Econometrica Supplementary Material

SUPPLEMENT TO "MARKET COMPETITION AND POLITICAL INFLUENCE: AN INTEGRATED APPROACH"

(*Econometrica*, Vol. 90, No. 6, November 2022, 2723–2753)

STEVEN CALLANDER
Graduate School of Business, Stanford University

DANA FOARTA
Graduate School of Business, Stanford University

TAKUO SUGAYA
Graduate School of Business, Stanford University

APPENDIX A: PROOFS FROM SECTION 3

A.1. *Preliminaries*

LET $l^{\max}$ be the smallest l such that there does not exist $f \in [0, l]$ for which

$$\hat{\pi}^M(l+1) - c(l) \geq \pi^L(l, f). \tag{A.1}$$

The myopic leader never invests if $l \geq l^{\max}$. Hence, we will focus on $(l, f) \in \mathcal{L}^* := \{(l', f') \in \mathbb{Z}_+^2 : l^{\max} \geq l' \geq f'\}$. We write $l \in \mathcal{L}_1^* := \{l' \in \mathbb{Z}_+ : \exists f \geq 0 \text{ such that } l^{\max} \geq l' \geq f\} = \{l' \in \mathbb{Z}_+ : l' \leq l^{\max}\}$, and $f \in \mathcal{L}_2^* := \{f' \in \mathbb{Z}_+ : \exists l \text{ such that } l^{\max} \geq l \geq f'\} = \{f' \in \mathbb{Z}_+ : f \leq l^{\max}\}$. As mentioned in the main text, we define $\pi(l, f)$ for noninteger values. Thus, it is useful to also define $\bar{\mathcal{L}}^* := \{(l, f) \in \mathbb{R}_+^2 : l^{\max} \geq l \geq f\}$ by replacing $\mathbb{Z}$ with $\mathbb{R}$. $\bar{\mathcal{L}}_1^*$ and $\bar{\mathcal{L}}_2^*$ are similarly defined.

As stated in footnote 16 in the main text, we define $\pi^P(l, l) = \rho(\hat{\pi}^M(l) - \pi^L(l, l))$. Although protection is not feasible at (l, l), defining π^P for (l, l) eases the notation.

A.2. *Proof of Lemma 1*

For each $l \in \mathcal{L}_1^*$, we define $IC_D(l)$ as follows: (i) If

$$\pi^L(l+1, f) - c(l) \geq \pi^L(l, f) \tag{A.2}$$

for all $f \in [0, l]$ (i.e., investment is always profitable), then we define $IC_D(l) = l^{\max}$. (ii) If there is no $f \in [0, l]$ such that (A.2) holds (i.e., investment is always unprofitable), define $IC_D(l) = -1$. (iii) Otherwise, let $IC_D(l)$ be the largest $f \in [0, l]$ such that (A.2) holds.

Then Assumption 1 and inequality (A.2) imply that the leader invests if $f < IC_D(l)$ and only if $f \leq IC_D(l)$. In addition, for each $(l, f) \in \mathcal{L}^*$, $\pi^L(l+2, f) - c(l+1) \geq \pi^L(l+1, f)$ implies $\pi^L(l+1, f) - c(l) \geq \pi^L(l, f)$ given increasing $c(l)$ and condition (2) of Assumption 1. Thus, for each $l \in \mathcal{L}_1^*$,

$$IC_D(l+1) - IC_D(l) \leq 0. \tag{A.3}$$

Steven Callander: sjc@stanford.edu
Dana Foarta: ofoarta@stanford.edu
Takuo Sugaya: tsugaya@stanford.edu

 https://doi.org/10.3982/ECTA19775

A.3. *Proof of Lemma 2*

Given $\pi^P(l,f)=\rho(\hat{\pi}^M(l)-\pi^L(l,f))$, and hence $\pi^M(l,f)=(1-\rho)\hat{\pi}^M(l)+\rho\pi^L(l,f)$, Assumptions 1 and 2 imply that, for each $l\geq f\geq 0$, we have

$$\frac{\partial^2}{\partial l\partial f}\pi^M(l,f)\leq 0 \quad \text{and} \quad \frac{\partial^2}{\partial l^2}\pi^M(l,f)\leq 0. \tag{A.4}$$

For each $l\in\mathcal{L}_1^*$, we define $IC_M(l)$ as follows: (i) If

$$\pi^M(l+1,f)-c(l)\geq\pi^M(l,f). \tag{A.5}$$

for all $f\in[0,l]$ (i.e., investment is always profitable), then we define $IC_M(l)=l^{\max}$. If there is no $f\in[0,l]$ such that (A.5) holds (i.e., investment is always unprofitable), define $IC_M(l)=-1$. (iii) Otherwise, let $IC_M(l)$ be the largest $f\in[0,l]$ such that (A.5) holds.

