Econometrica Supplementary Material

SUPPLEMENT TO "EXCHANGE DESIGN AND EFFICIENCY"

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This supplement to "Exchange Design and Efficiency" gives the remaining proofs and additional results for the general design (part B) and results and proofs for symmetric markets (part C). It also includes a color version of Figure 2.

APPENDIX B: OTHER PROOFS AND ADDITIONAL RESULTS: GENERAL DESIGN

LEMMA S1—Woodbury Matrix Identity: Suppose that $\mathbf{S} \in \mathbb{R}^{K\times K}$ and $\mathbf{T} \in \mathbb{R}^{L\times L}$ are square matrices, and $\mathbf{U} \in \mathbb{R}^{K\times L}$ and $\mathbf{V} \in \mathbb{R}^{L\times K}$ are real matrices. When $\mathbf{S}^{-1}$ and $\mathbf{T}^{-1}$ are (psedo)inverses of $\mathbf{S}$ and $\mathbf{T}$, respectively, the following matrix identity holds:

$$(\mathbf{S}+\mathbf{UTV})^{-1} = \mathbf{S}^{-1} - \mathbf{S}^{-1}\mathbf{U}\big(\mathbf{T}^{-1}+\mathbf{VS}^{-1}\mathbf{U}\big)^{-1}\mathbf{VS}^{-1}.$$

We define demand $q^{i*}_{k,n}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}$ as a function of residual supply intercept $\mathbf{s}^{-i}_{K(n)}$ (rather than price $\mathbf{p}_{K(n)}$) for each $k \in K(n)$ and n.

LEMMA S2—Asset by Asset Optimization: Consider a market structure $N = \{K(n)\}_n$. Given the residual supply of trader i, that is, price impact $\mathbf{\Lambda}^i$ and intercept distribution $F(\mathbf{s}^{-i}|\mathbf{q}^i_0)$, the following optimization problems are equivalent:

(1) a profile of demands $\{\{q^i_{k,n}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}\}_{k\in K(n)}\}_n$ maximizes the expected payoff (2);
(2) a profile of demands $\{\{q^{i*}_{k,n}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}\}_{k\in K(n)}\}_n$ maximizes the expected payoff (2);
(3) for each n and $k \in K(n)$, demand $q^{i*}_{k,n}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}$ maximizes the expected payoff (2), given trader i's demands for other assets $\{q^{i*}_{\ell,n}(\cdot)\}_{\ell\in K(n),\ell\neq k}$ in exchange n and other exchanges $\{\{q^{i*}_{\ell,n'}(\cdot)\}_{\ell\in K(n')}\}_{n'\neq n}$.

PROOF OF LEMMA S2 (ASSET BY ASSET OPTIMIZATION): Consider a Banach space $\mathcal{X}$ of profiles of twice continuously differentiable downward-sloping demands $q^i_{k,n}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}$ for all $k \in K(n)$ and n. Similarly, we consider a Banach space $\mathcal{X}^*$ of profiles of twice continuously differentiable downward-sloping demands $q^{i*}_{k,n}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}$ for all $k \in K(n)$ and n. Specifically, the Jacobians of demands $\frac{\partial \mathbf{q}^i_{K(n)}(\cdot)}{\partial \mathbf{p}_{K(n)}} = (\frac{\partial q^i_{k,n}(\cdot)}{\partial p_{\ell,n}})_{k,\ell} \in \mathbb{R}^{K(n)\times K(n)}$ and $\frac{\partial \mathbf{q}^{i*}_{K(n)}(\cdot)}{\partial \mathbf{s}^{-i}_{K(n)}} = (\frac{\partial q^i_{k,n}(\cdot)}{\partial s^{-i}_{\ell,n}})_{k,\ell} \in \mathbb{R}^{K(n)\times K(n)}$ are negative definite for all n; they are negative semi-definite if some assets in exchange n are perfectly correlated.

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(Part (1) ⇔ (2)). We first show that $\mathbf{q}^i(\cdot) \equiv \{\{q^i_{k,n}(\cdot)\}_{k\in K(n)}\}_n \in \mathcal{X}$ maps one-to-one to $\mathbf{q}^{i*}(\cdot) \equiv \{\{q^{i*}_{k,n}(\cdot)\}_{k\in K(n)}\}_n \in \mathcal{X}^*$ that yields the same expected payoff (2), and endow the spaces $\mathcal{X}$ and $\mathcal{X}^*$ with a norm $\|\cdot\|_\infty$ that assign the same norm to $\mathbf{q}^i(\cdot)$ and $\mathbf{q}^{i*}(\cdot)$ when they are mapped. Then, the equivalence between problems (1) and (2) is immediate.

Central to the equivalence between problems (1) and (2)—equivalently, the existence of the one-to-one mapping between $\mathcal{X}$ and $\mathcal{X}^*$—is that $\mathbf{p}_{K(n)}$ maps one-to-one to $\mathbf{s}^{-i}_{K(n)}$ in each n. A function of $\mathbf{p}_{K(n)}$ ($q^i_{k,n}(\cdot)$) is measurable with respect to $\mathbf{s}^{-i}_{K(n)}$, and a function of $\mathbf{s}^{-i}_{K(n)}$ ($q^{i*}_{k,n}(\cdot)$) is measurable with respect to $\mathbf{p}_{K(n)}$.

To construct a mapping between $\mathbf{q}^i(\cdot)$ and $\mathbf{q}^{i*}(\cdot)$, we first characterize the mapping between $\mathbf{p}_{K(n)}$ and $\mathbf{s}^{-i}_{K(n)}$, given residual supply and market clearing: The price vector $\mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}$ is determined applying market clearing to the demand of trader i and his residual supply in each exchange n:

$$\mathbf{q}^i_{K(n)}(\mathbf{p}_{K(n)}) = \mathbf{s}^{-i}_{K(n)} + \left(\left(\boldsymbol{\Lambda}^i_{K(n)}\right)'\right)^{-1}\mathbf{p}_{K(n)} \quad \forall \mathbf{s}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}. \tag{S1}$$

By the continuity of the downward-sloping demand $\mathbf{q}^i_{K(n)}(\cdot)$, Eq. (S1) uniquely determines price as continuous functions of intercepts' realizations $\mathbf{s}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}$, which we denote by $\mathbf{p}^*_{K(n)}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}^{K(n)}$. Then, trader i's quantity demanded is uniquely determined by $\mathbf{q}^{i*}_{K(n)}(\cdot) = \mathbf{q}^i_{K(n)} \circ \mathbf{p}^*_{K(n)}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}^{K(n)}$ in each n. Conversely, given $\mathbf{q}^{i*}(\cdot)$, a profile of demands is uniquely determined by $\mathbf{q}^i_{K(n)}(\cdot) = \mathbf{q}^{i*}_{K(n)} \circ (\mathbf{p}^*_{K(n)}(\cdot))^{-1}$ in each n when $(\mathbf{p}^*_{K(n)}(\cdot))^{-1}$ is the inverse of price function $\mathbf{p}^*_{K(n)}(\cdot)$. $\mathbf{q}^{i*}(\cdot)$ is downward-sloping if and only if $\mathbf{q}^i(\cdot)$ is downward-sloping, given the downward-sloping demands of traders $j \neq i$ (i.e., $\boldsymbol{\Lambda}^i_{K(n)}$ is positive semi-definite in Eq. (S1)).

Moreover, the system of equations (2) and (S1) that characterizes the expected payoff reduces to a single equation for $\mathbf{q}^{i*}(\cdot)$ (Eq. (S3)). Market clearing (Eq. (S1)) defines price as a function of trader i's quantity demanded $\mathbf{q}^{i*} \in \mathbb{R}^{\sum_n K(n)}$ and intercepts' realizations $\mathbf{s}^{-i} \in \mathbb{R}^{\sum_n K(n)}$:

$$\mathbf{p}^i(\mathbf{q}^{i*}, \mathbf{s}^{-i}) \equiv (\boldsymbol{\Lambda}^i)'(\mathbf{q}^{i*} - \mathbf{s}^{-i}) \quad \forall \mathbf{q}^{i*} = (\mathbf{q}^{i*}_{K(n)})_n \in \mathbb{R}^{\sum_n K(n)}\ \forall \mathbf{s}^{-i} = (\mathbf{s}^{-i}_{K(n)})_n \in \mathbb{R}^{\sum_n K(n)}. \tag{S2}$$

Substituting $\mathbf{p}^*(\cdot) = (\mathbf{p}^i \circ \mathbf{q}^{i*})(\cdot)$ into the system of equations (2) and (S2) characterizes the expected payoff as a function of $\mathbf{q}^{i*}(\cdot)$:

$$U(\mathbf{q}^{i*}(\cdot)) = E\left[\boldsymbol{\delta}^+ \cdot (\mathbf{q}^{i*} + \mathbf{q}^i_0) - \frac{\alpha^i}{2}(\mathbf{q}^{i*} + \mathbf{q}^i_0) \cdot \boldsymbol{\Sigma}^+(\mathbf{q}^{i*} + \mathbf{q}^i_0) - (\mathbf{q}^{i*} - \mathbf{s}^{-i}) \cdot \boldsymbol{\Lambda}^i \mathbf{q}^{i*} | \mathbf{q}^i_0\right]$$
$$\forall \mathbf{q}^{i*}(\cdot) \in \mathcal{X}^*. \tag{S3}$$

The expected payoff $U(\mathbf{q}^i(\cdot))$ in the system of equations (2) and (S2) satisfies $U(\mathbf{q}^i(\cdot)) = U(\mathbf{q}^{i*}(\cdot))$, given the mapping from $\mathbf{q}^i(\cdot) \in \mathcal{X}$ to $\mathbf{q}^{i*}(\cdot) \in \mathcal{X}^*$ defined by Eq. (S1).

