conditional second moments. Following Campbell and Viceira (2002), we treat (x^*_{t+1}, π^*_{t+1}) as discrete-interval differences of the continuous process, $(\mathcal{X}^*_{t+1} - \mathcal{X}^*_t, \mathcal{P}^*_{t+1} - \mathcal{P}^*_t)$.

With short time periods, the solution to (A18) is equivalent to

$$\max_{d^*} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \big(1 - e^{\mathrm{d}\mathcal{X}^*_t}\big) e^{-\mathrm{d}\mathcal{P}^*_t} \frac{d^*}{P^*_t} \right) \right\}, \tag{A20}$$

where $(\mathrm{d}\mathcal{X}^*_t, \mathrm{d}\mathcal{P}^*_t)$ follow (A19). Using Ito's Lemma, we rewrite the objective as:

$$\begin{aligned}
&\mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega\Big(- \mathrm{d}\mathcal{X}^*_t - \tfrac{1}{2}(\mathrm{d}\mathcal{X}^*_t)^2 \Big) \Big(1 - \mathrm{d}\mathcal{P}^*_t + \tfrac{1}{2}(\mathrm{d}\mathcal{P}^*_t)^2 \Big) \frac{d^*}{P^*_t} \right) \right\} \\
&= \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega\Big(- \mathrm{d}\mathcal{X}^*_t - \tfrac{1}{2}(\mathrm{d}\mathcal{X}^*_t)^2 + \mathrm{d}\mathcal{X}^*_t \mathrm{d}\mathcal{P}^*_t \Big) \frac{d^*}{P^*_t} \right) \right\} \\
&= -\frac{1}{\omega} \exp\left(\left[\omega\big(\mu_{1,t} + \tfrac{1}{2}\sigma^2_e + \sigma_{e\pi^*}\big) \frac{d^*}{P^*_t} + \frac{\omega^2 \sigma^2_e}{2} \left(\frac{d^*}{P^*_t} \right)^2 \right] \mathrm{d}t \right),
\end{aligned}$$

where the last line uses the facts that $(\mathrm{d}\mathcal{X}^*_t)^2 = \sigma^2_e \mathrm{d}t$ and $\mathrm{d}\mathcal{X}^*_t \mathrm{d}\mathcal{P}^*_t = -\sigma_{e\pi^*}\mathrm{d}t$, as well as the property of the expectation of an exponent of a normally distributed random variable; $\mu_{1,t}$ denotes the first component of the drift vector $\boldsymbol{\mu}_t$. Therefore, maximization in (A20) is equivalent to:

$$\max_{d^*} \left\{ -\omega\big(\mu_{1,t} + \tfrac{1}{2}\sigma^2_e + \sigma_{e\pi^*}\big) \frac{d^*}{P^*_t} - \tfrac{1}{2}\omega^2 \sigma^2_e \left(\frac{d^*}{P^*_t} \right)^2 \right\} \quad \text{w/solution} \quad \frac{d^*}{P^*_t} = -\frac{\mu_{1,t} + \frac{1}{2}\sigma^2_e + \sigma_{e\pi^*}}{\omega \sigma^2_e}.$$

This is the portfolio choice equation (A17), which obtains under CARA utility in the limit of short time periods, but note it is also equivalent to the exact solution under mean-variance preferences. The extra terms in the numerator correspond to Jensen's Inequality corrections to the expected real log return on the carry trade. Assuming $\boldsymbol{\sigma} \to 0$, yet $\omega \to \infty$ such that $\omega\sigma_e^2$ stays bounded away from zero, this solution converges to the policy function in (24), as we discuss below ■

(b) Equilibrium condition: To derive the modified UIP condition (25), we combine the portfolio choice solution (A17) with the market clearing condition (20) and the noise-trader currency demand $N_{t+1}^* = n\psi_t$ to obtain:

$$B_{t+1}^* + P_t^* n\psi_t - mP_t^* \frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2} = 0. \tag{A21}$$

The market clearing conditions in (20) together with the fact that both intermediaries and noise traders take zero capital positions, that is $\frac{D_{t+1}+N_{t+1}}{R_t} = -\mathcal{E}_t \frac{D_{t+1}^*+N_{t+1}^*}{R_t^*}$. This results in the equilibrium balance between home and foreign household asset positions, $\frac{B_{t+1}}{R_t} = -\mathcal{E}_t \frac{B_{t+1}^*}{R_t^*}$. Therefore, we can rewrite (A21) as:

$$\frac{i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1} + \frac{1}{2}\sigma_e^2 + \sigma_{e\pi^*}}{\omega\sigma_e^2/m} = n\psi_t - \frac{R_t^*}{R_t}\frac{Y_t}{\mathcal{Q}_t}\frac{B_{t+1}}{P_t Y_t},$$