Conditions (A.4) and (A.5) imply that the leader invests if $f<IC_M(l)$ and only if $f\leq IC_M(l)$. In addition, for each $(l,f)\in\mathcal{L}^*$, $\pi^M(l+2,f)-c(l+1)\geq\pi^M(l+1,f)$ implies $\pi^M(l+1,f)-c(l)\geq\pi^M(l,f)$ given increasing $c(l)$ and (A.4). Thus, for each $l\in\mathcal{L}_1^*$,

$$IC_M(l+1)-IC_M(l)\leq 0. \tag{A.6}$$

APPENDIX B: PROOF FROM SECTION 4

B.1. *Proof of Lemma 3*

Note that conditions (10) and (11) are equivalent to

$$(1-\delta)\frac{\partial}{\partial l}\pi^P(l,f)-\delta\min_{s\in[0,1]}\left(\frac{\partial^2}{\partial l\partial f}\pi^P(l,f+s)\right)\leq 0. \tag{B.1}$$

$$\begin{aligned}&(1-\delta)\left[\frac{\partial}{\partial l}\pi^P(l,f)+\frac{\partial}{\partial f}\pi^P(l,f)\right]\\&\quad-\delta\max_{s\in[0,1]}\left[\frac{\partial^2}{\partial l\partial f}\pi^P(l,f+s)+\frac{\partial^2}{\partial f^2}\pi^P(l,f+s)\right]\geq 0.\end{aligned} \tag{B.2}$$

First, we derive the following two inequalities: For each $(l+t,f)\in\bar{\mathcal{L}}^*$ with $l+t>f+1$, we have

$$\frac{d}{dt}\big(\pi^P(l+t,f)-\delta\pi^P(l+t,f+1)\big)\leq 0, \tag{B.3}$$

and, for each $(l+t,f+t)\in\bar{\mathcal{L}}^*$ with $l+t>f+1+t$, we have

$$\frac{d}{dt}\big(\pi^P(l+t,f+t)-\delta\pi^P(l+t,f+1+t)\big)\geq 0. \tag{B.4}$$

Note that the derivative is well-defined. Then (B.3) can be written as

$$(1-\delta)\frac{\partial}{\partial l}\pi^P(l+t,f)-\delta\left(\frac{\partial}{\partial l}\pi^P(l+t,f+1)-\frac{\partial}{\partial l}\pi^P(l+t,f)\right)\leq 0.$$

By the intermediate value theorem,

$$\frac{\partial}{\partial l}\pi^P(l+t, f+1) - \frac{\partial}{\partial l}\pi^P(l+t, f) \geq \min_{s\in[0,1]}\left(\frac{\partial^2}{\partial l \partial f}\pi^P(l+t, f+s)\right).$$

Hence, (B.1) implies the result; similarly, (B.4) can be written as

$$\begin{aligned}
&(1-\delta)\left(\frac{\partial}{\partial l}\pi^P(l+t, f+t) + \frac{\partial}{\partial f}\pi^P(l+t, f+t)\right) \\
&\quad + \delta\left(\frac{\partial}{\partial l}\pi^P(l+t, f+t) + \frac{\partial}{\partial f}\pi^P(l+t, f+t) - \frac{\partial}{\partial l}\pi^P(l+t, f+1+t)\right. \\
&\quad \left. - \frac{\partial}{\partial f}\pi^P(l+t, f+1+t)\right) \geq 0. \qquad \text{(B.5)}
\end{aligned}$$

By the intermediate value theorem,

$$\begin{aligned}
&\frac{\partial}{\partial l}\pi^P(l+t, f+t) + \frac{\partial}{\partial f}\pi^P(l+t, f+t) \\
&\qquad - \frac{\partial}{\partial l}\pi^P(l+t, f+1+t) - \frac{\partial}{\partial f}\pi^P(l+t, f+1+t) \\
&\quad \geq \min_{s\in[0,1]} -\left(\frac{\partial^2}{\partial l \partial f}\pi^P(l, f+s) + \frac{\partial^2}{\partial f^2}\pi^P(l, f+s)\right) \\
&\quad = -\max_{s\in[0,1]}\left(\frac{\partial^2}{\partial l \partial f}\pi^P(l, f+s) + \frac{\partial^2}{\partial f^2}\pi^P(l, f+s)\right). \qquad \text{(B.6)}
\end{aligned}$$

Thus, (B.2) implies the result.

Second, we prove that, for each $l \geq \underline{l}$,[1] both $IC_P(l)$ and $IC_P(l+1)$ are well-defined. To see why, it suffices to show that (i) $\pi^P(l, l-1) \geq \delta\pi^P(l, l)$ and (ii) if there exists $f \in [0, l-1]$ satisfying $\pi^P(l, f) = \delta\pi^P(l, f+1)$, then such f is unique.