Endow the space $\mathcal{X}^*$ with a norm $\|\cdot\|_\infty$ defined by

$$\|\mathbf{q}^{i*}(\cdot)\|_\infty \equiv \max_{k\in K(n),n} \|q^{i*}_{k,n}(\cdot)\| = \max_{k\in K(n),n} \left(E\left[\left|q^{i*}_{k,n}(\mathbf{s}^{-i}_{K(n)})\right|^2 | \mathbf{q}^i_0\right]\right)^{1/2}, \tag{S4}$$

given $\mathbf{\Lambda}^i$ and $F(\mathbf{s}^{-i}|\mathbf{q}_0^i)$. Because $\mathbf{q}^{i*}(\cdot) \in \mathcal{X}^*$ maps one-to-one to $\mathbf{q}^i(\cdot) \in \mathcal{X}$,[1] given $\mathbf{\Lambda}^i$ and $F(\mathbf{s}^{-i}|\mathbf{q}_0^i)$, the maximization of the expected payoff (2) with respect to a profile $\mathbf{q}^i(\cdot) = \{\{q_{k,n}^i(\cdot)\}_k\}_n \in \mathcal{X}$ subject to market clearing (S1) is equivalent to the maximization of the expected payoff (S3) with respect to a profile $\mathbf{q}^{i,*}(\cdot) = \{\{q_{k,n}^{i*}(\cdot)\}_k\}_n \in \mathcal{X}^*$.

(Part (2) ⇔ (3)). We want to show that the maximization of expected payoff (S3) with respect to a profile of demands $\{\{q_{k,n}^{i*}(\cdot)\}_k\}_n$ is equivalent to the maximization with respect to the demand $q_{k,n}^{i*}(\cdot)$, given the trader's demands for other assets, for all $k \in K(n)$ and n. By the Second Partial Derivative Test, to show the equivalence between problems (2) and (3) in the lemma, it suffices to show that the mapping $U(\cdot) : \mathcal{X}^* \to \mathbb{R}$ is twice (Fréchet) differentiable[2] and satisfies the second-order condition.

(Differentiability of expected payoff with respect to demand schedules). First, we will show that $DU(\cdot) : \mathcal{X}^* \to \mathbb{R}^{\sum_n K(n)}$:

$$DU\big(\mathbf{q}^{i*}(\cdot)\big) = E\big[\boldsymbol{\delta}^+ - \alpha^i \mathbf{\Sigma}^+\big(\mathbf{q}^{i*} + \mathbf{q}_0^i\big) - \mathbf{p}^* - \mathbf{\Lambda}^i \mathbf{q}^{i*}|\mathbf{q}_0^i\big] \quad \forall \mathbf{q}^{i*}(\cdot) \in \mathcal{X}^* \tag{S5}$$

is the Fréchet derivative of $U(\cdot)$ with respect to $\mathbf{q}^{i*}(\cdot)$. Consider a demand change $\Delta\mathbf{q}^{i*}(\cdot) \equiv \{\{\Delta q_{k,n}^{i*}(\cdot)\}_k\}_n$ such that $\widetilde{\mathbf{q}}^{i*}(\cdot) \equiv \mathbf{q}^{i*}(\cdot) + \Delta\mathbf{q}^{i*}(\cdot)$ is in $\mathcal{X}^*$. Because $\widetilde{\mathbf{q}}^{i*}(\cdot)$ is downward-sloping, by the same argument as in (Part (1) ⇔ (2)), we can define price $\widetilde{\mathbf{p}}^*(\cdot)$, that is a function of $\mathbf{s}^{-i}$, analogously to Eqs. (S1)–(S2). Substituting $\widetilde{\mathbf{p}}^*(\cdot)$ into Eq. (S3) gives the expected payoff change (S3) for a demand change $\Delta\mathbf{q}^i(\cdot)$:

$$\begin{aligned} &U\big(\widetilde{\mathbf{q}}^{i*}(\cdot)\big) - U\big(\mathbf{q}^{i*}(\cdot)\big) \\ &\quad = E\bigg[\big(\boldsymbol{\delta}^+ - \alpha^i \mathbf{\Sigma}^+\big(\mathbf{q}^{i*} + \mathbf{q}_0^i\big) - \mathbf{p}^* - \mathbf{\Lambda}^i \mathbf{q}^{i*}\big) \cdot \big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big) \\ &\qquad - \big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big) \cdot \left(\frac{\alpha^i}{2}\mathbf{\Sigma}^+ + \mathbf{\Lambda}^i\right)\big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big)|\mathbf{q}_0^i\bigg]. \end{aligned} \tag{S6}$$

By the convexity of the quadratic matrix function $(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}) \cdot (\frac{\alpha^i}{2}\mathbf{\Sigma}^+ + \mathbf{\Lambda}^i)(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*})$, the Jensen's inequality implies an upper bound on the change in the expected payoff:

$$\begin{aligned} &\big|U\big(\widetilde{\mathbf{q}}^{i*}(\cdot)\big) - U\big(\mathbf{q}^{i*}(\cdot)\big) - E\big[\big(\boldsymbol{\delta}^+ - \alpha^i \mathbf{\Sigma}^+\big(\mathbf{q}^{i*} + \mathbf{q}_0^i\big) - \mathbf{p}^* - \mathbf{\Lambda}^i \mathbf{q}^{i*}\big) \cdot \big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big)|\mathbf{q}_0^i\big]\big| \\ &\quad = \left|E\left[\big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big) \cdot \left(\frac{\alpha^i}{2}\mathbf{\Sigma}^+ + \mathbf{\Lambda}^i\right)\big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big)|\mathbf{q}_0^i\right]\right| \\ &\quad \leq \left(\mathbf{1} \cdot \left|\frac{\alpha^i}{2}\mathbf{\Sigma}^+ + \mathbf{\Lambda}^i\right|\mathbf{1}\right)\Big(\max_{k \in K(n),n}\big\{E\big[\big|\widetilde{q}_{k,n}^{i*} - q_{k,n}^{i*}\big|^2|\mathbf{q}_0^i\big]\big\}\Big) \end{aligned}$$

[1]We endow the space $\mathcal{X}$ (rather than $\mathcal{X}^*$) with a norm $\|\cdot\|_\infty$ defined by

$$\|\mathbf{q}^i(\cdot)\|_\infty \equiv \max_{k \in K(n),n}\|q_{k,n}^i(\cdot)\| = \max_{k \in K(n),n}\big(E\big[\big|q_{k,n}^i\big(\mathbf{p}_{K(n)}^*\big)\big|^2|\mathbf{q}_0^i\big]\big)^{1/2},$$

given $\mathbf{\Lambda}^i$ and $F(\mathbf{s}^{-i}|\mathbf{q}_0^i)$ and market clearing (S1). By the definition of the norm in $\mathcal{X}^*$ in Eq. (S4), $\|\mathbf{q}^i(\cdot)\|_\infty = \|\mathbf{q}^{i*}(\cdot)\|_\infty$ when $\mathbf{q}_{K(n)}^{i*}(\cdot) = \mathbf{q}_{K(n)}^i \circ \mathbf{p}_{K(n)}^*(\cdot)$ in each n.