where we normalized net foreign assets by nominal output $P_t Y_t$ and used the definition of the real exchange rate $\mathcal{Q}_t$ in (8). We next log-linearize this equilibrium condition around a symmetric equilibrium with $\bar{R} = \bar{R}^* = 1/\beta$, $\bar{B} = \bar{B}^* = 0$, $\mathcal{Q} = 1$, and some $\bar{Y}$ and $\bar{P} = \bar{P}^*$. As shocks become small, the (co)variances σ_e^2 and $\sigma_{e\pi^*}$ become second order and drop out from the log-linearization. We adopt the asymptotics in which as σ_e^2 shrinks, ω/m increases proportionally leaving the risk premium term $\omega\sigma_e^2/m$ constant, finite and separated from zero in the limit.[3] As a result, the log-linearized equilibrium condition is:

$$\frac{1}{\omega\sigma_e^2/m}\Big(i_t - i_t^* - \mathbb{E}_t \Delta e_{t+1}\Big) = n\psi_t - \beta\bar{Y} b_{t+1}, \tag{A22}$$

where $b_{t+1} = \frac{\bar{R}}{\bar{P}\bar{Y}} B_{t+1} = -\frac{\bar{R}^*}{\bar{P}\bar{Y}} B_{t+1}^*$. After rearranging, this yields the modified UIP condition (25), completing the proof of the lemma. ■

Income and losses in the financial market Consider the income and losses of the non-household participants in the financial market — the intermediaries and the noise traders:

$$\frac{D_{t+1}^* + N_{t+1}^*}{R_t^*}\tilde{R}_{t+1}^* = \big(md_{t+1}^* + n\psi_t\big)\big(1 - e^{x_{t+1}}\big),$$

[3] Note that σ_e^2/m is the quantity of risk per intermediary and ω is their aversion to risk; alternatively, ω/m can be viewed as the effective risk aversion of the whole sector of intermediaries who jointly hold all exchange rate risk. Our approach follows Hansen and Sargent (2011) and Hansen and Miao (2018), who consider the continuous-time limit in the models with ambiguity aversion. The economic rationale of this asymptotics is not that second moments are zero and effective risk aversion ω/m is infinite, but rather that risk premia terms, which are proportional to $\omega\sigma_e^2/m$, are finite and nonzero. Indeed, the first-order dynamics of the equilibrium system result in well-defined second moments of the variables, including σ_e^2, as in Devereux and Sutherland (2011) and Tille and van Wincoop (2010). An important difference of our solution concept is that it allows for a non-zero first-order component of the return differential, namely a non-zero expected Carry trade return. We characterize the equilibrium σ_e^2 below in Appendix A.6.

where we used the definition of $\tilde{R}^*_{t+1}$ and the log Carry trade return $x_{t+1} \equiv i_t - i^*_t - \mathbb{E}_t \Delta e_{t+1} = \log(R_t/R^*_t) - \Delta \log \mathcal{E}_{t+1}$. Using the same steps as in the proof of Lemma 1, we can approximate this income as:

$$\left(-m\frac{\mathbb{E}_t x_{t+1}}{\omega\sigma_e^2} + n\psi_t\right)\big(-x_{t+1}\big) = -\beta\bar{Y}b_{t+1}x_{t+1},$$

where the equality uses (A22). Therefore, while the UIP deviations (realized x_{t+1} and expected $\mathbb{E}_t x_{t+1}$) are first order, the income and losses in the financial markets are only second order, as $b_{t+1} = B_{t+1}/(\beta\bar{P}\bar{Y})$ is first order around $\bar{B} = 0$. Intuitively, the income and losses in the financial market are equal to the realized UIP deviation times the gross portfolio position — while both are first order, their product is second order, and hence negligible from the point of view of the country budget constraint.

A.6 Derivations and Proofs for Section 5

In order to prove Propositions 2 and 3, we first derive the equilibrium system and solve for the equilibrium exchange rate process. A lot of the derivations build on Itskhoki and Mukhin (2021) and we refer the reader to that paper for a more detailed description of the equilibrium conditions and log-linearization of the equilibrium system around a symmetric steady state.

Market clearing First, we derive (26). We combine together the linearized goods market clearing, $y_t = (1-\gamma)c_{Ht} + \gamma c^*_{Ht}$, with home and foreign demand for the home good in (6)–(7), which in the absence of taste shocks (ξ_t, ξ^*_t) can be written as:

$$c_{Ht} = -\theta(p_{Ht} - p_t) + c_t \qquad \text{and} \qquad c^*_{Ht} = -\theta(p^*_{Ht} - p^*_t) + c^*_t.$$

From the definitions of the price index, we obtain $p_t = (1-\gamma)p_{Ht} + \gamma p_{Ft}$ and $p^*_t = (1-\gamma)p^*_{Ft} + \gamma p^*_{Ht}$, and therefore:

$$p_{Ht} - p_t = \gamma(p_{Ht} - p_{Ft}) = -\gamma s_t \qquad \text{and} \qquad p^*_{Ht} - p^*_t = (1-\gamma)(p^*_{Ht} - p^*_{Ft}) = -(1-\gamma)s_t,$$

where, due to the law of one price ($p_{Ht} = p^*_{Ht} + e_t$ and $p_{Ft} = p^*_{Ft} + e_t$), the terms of trade are:

$$s_t = p_{Ft} - p^*_{Ht} - e_t = (p^*_t + e_t - p_t)/(1-2\gamma) = q_t/(1-2\gamma).$$

Substituting these expressions into the market clearing results in:

$$y_t = \frac{2\theta\gamma(1-\gamma)}{1-2\gamma}q_t + (1-\gamma)c_t + \gamma c^*_t,$$

which equalizes aggregate supply and aggregate demand for the home good. Combining it together with the foreign counterpart, we have:

$$y_t - y^*_t = \frac{2\gamma}{1-2\gamma}2\theta(1-\gamma)q_t + (1-2\gamma)(c_t - c^*_t), \tag{A23}$$

where the term with the real exchange rate is the expenditure switching term. Equations (A23) characterizes the locus of (relative) output and consumption combinations which clear the product market (for home and foreign goods).