To show (i), for each $\hat{l} \geq l$, we calculate (B.4) given $(\underline{l}+t, \underline{l}+t-1)$ for each t and then integrate it over $t \in [0, \hat{l} - \underline{l}]$ to yield

$$\begin{aligned}
0 &\leq \pi^P(\hat{l}, \hat{l}-1) - \delta\pi^P(\hat{l}, \hat{l}) - \left(\pi^P(\underline{l}, \underline{l}-1) - \delta\pi^P(\underline{l}, \underline{l})\right) \\
&= \pi^P(\hat{l}, \hat{l}-1) - \delta\pi^P(\hat{l}, \hat{l}).
\end{aligned}$$

For (ii), it suffices to show that $\pi^P(l, f) - \delta\pi^P(l, f+1)$ is increasing in f, which follows from

$$\begin{aligned}
&\frac{\partial}{\partial f}\left(\pi^P(l, f) - \delta\pi^P(l, f+1)\right) \\
&\quad = \frac{\partial}{\partial f}\left((1-\delta)\pi^P(l, f) - \delta\pi^P(l, f+1) - \pi^P(l, f)\right) \\
&\quad = (1-\delta)\frac{\partial}{\partial f}\pi^P(l, f) - \delta\frac{\partial}{\partial f}\left(\pi^P(l, f+1) - \pi^P(l, f)\right), \qquad \text{(B.7)}
\end{aligned}$$

which is nonnegative given Assumption 1.

[1] Where $\underline{l}$ is defined as the smallest l such that $\pi^P(l, l-1) \geq \delta\pi^P(l, l)$ with $\pi^P(l, l) = \rho(\hat{\pi}^M(l) - \pi^L(l, l)$.

Third, we prove $IC_P(l+1) - IC_P(l) \geq 0$. Integrating (B.3) over $t \in [0, 1]$ implies that $(\pi^P(l+1, f) - \delta\pi^P(l+1, f+1)) - (\pi^P(l, f) - \delta\pi^P(l, f+1)) \leq 0$, and hence

$$\pi^P(l+1, f) - \delta\pi^P(l+1, f+1) \geq 0 \quad \Rightarrow \quad \pi^P(l, f) - \delta\pi^P(l, f+1) \geq 0. \tag{B.8}$$

Thus, if $(l+1, f)$ is above the IC_P curve, then (l, f) is also above the IC_P curve (and hence the slope of $IC_P(l)$ is no less than zero).

Finally, we prove $IC_P(l+1) - IC_P(l) \leq 1$. Integrating (B.4) over $t \in [0, 1]$ implies that $(\pi^P(l+1, f+1) - \delta\pi^P(l+1, f+2)) - (\pi^P(l, f) - \delta\pi^P(l, f+1)) \geq 0$, and hence

$$\pi^P(l, f) \geq \delta\pi^P(l, f+1) \quad \Rightarrow \quad \pi^P(l+1, f+1) \geq \delta\pi^P(l+1, f+2). \tag{B.9}$$

Thus, if (l, f) is above the IC_P curve, then $(l+1, f+1)$ is also above the IC_P curve (and hence the slope of $IC_P(l)$ is no more than one).

B.2. *Proof of Proposition 1*

Below, we summarize the argument for the proof, using auxiliary results that we prove in auxiliary lemmas provided in the Supplementary Appendix, Section 2.

To simplify notation, we henceforth call the technology level profile at the beginning of the period (i.e., at the timing of the leader's decision) "*the ex ante state*," and we call the technology level profile after the leader's investment (i.e., at the timing of the policy-maker's decision) "*the interim state*."

Policymaker Threshold. For each l, we define $IC_P(l)$ as the smallest $f \in \mathcal{L}_2^*$ such that $f \leq l$ and

$$\pi^P(l, f) \geq \delta\pi^P(l, f+1). \tag{B.10}$$

If such f does not exist, then define $IC_P(l) = -1$. We derive the two results about $IC_P(l)$. First, by auxiliary Lemma 2.1, $IC_P(l)$ is a proper threshold (the above inequality is satisfied if and only if $f \geq IC_P(l)$) and by Lemma 3 has a slope less than one.

Second, by auxiliary Lemma 2.2, if the current state (l, f) satisfies $l \geq f+1$ and $f \geq IC_P(l)$, then the policymaker prefers to stay in the current state: that is, for each t and each $(l', f') \in \mathcal{L}^*$ such that there exists a feasible path from (l, f) to (l', f') that can be completed in t periods, we have

$$\pi^P(l, f) > 1_{\{l' > f'\}}\delta^t\pi^P(l', f'). \tag{B.11}$$

Denote by $V(l, f)$ the policymaker's value function given the ex ante state (l, f). In particular, since no protection is feasible if $f = l$, (B.11) implies that, for each Markov perfect equilibrium and each $(l, f) \in \mathcal{L}^*$ with $l \geq f+1$ and $f \geq IC_P(l)$,

$$\frac{\pi^P(l, f)}{1-\delta} \geq \max\{V(l+1, f), \delta V(l, f+1)\}. \tag{B.12}$$

Regions. The ex ante state $(l, f) \in \mathcal{L}^*$ may fall into one of the three regions:

1. Region 1: $f \leq IC_P(l)$ and $f \geq IC_D(l)$. In this region, as will be seen, the leader does not invest (NI) and the policymaker does not protect (NP), except near the 45-degree line.
2. Region 2: $f \geq IC_P(l)$ and $f \geq IC_M(l)$. In this region, as will be seen, the leader does not invest (NI) and the policymaker protects (P) whenever $l > f$.
3. Region 3: $f \leq IC_P(l)$ and $f \leq IC_D(l)$ or $f \geq IC_P(l)$ and $f \leq IC_M(l)$.