[2]Let V and W be normed vector spaces, and $U \subset V$ be an open subset of V. A function $f : U \to W$ is Fréchet differentiable at $x \in U$ if there exists a bounded linear operator $A : V \to W$ such that $\lim_{\|h\| \to 0} \frac{\|f(x+h) - f(x) - Ah\|_W}{\|h\|_V} = 0$. If such an operator A exists, it is unique. $Df(x) = A$ is the Fréchet derivative of f at x.

$$\leq \left(\mathbf{1} \cdot \left|\frac{\alpha^i}{2}\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^i\right|\mathbf{1}\right)\left(\left\|\widetilde{\mathbf{q}}^{i*}(\cdot) - \mathbf{q}^{i*}(\cdot)\right\|_\infty\right)^2. \tag{S7}$$

Finally, taking the limit of the payoff change (S7) as $\|\widetilde{\mathbf{q}}^{i*}(\cdot) - \mathbf{q}^{i*}(\cdot)\|_\infty \to 0$, we have

$$\begin{aligned}
&\lim_{\|\widetilde{\mathbf{q}}^{i*}(\cdot)-\mathbf{q}^{i*}(\cdot)\|_\infty \to 0}\Big(\big|U\big(\widetilde{\mathbf{q}}^i(\cdot)\big) - U\big(\mathbf{q}^i(\cdot)\big) \\
&\quad - E\big[\big(\boldsymbol{\delta}^+ - \alpha^i\boldsymbol{\Sigma}^+\big(\mathbf{q}^{i*} + \mathbf{q}_0^i\big) - \mathbf{p}^* - \boldsymbol{\Lambda}^i\mathbf{q}^{i*}\big) \cdot \big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big)|\mathbf{q}_0^i\big]\big|\Big)/\big(\left\|\widetilde{\mathbf{q}}^{i*}(\cdot) - \mathbf{q}^{i*}(\cdot)\right\|_\infty\big) \\
&\leq \lim_{\|\widetilde{\mathbf{q}}^{i*}(\cdot)-\mathbf{q}^{i*}(\cdot)\|_\infty \to 0}\left(\mathbf{1} \cdot \left|\frac{\alpha^i}{2}\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^i\right|\mathbf{1}\right)\left\|\widetilde{\mathbf{q}}^{i*}(\cdot) - \mathbf{q}^{i*}(\cdot)\right\|_\infty = 0.
\end{aligned}$$

Given that all elements of $|\frac{\alpha^i}{2}\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^i|$ are bounded, (S5) is bounded (i.e., $|DU(\mathbf{q}^{i*}(\cdot))| < \infty$) for any $\mathbf{q}^{i*}(\cdot) \in \mathcal{X}^*$ such that $\|\mathbf{q}^{i*}(\cdot)\|_\infty < \infty$, and (S5) is the Fréchet derivative of $U(\cdot)$.

(Second-order condition). We show that the second-order condition of the optimization problem (S3) holds. The Hessian of $U(\cdot)$, $D^2U(\cdot) : \mathcal{X}^* \to \mathbb{R}^{(\sum_n K(n)) \times (\sum_n K(n))}$, is

$$D^2U\big(\mathbf{q}^{i*}(\cdot)\big) = -\alpha^i\boldsymbol{\Sigma}^+ - \boldsymbol{\Lambda}^i - \big(\boldsymbol{\Lambda}^i\big)' \quad \forall \mathbf{q}^{i*}(\cdot) \in \mathcal{X}^*.$$

This is because, by the definition of the Fréchet derivative of $DU(\cdot)$, we have

$$\begin{aligned}
&\lim_{\|\widetilde{\mathbf{q}}^{i*}(\cdot)-\mathbf{q}^{i*}(\cdot)\|_\infty \to 0} \frac{\left\|DU\big(\widetilde{\mathbf{q}}^{i*}(\cdot)\big) - DU\big(\mathbf{q}^{i*}(\cdot)\big) - D^2U\big(\mathbf{q}^{i*}(\cdot)\big)\Delta\mathbf{q}^i(\cdot)\right\|}{\left\|\widetilde{\mathbf{q}}^{i*}(\cdot) - \mathbf{q}^{i*}(\cdot)\right\|_\infty} \\
&\quad = \lim_{\|\widetilde{\mathbf{q}}^{i*}(\cdot)-\mathbf{q}^{i*}(\cdot)\|_\infty \to 0}\Big(\big|E\big[-\alpha^i\boldsymbol{\Sigma}^+\big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big) - \boldsymbol{\Lambda}^i\big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big) - \big(\boldsymbol{\Lambda}^i\big)'\big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big) \\
&\qquad + \big(\alpha^i\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^i + \big(\boldsymbol{\Lambda}^i\big)'\big)\big(\widetilde{\mathbf{q}}^{i*} - \mathbf{q}^{i*}\big)|\mathbf{q}_0^i\big]\big|\Big)/\big(\left\|\widetilde{\mathbf{q}}^{i*}(\cdot) - \mathbf{q}^{i*}(\cdot)\right\|_\infty\big) = 0.
\end{aligned}$$

$D^2U(\cdot)$ is a constant (matrix) function on $\mathcal{X}^*$. Given the downward-sloping demands of traders $j \neq i$ (i.e., $\boldsymbol{\Lambda}^i$ is positive semi-definite), $D^2U(\cdot)$ is negative semi-definite. Hence, the second-order condition of the maximization problem (2) holds. The Second Partial Derivative Test then implies the equivalence between a trader's optimization with respect to a profile of demands $\{\{q_{k,n}^{i*}(\cdot)\}_{k\in K(n)}\}_n \in \mathcal{X}^*$ and asset by asset optimization with respect to $q_{k,n}^{i*}(\cdot)$, given his demands for assets $\ell \neq k$, for all k and n.

It is immediate that the second-order conditions in problems (1) and (3) hold, given that the second-order condition holds in problem (2). *Q.E.D.*

PROOF OF PROPOSITION 2 (EQUILIBRIUM: UNCONTINGENT TRADING): Let the market structure be $N = \{K(n)\}_n$. Consider a trader who optimizes against a residual market $\{\{\mathbf{q}_{K(n)}^j(\cdot)\}_n\}_{j\neq i}$, for which the residual supply is the sufficient statistic. Assuming the linearity of other traders' demands, the trader's residual supply in each exchange n is parameterized as a linear function of the price vector $\mathbf{p}_{K(n)}$:

$$\mathbf{S}_{K(n)}^{-i}(\mathbf{p}_{K(n)}) = \mathbf{s}_{K(n)}^{-i} + \big(\big(\boldsymbol{\Lambda}_{K(n)}^i\big)^{-1}\big)'\mathbf{p}_{K(n)} \quad \forall \mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)},$$

where $\mathbf{s}_{K(n)}^{-i} \equiv \mathbf{S}_{K(n)}^{-i}(0) \in \mathbb{R}^{K(n)}$ is the intercept of the trader's residual supply and $\boldsymbol{\Lambda}_{K(n)}^i = ((\frac{\partial \mathbf{S}_{K(n)}^{-i}(\cdot)}{\partial \mathbf{p}_{K(n)}})^{-1})' \in \mathbb{R}^{K(n)\times K(n)}$ is the transpose of the Jacobian of inverse residual supply.

(Part (i): "Only if"). Suppose that a profile of (net) demands of trader i $\{\{q^i_{k,n}(\cdot)\}_{k\in K(n)}\}_n$ satisfies the first-order condition: for each $k \in K(n)$ and n,

$$\delta_k - \alpha^i \mathbf{\Sigma}_k \mathbf{q}^i_0 - \alpha^i \mathbf{\Sigma}^+_k E\big[\mathbf{q}^i|\mathbf{s}^{-i}_{K(n)}, \mathbf{q}^i_0\big] = p_{k,n} + \big(\mathbf{\Lambda}^i_{K(n)}\big)_k \mathbf{q}^i_{K(n)} \quad \forall \mathbf{s}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}. \tag{S8}$$

When written in matrix form, the first-order condition (S8) in each exchange n becomes a single matrix equation:

$$\boldsymbol{\delta}^+_{K(n)} - \alpha^i \mathbf{\Sigma}_{K(n)} \mathbf{q}^i_0 - \alpha^i \mathbf{\Sigma}^+_{K(n)} E\big[\mathbf{q}^i|\mathbf{s}^{-i}_{K(n)}, \mathbf{q}^i_0\big] = \mathbf{p}_{K(n)} + \mathbf{\Lambda}^i_{K(n)} \mathbf{q}^i_{K(n)} \quad \forall \mathbf{s}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}, \tag{S9}$$

where $\mathbf{\Lambda}^i_{K(n)} \equiv \frac{d\mathbf{p}_{K(n)}}{d\mathbf{q}^i_{K(n)}} \in \mathbb{R}^{K(n)\times K(n)}$ is his price impact in exchange n.

