Econometrica Supplementary Material

SUPPLEMENT TO "THE ANATOMY OF SORTING—EVIDENCE FROM DANISH DATA"

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APPENDIX D: NUMERICAL IMPLEMENTATION

THE IMPLEMENTATION OF THE ESTIMATION ALLOWS the estimation to be scaled up to larger data sets by expansion of the number of CPUs in the computing cluster. The following describes how the storage and computation requirements of the estimation are delegated across CPUs in a parallel computing environment. The coding is done in Fortran and parallelization is performed with OpenMPI.

D.1. *Data Structure*

The Danish Matched Employer–Employee (MEE) data comprise approximately $I = 4{,}000{,}000$ workers and $J = 400{,}000$ firms observed at a weekly frequency from 1985 to 2013. The fundamental observation in the data is a spell (either employment or unemployment).

A worker history consists of a series of employment and unemployment spells. It is stored as a linked list. Each object in the list is a spell. The spell object contains

- Start and end weeks of the spell.
- ID's of the worker and firm (unemployment has firm ID 0).
- A vector of wage observations for each year of the spell.
- Pointers to the previous and next spell in the worker's history.
- Pointers to the previous and next spell in the firm's spell list (unlike the worker's linked list, the firm list is not necessarily chronological).

In addition, the data structure holds the observable characteristics of each worker and firm separate from the list of spells. The worker i object holds the worker's observable characteristics (gender, education, birth year, year of entry into labor market, etc.) as well as pointers to the first and last spells in the worker's labor history. The firm j object holds observable characteristics (public–private) and pointers to the first and last spell in its list of spells. The firm $j = 0$ list holds all the unemployment spells in the data.

The data storage is divided across CPUs so that each CPU holds its own subset of worker histories. Denote by ι_c the set of worker IDs assigned to CPU c. Each CPU holds

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the entire set of firms, but CPU c's list of employment spells in firm j consists only of those that are contributed by workers in the subset ι_c.

The Danish MEE data set is relatively small by international comparison (small size of the Danish population). Nevertheless, it does place significant demands on computer memory. Needless to say, this issue only becomes more acute for MEE data from larger countries. It is a virtue of the code that the memory requirement associated with each CPU is roughly $1/C$ of the total size of the data given a total of C CPUs. Thus, the memory pool available for the estimation is the combined memory of the nodes in the cluster, which is trivially scaled up by adding more nodes, by opposition to a data structure where each CPU holds the entire data set, which would place heavy memory requirements on multi-CPU nodes.

D.2. *E-Step*

D.2.1. *Likelihood Evaluation for a Given* $(\beta, \mathcal{L})$

Each CPU holds its own copy of the firm classification, $\mathcal{L}$. With this, CPU c evaluates $L_i(\beta, \mathcal{L}) = \sum_{k=1}^{K} L_i(k; \beta, \mathcal{L})$ for any $i \in \iota_c$ by walking through the worker i linked list of spells. CPU c calculates $L^c = \sum_{i\in\iota_c} \ln L_i(\beta, \mathcal{L})$. The likelihood of the data is then found by summing L^c across CPUs, $L(\beta, \mathcal{L}) = \exp(\sum_{c=1}^{C} L^c)$. This is a modest communication of a single double precision number across the C CPUs. The calculation of the overall likelihood is not necessary for the execution of the E-step, but serves as an useful check that the algorithm is indeed proceeding to increase the likelihood in each iteration.

D.2.2. *Worker Posterior Update for a Given* $(\beta, \mathcal{L})$

CPU c updates worker posteriors for all $i \in \iota_c$ by, $p_i(k; \beta, \mathcal{L}) = L_k(k; \beta, \mathcal{L})/L_i(\beta, \mathcal{L})$. No communication across CPUs is necessary for this and CPU c knows only the posteriors for workers $i \in \iota_c$. Nowhere in the CEM algorithm does CPU c need to know the worker posterior for workers outside ι_c. This is a significant saving in communication, which would otherwise involve the communication of $I \times K$ double precision numbers across the C CPUs in each E-step.