Investment Threshold. Given Lemma 3, the IC_P curve intersects the 45-degree line at most once. Thus, depending on the location of the IC_P and IC_M curves, the following two cases are possible:

Case 1. The IC_P curve intersects with the l-axis at $(\hat{l}, 0)$ with $\hat{l} \geq 1$:

(a) IC_P and IC_M intersect in $\mathcal{L}^*$. In this case, let l_0 be the smallest l such that there exists $f \leq l-1$ for which (l, f) is in Region 2 and $(l, f-1)$ is below the IC_P curve: $f \geq IC_P(l)$, $f \geq IC_M(l)$, and $f-1 \leq IC_P(l)$. Then take f_0 satisfying $l_0 \geq f_0+1$, $f_0 \geq IC_P(l_0)$, $f_0 \geq IC_M(l_0)$, and $f_0-1 \leq IC_P(l_0)$. By definition, (l_0, f_0-1) is *below* the IC_P curve and (l_0, f_0) is *above* the IC_P curve.

(b) IC_P and IC_M do not intersect in $\mathcal{L}^*$. Since the slope of the IC_P curve is less than one by Lemma 3, it means that $IC_M^{-1}(0) \leq \hat{l}$. Therefore, $(\lceil IC_M^{-1}(0) \rceil, 0)$ is in Region 2.

Case 2. The IC_P curve intersects with the l-axis at $(\hat{l}, 0)$ with $\hat{l} < 1$:

(a) IC_P and IC_M intersect in $\mathcal{L}^*$. In this case, let l_0 be the smallest l such that there exists $f \leq l-1$ so that (l, f) is in Region 2 and $(l, f-1)$ is below the IC_P curve: $f \geq IC_P(l)$, $f \geq IC_M(l)$, and $f-1 \leq IC_P(l)$. Then take f_0 satisfying $l_0 \geq f_0+1$, $f_0 \geq IC_P(l_0)$, $f_0 \geq IC_M(l_0)$, and $f_0-1 \leq IC_P(l_0)$. By definition, (l_0, f_0-1) is *below* the IC_P curve and (l_0, f_0) is *above* the IC_P curve.

(b) IC_P and IC_M do not intersect in $\mathcal{L}^*$. In this case, let f_0 be the smallest f_0 such that $f_0 \geq IC_P(f_0+1)$. Define $l_0 = f_0+1$. Again, (l_0, f_0) is *above* the IC_P curve.

Given (l_0, f_0), for each f, define $L^*(f) = l_0 - (f_0 - f)$. In addition, let $\underline{f} \in \mathbb{Z}_+$ be the smallest $f \geq 0$ such that $(f+1, f)$ is above the IC_P curve. In Case 1, $\underline{f} = 0$, and in Case 2, $\underline{f} \geq 1$. In Proposition 1, we focus on Case 1. In Case 1(a), we define $(l^I, f^I) = (l_0, f_0)$. In the Supplementary Appendix, we also cover Case 2. All the lemmas without reference to a specific case hold for all cases.

Equilibrium Uniqueness. The following lemma establishes equilibrium uniqueness given the form of renegotiation proofness described in the main text.

LEMMA B.1: *The set of subgame perfect equilibrium (SPE) payoffs that satisfy renegotiation proofness exists and is a singelton at each ex ante state $(l, f) \in \mathcal{L}^*$ and also at each interim state $(l, f) \in \mathcal{L}^*$. In this renegotiation-proof subgame perfect equilibrium, the strategy is Markov: the leader's investment decision depends only on the ex ante state, and the policymaker's protection decision depends only on the interim state. Moreover, for each $(l, f) \in \mathcal{L}^*$, suppose (NI, P) is incentive compatible;*[2] *then, the unique on-path outcome in the subgame starting with the ex ante state (l, f) is to repeat (NI, P) forever.*

For $l \geq l^{\max}$, since the policymaker is a single decision maker, the result holds. By backward induction, we can also show that, except for (NI, P), the dynamic-game payoff profile of taking a certain action profile is determined, since the state transits to another state with higher l or f. For the action profile (NI, P), the next state will be the same as the current one, and so the payoff profile depends on the expectation of the continuation play. We apply Pareto criteria to select the equilibrium.

[2]More precisely, given the unique renegotiation-proof subgame perfect equilibrium payoffs $V(l+1, f)$ at ex ante state $(l+1, f)$ and $V(l+1, f+1)$ at ex ante state $(l+1, f+1)$, the policymaker protects the leader at the interim state $(l+1, f)$ if and only if $\pi^P(l+1, f) + \delta V(l+1, f) \geq \delta V(l+1, f+1)$. Given this strategy of the policymaker at interim state $(l+1, f)$, suppose that the leader weakly prefers NI if NI leads to protection. In addition, suppose that the policymaker's payoff satisfies $\pi^P(l, f)/(1-\delta) \geq \delta V(l+1, f)$.

Given this result, in what follows, we refer to "equilibrium" as the unique renegotiation proof SPE. Let $\text{eqm}(l, f) \in \{I, NI\} \times \{P, NP\}$ be the equilibrium outcome given the ex ante state $(l, f) \in \mathcal{L}^*$.