To demonstrate that the first-order conditions (S9) computed pointwise with respect to each realization of $\mathbf{s}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}$ are sufficient to the optimization of demand schedules $\mathbf{q}^i_{K(n)}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}^{K(n)}$,[3] we show that a demand change $\Delta\mathbf{q}^i_{K(n)}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}^{K(n)}$ does not increase the trader's payoff (2).[4] The payoff change following an arbitrary demand change $\Delta\mathbf{q}^i_{K(n)}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}^{K(n)}$ that is a twice continuously differentiable function in $\mathbf{s}^{-i}_{K(n)}$ is (as characterized in the proof of Lemma S2, Eq. (S6)):

$$E\big[\big(\boldsymbol{\delta}^+_{K(n)} - \alpha^i \mathbf{\Sigma}_{K(n)} \mathbf{q}^i_0 - \alpha^i \mathbf{\Sigma}^+_{K(n)} \mathbf{q}^i - \mathbf{p}_{K(n)} - \mathbf{\Lambda}^i_{K(n)} \mathbf{q}^i_{K(n)}\big) \cdot \Delta\mathbf{q}^i_{K(n)} | \mathbf{q}^i_0\big] - o\big(\big\|\Delta\mathbf{q}^i_{K(n)}\big\|^2_\infty\big). \tag{S10}$$

Denoting the intercept distribution by $F(\mathbf{s}^{-i}_{K(n)}|\mathbf{q}^i_0)$, the payoff change (S10) can be written as follows:

$$\int E\big[\big(\boldsymbol{\delta}^+_{K(n)} - \alpha^i \mathbf{\Sigma}_{K(n)} \mathbf{q}^i_0 - \alpha^i \mathbf{\Sigma}^+_{K(n)} \mathbf{q}^i - \mathbf{p}_{K(n)} - \mathbf{\Lambda}^i_{K(n)} \mathbf{q}^i_{K(n)}\big) | \mathbf{s}^{-i}_{K(n)}, \mathbf{q}^i_0\big] \cdot \Delta\mathbf{q}^i_{K(n)}\, dF\big(\mathbf{s}^{-i}_{K(n)}|\mathbf{q}^i_0\big)$$
$$- o\big(\big\|\Delta\mathbf{q}^i_{K(n)}\big\|^2_\infty\big). \tag{S11}$$

If the integrand is zero for all intercept realizations $\mathbf{s}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}$, that is, if the pointwise first-order condition (S9) holds, then the payoff change (S11) is nonpositive for any demand change $\Delta\mathbf{q}^i_{K(n)}(\cdot)$. Given the one-to-one mapping between $\mathbf{p}_{K(n)}$ and $\mathbf{s}^{-i}_{K(n)}$ (i.e., $\mathbf{\Lambda}^i_{K(n)} > 0$ in Eq. (S1)), the first-order condition (S9) is pointwise with respect to $\mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}$:

$$\boldsymbol{\delta}^+_{K(n)} - \alpha^i \mathbf{\Sigma}_{K(n)} \mathbf{q}^i_0 - \alpha^i \mathbf{\Sigma}^+_{K(n)} E\big[\mathbf{q}^i|\mathbf{p}_{K(n)}, \mathbf{q}^i_0\big] = \mathbf{p}_{K(n)} + \mathbf{\Lambda}^i_{K(n)} \mathbf{q}^i_{K(n)} \quad \forall \mathbf{p}_{K(n)} \in \mathbb{R}^{K(n)}.$$

Given that the second-order condition $-\alpha^i \mathbf{\Sigma}^+ - \mathbf{\Lambda}^i - (\mathbf{\Lambda}^i)' < \mathbf{0}$ holds (Lemma S2), pointwise optimization (S9) is also sufficient for optimization with respect to $\mathbf{q}^i_{K(n)}(\cdot)$.

[3]As seen in the proof of Lemma S2, given downward-sloping and continuous $\mathbf{q}^i_{K(n)}(\cdot)$, Eq. (S1) uniquely determines trader i's quantity demanded in each n as continuous functions of a realization of intercepts $\mathbf{s}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}$, which we denote by $\mathbf{q}^{i*}_{K(n)}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}^{K(n)}$. For simplicity, we omit the superscript '$*$' from the proof of Proposition 2.

[4]A unilateral demand change of trader i is understood as a profile of arbitrary twice continuously differentiable functions $\{\Delta q^i_k(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}\}_k$ so that $\mathbf{q}^i_{K(n)}(\cdot) + \Delta\mathbf{q}^i_{K(n)}(\cdot)$ are downward-sloping with respect to the contingent variables, that is, the Jacobian $\frac{\partial(\mathbf{q}^i_{K(n)}(\cdot)+\Delta\mathbf{q}^i_{K(n)}(\cdot))}{\partial\mathbf{p}_{K(n)}} \in \mathbb{R}^{K(n)\times K(n)}$ is negative semi-definite.

(Part (i): "If"). We prove by contradiction that condition (i) is necessary for each trader's optimality of demand schedules in problem (2). Suppose that for some realization $\bar{\mathbf{s}}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}$,

$$E\big[\big(\boldsymbol{\delta}^{+}_{K(n)} - \alpha^i \boldsymbol{\Sigma}_{K(n)}\mathbf{q}^i_0 - \alpha^i \boldsymbol{\Sigma}^{+}_{K(n)}\mathbf{q}^i - \mathbf{p}_{K(n)} - \boldsymbol{\Lambda}^i_{K(n)}\mathbf{q}^i_{K(n)}\big)|\bar{\mathbf{s}}^{-i}_{K(n)}, \mathbf{q}^i_0\big] > 0. \tag{S12}$$

The marginal payoff (i.e., the LHS of Eq. (S12)) is continuous in $\mathbf{s}^{-i}_{K(n)}$ by the continuity of $\mathbf{q}^i_{K(n)}(\cdot)$ and $\mathbf{p}_{K(n)}(\cdot)$ with respect to $\mathbf{s}^{-i}_{K(n)}$. Hence, the marginal payoff is positive for all prices in a neighborhood of $\bar{\mathbf{s}}^{-i}_{K(n)}$: that is, there exists $\varepsilon > 0$ such that

$$E\big[\big(\boldsymbol{\delta}^{+}_{K(n)} - \alpha^i \boldsymbol{\Sigma}_{K(n)}\mathbf{q}^i_0 - \alpha^i \boldsymbol{\Sigma}^{+}_{K(n)}\mathbf{q}^i - \mathbf{p}_{K(n)} - \boldsymbol{\Lambda}^i_{K(n)}\mathbf{q}^i_{K(n)}\big)|\mathbf{s}^{-i}_{K(n)}, \mathbf{q}^i_0\big] > 0 \quad \forall \mathbf{s}^{-i}_{K(n)} \in R_\varepsilon\big(\bar{\mathbf{s}}^{-i}_{K(n)}\big),$$

where $R_\varepsilon(\bar{\mathbf{s}}^{-i}_{K(n)}) \equiv \{\mathbf{s}^{-i}_{K(n)} | \|\mathbf{s}^{-i}_{K(n)} - \bar{\mathbf{s}}^{-i}_{K(n)}\|_\infty < \varepsilon\}$ is an open set that contains $\bar{\mathbf{s}}^{-i}_{K(n)}$. Because the measure of $R_\varepsilon(\bar{\mathbf{s}}^{-i}_{K(n)})$ is nonzero, one can construct a demand change $\Delta\mathbf{q}^i_{K(n)}(\cdot)$ with a positive payoff change in Eq. (S11): Pick a twice continuously differentiable, positive, bounded, and downward-sloping function $\boldsymbol{\eta}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}^{K(n)}_{+}$ such that the Jacobian $\frac{\partial \boldsymbol{\eta}(\cdot)}{\partial \mathbf{s}^{-i}_{K(n)}} \in \mathbb{R}^{K(n)\times K(n)}$ is negative semi-definite, $\boldsymbol{\eta}(\mathbf{s}^{-i}_{K(n)}) = 0$ for all $\mathbf{s}^{-i}_{K(n)} \notin R_\varepsilon(\bar{\mathbf{s}}^{-i}_{K(n)})$, and $\overline{\eta} \equiv \|\boldsymbol{\eta}(\cdot)\|_\infty < \infty$. For a demand change $\Delta\mathbf{q}^i_{K(n)}(\cdot) = \nu\boldsymbol{\eta}(\cdot)$ by trader i for some $\nu \in \mathbb{R}_+$, the payoff change (S11) is

$$\begin{aligned}\nu \int_{R_\varepsilon(\bar{\mathbf{s}}^{-i}_{K(n)})} & E\big[\big(\boldsymbol{\delta}^{+}_{K(n)} - \alpha^i \boldsymbol{\Sigma}_{K(n)}\mathbf{q}^i_0 - \alpha^i \boldsymbol{\Sigma}^{+}_{K(n)}\mathbf{q}^i - \mathbf{p}_{K(n)} - \boldsymbol{\Lambda}^i_{K(n)}\mathbf{q}^i_{K(n)}\big)|\mathbf{s}^{-i}_{K(n)}, \mathbf{q}^i_0\big] \\ & \times \boldsymbol{\eta}\big(\mathbf{s}^{-i}_{K(n)}\big)\, dF\big(\mathbf{s}^{-i}_{K(n)}|\mathbf{q}^i_0\big) - \nu^2 o(\overline{\eta}).\end{aligned} \tag{S13}$$

Equation (S13) is quadratic in ν with a negative quadratic coefficient and a *positive* linear coefficient. It follows that, for this $\nu > 0$, the payoff change (S13) is strictly positive, and thus the demand increase $\Delta\mathbf{q}^i_{K(n)}(\cdot) = \nu\boldsymbol{\eta}(\cdot)$ is a strictly profitable deviation. This contradicts the optimality of $\mathbf{q}^i_{K(n)}(\cdot)$.