D.3. *M-Step*

The M-step uses the updated posterior $p_i(k; \beta, \mathcal{L})$ from the E-step. Each part of the M-step requires only modest communication between nodes.

D.3.1. $\pi_k(z)$ *Update for Given* $(\beta, \mathcal{L})$

With the worker posteriors in hand, CPU c calculates $\pi_{k,c}(z) = \sum_{i\in\iota_c} p_i(k; \beta, \mathcal{L})\mathbf{1}\{z_i = z\}$, which is communicated across the CPUs. This is a $K \times Z$ dimension double precision array communication across C CPUs where each CPU receives $\sum_{c=1}^{C} \pi_{k,c}(z)$.[28] Each CPU then calculates $\pi_k(z) = \sum_{c=1}^{C} \pi_{k,c}(z)/[\sum_{k=1}^{K} \sum_{c=1}^{C} \pi_{k,c}(z)]$.

D.3.2. $m_{k\ell}(x)$ *Update for Given* $(\beta, \mathcal{L})$

CPU c calculates $m_{k\ell,c}(x) = \sum_{i\in\iota_c} p_i(k; \beta, \mathcal{L})\mathbf{1}\{x_{i1} = x, \ell_{i1} = \ell\}$, which is communicated across the CPUs with each CPU receiving $\sum_{c=1}^{C} m_{k\ell,c}(x)$. This is a $K \times L \times X_{\text{ini}}$

[28]Using mpi_allreduce.

double precision array where X_{ini} is the number of x categories in the initial distribution. Each CPU then calculates $m_{k\ell}(x) = \sum_{c=1}^{C} m_{k\ell,c}(x)/[\sum_{\ell=1}^{L}\sum_{c=1}^{C} m_{k\ell,c}(x)]$.

D.3.3. *Wage Parameters for Given* $(\beta, \mathcal{L})$

CPU c calculates

$$\mu_{k\ell,c}(x) = \sum_{i\in\iota_c} p_i(k;\beta,\mathcal{L}) \sum_{t=1}^{T} \mathbf{1}\{\ell_{it} = \ell, x_{it} = x\} w_{it}$$

$$d_{k\ell,c}(x) = \sum_{i\in\iota_c} p_i(k;\beta,\mathcal{L}) \sum_{t=1}^{T} \mathbf{1}\{\ell_{it} = \ell, x_{it} = x\}.$$

These 2 $K \times L \times X$ arrays are communicated across CPUs to form $\sum_{c=1}^{C} \mu_{k\ell,c}(x)$ and $\sum_{c=1}^{C} d_{k\ell,c}(x)$, where X is the number of relevant x categories for the wage parameters as well as γ and λ mobility parameters. Each CPU proceeds to calculate $\mu_{k\ell}(x) = \sum_{c=1}^{C} \mu_{k\ell,c}(x)/\sum_{c=1}^{C} d_{k\ell,c}(x)$.

Moving to the variance, CPU c calculates $\sigma_{k\ell,c}(x) = \sum_{i=1}^{I} p_i(k;\beta,\mathcal{L}) \sum_{t=1}^{T} \mathbf{1}\{\ell_{it} = \ell, x_{it} = x\}[w_{it} - \mu_{k\ell}(x)]^2$. The $K \times L \times X$ array is communicated across CPUs to form $\sum_{c=1}^{C} \sigma_{k\ell,c}(x)$. Each CPU calculates $\sigma_{k\ell}(x) = \sqrt{\sum_{c=1}^{C} \sigma_{k\ell,c}(x)/\sum_{c=1}^{C} d_{k\ell,c}(x)}$.