Equilibrium Characterization. For simplicity, we assume that there is no $(l, f) \in \mathcal{L}^*$ such that $\pi^P(l, f) = \delta\pi^P(l, f + 1)$, $\pi^L(l + 1, f) - \pi^L(l, f) = c(l)$, or $\pi^M(l + 1, f) - \pi^M(l, f) = c(l)$.[3]

First, in Region 2, $\text{eqm}(l, f) = (NI, P)$ (auxiliary Lemma 2.7). This follows from (B.12) and renegotiation proofness (note that the myopic leader always prefers P).

Next, in Region 1, $\text{eqm}(l, f) = (NI, NP)$, except near the 45-degree line (auxiliary Lemma 2.8). Given that $f \geq IC_D(l)$, the leader does not invest if protection is not offered. Combined with the inductive argument, this implies that the policymaker does not protect the leader, without worrying about the effect of her current action on the leader's future investment decision.

Lastly, we analyze Region 3. In this region, we first show that, if the policymaker does not protect the leader at the interim state (l, f), then, for $(l-1, f)$, either $\text{eqm}(l-1, f) = (NI, P)$ or the policymaker does not protect the leader at the interim state $(l - 1, f)$ (auxiliary Lemma 2.9). The result follows from Assumption 3. This implies that, at ex ante state (l, f), if $f \geq IC_P(l)$ and the policymaker does not protect the leader at the interim state $(l+1, f)$, then the equilibrium outcome at (l, f) is (NI, P) (auxiliary Lemma 2.10).

Next, we show that, once the leader invests at the ex ante state (l, f) and the policymaker protects the leader at the interim state $(l + 1, f)$, then protection will always be offered in the on-path continuation play. Specfically, for each $(l, f) \in \mathcal{L}^*$, suppose either the policymaker protects the leader at the interim state $(l + 1, f)$ and $f \leq IC_M(l)$, or $\text{eqm}(l, f) = (I, P)$. Then, auxiliary Lemma 2.3 shows that there exists $l' \geq l + 1$ such that $\text{eqm}(\tilde{l}, f) = (I, P)$ for all $l \leq \tilde{l} \leq l' - 1$ and $\text{eqm}(l', f) = (NI, P)$. The result holds because Assumption 3 implies that the leader stops investing as soon as protection is not expected after investment. This in particular implies that, if the policymaker protects the leader at the interim state (l, f), then her payoff is bounded by $\frac{\pi^P(l,f)}{1-\delta}$ (auxiliary Lemma 2.11) since Assumption 4 implies that higher technology level for the leader reduces the policymaker's payoff.

We say *Condition (*) holds* for (l, f) if (i) $\text{eqm}(l, f) = (NI, P)$ and (ii) $(l, f - 1)$ is below IC_P curve. By definition, (l_0, f_0) satisfies Condition (*) in Cases 1(a) and 2(a).[4] Define $L(f_0) = l_0$. The following lemma establishes the key inductive argument.

LEMMA B.2: *In Cases* 1(*a*) *and* 2(*a*), *for each* $f \in \{f_0 - 1, f_0 - 2, \dots, \underline{f}\}$, *there exists* $L(f) \leq L(f + 1) - 1$ *such that Condition* (*) *holds for* $(L(f), f)$.

PROOF: Since Condition (*) holds for (l_0, f_0), it suffices to prove that, for each $f \in \{\underline{f} + 1, \dots, f_0\}$, if there exists $L(f)$ such that Condition (*) holds for $(L(f), f)$, then there exists $L(f - 1) \leq L(f) - 1$ such that Condition (*) holds for $(L(f - 1), f - 1)$.

First, note that no protection is offered at the interim state $(L(f), f - 1)$. Suppose otherwise: protection is offered at the interim state $(L(f), f - 1)$. Then auxiliary Lemma 2.11

[3]Without this assumption, all the proofs go through with more tedious tie-breaking analysis based on renegotiation proofness.

[4]Although Proposition 1 considers Case 1 only, we cover Case 2 in the Supplementary Appendix, so it is useful to include Case 2(a) here.

implies $V(L(f), f-1) \leq \frac{\pi^P(L(f),f-1)}{1-\delta}$. Since $(L(f), f-1)$ is below IC_P and $V(L(f), f) = \frac{\pi(L(f),f)}{1-\delta}$, protection is suboptimal.

Second, there exists $L(f-1) \leq L(f)-1$ such that $\text{eqm}(L(f-1), f-1) = (NI, P)$ and $(L(f-1), f-2)$ is below IC_P, that is, (i) and (ii) of Condition (*) hold for $(L(f-1), f-1)$.