Similarly, we can show that if, for some realization $\bar{\mathbf{s}}^{-i}_{K(n)} \in \mathbb{R}^{K(n)}$,

$$E\big[\big(\boldsymbol{\delta}^{+}_{K(n)} - \alpha^i \boldsymbol{\Sigma}_{K(n)}\mathbf{q}^i_0 - \alpha^i \boldsymbol{\Sigma}^{+}_{K(n)}\big(\mathbf{q}^i + \mathbf{q}^i_0\big) - \mathbf{p}_{K(n)} - \boldsymbol{\Lambda}^i_{K(n)}\mathbf{q}^i_{K(n)}\big)|\bar{\mathbf{s}}^{-i}_{K(n)}, \mathbf{q}^i_0\big] < 0,$$

then trader i can increase his payoff by reducing his demands around $\bar{\mathbf{s}}^{-i}_{K(n)}$ by $\Delta\mathbf{q}^i_{K(n)}(\cdot) = \nu\boldsymbol{\eta}(\cdot)$ for some $\nu < 0$ and $\boldsymbol{\eta}(\cdot) : \mathbb{R}^{K(n)} \to \mathbb{R}^{K(n)}_{+}$ with the same properties as above.

(Part (ii): "Only if"). We show that condition (ii) is sufficient for equilibrium (Definition 2): given each trader's optimization problem that takes the residual supply as given (condition (i)), the requirement that the residual supply is correct (condition (ii)) is sufficient for each trader's optimization problem that takes other traders' demands as given (Definition 2).

In trader i's optimization problem, other traders' demands $\{\{\mathbf{q}^i_{K(n)}(\cdot)\}_n\}_{j\neq i}$ are payoff-relevant to his expected payoff (2) only via the price distribution $F(\mathbf{p}|\mathbf{q}^i_0)$ and price impact $\boldsymbol{\Lambda}^i \equiv \frac{d\mathbf{p}}{d\mathbf{q}^i}$. Because $F(\mathbf{p}|\mathbf{q}^i_0)$ and $\boldsymbol{\Lambda}^i$ are determined by applying market clearing to demand schedules $\mathbf{q}^i_{K(n)}(\cdot) + \sum_{j\neq i}\mathbf{q}^j_{K(n)}(\cdot) = \mathbf{0}$ in each exchange n, the sum of other traders' demands $\{\sum_{j\neq i}\mathbf{q}^j_{K(n)}(\cdot)\}_n$—equivalently, the residual supply $\{\mathbf{S}^{-i}_{K(n)}(\cdot) \equiv -\sum_{j\neq i}\mathbf{q}^j_{K(n)}(\cdot)\}_n$—is the sufficient statistic for $F(\mathbf{p}|\mathbf{q}_0^i)$ and $\mathbf{\Lambda}^i$, and thus the sufficient statistic for the profile of other traders' demands in trader i's optimization problem.

(Part (ii): "If"). We show that condition (ii) is necessary for equilibrium: If the residual supply satisfies $\widetilde{\mathbf{S}}_{K(n)}^{-i}(\cdot) \neq -\sum_j \mathbf{q}_{K(n)}^j(\cdot)$ for some i and n for some realization of $\{\mathbf{q}_0^j\}_{j\neq i} \in \mathbb{R}^{(I-1)K}$, then trader i's demand $\widetilde{\mathbf{q}}_{K(n)}^i(\cdot)$ that is optimized when taking $\widetilde{\mathbf{S}}_{K(n)}^{-i}(\cdot)$ as given is not an equilibrium demand. The argument is by contradiction, and mimics the proof of Lemma 1 in Rostek and Weretka (2015).

Suppose that trader i submits demand functions $\{\widetilde{\mathbf{q}}_{K(n)}^i(\cdot) \equiv \mathbf{q}_{K(n)}^i(\cdot; \widetilde{\mathbf{S}}^{-i}(\cdot))\}_n$ for which $\widetilde{\mathbf{S}}_{K(n')}^{-i}(\cdot) \neq -\sum_j \mathbf{q}_{K(n')}^j(\cdot)$ for some n'. Then, either $\widetilde{\mathbf{\Lambda}}_{K(n')}^i \neq -((\sum_{j\neq i} \frac{\partial \mathbf{q}_{K(n')}^j(\cdot)}{\partial \mathbf{p}_{K(n')}})^{-1})'$ or the residual supply intercept that defines $F(\widetilde{\mathbf{s}}_{K(n')}^{-i}|\mathbf{q}_0^i)$ is such that $\widetilde{\mathbf{s}}_{K(n')}^{-i} \neq -\sum_{j\neq i} \mathbf{q}_{K(n')}^j(0)$ for some realization of $\{\overline{\mathbf{q}}_0^j\}_{j\neq i} \in \mathbb{R}^{(I-1)K}$. Then, the first-order condition (S9) of trader i in exchange n' that takes as given other traders' demands $\{\mathbf{q}^j(\cdot)\}_{j\neq i}$—rather than function $\widetilde{\mathbf{S}}^{-i}(\cdot)$—is violated at the realization $\{\overline{\mathbf{q}}_0^j\}_{j\neq i}$:

$$\boldsymbol{\delta}_{K(n')}^+ - \alpha^i \mathbf{\Sigma}_{K(n')} \mathbf{q}_0^i - \alpha^i \mathbf{\Sigma}_{K(n')}^+ E\big[\widetilde{\mathbf{q}}^i | \mathbf{s}_{K(n')}^{-i}, \mathbf{q}_0^i\big] - \mathbf{p}_{K(n')} - \mathbf{\Lambda}_{K(n')}^i \widetilde{\mathbf{q}}_{K(n')}^i \neq \mathbf{0},$$

where $\mathbf{\Lambda}_{K(n')}^i \equiv -((\sum_{j\neq i} \frac{\partial \mathbf{q}_{K(n')}^j(\cdot)}{\partial \mathbf{p}_{K(n)}})^{-1})'$ is the inverse of the transpose of the Jacobian of $-(\sum_{j\neq i} \mathbf{q}_{K(n')}^j(\cdot))$, and $\mathbf{s}_{K(n')}^{-i} \equiv -\sum_{j\neq i} \mathbf{q}_{K(n')}^j(0)$ is its intercept. Following the same argument as in the proof of (Part (i): Part If), one can construct a deviation $\Delta \mathbf{q}_{K(n')}^i(\cdot) = \nu \boldsymbol{\eta}(\cdot)$ for which the expected payoff change (Eq. (S13)) is positive. It follows that $\{\mathbf{q}_{K(n)}^i(\cdot; \widetilde{\mathbf{S}}^{-i}(\cdot))\}_n$ is not a best response to the profile of other traders' demands $\{\mathbf{q}^j(\cdot)\}_{j\neq i}$, and hence is not an equilibrium. *Q.E.D.*

PROOF OF COROLLARY 1 (EQUILIBRIUM PRICES AND ALLOCATIONS): We characterize equilibrium prices and allocations as functions of the equilibrium demand coefficients $\{\mathbf{B}^i, \mathbf{C}^i\}_i$ and price impacts $\{\mathbf{\Lambda}^i\}_i$. Applying market clearing to demand schedules (39) yields the price vector:

$$\mathbf{p} = \Big(\sum_i \mathbf{C}^i\Big)^{-1} \Big(\sum_i \boldsymbol{a}^i - \sum_i \mathbf{B}^i \mathbf{q}_0^i\Big). \tag{S14}$$

Summing the intercepts $\{\boldsymbol{a}^i\}_i$ in Eq. (27), we have

$$\sum_i \boldsymbol{a}^i = \Big(\sum_i \mathbf{C}^i\Big)\Big(\boldsymbol{\delta}^+ - \Big(\sum_j (\alpha^j \mathbf{\Sigma}^+ + \mathbf{\Lambda}^j)^{-1}\Big)^{-1} \sum_j (\alpha^j \mathbf{\Sigma}^+ + \mathbf{\Lambda}^j)^{-1} \mathbf{W} \alpha^j \mathbf{\Sigma} E\big[\mathbf{q}_0^j\big]\Big) + \sum_i \mathbf{B}^i E\big[\mathbf{q}_0^i\big]. \tag{S15}$$

Substituting for $\sum_i \boldsymbol{a}^i$ from Eq. (S15), the price equation (S14) becomes

$$\mathbf{p} = \boldsymbol{\delta}^+ - \underbrace{\Big(\sum_j (\alpha^j \mathbf{\Sigma}^+ + \mathbf{\Lambda}^j)^{-1}\Big)^{-1} \sum_j (\alpha^j \mathbf{\Sigma}^+ + \mathbf{\Lambda}^j)^{-1} \mathbf{W} \alpha^j \mathbf{\Sigma} E\big[\mathbf{q}_0^j\big]}_{\equiv E[\mathbf{Q}^c]}$$

$$-\underbrace{\left(\sum_j \mathbf{C}^j\right)^{-1}\sum_j \mathbf{B}^j\left(\mathbf{q}_0^j - E\left[\mathbf{q}_0^j\right]\right)}_{\equiv \mathbf{Q}-E[\mathbf{Q}]}. \tag{S16}$$

$\mathbf{Q} \equiv (\sum_j \mathbf{C}^j)^{-1}\sum_j \mathbf{B}^j\mathbf{q}_0^j$ is the aggregate risk in the uncontingent market, while $\mathbf{Q}^c \equiv (\sum_j(\alpha^j\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^j)^{-1})^{-1}\sum_j(\alpha^j\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^j)^{-1}\mathbf{W}\alpha^j\boldsymbol{\Sigma}\mathbf{q}_0^j$ is the aggregate risk in the contingent markets, where we used $\mathbf{C}^{j,c} = (\alpha^j\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^j)^{-1}$ and $\mathbf{B}^{j,c} = (\alpha^j\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^j)^{-1}\mathbf{W}\alpha^j\boldsymbol{\Sigma}$ for all j.