D.3.4. *Mobility Parameters for Given* $(\beta, \mathcal{L})$

Running through worker spell lists, each CPU calculates mobility counts,

$$\overline{n}_{k\ell,c}(x) = \sum_{i\in\iota_c} p_i(k;\beta,\mathcal{L}) \#\{t : D_{it} = 0, \ell_{it} = \ell, x_{it} = x\}$$

and

$$n_{k\ell\ell',c}(x) = \sum_{i\in\iota_c} p_i(k;\beta,\mathcal{L}) \#\{t : D_{it} = 1, \ell_{it} = \ell, \ell_{i(t+1)} = \ell', x_{it} = x\}.$$

These two integer arrays (of size $K \times (L+1) \times X$ and $K \times (L+1)^2 \times X$, resp.) are communicated across CPUs to form $\overline{n}_{k\ell}(x) = \sum_{c=1}^{C} \overline{n}_{k\ell,c}(x)$ and $n_{k\ell\ell'}(x) = \sum_{c=1}^{C} n_{k\ell\ell',c}(x)$. With these counts, each CPU updates $\gamma_{k\ell}(x)$, $\lambda_\ell(x)$, and $\nu_\ell(x)$ according to Section B.

D.4. *C-Step*

The C-step reassigns firm types in such a way as to increase the value of the expected log-likelihood function, thereby increasing the likelihood of the data. The C-step can be viewed as a simple extension of the M-step where the firm classification is just another set of parameters to be chosen so as to improve on the expected log likelihood. While the M-step requires very modest communication, the C-step does involve J separate communications of size L arrays within the cluster. This is a significant communication load, and consequently, it is advantageous to do multiple EM iterations between C-steps.

The firm IDs have been chosen so that firms are ordered by size ($j = 1$ is the largest firm where size is the number of wage observations throughout the panel). The algorithm reassigns firm type j by

$$\ell_j^{(s+1)} = \arg\max_{\ell} \sum_{i=1}^{I} \sum_{k=1}^{K} p_i\big(k; \widehat{\beta}^{(s)}, \mathcal{L}^{(s)}\big) \ln L_i\big(k; \widehat{\beta}^{(s)}, \mathcal{L}_{-j}^{(s)}(\ell)\big), \tag{14}$$

where $\mathcal{L}_{-j}^{(s)}(\ell)$ is the firm classification that is obtained by taking the $\mathcal{L}^{(s)}$ classification where all firm types $j' = 1, \dots, j-1$ have already been reassigned and, furthermore, replace the j'th element with ℓ. Do the reassignment in order. This step increases the expected log likelihood.

Done naively, the step is expensive since it involves $L \times J$ expected likelihood evaluations of the data. But the expected log likelihood varies with firm j's type only through the spells that directly involve firm j and through firm j's type's impact on the $q(\ell, \mathcal{L}_{-j}^{(s)}(\ell))$ distribution. The latter does involve all spells but in a way that allows simplification. Define by $\Omega(\mathcal{L})$, the contribution to the expected log likelihood from the $q(\cdot|\mathcal{L})$ related terms,

$$\Omega(\beta, \mathcal{L}) = -\sum_{i=1}^{I} \sum_{k=1}^{K} p_i(k; \beta, \mathcal{L}) \left[\ln q(\ell_{i1}|\mathcal{L}) + \sum_{t=1}^{T} D_{it} \ln q(\ell_{i(t+1)}|\mathcal{L}) \right]$$

Define

$$n_\ell^q(\mathcal{L}) = \sum_{i=1}^{I} \big[\mathbf{1}\{\ell_{i1} = \ell\} + \#\{t : D_{it} = 1, \ell_{i(t+1)} = \ell\}\big]$$

with which we can write

$$\Omega(\beta, \mathcal{L}) = -\sum_{\ell=1}^{L} \ln q(\ell|\mathcal{L}) n_\ell^q(\mathcal{L}).$$

It is worth noting that another way of calculating Ω is by adding up spells at the firm level. Denote by $\hat{n}_j$ the number of employment spells in firm j,

$$\hat{n}_j = \sum_{i=1}^{I} \left[\mathbf{1}\big[j(i,1) = j\big] + \sum_{t=1}^{T} \mathbf{1}\big[D_{it} = 1, j(i, t+1) = j\big] \right],$$

with this, Ω can be written as

$$\Omega(\beta, \mathcal{L}) = -\sum_{\ell=1}^{L} \ln q(\ell|\mathcal{L}) \sum_{j=1}^{J} \hat{n}_j \mathbf{1}[\ell_j = \ell] = -\sum_{\ell=1}^{L} \ln q(\ell|\mathcal{L}) \hat{n}(\ell|\mathcal{L}),$$

where the number of spells in type ℓ firms is

$$\hat{n}(\ell|\mathcal{L}) \equiv \sum_{j=1}^{J} \hat{n}_j \mathbf{1}[\ell_j = \ell]. \tag{15}$$

This firm-centric formulation of Ω is the preferable one for the firm reclassification algorithm.