The second step is to use the labor market clearing condition to solve out aggregate output. Labor market clears when ℓ_t satisfies simultaneously the household labor supply, $\sigma c_t + \frac{1}{\nu}\ell_t = w_t - p_t$, and the firm labor demand given by the production function, $y_t = a_t + \ell_t$, which together result in:

$$y_t + \sigma\nu c_t = \nu(w_t - p_t) + a_t.$$

Combining this with its foreign counterpart, we have:

$$(y_t - y_t^*) + \sigma\nu(c_t - c_t^*) = \nu(q_t - q_t^W) + (a_t - a_t^*) = -\frac{2\gamma\nu}{1-2\gamma}q_t + (1+\nu)(a_t - a_t^*), \qquad \text{(A24)}$$

where $q_t^W = w_t^* + e_t - w_t$ is the wage-based real exchange rate and we used the relationship between $q_t = (1-2\gamma)[q_t^W + (a_t - a_t^*)]$.[4] Equation (A24) characterizes the locus of output and consumption combinations which clear the labor market. Combined together with (A23), the two conditions characterize the general labor and product market clearing, which we rewrite in the relative consumption and real exchange rate space as:

$$(1 - 2\gamma + \sigma\nu)(c_t - c_t^*) = -\frac{2\gamma}{1-2\gamma}\left[2\theta(1-\gamma) + \nu\right] q_t + (1+\nu)(a_t - a_t^*),$$

which is equivalent to (26) in the text after noting that $\varphi = 1/\nu$ is the inverse Frisch elasticity.

Equilibrium exchange rate process We next use (26) to solve out relative consumption, $c_t - c_t^*$, from the dynamic system (19) and (23), which results in two equations in (q_t, b_t):[5]

$$\begin{aligned} -(1+\gamma\sigma\kappa_q)\mathbb{E}_t\Delta q_{t+1} &= -\sigma\kappa_a\mathbb{E}_t\Delta\tilde{a}_{t+1} + \chi_1\psi_t - \chi_2 b_{t+1}, \\ \beta b_{t+1} - b_t &= \gamma[(\hat{\theta} + \gamma\kappa_q)q_t - \kappa_a\tilde{a}_t], \end{aligned}$$

where $\tilde{a}_t \equiv a_t - a_t^*$ and $\mathbb{E}_t\Delta\tilde{a}_{t+1} = -(1-\rho)\tilde{a}_t$ as (a_t, a_t^*) follow AR(1)s with persistence ρ.

We next rewrite this dynamic system in matrix form:

$$\begin{pmatrix} 1 & -\hat{\chi}_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathbb{E}_t q_{t+1} \\ \hat{b}_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1/\beta \end{pmatrix}\begin{pmatrix} q_t \\ \hat{b}_t \end{pmatrix} - \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix},$$

where for brevity we make the following substitution of variables:

$$\begin{aligned} &\hat{b}_t \equiv \frac{\beta}{\gamma(\hat{\theta} + \gamma\kappa_q)}b_t, \qquad \hat{a}_t \equiv \frac{\kappa_a}{\hat{\theta} + \gamma\kappa_q}\tilde{a}_t, \\ &\hat{\chi}_1 \equiv \frac{\chi_1}{1+\gamma\sigma\kappa_q}, \qquad \hat{\chi}_2 \equiv \frac{\gamma(\hat{\theta} + \gamma\kappa_q)}{\beta(1+\gamma\sigma\kappa_q)}\chi_2, \qquad k \equiv \frac{\sigma(\hat{\theta} + \gamma\kappa_q)}{1+\gamma\sigma\kappa_q}. \end{aligned} \qquad \text{(A25)}$$

[4]Under constant-markup pricing, the linearized pricing equations are $p_{Ht} = w_t - a_t$ and $p_{Ft} = w_t^* + e_t - a_t^*$, so that $p_t = (1-\gamma)(w_t - a_t) + \gamma(w_t^* + e_t - a_t^*)$. Together with the foreign counterpart, it results in the relationship between q_t and q_t^W in the text. See Itskhoki and Mukhin (2021) and Itskhoki (2021) for derivations of these relationships in a more general model with variable markups, pricing to market and Balassa-Samuelson terms.