To see why, let $\tilde{L}(f-1)$ be the smallest $l \geq f$ such that $(l, f-1)$ is below the IC_P curve. We make the following three observations: (a) such l exists since we have assumed that $f-1 \geq \underline{f}$; (b) we have $\tilde{L}(f-1) \leq L(f)$ since $(L(f), f-1)$ is below IC_P curve by (ii) of Condition (*) for f (inductive hypothesis); (c) since $(\tilde{L}(f-1), f-1)$ is below the IC_P curve and Lemma 3 implies that the slope of the IC_P curve is less than one, $(\tilde{L}(f-1)-1, f-2)$ is below the IC_P curve. Therefore, it remains to show that there exists $\tilde{l}$ such that $L(f)-1 \geq \tilde{l} \geq \tilde{L}(f-1)-1$ and the equilibrium outcome at $(\tilde{l}, f-1)$ is (NI, P) (once we show this, we can take $L(f-1)$ equal to such $\tilde{l}$).

Consider the following three cases:

(1) If $\tilde{L}(f-1) = L(f)$, since no protection is offered at interim state $(L(f), f-1)$, Auxiliary Lemma 2.10 implies $\text{eqm}(\tilde{L}(f-1)-1, f-1) = (NI, P)$. To see this, note that $\tilde{L}$ is defined as the smallest l such that $(l-1, f)$ is below the IC_P curve; hence, $(\tilde{L}(f-1)-1, f-1)$ is above IC_P, and $\tilde{l} = \tilde{L}(f-1)-1$ satisfies the claim.

(2) If $\tilde{L}(f-1) \leq L(f)-1$ and $\text{eqm}(l, f-1) = (NI, P)$ for some $l = \tilde{L}(f-1)-1, \ldots, L(f)-1$, then we can take $\tilde{l}$ equal to such l.

(3) If $\tilde{L}(f-1) \leq L(f)-1$ and $\text{eqm}(l, f-1) \neq (NI, P)$ for each $l = \tilde{L}(f-1)-1, \ldots, L(f)-1$, then by Auxiliary Lemma 2.9, no protection is offered at the interim state $(\tilde{L}(f-1), f-1)$. Since $(\tilde{L}(f-1)-1, f-1)$ is above the IC_P curve, auxiliary Lemma 2.10 implies that $\text{eqm}(\tilde{L}(f-1)-1, f-1) = (NI, P)$, which is a contradiction. *Q.E.D.*

In Cases 1(a) and 2(a), for each $f \geq \underline{f}$, let $L^{**}(f)$ be the smallest l with $\text{eqm}(l, f) = (NI, P)$. For $f \in \{\underline{f}, \ldots, f_0\}$, such l exists and $L^{**}(f) \leq L(f)$ by Lemma B.2. For $f \geq f_0$, since (l_0, f_0) is in Region 2, for each $f \geq f_0$, there exists l such that (l, f) is in Region 2, and hence $\text{eqm}(l, f) = (NI, P)$. Thus, for each $f \geq \underline{f}$, the cutoff $L^{**}(f)$ is well-defined.

Finally, auxiliary Lemma 2.12 shows that, for each $f \geq \underline{f}$, given the smallest $l \geq f+1$ with $\text{eqm}(l, f) = (NI, P)$, protection is offered at interim state (l', f) with $f+1 \leq l' \leq l-1$. To see why, if it were not the case, then auxiliary Lemma 2.10 implies that, as soon as l' becomes sufficiently small so that $(l'-1, f)$ is below IC_P curve, we have $\text{eqm}(l'-1, f) = (NI, P)$; but this is a contradiction to l being the smallest technology level with $\text{eqm}(l, f) = (NI, P)$ (the only complication is when $l' = f+1$, and hence protection is not feasible at $(l'-1, f)$.)

Note that Proposition 1 considers Case 1. If we have Case 1(a), since $\underline{f} = 0$, then $L^{**}(0)$ is the smallest $l \geq 1$ with $\text{eqm}(l, 0) = (NI, P)$. By auxiliary Lemma 2.12, at the ex ante state $(0, 0)$, we have either $\text{eqm}(0, 0) = (NI, NP)$ (and $(0, 0)$ is the steady state) or $\text{eqm}(0, 0) = (I, P)$. By auxiliary Lemma 2.3, the latter implies that the equilibrium path is $(0, 0) \to (1, 0) \to \cdots \to (L^{**}(0), 0)$. Together with $L^{**}(f) \leq L(f) \leq L^*(f) \leq l_0 - f_0 = l^I - f^I$, Proposition 1 holds.

Consider next Case 1(b). By auxiliary Lemma 2.7, $\text{eqm}(\lceil IC_M^{-1}(0) \rceil, 0) = (NI, P)$. By auxiliary Lemma 2.3, there exists $L^{**}(f) \leq \lceil IC_M^{-1}(0) \rceil$ such that the equilibrium path is $(0, 0) \to (1, 0) \to \cdots \to (L^{**}(0), 0)$. Thus, Proposition 1 holds.