Continuing the firm-centric formulation of the log likelihood, denote by $\iota(j) = \{(i,t) | j(i,t) = j\}$, that is, all worker-time pairs with firm j. We can then write the the firm j classification update as

$$\ell_j^{(s+1)} = \arg\max_{\ell} \Bigg[\sum_{(i,t)\in\iota(j)} \sum_{k=1}^{K} p_i\big(k; \widehat{\beta}^{(s)}, \mathcal{L}^{(s)}\big) \times \big[f_{k\ell}(w_{it}|x_{it}) + (1 - D_{it}) \ln \overline{M}_{k\ell_{it}}(x_{it}) + D_{i(t-1)} \ln M_{k\ell_{i(t-1)}\ell} + D_{it} \ln M_{k\ell\ell_{i(t+1)}} \big] + \Omega\big(\beta, \mathcal{L}_{-j}^{(s)}(\ell)\big) \Bigg]. \tag{16}$$

The algorithm is then as follows:

1. The firm j spell counts, $\hat{n}_j$, are determined at the outset of the overall estimation where all processors count how many spells they each have for each given firm j. $\hat{n}_j$ is then found by a communication of a size J integer vector across all processors. Furthermore, the firm IDs $j = 1, \dots, J$, are ordered by firm size—specifically the size of $\iota(j)$. These steps are not done in the C-step but rather just once at the outset of the full CEM algorithm.
2. The firm classification at the outset of the C-step is $\mathcal{L}^{(s)}$. Denote by $\mathcal{L}^{(s),0} = \mathcal{L}^{(s)}$, where $\mathcal{L}^{(s),j}$ is the firm classification in the jth substep of the C-step. Initialize the C-step by the determination of $\hat{n}(\ell|\mathcal{L}^{(s)})$ by equation (15).
3. Take firm $j = 1$. Find the optimal firm type for firm j according to equation (16) and firm classification $\mathcal{L}^{(s),j-1}$. The (i,t) pairs in $\iota(j)$ are by the data delegation spread out across different CPUs. Each CPU evaluates the summation in equation (16) for its own (i,t) pairs for each firm type $\ell = 1, \dots, L$. The data structure has for each firm defined a linked list of its spells held by CPU c, which allows quick within CPU evaluation of each CPU's contribution to equation (16). The full sum for each ℓ is then obtained by a summation across all CPUs to the master process. This is a communication of an L size array from each node to the master node. The master process resolves the maximization problem in equation (16), and communicates the optimal firm type $\ell_j^{(s+1)}$ to all CPUs, a single integer.
4. Update the firm classification $\mathcal{L}^{(s),j} = \mathcal{L}_{-j}^{(s),(j-1)}(\ell_j^{(s+1)})$. Thus, as the algorithm steps through $j = 1, \dots, J$, the firm classification is updated sequentially with a new firm type for firm j. Also, update $\hat{n}^j(\ell) = \hat{n}(\ell|\mathcal{L}^{(s),j})$ and the type frequencies $q(\ell|\mathcal{L}^{(s),j})$. This is done by the simple algorithm (stated just for $\hat{n}^j$)
   (a) If $\ell_j^{(s+1)} = \ell_j^{(s)}$, then $\hat{n}^j(\ell) = \hat{n}^{(j-1)}(\ell)$ for all ℓ.
   (b) Else, $\hat{n}^j(\ell_j^{(s+1)}) = \hat{n}^{(j-1)}(\ell_j^{(s+1)}) + \hat{n}_j$ and $\hat{n}^j(\ell_j^{(s)}) = \hat{n}^{j-1}(\ell_j^{(s)}) - \hat{n}_j$. For all other firm types, $\hat{n}^j(\ell) = \hat{n}^{(j-1)}(\ell)$.
5. Loop back to step 3 for next j. Exit when $j = J$ is completed. Denote by $\mathcal{L}^{(s+1)} = \mathcal{L}^{(s),J}$.