[5]Recall that $\frac{NX_t}{GDP} = \gamma[\theta q_t + (\theta - 1)s_t - (c_t - c_t^*)]$ in the absence of taste shocks, $\xi_t = \xi_t^* = 0$, and since $s_t = q_t/(1-2\gamma)$ as derived above, we have $\frac{NX_t}{GDP} = \gamma[\hat{\theta}q_t - (c_t - c_t^*)]$ with $\hat{\theta} = \theta + \frac{\theta-1}{1-2\gamma} = \frac{2\theta(1-\gamma)-1}{1-2\gamma}$, as stated in the text.

Diagonalizing the dynamic system, we have:

$$\mathbb{E}_t x_{t+1} = B x_t - C \begin{pmatrix} \psi_t \\ \hat{a}_t \end{pmatrix}, \quad \text{where} \quad B \equiv \begin{pmatrix} 1+\hat{\chi}_2 & \hat{\chi}_2/\beta \\ 1 & 1/\beta \end{pmatrix}, \quad C \equiv \begin{pmatrix} \hat{\chi}_1 & (1-\rho)k + \hat{\chi}_2 \\ 0 & 1 \end{pmatrix},$$

and we denoted $x_t \equiv (q_t, \hat{b}_t)'$. The eigenvalues of B are:

$$\mu_{1,2} = \frac{(1+\hat{\chi}_2+1/\beta) \mp \sqrt{(1+\hat{\chi}_2+1/\beta)^2 - 4/\beta}}{2/\beta} \qquad \text{such that} \qquad 0 < \mu_1 \le 1 < \frac{1}{\beta} \le \mu_2,$$

and $\mu_1 + \mu_2 = 1 + \hat{\chi}_2 + 1/\beta$ and $\mu_1 \cdot \mu_2 = 1/\beta$. Note that when $\chi_2 = 0$, and hence $\hat{\chi}_2 = 0$, the two roots are simply $\mu_1 = 1$ and $\mu_2 = 1/\beta$.

The left eigenvalue associated with $\mu_2 > 1$ is $v = (1, 1/\beta - \mu_1)$, such that $vB = \mu_2 v$. Therefore, we can pre-multiply the dynamic system by v and rearrange to obtain:

$$v x_t = \frac{1}{\mu_2} \mathbb{E}_t\{v x_{t+1}\} + \frac{1}{\mu_2} \hat{\chi}_1 \psi_t + \left[\frac{(1-\rho)k + \hat{\chi}_2}{\mu_2} + \frac{1/\beta - \mu_1}{\mu_2} \right] \hat{a}_t.$$

Using the facts that $\hat{\chi}_2 + 1/\beta - \mu_1 = \mu_2 - 1$ and $1/\mu_2 = \beta\mu_1$, we solve this dynamic equation forward to obtain the equilibrium cointegration relationship:

$$v x_t = q_t + (1/\beta - \mu_1)\hat{b}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{1-\beta\mu_1+\beta(1-\rho)k\mu_1}{1-\beta\rho\mu_1}\hat{a}_t. \tag{A26}$$

Combining this with the second dynamic equation for $\hat{b}_{t+1}$, we solve for:

$$\hat{b}_{t+1} - \mu_1 \hat{b}_t = \overbrace{q_t + \left(\tfrac{1}{\beta} - \mu_1\right)\hat{b}_t}^{=v x_t} - \hat{a}_t = \frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t, \tag{A27}$$

Note that $\hat{b}_{t+1}$ in (A27) follows a stationary AR(2) with roots ρ and μ_1.

Finally, we apply the lag operator $(1-\mu_1 L)$ to (A26) and use (A27) to solve for:

$$\begin{aligned}(1-\mu_1 L) q_t &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)(k-1)\mu_1}{1-\beta\rho\mu_1}\hat{a}_t\right] + (1-\mu_1 L)\hat{a}_t \\ &= (1-\beta^{-1}L)\left[\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t + \frac{\beta(1-\rho)\mu_1}{1-\beta\rho\mu_1}k\hat{a}_t\right] + \frac{1-\beta\mu_1}{1-\beta\rho\mu_1}(1-\rho\mu_1 L)\hat{a}_t, \end{aligned} \tag{A28}$$

where L is the lag operator such that $Lq_t = q_{t-1}$. Therefore, equilibrium RER q_t follows a stationary ARMA(2,1) with autoregressive roots $\delta = \mu_1$ and ρ. In the limit $\chi_2 \to 0$, which implies $\mu_1 \to 1$, this process for q_t becomes an ARIMA(1,1,1), which nonetheless has impulse responses that are arbitrarily close to a stationary ARMA(2,1) with a large $\mu_1 \lesssim 1$.