To characterize the equilibrium quantity traded of trader i, substitute equilibrium price $\mathbf{p}$ and demand coefficient $\boldsymbol{a}^i$ into trader i's demand (39): for each i,

$$\mathbf{q}^i = \left(\alpha^i\boldsymbol{\Sigma}^+ + \boldsymbol{\Lambda}^i\right)^{-1}\left(E\left[\mathbf{Q}^c\right] - \mathbf{W}\alpha^i\boldsymbol{\Sigma}E\left[\mathbf{q}_0^i\right]\right) + \mathbf{C}^i\left(\mathbf{Q} - E[\mathbf{Q}]\right) - \mathbf{B}^i\left(\mathbf{q}_0^i - E\left[\mathbf{q}_0^i\right]\right). \tag{S17}$$

In the symmetric equilibrium (i.e., assuming $\alpha^i = \alpha$ for all i), the equilibrium price (S16) and quantity traded (S17) become Eqs. (17) and (18), respectively. *Q.E.D.*

LEMMA S3—Price Impact in Competitive Markets: *Consider a market structure $N = \{K(n)\}_n$, let $\{\alpha^i\}_i$ be the profile of traders' risk aversions, and suppose $\{\boldsymbol{\Lambda}^{i,I}\}_i$ is a profile of the equilibrium price impacts for all $I < \infty$ and in the limit market as $I \to \infty$. The equilibrium price impact becomes zero as $I \to \infty$ if $\alpha^{i,I} = \alpha^i\gamma^I \in \mathbb{R}_+$ increases slower than linearly by a common factor $\gamma^I \sim o(I^{1-\varepsilon})$ for some $\varepsilon \in (0,1)$: for each i, $\boldsymbol{\Lambda}^i = \lim_{I\to\infty}\boldsymbol{\Lambda}^{i,I} = \mathbf{0}$.*

PROOF OF LEMMA S3 (PRICE IMPACT IN COMPETITIVE MARKETS): By Definition 3, the equilibrium price impact in the competitive market is $\boldsymbol{\Lambda}^i = \lim_{I\to\infty}\boldsymbol{\Lambda}^{i,I}$. Theorem 5 characterizes the fixed point equation for price impact matrices $\{\boldsymbol{\Lambda}^{i,I}\}_i$ for $I < \infty$. We show that price impact $\boldsymbol{\Lambda}^{i,I}$ is proportional to a factor $\gamma^I \in \mathbb{R}_+$ that is common to all traders $\alpha^{i,I} = \alpha^i\gamma^I$: that is, $\{\boldsymbol{\Lambda}^{i,I}\}_i$ is a profile of equilibrium price impacts when traders' risk aversions are $\{\alpha^i\gamma^I\}_i$ if and only if $\{\widetilde{\boldsymbol{\Lambda}}^{i,I} \equiv \frac{1}{\gamma^I}\boldsymbol{\Lambda}^{i,I}\}_i$ is a profile of equilibrium price impacts when traders' risk aversions are $\{\alpha^i\}_i$ independently of the number of traders I. This can be shown substituting $\alpha^{i,I} = \alpha^i\gamma^I$ for all i into Eqs. (28)–(30):

$$\begin{aligned}
\mathbf{B}^{i,I} &= \left(\alpha^i\boldsymbol{\Sigma}^+ + \frac{1}{\gamma^I}\boldsymbol{\Lambda}^{i,I}\right)^{-1}\mathbf{W}\alpha^i\boldsymbol{\Sigma} \\
&\quad - \left(\left(\alpha^i\boldsymbol{\Sigma}^+ + \frac{1}{\gamma^I}\boldsymbol{\Lambda}^{i,I}\right)^{-1} - \gamma^I\mathbf{C}^{i,I}\right)\left(\gamma^I\overline{\mathbf{C}}^I\right)^{-1}\left(\frac{\sigma_{pv}}{I(\sigma_{cv}+\sigma_{pv})}\mathbf{B}^{i,I} + \frac{\sigma_{cv}}{\sigma_{cv}+\sigma_{pv}}\overline{\mathbf{B}}^I\right),
\end{aligned}$$

$$\left[\left(\mathbf{Id} - \left(\alpha^i\boldsymbol{\Sigma}^+ + \frac{1}{\gamma^I}\boldsymbol{\Lambda}^{i,I}\right)\gamma^I\mathbf{C}^{i,I}\right)\left(\gamma^I\overline{\mathbf{C}}^I\right)^{-1}\left(\sum_{j\neq i}\mathbf{B}^{j,I}\boldsymbol{\Omega}\left(\mathbf{B}^{j,I} + \frac{\sigma_{cv}}{\sigma_{cv}+\sigma_{pv}}\sum_{h\neq i}\mathbf{B}^{h,I}\right)'\right)\right]_N = 0,$$

$$\frac{1}{\gamma^I}\boldsymbol{\Lambda}^{i,I} = \left(\sum_{j\neq i}\gamma^I\mathbf{C}^j\right)^{-1}.$$

Hence, with $\{\alpha^i\}_i$, the equilibrium demand coefficients and price impacts are $\widetilde{\mathbf{C}}^{i,I} \equiv \gamma^I\mathbf{C}^{i,I}$, $\widetilde{\mathbf{B}}^{i,I} \equiv \mathbf{B}^{i,I}$, and $\widetilde{\boldsymbol{\Lambda}}^{i,I} \equiv \frac{1}{\gamma^I}\boldsymbol{\Lambda}^{i,I}$ for all i. The proportionality of the price impact matrix $\boldsymbol{\Lambda}^{i,I}$ to the common factor γ^I gives lower and upper bounds for the limit of the price impact matrix:

$$\mathbf{0} \leq \lim_{I\to\infty} \mathbf{\Lambda}^{i,I} = \lim_{I\to\infty} \gamma^I \lim_{I\to\infty} \widetilde{\mathbf{\Lambda}}^{i,I} = \lim_{I\to\infty} \gamma^I \lim_{I\to\infty}\left(\left(\sum_{j\neq i} \widetilde{\mathbf{C}}^{i,I}\right)^{-1}\right)' \leq \lim_{I\to\infty} \frac{\gamma^I}{I-1} \max_i\left(\left(\widetilde{\mathbf{C}}^{i,I}\right)^{-1}\right)'.$$

Given that an equilibrium exists in the limit market ($I \to \infty$), the Jacobian of each trader's demand schedule is bounded: $\lim_{I\to\infty}(\widetilde{\mathbf{C}}^{i,I})^{-1}_{k\ell} < \infty$ for all k, ℓ, and i. We have $\lim_{I\to\infty} \mathbf{\Lambda}^{i,I} = \mathbf{0}$ when $\frac{\gamma^I}{I-1} \sim o(I^{-\varepsilon})$ decreases to zero as $I \to \infty$. We conclude that $\mathbf{\Lambda}^i = \mathbf{0}$ for all i. *Q.E.D.*

In what follows, we assume that risk preferences are symmetric across traders, and endowments are independent across assets k unless specified otherwise. Then, $\mathbf{B}^i = \mathbf{B}$, $\mathbf{C}^i = \mathbf{C}$, and $\mathbf{\Lambda}^i = \mathbf{\Lambda}$ for all i, and $\mathbf{\Omega} = \mathbf{Id}$.

ASSSUMPTION—Symmetric Risk Preferences: *Let $\alpha^i = \alpha$ for all i.*

In a symmetric equilibrium, Eqs. (27)–(30) in Theorem 5 simplify as summarized by Corollary S1.