APPENDIX E: ADDITIONAL RESULTS

TABLE E.I

FIRM CHARACTERISTICS BY TYPE IN PERIOD 2.

ℓ	No Info	Public	Private	Avg. Spell All ℓ-Firms	No. Firm	Avg Size/Yr.	Avg. Inflow/Yr	Avg. Outflow/Yr.	Avg. Age in Yr.
1	0.02	0.33	0.65	7960	6226	1.28	0.72	0.70	4.94
2	0.02	0.40	0.57	23,369	4781	4.89	0.58	0.53	8.47
3	0.03	0.25	0.72	12,526	9233	1.36	0.62	0.60	5.08
4	0.02	0.31	0.67	66,569	7073	9.41	0.52	0.46	9.45
5	0.04	0.06	0.90	20,176	14,742	1.37	0.77	0.76	3.94
6	0.02	0.10	0.88	31,959	20,378	1.57	0.49	0.48	6.78
7	0.00	0.04	0.95	47,013	22,204	2.12	0.49	0.47	9.97
8	0.23	0.50	0.27	282,366	150	1882.44	0.36	0.03	12.91
9	0.02	0.17	0.81	434,100	5868	73.98	0.39	0.25	11.29
10	0.00	0.11	0.89	102,101	19,693	5.18	0.35	0.31	12.04
11	0.02	0.04	0.95	94,658	14,175	6.68	0.40	0.37	6.89
12	0.03	0.06	0.92	34,998	31,460	1.11	0.47	0.46	7.00
13	0.03	0.04	0.93	69,139	22,599	3.06	0.53	0.52	4.47
14	0.04	0.03	0.92	102,721	5182	19.82	0.50	0.41	5.28

TABLE E.II

FIRM CHARACTERISTICS BY TYPE IN PERIOD 3.

ℓ	No Info	Public	Private	Avg. Spell All ℓ-Firms	No. Firms	Avg. Size/Yr.	Avg. Inflow/Yr.	Avg. Outflow/Yr.	Avg. Age in Yr.
1	0.05	0.33	0.62	4662	3778	1.23	0.76	0.70	6.36
2	0.05	0.55	0.41	4464	666	6.70	0.71	0.56	11.88
3	0.06	0.28	0.66	5221	5168	1.01	0.87	0.83	5.74
4	0.05	0.12	0.83	11,679	4685	2.49	0.72	0.64	5.98
5	0.08	0.09	0.83	22,360	300	74.53	0.74	0.33	7.00
6	0.02	0.46	0.51	12,979	7392	1.76	0.64	0.55	12.24
7	0.07	0.10	0.83	11,761	9475	1.24	0.85	0.82	5.25
8	0.02	0.66	0.32	26,760	2994	8.94	0.55	0.43	15.31
9	0.05	0.06	0.89	49,036	6651	7.37	0.65	0.55	7.37
10	0.06	0.06	0.88	23,153	12,956	1.79	0.56	0.54	6.52
11	0.22	0.56	0.23	285,184	200	1425.92	0.39	0.02	16.48
12	0.00	0.12	0.88	41,711	14,645	2.85	0.35	0.31	15.07
13	0.11	0.07	0.82	71,019	15,276	4.65	0.42	0.38	7.99
14	0.01	0.22	0.78	237,283	6292	37.71	0.41	0.25	14.94
15	0.02	0.06	0.92	25,993	14,381	1.81	0.53	0.49	11.22
16	0.10	0.04	0.86	36,182	22,943	1.58	0.47	0.46	6.97
17	0.04	0.02	0.95	29,515	8721	3.38	0.72	0.62	7.02
18	0.07	0.02	0.91	120,541	11,822	10.20	0.40	0.32	9.20
19	0.03	0.05	0.92	94,202	11,143	8.45	0.52	0.44	9.77
20	0.06	0.03	0.90	15,804	14,494	1.09	0.79	0.77	5.71
21	0.10	0.03	0.87	220,533	1949	113.15	0.42	0.19	10.86
22	0.15	0.03	0.82	41,427	16,187	2.56	0.54	0.51	5.49

TABLE E.III

FIRM CHARACTERISTICS BY TYPE IN PERIOD 4.