Furthermore, one can partition the components of q_t in (A28) driven by ψ_t and $\tilde{a}_t$ into two subprocesses q_t^ψ and q_t^a such that $q_t = q_t^\psi + q_t^a$:

$$(1-\mu_1 L) q_t^\psi = (1-\beta^{-1}L)\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\psi_t, \tag{A29}$$

$$(1-\mu_1 L) q_t^a = \left[(1-\beta^{-1}L)\frac{\beta(1-\rho)\mu_1}{1-\beta\rho\mu_1}k + \frac{1-\beta\mu_1}{1-\beta\rho\mu_1}(1-\rho\mu_1 L)\right]\hat{a}_t. \tag{A30}$$

Note that:

(i) as $\chi_1 \to 0$ (and hence $\hat{\chi}_1 \to 0$), $q_t^\psi \to 0$ and $q_t = q_t^a$;

(ii) the two components in q_t^a correspond to the effects of productivity shocks on the Euler equation and the budget constraint respectively, with the former component disappearing in the limit of persistent shocks $\rho \to 1$, such that the productivity component of the real exchange rate is simply $q_t^a = \hat{a}_t = \frac{\kappa_a}{\hat{\theta}+\gamma\kappa_q}\tilde{a}_t$, a random walk that does not depend on χ_1 or χ_2. As a result, in this case, $\chi_1 \to 0$ implies $q_t = q_t^a = \hat{a}_t$.

Equilibrium variance of the exchange rate Solution (A28) characterizes the behavior of q_t for given values of χ_1 and χ_2 (and hence μ_1, μ_2), which from (25) themselves depend on $\sigma_e^2 = \text{var}_t(\Delta e_{t+1})$. Under the peg, $\sigma_e^2 = 0$ and hence $\chi_1 = \chi_2 = 0$. Under the float, monetary policy stabilizes inflation, ensuring $e_t = q_t$, and hence we have $\sigma_e^2 = \text{var}_t(\Delta q_{t+1})$. We now solve for the equilibrium value of σ_e^2, and thus of $(\chi_1, \chi_2, \mu_1, \mu_2)$.

Using (A28), we calculate $\sigma_e^2 = \text{var}_t(\Delta q_{t+1})$ for given χ_1 and χ_2:

$$\sigma_e^2 = \text{var}_t(\Delta q_{t+1}) = \left(\frac{\beta\mu_1\hat{\chi}_1}{1-\beta\rho\mu_1}\right)^2\sigma_\psi^2 + \left(\frac{\beta(1-\rho)\mu_1 k+(1-\beta\mu_1)}{1-\beta\rho\mu_1}\right)^2\sigma_a^2 = \frac{\hat{\chi}_1^2\sigma_\psi^2 + \big((1-\rho)k+(\mu_2-1)\big)^2\sigma_a^2}{(\mu_2-\rho)^2},$$

where the second line used the fact that $\beta\mu_1 = 1/\mu_2$. In addition, recall that:

$$\hat{\chi}_1 = \frac{n}{1+\gamma\sigma\kappa_q}\frac{\omega\sigma_e^2}{m}, \qquad \hat{\chi}_2 \equiv \frac{\hat{\theta}+\gamma\kappa_q}{1+\gamma\sigma\kappa_q}\gamma\bar{Y}\frac{\omega\sigma_e^2}{m} \quad \text{and} \quad \mu_2 = \frac{(1+\beta\hat{\chi}_2+\beta)+\sqrt{(1+\beta\hat{\chi}_2+\beta)^2-4\beta}}{2\beta}.$$

We therefore can rewrite the fixed point equation for $\sigma_e^2 > 0$ as follows:

$$F(x,\tilde{\omega}) = \big(\mu_2(\tilde{\omega}x)-\rho\big)^2 x - b(\tilde{\omega}x)^2 - c = 0, \tag{A31}$$

where we used the following notation:

$$x \equiv \sigma_e^2 \geq 0, \qquad \tilde{\omega} = \frac{\omega}{m}, \qquad b \equiv \left(\frac{n}{1+\gamma\sigma\kappa_q}\right)^2\sigma_\psi^2, \qquad c \equiv \big((1-\rho)k+(\mu_2-1)\big)^2\sigma_a^2 \geq 0,$$

and $\mu_2(\cdot)$ is a function which gives the equilibrium values of μ_2 defined above as a function of $\tilde{\omega}\sigma_e^2$ for given values of the model parameters. Note that for any given $\tilde{\omega} > 0$:

$$\lim_{x\to 0} F(x,\tilde{\omega}) = -c \leq 0,$$
$$\lim_{x\to\infty}\frac{F(x,\tilde{\omega})}{x^3} = \lim_{x\to\infty}\left(\frac{\mu_2(\tilde{\omega}x)}{x}\right)^2 = \left(\frac{\beta\hat{\chi}_2^2}{\sigma_e^2}\right) = \left(\frac{\hat{\theta}+\gamma\kappa_q}{1+\gamma\sigma\kappa_q}\gamma\bar{Y}\tilde{\omega}\right)^2 > 0.$$

Therefore, by continuity at least one fixed-point $F(\sigma_e^2,\tilde{\omega}) = 0$ with $\sigma_e^2 \geq 0$ exists, and all such that $\sigma_e^2 > 0$ whenever $c > 0$ (that is, when $\sigma_a > 0$). One can further show that for large enough σ_a and $\sigma_\psi/(1-\beta\rho)$, the high volatility $\sigma_e^2 > 0$ equilibrium is unique (see Figure A10 for illustration).[6]