COROLLARY S1—Symmetric Equilibrium; General Design: *Consider a market structure $N = \{K(n)\}_n$. In a symmetric equilibrium, (net) demand schedules, defined by matrix coefficients $\{\boldsymbol{a}^i\}_i$, $\mathbf{B}$, and $\mathbf{C}$, and price impact $\mathbf{\Lambda}$ are characterized by the following conditions: for each i,*

(i) (Optimization, given price impact) *Given price impact matrix $\mathbf{\Lambda}$, best-response coefficients $\boldsymbol{a}^i$, $\mathbf{B}$, and $\mathbf{C}$ are characterized by*

$$\boldsymbol{a}^i = \mathbf{C}\big(\boldsymbol{\delta}^+ - \big(\mathbf{W}\alpha\mathbf{\Sigma} - \mathbf{C}^{-1}\mathbf{B}\big)E[\overline{\mathbf{q}}_0]\big) + \big(\big(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}\big)^{-1}\mathbf{W}\alpha\mathbf{\Sigma} - \mathbf{B}\big)\big(E[\overline{\mathbf{q}}_0] - E\big[\mathbf{q}_0^i\big]\big), \tag{S18}$$

$$\mathbf{B} = \big((1-\sigma_0)\big(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}\big) + \sigma_0\mathbf{C}^{-1}\big)^{-1}\mathbf{W}\alpha\mathbf{\Sigma}, \tag{S19}$$

$$\mathbf{C} = \big[\big(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}\big)\big(\mathbf{B}\mathbf{B}'\big)\big[\mathbf{B}\mathbf{B}'\big]_N^{-1}\big]_N^{-1}, \tag{S20}$$

where $\sigma_0 \equiv \frac{\sigma_{cv}+\frac{1}{I}\sigma_{pv}}{\sigma_{cv}+\sigma_{pv}}$.

(ii) (Correct price impact) *Price impact $\mathbf{\Lambda}$ equals the transpose of the Jacobian of the trader's inverse residual supply function*:

$$\mathbf{\Lambda} = \frac{1}{I-1}\big(\mathbf{C}^{-1}\big)'. \tag{S21}$$

Note that the price slope $\mathbf{C}^{-1} = \text{diag}(\mathbf{C}^{-1}_{K(n)})_n$ is a block-diagonal matrix.

Endogenous price covariance. Underlying this result is the lack of proportionality between the equilibrium price impact, and hence price covariance, and asset covariance $\mathbf{\Sigma}$ with limited demand conditioning, as seen in Eq. (17). Consequently, $\mathbf{\Lambda}$ and $\text{Cov}[p_k, p_\ell]$ depend on the covariance of *all* assets and, in fact, need not match the sign of asset correlation (i.e., $\sigma_{k\ell}$); for example, prices of complementary assets ($\sigma_{k\ell} < 0$) can be positively correlated ($\text{Cov}[p_k, p_\ell] > 0$). The intuition can be seen in the price equation (17):

$\text{Cov}[p_k, p_\ell]$ is determined by

$$(\mathbf{C}^{-1}\mathbf{B})_{k\ell} = \left(\left(\mathbf{C} + \kappa(\alpha\mathbf{\Sigma})^{-1}\right)^{-1}\right)_{k\ell} = \alpha\sigma_{k\ell} - \alpha\mathbf{\Sigma}_k\left(\alpha\mathbf{\Sigma} + \kappa\mathbf{C}^{-1}\right)^{-1} \cdot \alpha\mathbf{\Sigma}_\ell, \tag{S22}$$

where $\kappa = \frac{1+(I-2)\sigma_0}{(I-1)(1-\sigma_0)} \in \mathbb{R}_+$. When demand coefficient $\mathbf{C}$ is not proportional to $(\alpha\mathbf{\Sigma})^{-1}$, one can have $\text{sign}(\frac{\partial E[p_\ell|p_k,\mathbf{q}_0^j]}{\partial p_k}) = \text{sign}(\text{Cov}[p_k, p_\ell]) \neq \text{sign}(\sigma_{k\ell})$ for some $\ell \neq k$, and as a result, $\lambda_k < \lambda_k^c$ by Eq. (20) (Figure 1(B)). In the contingent market, the price covariance matrix is proportional to the asset covariance: substituting $\mathbf{C}^c = (\alpha\mathbf{\Sigma} + \mathbf{\Lambda}^c)^{-1} = \frac{I-2}{I-1}(\alpha\mathbf{\Sigma})^{-1}$ in Eq. (S22), we have

$$\left(\left(\mathbf{C}^c + \kappa(\alpha\mathbf{\Sigma})^{-1}\right)^{-1}\right)_{k\ell} = \frac{(I-1)\kappa}{(I-1)\kappa + (I-2)}\alpha\sigma_{k\ell} \quad \forall k\ \forall \ell;$$

hence, $\text{sign}(\text{Cov}[p_k, p_\ell]) = \text{sign}(\sigma_{k\ell})$.

PROOF OF THEOREM 2 (EXISTENCE OF SYMMETRIC EQUILIBRIUM): Consider a market structure $N = \{K(n)\}_n$ in which all assets are traded in one exchange (i.e., $\sum_n K(n) = K$). Let $\mathcal{M}$ be the set of all $(\sum_n K(n))$-dimensional block-diagonal matrices, such that, for any $\mathbf{M} \in \mathcal{M}$, $\mathbf{M}_{K(n),K(n')} = 0$ for distinct exchanges $n \neq n'$. Given that the price impact matrices are block-diagonal, we introduce a partial order on $\mathcal{M}$: $\mathbf{M}^1 \leq \mathbf{M}^2$ if $(\mathbf{M}^2 - \mathbf{M}^1)$ is positive semi-definite, or equivalently, if $\mathbf{M}^1_{K(n),K(n)} \leq \mathbf{M}^2_{K(n),K(n)}$ for all n.

(*Existence*). Substituting $\mathbf{B}$ from Eq. (S19) into Eq. (S20), the fixed point equation (S21) for $\mathbf{\Lambda}$ becomes

$$\underbrace{\left[(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda} - (I-1)\mathbf{\Lambda}')\left(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda} + \frac{(I-1)\sigma_0}{1-\sigma_0}\mathbf{\Lambda}'\right)^{-1}\mathbf{W}\alpha\mathbf{\Sigma}\alpha\mathbf{\Sigma}\mathbf{W}'\left(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}' + \frac{(I-1)\sigma_0}{1-\sigma_0}\mathbf{\Lambda}\right)^{-1}\right]_N}_{\equiv L(\mathbf{\Lambda})}$$

$$= \mathbf{0}. \tag{S23}$$

Define a mapping $L(\cdot) : \mathcal{M} \to \mathcal{M}$ using the LHS of Eq. (S23). We want to show that there exists $\mathbf{\Lambda}$ such that $L(\mathbf{\Lambda}) = \mathbf{0}$. We first show that there exist two bounds $(\underline{\mathbf{\Lambda}}, \overline{\mathbf{\Lambda}})$, such that $L(\underline{\mathbf{\Lambda}}) \geq \mathbf{0}$ and $L(\overline{\mathbf{\Lambda}}) \leq \mathbf{0}$. Then, by the Brouwer fixed point theorem,[5] since the set of block-diagonal matrices defined by the bounds $(\underline{\mathbf{\Lambda}}, \overline{\mathbf{\Lambda}})$ is compact and the mapping $L(\mathbf{\Lambda})$ is continuous, there exists a solution $\mathbf{\Lambda}$ to the fixed point problem $L(\mathbf{\Lambda}) = \mathbf{0}$.

Let $\underline{\mathbf{\Lambda}} \equiv \mathbf{0}$ and $\overline{\mathbf{\Lambda}} \equiv \frac{\alpha}{I-2}N[\mathbf{\Sigma}^+]_N$. It is immediate that $\underline{\mathbf{\Lambda}}$ satisfies the desired condition: evaluating $L(\mathbf{\Lambda})$ at $\underline{\mathbf{\Lambda}} = \mathbf{0}$, we have $L(\underline{\mathbf{\Lambda}}) = [\alpha\mathbf{\Sigma}^+]_N \geq \mathbf{0}$ by the positive semi-definiteness of $\alpha\mathbf{\Sigma}^+$.