B.3. *Proof of Proposition 2*

Thresholds and Conditions. Let $\mathcal{IC}_{EA}$ be the set of $(l, f) \in \mathcal{L}^*$ such that $\pi^M(l+1, f) - c(l) - \pi^L(l, f) \geq 0$. Note that the set $\mathcal{IC}_{EA}$ does not necessarily have a cutoff structure. That is, even if (l, f) is in $\mathcal{IC}_{EA}$, $(l, f-1)$ may not be in $\mathcal{IC}_{EA}$.[5]

Let IC_{EA} be the largest technology level $l \in \mathcal{L}^*$ such that $\pi^M(l'+1, l') - \pi^L(l', l') - c(l') \geq 0$ for all $l' \leq l$. Note that the case considered in Proposition 1, where $\widehat{l} \geq 1$ implies

$$IC_P(l) \leq l - 1, \tag{B.13}$$

for all $1 \leq l \leq l^{\max}$. In addition, to avoid a tedious tie-breaking, we assume that, for each $(l, f) \in \mathcal{L}^*$ and $y, y' \in \{L, M\}$, we have

$$\pi^y(l+1, f) - \pi^{y'}(l, f) \neq c(l). \tag{B.14}$$

Next, as in (B.11), for each $(l, f) \in \mathcal{L}^*$ satisfying $l \geq f+1$ and $f \geq IC_P(l)$, t, and $(l', f') \in \mathcal{L}^*$ such that there exists a feasible path from (l, f) to (l', f') that can be completed in t periods:

$$\pi^P(l, f) > 1_{\{l' > f'\}} \delta^t \pi^P(l', f'). \tag{B.15}$$

Finally, Lemma 1 shows that Assumptions 1 and 4 imply that for each $l \in \mathcal{L}_1^*$,

$$IC_D(l+1) - IC_D(l) \leq 0 \quad \text{and} \quad IC_D(l) \geq IC_M(l). \tag{B.16}$$

Equilibrium Steady-State Characterization. Let (l, f, k) be the tuple of payoff-relevant states, where $k \in \{0, \dots, \kappa\}$ indicates how many consecutive periods the leader has been protected. Having $k = \kappa$ means that the follower disappeared. The ex ante state (l, f, k) represents the state at the beginning of the period, while the interim state (l, f, k) represents the state after the leader's investment decision.

We say that the subgame perfect equilibrium is *weakly renegotiation-proof* if the policymaker breaks her indifferent between two actions by taking the action that gives the higher continuation payoff for the leader. We use weak renegotiation proof subgame perfect equilibrium as our equilibrium concept. In equilibrium, we show that the steady state technology level is no less than IC_{EA} (Proposition 2). To prove this result, we first provide a counterpart of Lemma B.1.

LEMMA B.3: *The weak-renegotiation-proof subgame perfect equilibrium exists and is unique and Markov perfect*.

The formal proofs to this lemma and all auxiliary lemmas in this section are provided in the Supplementary Appendix Section 3.

In Lemma B.1, given the ex ante state (l, f), if the leader does not invest and the policymaker protects the leader, the next ex ante state is again (l, f). Here, given the ex ante state (l, f, k), if the leader does not invest and the policymaker protects the leader, the next ex ante state is $(l, f, k+1)$. Thus, the state is always "moving up" unless the follower

[5] To see why, rewrite $\pi^M(l+1, f) - c(l) - \pi^L(l, f) \geq 0$ as $\pi^M(l+1, f) - \pi^L(l+1, f) + \pi^L(l+1, f) - c(l) - \pi^L(l, f) \geq 0$, or equivalently, $(1-\rho)[\hat{\pi}^M(l+1) - \pi^L(l+1, f)] + \pi^L(l+1, f) - \pi^L(l, f) - c(l) \geq 0$. The last three terms represent the benefit of investment in duopoly, which is decreasing in f. By contrast, the first term is proportional to the benefit of protection, which is increasing in f. Thus, it is not clear if the incentive to invest is higher or lower with higher f.

has disappeared or the competition is head to head ($l = f$), $k = 0$, and the leader does not invest. Since the policymaker has no choice in these exceptional cases, simple subgame perfection and backward induction implies uniqueness, except for tie-breaking. Given this lemma, we write the policymaker's value function at ex ante state (l, f, k) as $V(l, f, k)$.

We next pin down the state transition for (l, f) with $f \geq IC_P(l)$ and $f > IC_D(l)$. In this case, the leader does not invest unless investment leads to protection and no investment leads to no protection. Thus, the policymaker protects the leader if $l > f$ and $k < \kappa - 1$, and the leader does not invest unless the current state profile is on the 45-degree line. If the current state profile is on the 45-degree line, then $k = 0$. This is because the follower must be in the market in order to catch up to the leader for the state profile to reach the 45-degree line; once the state profile stays on the 45-degree line, protection is no longer feasible. Thus, no investment leads to no protection by feasibility, while investment leads to protection. Therefore, the firm with an investment opportunity invests if and only if $(l, f) \in \mathcal{IC}_{EA}$.