Finally, we consider the limit of log-linearization in Lemma 1, where $(\sigma_a, \sigma_\psi) = \sqrt{\epsilon}\cdot(\bar{\sigma}_a, \bar{\sigma}_\psi) = \mathcal{O}(\sqrt{\epsilon})$ as $\epsilon \to 0$, where $(\bar{\sigma}_a, \bar{\sigma}_\psi)$ are some fixed numbers. Then in (A31), $(b,c) = \mathcal{O}(\epsilon)$, as (b,c) are linear in $(\sigma_a^2, \sigma_\psi^2)$. This implies that for any given fixed point $(\bar{\sigma}_e^2, \bar{\omega})$, with $F(\bar{\sigma}_e^2, \bar{\omega}; \bar{\sigma}_a^2, \bar{\sigma}_\psi^2) = 0$, there

[6]For small $\sigma_a > 0$, there typically exist three equilibria with $\sigma_e^2 > 0$. When $\sigma_a = 0$, there always exists an equilibrium with $\sigma_e^2 = \chi_{1,2} = 0$ and one or two additional equilibria with $\sigma_e > 0$, provided $\sigma_\psi > 0$.

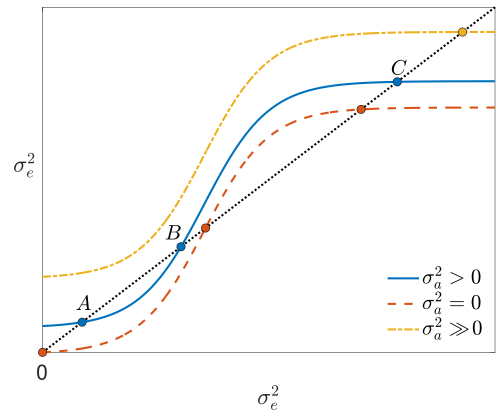


Figure A10: Equilibrium volatility of the exchange rate, σ_e^2

Note: the figure plots the implied conditional exchange rate volatility $\sigma_e^2 = \text{var}_t(\Delta e_{t+1})$ for the corresponding values of $\hat{\chi}_1(\sigma_e^2)$ and $\hat{\chi}_2(\sigma_e^2)$ as a function of σ_e^2 on the x-axis. The intersections with the diagonal 45° line A, B, C are the equilibrium fixed point values of σ_e^2. The red dashed line corresponds to the case of no macro fundamental shocks, $\sigma_a = 0$, featuring an equilibrium with $\sigma_e^2 = 0$, while the other two lines correspond to $\sigma_a > 0$, and thus feature either three equilibria with $\sigma_e^2 > 0$ (when σ_a and σ_ψ are small) or a unique high volatility equilibrium with $\sigma_e^2 \gg 0$ (when σ_a and σ_ψ are larger).

exists a sequence of fixed points $F(\epsilon\bar{\sigma}_e^2, \bar{\omega}/\epsilon; \epsilon\bar{\sigma}_a^2, \epsilon\bar{\sigma}_\psi^2) = 0$ as $\epsilon \to 0$, for which $\sigma_e^2 = \epsilon\bar{\sigma}_e^2 = \mathcal{O}(\epsilon)$, $\tilde{\omega} = \bar{\omega}/\epsilon = \mathcal{O}(1/\epsilon)$ and $\tilde{\omega}\sigma_e^2 = \bar{\omega}\bar{\sigma}_e^2 = const.$ To verify this, one can simply divide (A31) by ϵ and note that, for a given $\tilde{\omega}x$, $F(x, \tilde{\omega})$ is linear in (x, b, c), which means that the fixed point x scales with (b, c) provided that $\tilde{\omega}x$ stays constant. This confirms the conjecture used in the proof of Lemma 1. ■

Proof of Proposition 2 The proof follows directly from results above. First, the existence of equilibria under both the float and the peg follows from the equilibrium exchange rate process (A28) together with the fixed point argument for σ_e^2 established above. Part (a) of the proposition follows from the decomposition of $q_t = q_t^\psi + q_t^a$ in (A29)–(A30), which implies:

$$\text{var}(\Delta q_t) = \text{cov}(\Delta q_t^\psi, \Delta q_t) + \text{cov}(\Delta q_t^a, \Delta q_t),$$

with $\text{cov}(\Delta q_t^\psi, \Delta q_t) = 0$ under the peg as $q_t^\psi \equiv 0$. Thus, it is sufficient to require that $\text{cov}(\Delta q_t^\psi, \Delta q_t) \gg \text{cov}(\Delta q_t^a, \Delta q_t)$ under the float, which is the case for a sufficiently large σ_ψ/σ_a, and thus can be always guaranteed.