ℓ	No Info	Public	Private	Avg. Spell All ℓ-Firms	No. Firms	Avg. Size/Yr.	Avg. Inflow/Yr.	Avg. Outflow/Yr.	Avg. Age in Yr.
1	0.16	0.38	0.46	3895	3075	1.27	0.79	0.71	8.64
2	0.18	0.23	0.60	6941	5571	1.25	0.84	0.80	6.85
3	0.16	0.49	0.35	7423	1157	6.42	0.73	0.60	14.34
4	0.22	0.21	0.57	10,819	6572	1.65	0.60	0.56	8.11
5	0.32	0.03	0.65	47,178	7471	6.31	0.66	0.59	7.46
6	0.19	0.10	0.71	13,240	9426	1.40	0.85	0.83	5.39
7	0.01	0.69	0.30	18,706	4593	4.07	0.52	0.45	19.52
8	0.35	0.08	0.58	33,863	12,731	2.66	0.55	0.53	6.58
9	0.06	0.12	0.82	8408	10,803	0.78	0.67	0.64	11.32
10	0.07	0.07	0.86	23,614	11,933	1.98	0.51	0.48	13.92
11	0.37	0.09	0.54	62,922	18,688	3.37	0.37	0.36	9.65
12	0.82	0.06	0.12	433,508	94	4611.78	0.45	0.01	10.24
13	0.00	0.43	0.57	174,620	1353	129.06	0.39	0.17	20.40
14	0.00	0.13	0.87	47,447	11,554	4.11	0.37	0.33	17.75
15	0.30	0.06	0.63	106,902	10,459	10.22	0.55	0.48	10.06
16	0.34	0.04	0.62	49,422	24,507	2.02	0.45	0.44	6.67
17	0.23	0.06	0.72	375,017	5034	74.50	0.44	0.29	13.67
18	0.17	0.01	0.81	32,934	7711	4.27	0.70	0.63	7.48
19	0.38	0.02	0.60	132,247	12,475	10.60	0.40	0.36	9.64
20	0.18	0.04	0.78	21,437	15,244	1.41	0.75	0.74	4.79
21	0.38	0.01	0.61	38,556	11,903	3.24	0.40	0.38	8.62
22	0.33	0.03	0.64	20,837	12,042	1.73	0.66	0.65	4.00

TABLE E.IV

FIRM CHARACTERISTICS BY TYPE IN PERIOD 5.