LEMMA B.4: *For each (l, f, k) with $(l, f) \in \mathcal{L}^*$ and $k \in \{0, \dots, \kappa\}$, the leader's equilibrium strategy satisfies the following*:

1. *If $l > f$ at the ex ante state (l, f, k), the leader does not invest if $f \geq IC_P(l)$ and $f \geq IC_D(l)$.*
2. *If $l = f$ at the ex ante state (l, f, k), then the firm with an investment opportunity invests if and only if $(l, f) \in \mathcal{IC}_{EA}$.*

The policymaker's equilibrium strategy satisfies the following:

3. *If $k = \kappa - 1$, then the policymaker does not protect the leader at the interim state (l, f, k) if and only if either $l - 1 > f$ or $(l, l) \in \mathcal{IC}_{EA}$.*
4. *If $f \geq IC_P(l)$, $f \geq IC_D(l)$, and $k < \kappa - 1$,*
   (a) *If $f = l$, the policymaker protects the leader at interim state $(l + 1, f, k)$.*
   (b) *If $f < l$, the policymaker protects the leader at interim state (l, f, k).*
5. *If $f \geq IC_P(l)$, then the value $V(l, f, k)$ at the ex ante state (l, f, k) is decreasing in k and $V(l, f, k) \leq \frac{\pi^P(l+1,l)}{1-\delta}$ if $f = l$ and $V(l, f, k) \leq \frac{\pi^P(l,f)}{1-\delta}$ if $f \leq l - 1$.*

Given Lemma B.4, to show that the leader's technology level is no less than IC_{EA} in the long run, it suffices to show that the equilibrium path reaches a state (l, f, k) with $f \geq IC_P(l)$, $f \geq IC_D(l)$, and $k \leq \kappa - 1$.

First, in Auxiliary Lemma 3.1, we consider an ex ante state $(l, f, \kappa - 1)$ and leader's investment decision $\iota \in \{0, 1\}$ at $(l, f, \kappa - 1)$ and show that, if the leader invests at the ex ante state $(l + \iota, f + 1, 0)$ or $l + \iota > f + 1$, then not protecting is optimal at the interim state $(l + \iota, f, \kappa - 1)$ given equation (12) in the main text. This is because protection is feasible in the continuation play after the policymaker does not protect the leader at the interim state $(l + \iota, f, \kappa - 1)$.

Second, Auxiliary Lemma 3.2 shows that the equilibrium path reaches a state (l, f, k) either with $l \geq IC_{EA}$ or with $f \geq IC_P(l)$, $f \geq IC_D(l)$, and $k \leq \kappa - 1$. The result is obtained by noting that the steady-state (l, f, k) satisfies either $l = f$ or $k = \kappa$ since otherwise either f increases without protection or k increases with protection. If the steady state is $(l, l, 0)$,[6] consider the last interim state $(l, l - 1, k)$ before reaching (l, l, k). At that interim state, the policymaker would be better off by protecting the leader, as her payoff would be zero once the ex ante state reaches $(l, l, 0)$. This is a contradiction.

[6]See above why $k = 0$ on the 45-degree line.

If the steady state is (l, f, κ), then consider the last interim state $(l, f, \kappa - 1)$ before reaching (l, f, κ). By Auxiliary Lemma 3.1, we have $l = f - 1$ and the leader does not invest at the ex ante state $(l, f + 1, 0)$. Since protection is not feasible given $l = f + 1$, the leader not investing implies $f + 1 \geq IC_D(l) \geq IC_D(l + 1)$. Moreover, by (B.13), once the leader invests at the ex ante state $(l, f + 1, 0)$, the interim state $(l + 1, f + 1, 0)$ satisfies $f + 1 \geq IC_P(l + 1)$, and Lemma B.4 implies that the policymaker will protect the leader. Nonetheless, the leader does not invest. Thus, $(l, f + 1) \notin \mathcal{IC}_{EA}$. Given $l = f + 1$, we have $l \geq IC_{EA}$. This concludes the proof that in the steady state, the leader's technology level is no less than IC_{EA}.

B.4. *Proof of Lemma 6*

If ρ is equal to zero, then $\pi^M(l + 1, f) - \pi^M(l, f) = \hat{\pi}^M(l + 1) - \hat{\pi}^M(l)$, and hence the result holds from Assumption 4 with a strict inequality. Since the state space is finite, by the continuity of the payoff function with respect to ρ, the result holds.

B.5. *Proof of Proposition 3*

By Propositions 1 and 2, l^* is no higher than the solution to $IC_M(l) = 0$. Thus, $\pi^M(l^* + 1, 0) - \pi^M(l^*, 0) \geq c(l^*)$. On the other hand, l^{**} is no less than the solution to $IC_D(l, f) = l$, and hence $\pi^L(l^{**} + 1, l^{**}) - \pi^L(l^{**}, l^{**}) \leq c(l^{**})$.

If $l^* \geq l^{**}$, then since c is increasing, we have $\pi^M(l^* + 1, 0) - \pi^M(l^*, 0) \geq \pi^L(l^{**} + 1, l^{**}) - \pi^L(l^{**}, l^{**})$. Given Lemma 6, for $\rho < \rho'$, we have $\pi^M(l^* + 1, 0) - \pi^M(l^*, 0) \leq \pi^L(l^* + 1, l^{**}) - \pi^L(l^*, l^{**})$. By Lemma 6, this implies $l^{**} \geq l^*$, as desired.

Co-editor Alessandro Lizzeri handled this manuscript.

Manuscript received 26 May, 2021; final version accepted 25 June, 2022; available online 15 July, 2022.