Part (b) of the proposition follows from (26): as $\gamma \to 0$, $c_t - c_t^* \to \frac{1+\varphi}{\sigma+\varphi}(a_t - a_t^*)$, independently of the process for q_t and the exchange rate regime. The same applies for output, with $y_t - y_t^* \to \frac{1+\varphi}{\sigma+\varphi}(a_t - a_t^*)$. Finally, inflation $\pi_t - \pi_t^* \equiv 0$ under the float, and under the peg $\pi_t - \pi_t^* = -\Delta q_t = -\Delta q_t^a$, with volatility arbitrary close to zero relative to the volatility of Δq_t under the float, as follows from part (a). ■

A.7 Model of a Swiss Peg of 2011–2015

Model We adopt a simple quantitative version of the model in Section 5 to analyze the dynamics of the Swiss franc during 2000–2020, which features a peg from the end of 2011 till the beginning of 2015. The world consists of two asymmetric economies: Home (Switzerland) is a small economy that accounts for infinitesimal share of Foreign (Euro Area, EA) consumption and output. There are three shocks: to output $y_t - y_t^*$, preferences $\xi_t - \xi_t^*$, and currency demand ψ_t; and two policy instruments — FXI f_t and interest rate i_t — determine the volatility of the nominal exchange rate σ_e^2.

To introduce FXI, we generalize the financial market clearing conditions (20) to additionally feature official home-currency and FX reserves F_{t+1} and F^*_{t+1}, respectively. In particular, $F_{t+1} < 0$ corresponds to the home-currency (franc) liabilities issued by the government in exchange for foreign-currency assets $F^*_{t+1} > 0$. Following the same steps as in the proof of Lemma 1 in Appendix A.5, we obtain the following international risk-sharing condition:

$$\mathbb{E}_t\{\sigma(\Delta c_{t+1} - \Delta c^*_{t+1}) - \Delta q_{t+1}\} = \chi(\sigma_e^2) \cdot (\psi_t + f_t - \iota b_{t+1}). \tag{A32}$$

The country budget constraint is still given by:

$$\beta b_{t+1} - b_t = \gamma \left[\hat{\theta} q_t - (c_t - c^*_t) - (\xi_t - \xi^*_t)\right] \equiv 2\gamma \cdot nx_t, \tag{A33}$$

and the goods market clearing condition is:

$$c_t - c^*_t = \kappa_y(y_t - y^*_t) - \kappa_q q_t + \kappa_\xi(\xi_t - \xi^*_t), \tag{A34}$$

where $c^*_t = y^*_t$, $\hat{\theta} = \frac{\theta(2-\gamma)-1}{1-\gamma}$, and $\kappa_y \equiv \frac{1}{1-\gamma}$, $\kappa_\xi \equiv \frac{\gamma}{1-\gamma}$ and $\kappa_q \equiv \frac{\gamma(2-\gamma)\theta}{(1-\gamma)^2}$.[7] We normalize ψ_t, f_t, b_t by home GDP and hence $\chi(\sigma_e^2) = \bar{Y}\omega\sigma_e^2/m$. As before, γ is the home openness and net exports are normalized by total trade $nx_t \equiv \frac{EX_t - IM_t}{EX_t + IM_t}$.

Data We use quarterly data for Switzerland and the EA from 2000–2020. The periods from 2000:Q1-2011:Q3 and from 2015:Q1-2019:Q3 correspond to a float and the period from 2011:Q4-2014:Q4 is a peg. We normalized log-deviations to zero in the first period: $b_0 = f_0 = q_0 = y_0 - y^*_0 = c_0 - c^*_0 = 0$. The real exchange rate refers to the bilateral CHF-EUR rate. The relative consumption $c_t - c^*_t$ corresponds to the difference in log seasonally-adjusted real consumption in each country. Since we exclude investment and government spendings and $Y = C$ in the steady state, we compute nominal net exports as $2\gamma nx_t$ times nominal consumption and add it up with nominal consumption to get Home real output. Foreign real output coincides with real consumption. FX reserves and noise trader shocks are normalized by nominal consumption to solve the model, while the estimates in all figures are re-normalized by nominal GDP for presentation purposes. The data comes from IFS (2024), FRED (2024) and SECO (2024).

Calibration We use the standard values of $\beta = 0.995$ and $\sigma = 2$. For simplicity, we adopt an approximation with $\iota = 0$ to reflect the fact that b_{t+1} is a persistent slow moving macro variable relative to volatile jump variables such as gross capital flows (affecting currency demand ψ_t) and offsetting FX interventions f_t. We calibrate the value of $\chi(\sigma_e^2)$ in the floating regime to match a 1% depreciation of the real exchange rate from a purchase of foreign reserves equal to 10% of GDP. This elasticity endogenously declines nine-fold under the peg reflecting the change in the volatility of the nominal exchange rate σ_e^2 in the data. The shocks are assumed to follow AR(1) processes with persistence parameters $(\rho_y, \rho_\xi, \rho_f, \rho_\psi) = (0.9, 0.9, 1, 1)$ consistent with the estimated autocorrelations for the shocks recovered from the data.

We calibrate the openness of Switzerland to $\gamma = 0.5$ motivated by trade-to-GDP ratio (imports + exports) for Switzerland between 90–130% over this period.[8] The value of γ is sufficient to estimate κ_y and κ_ξ. In contrast, we calibrate coefficients κ_q and $\hat{\theta}$ leveraging the pass-through estimates for Switzerland from Auer, Burstein, and Lein (2021). From the reported elasticity of market shares of -0.8 and the share of foreign goods of 0.25, we get that $\theta = 0.2$ at the retail level and therefore,

[7] Notice a slight difference in notation relative to baseline: κ_y and κ_ξ now include γ and $\xi_t - \xi^*_t$ includes $1-\gamma$.