ℓ	No. Firms	No. Workers	Avg. Size/Yr.	Legal Status			Avg. Inflow/Yr.	Avg. Outflow/Yr.	Avg. Age
				Private	Public	Mixed			
1	5442	5655	1.04	0.60	0.07	0.33	0.79	0.77	7.71
2	2425	32,555	13.42	0.62	0.01	0.37	0.69	0.58	7.52
3	7091	20,681	2.92	0.70	0.01	0.29	0.66	0.61	7.14
4	4866	8058	1.66	0.44	0.36	0.20	0.60	0.53	17.23
5	9067	12,432	1.37	0.66	0.02	0.32	0.86	0.84	5.95
6	2554	18,154	7.11	0.36	0.62	0.02	0.52	0.44	22.79
7	10,150	70,578	6.95	0.58	0.01	0.41	0.54	0.48	9.59
8	13,781	25,247	1.83	0.71	0.02	0.27	0.52	0.51	7.42
9	126	347,968	2761.65	0.12	0.07	0.81	0.37	0.02	14.80
10	11,313	18,457	1.63	0.81	0.13	0.06	0.47	0.45	16.19
11	20,705	54,578	2.64	0.56	0.01	0.43	0.33	0.32	10.95
12	2200	213,090	96.86	0.48	0.03	0.49	0.45	0.26	12.35
13	6669	16,025	2.40	0.74	0.02	0.24	0.71	0.67	8.13
14	8519	32,806	3.85	0.79	0.20	0.01	0.35	0.31	21.73
15	22,301	43,666	1.96	0.72	0.00	0.27	0.46	0.45	7.00
16	6781	18,851	2.78	0.67	0.00	0.32	0.71	0.64	9.07
17	10,267	101,446	9.88	0.60	0.01	0.38	0.50	0.44	11.21
18	14,599	15,193	1.04	0.65	0.01	0.34	0.79	0.78	5.82
19	3408	113,882	33.42	0.80	0.20	0.00	0.36	0.23	23.65
20	11,098	83,863	7.56	0.55	0.01	0.44	0.36	0.32	10.78
21	15,429	20,233	1.31	0.63	0.00	0.37	0.52	0.52	7.31
22	5368	24,685	4.60	0.71	0.00	0.28	0.45	0.41	13.63

APPENDIX F: THE VALUE OF A MATCH

Denote by $V_{k\ell}(x)$ the net present value of a type (k, x) worker's future utility stream given a current match with a type ℓ firm. Unemployment is denoted by $V_{k0}(x)$. Let $y_{k\ell}(x)$ be the per period utility flow in such a match. When a worker is faced with a choice between a match $V_{k\ell}(x)$ and another $V_{k\ell'}(x)$ make one of two isomorphic assumptions: (1) A random utility model where each option in the binary match choice is associated with an iid Gumbel distributed taste shock with variance parameter ν_{kx}, or (2) a random mobility cost model where the worker realizes a random mobility cost from a logistic distribution with mean zero and variance parameter ν_{kx}. In either case, the probability that the worker makes the move to the ℓ' firm is $\exp(V_{k\ell'}(x)/\nu_{kx})/[\exp(V_{k\ell'}(x)/\nu_{kx}) + \exp(V_{k\ell}(x)/\nu_{kx})]$.

Assume the model's job offer and layoff events are mutually exclusive. Denote the discount factor by β. With this and following Rust (1981), the value of a match is given by the recursive formulation,

$$V_{k\ell}(x) = y_{k\ell}(x) + \beta E\Bigg[\nu_{kx'} \sum_{\ell'=1}^{L} \lambda_{k\ell'}(x') \ln\big(\exp\big(V_{k\ell}(x')/\nu_{kx'}\big) + \exp\big(V_{k\ell'}(x')/\nu_{kx'}\big)\big)$$
$$+ \nu_{kx'}\lambda_k(x')\vartheta + \delta_{k\ell}(x')V_{k0}(x') + \big(1 - \lambda_k(x') - \delta_{k\ell}(x')\big)V_{k\ell}(x')\Bigg],$$

where x' follows the law of motion in the model, and $\vartheta \approx 0.577$ is the Euler–Mascheroni constant. Also, $\lambda_k(x) = \sum_{\ell'} \lambda_{k\ell'}(x)$. With this interpretation, the estimated job preferences, $\gamma_{k\ell}(x)$, in the model are related to job values by $\gamma_{k\ell}(x) = \exp(V_{k\ell}(x)/\nu_{kx})$.

In this interpretation of the model, the structure implies that tenure categories may change across job comparisons if the worker is faced with a move from a long tenure job since the new job necessarily starts as short tenure. The estimation does not impose this constraint on the model, which may be interpreted as a kind of myopia on the part of workers. One can ex post evaluate the importance of the restriction: Using the estimated long-term mobility patterns, the constraint can be imposed using the short-tenure job preferences to obtain modified long-tenure job preferences consistent with the restriction that a new job is understood to be short tenure. We have done so and the modified long-tenure preferences are very closely correlated (> 0.99) with the estimated ones (note available upon request).

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