[8] Focusing on trade flows against the EA and replacing GDP with the sum of consumption and net exports to the EA results in a similar trade-to-output ratio of 110–140%.

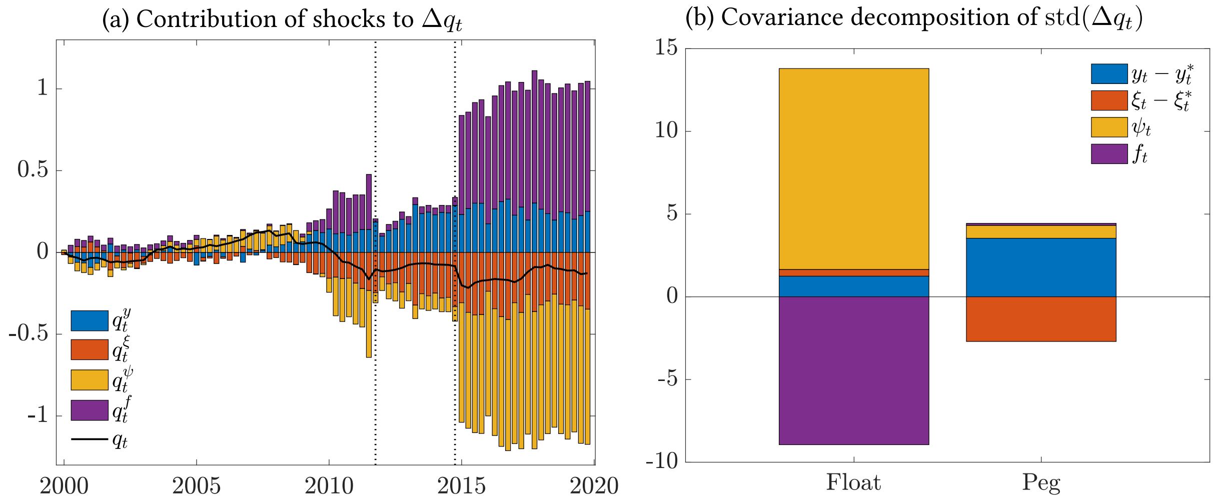


Figure A11: Decomposition of the Swiss exchange rate

Note: The left panel shows the decomposition of the Swiss real exchange rate into four shocks: relative output, trade shocks, financial shocks and FXI. The right panel shows the (covariance) decomposition of the standard deviation of the Swiss real exchange rate for the periods of the float (2000-11 and 2015-20) and the peg (2011-15).

$\kappa_q = 0.6$. At the same time, the exchange rate pass-through (ERPT) of 0.5 and 0.3 into import and export prices (at the border level) implies that $s_t = 0.2q_t$ consistent also with estimates from Obstfeld and Rogoff (2000) and Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller (2020). Combining the expenditure switching effect with the terms-of-trade effect, we arrive at $\hat{\theta} = 0.4$.

The paths of FX reserves f_t and output shocks $y_t - y_t^*$ are estimated directly from the data. Given parameter values and the observed paths of $c_t - c_t^*, q_t$ and nx_t, we then back out taste shocks $\xi_t - \xi_t^*$. Finally, we use the model's full solution, which differs between periods of floats and peg, to recover ψ_t. The procedure relies on updating recursively the values of b_t matching the path of net exports.

Decomposition Figure A11 shows the contribution of the four shocks to the real exchange rate. The left panel decomposes the dynamics of the Swiss real exchange rate into four components driven by output, taste, financial and FXI shocks:

$$\Delta q_t = \Delta q_t^y + \Delta q_t^\xi + \Delta q_t^\psi + \Delta q_t^f,$$

using the linear structure of the model. The right panel presents the covariance decomposition of changes in the real exchange rate:

$$\mathrm{var}(\Delta q_t) = \mathrm{cov}(\Delta q_t, \Delta q_t^y) + \mathrm{cov}(\Delta q_t, \Delta q_t^\xi) + \mathrm{cov}(\Delta q_t, \Delta q_t^\psi) + \mathrm{cov}(\Delta q_t, \Delta q_t^f),$$

separately for the periods with a floating and pegged regimes, and plotting in units of standard deviation $\mathrm{std}(\Delta q_t)$. Consistent with the main point of the paper, financial shocks dominate under a floating regime, but play much smaller role under the peg. Furthermore, the noise trader shocks under the float are largely offset by FXI resulting in much smaller net effects of currency demand shocks on the exchange rate. The contribution of shocks collapses during the peg because of a more elastic arbitrage by intermediaries and therefore, a lower pass-through of currency demand shocks into the equilibrium exchange rate, eliminating the need for FXI. Throughout the period, the combination of macro and trade shocks puts an appreciation pressure on the franc, rationalizing the observed trade surpluses. Financial shocks generate a depreciation pressure pre-2008, which turns into an appreciation pressure thereafter, despite active FXI to counteract the appreciation during the periods of the float.