Evaluating $L(\mathbf{\Lambda})$ at $\overline{\mathbf{\Lambda}}$, we have

$$L(\overline{\mathbf{\Lambda}}) = \alpha\Bigg[\left(\mathbf{Id} + \frac{\kappa N}{I-2}[\mathbf{\Sigma}^+]_N(\mathbf{\Sigma}^+)^{-1}\right)^{-1}(\mathbf{\Sigma}^+ - N[\mathbf{\Sigma}^+]_N) \times \left(\mathbf{Id} + \frac{\kappa N}{I-2}[\mathbf{\Sigma}^+]_N(\mathbf{\Sigma}^+)^{-1}\right)^{-1}\Bigg]_N, \tag{S24}$$

[5]More precisely, the Brouwer fixed point theorem is applied to the equation $\mathbf{\Lambda} = L(\mathbf{\Lambda}) + \mathbf{\Lambda}$ on the partially ordered compact set $\{\mathbf{\Lambda}|\underline{\mathbf{\Lambda}} \leq \mathbf{\Lambda} \leq \overline{\mathbf{\Lambda}}\} \subset \mathbb{R}^{(\sum_n K(n))\times(\sum_n K(n))}$.

where $\kappa \equiv \frac{1+(I-2)\sigma_0}{1-\sigma_0} \in \mathbb{R}_+$. Given that $\mathbf{\Sigma}^+$ is positive semi-definite, $(\mathbf{Id}+\frac{\kappa N}{I-2}[\mathbf{\Sigma}^+]_N(\mathbf{\Sigma}^+)^{-1})^{-1}$ is positive definite, where we used that the inverse of a positive definite matrix is positive definite. In turn, matrix $(\mathbf{\Sigma}^+ - N[\mathbf{\Sigma}^+]_N)$ in Eq. (S24) is negative definite. It is negative semi-definite if and only if either some assets in an exchange are perfectly correlated or all $\sum_n K(n)$ assets are perfectly correlated. To prove this, observe that for any vector $\mathbf{v} \equiv (\mathbf{v}_{K(n)})_n \in \mathbb{R}^{\sum_n K(n)}$, we have

$$\operatorname{Cov}[\mathbf{v}_{K(n)} \cdot \mathbf{r}_{K(n)}, \mathbf{v}_{K(n')} \cdot \mathbf{r}_{K(n')}] \le \frac{1}{2}\operatorname{Var}[\mathbf{v}_{K(n)} \cdot \mathbf{r}_{K(n)}] + \frac{1}{2}\operatorname{Var}[\mathbf{v}_{K(n')} \cdot \mathbf{r}_{K(n')}] \quad \forall n, n' \in N. \tag{S25}$$

Using that $\mathbf{\Sigma}^+$ is the covariance matrix of the distribution of asset returns $\mathbf{r} = (\mathbf{r}_{K(n)})_n$, inequality (S25) is equivalent to

$$\mathbf{v}'_{K(n)}\mathbf{\Sigma}^+_{K(n),K(n')}\mathbf{v}_{K(n')} \le \frac{1}{2}\mathbf{v}'_{K(n)}\mathbf{\Sigma}^+_{K(n),K(n)}\mathbf{v}_{K(n)} + \frac{1}{2}\mathbf{v}'_{K(n')}\mathbf{\Sigma}^+_{K(n'),K(n')}\mathbf{v}_{K(n')} \quad \forall n, n' \in N. \tag{S26}$$

Summing each side of Eq. (S26) over n and n' gives

$$\mathbf{v}'\mathbf{\Sigma}^+\mathbf{v} = \sum_{n,n'}\mathbf{v}'_{K(n)}\mathbf{\Sigma}^+_{K(n),K(n')}\mathbf{v}_{K(n')} \le N\sum_{n}\mathbf{v}'_{K(n)}\mathbf{\Sigma}^+_{K(n),K(n)}\mathbf{v}_{K(n)} = \mathbf{v}'\big(N\big[\mathbf{\Sigma}^+\big]_N\big)\mathbf{v},$$

and hence, $\mathbf{v}'(\mathbf{\Sigma}^+ - N[\mathbf{\Sigma}^+]_N)\mathbf{v} \le 0$ for any vector $\mathbf{v}$.

By the positive semi-definiteness of $(\mathbf{Id} + \frac{\kappa N}{I-2}[\mathbf{\Sigma}^+]_N(\mathbf{\Sigma}^+)^{-1})^{-1}$ and the negative semi-definiteness of $(\mathbf{\Sigma}^+ - N[\mathbf{\Sigma}^+]_N)$, it follows that the RHS of Eq. (S24) becomes

$$\left(\mathbf{Id} + \frac{\kappa N}{I-2}\big[\mathbf{\Sigma}^+\big]_N\big(\mathbf{\Sigma}^+\big)^{-1}\right)^{-1}\big(\mathbf{\Sigma}^+ - N\big[\mathbf{\Sigma}^+\big]_N\big)\left(\mathbf{Id} + \frac{\kappa N}{I-2}\big[\mathbf{\Sigma}^+\big]_N\big(\mathbf{\Sigma}^+\big)^{-1}\right)^{-1} \le \mathbf{0}. \tag{S27}$$

Consequently, $L(\overline{\mathbf{\Lambda}}) \le \mathbf{0}$; the equality holds if all $\sum_n K(n)$ assets are perfectly correlated.[6] This completes the argument.

(*Uniqueness for* $K = 2$). We show that the equilibrium in the uncontingent market for $K = 2$ is unique. As Appendix C.2 shows, the equilibrium fixed point equation (S23) for $\mathbf{\Lambda} = \operatorname{diag}(\lambda, \lambda)$ simplifies to

$$\lambda = \frac{\alpha}{I-2} + \frac{\alpha\rho}{I-2}\frac{2xy}{x^2+y^2}, \tag{S28}$$

where $x \equiv (1-\sigma_0)(1-\rho^2)\alpha + (1+(I-2)\sigma_0)\lambda$ and $y \equiv \rho(1+(I-2)\sigma_0)\lambda$ characterize each row of $\mathbf{B}$ in Eq. (S19): $\mathbf{b}_1 = (x, y)$ and $\mathbf{b}_2 = (y, x)$. Rearranging Eq. (S28) gives a third-order polynomial of λ:

$$\begin{aligned} 0 = {} & -(I-2)\big(1+\rho^2\big)\big(1+(I-2)\sigma_0\big)^2\lambda^3 + \big(4-\big(1-\rho^2\big)(2I-1) \\ & + (I-2)\big(3+\rho^2\big)\sigma_0\big)\big(1+(I-2)\sigma_0\big)\alpha\lambda^2 \\ & + \big(4-\big(1-\rho^2\big)I + (I-2)\big(3+\rho^2\big)\sigma_0\big)(1-\sigma_0)\big(1-\rho^2\big)\alpha^2\lambda \\ & + \alpha^3(1-\sigma_0)^2\big(1-\rho^2\big)^2. \end{aligned}$$

By the Descartes' sign rule, there exists a unique positive solution λ. *Q.E.D.*

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[6]The equality in (S27) implies that $\mathbf{\Lambda} = \frac{N}{I-2}[\mathbf{\Sigma}^+]_N$ is the solution to Eq. (S23) if and only if all $\sum_n K(n)$ assets are perfectly correlated.

PROOF OF PROPOSITION 4: (SUFFICIENT STATISTIC FOR EQUILIBRIUM PAYOFFS). Let $I < \infty$ and $K > 1$. Consider a market $N = \{K(n)\}_n$, represented by the indicator matrix $\mathbf{W} \in \{0, 1\}^{(\sum_n K(n))\times K}$ (Definition 6). Let $\{q^i_{k,n}(\cdot)\}_{i,k,n}$ be a profile of equilibrium demands. We characterize the equilibrium payoff of each trader (Eq. (S30)) as a function of per-unit price impact $\widehat{\mathbf{\Lambda}} \in \mathbb{R}^{K\times K}$ and the endowment coefficient of total demand $\widehat{\mathbf{B}} \in \mathbb{R}^{K\times K}$ (Eq. (49)).

(Part (1)). Substituting the equilibrium prices and trades from Eqs. (17) and (18) into $u^i(\cdot) - \mathbf{p}\cdot\mathbf{q}^i$ gives the ex post equilibrium payoff of trader i in N:

$$\begin{aligned}
&u^i(\mathbf{q}^i) - \mathbf{p}\cdot\mathbf{q}^i \\
&= \left(\boldsymbol{\delta}\cdot\mathbf{q}^i_0 - \frac{\alpha}{2}\mathbf{q}^i_0\cdot\mathbf{\Sigma}\mathbf{q}^i_0\right) + \left(\boldsymbol{\delta}^+ - \mathbf{p} - \mathbf{W}\alpha\mathbf{\Sigma}\mathbf{q}^i_0\right)\cdot\mathbf{q}^i - \frac{\alpha}{2}\mathbf{q}^i\cdot\mathbf{\Sigma}^+\mathbf{q}^i \\
&= \left(\boldsymbol{\delta}\cdot\mathbf{q}^i_0 - \frac{\alpha}{2}\mathbf{q}^i_0\cdot\mathbf{\Sigma}\mathbf{q}^i_0\right) \\
&\quad + \frac{1}{2}\left(2\left(\mathbf{W}\alpha\mathbf{\Sigma} - \mathbf{C}^{-1}\mathbf{B}\right)\left(E[\overline{\mathbf{q}}_0] - \overline{\mathbf{q}}_0\right) - \left(\alpha\mathbf{\Sigma}^+\left(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}\right)^{-1}\mathbf{W}\alpha\mathbf{\Sigma} - \alpha\mathbf{\Sigma}^+\mathbf{B}\right)\right. \\
&\quad \times \left(E[\overline{\mathbf{q}}_0] - E\left[\mathbf{q}^i_0\right]\right) \\
&\quad + \left(2\mathbf{W}\alpha\mathbf{\Sigma} - \alpha\mathbf{\Sigma}^+\mathbf{B}\right)\left(\overline{\mathbf{q}}_0 - \mathbf{q}^i_0\right)\Big)\cdot\left(\left(\left(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}\right)^{-1}\mathbf{W}\alpha\mathbf{\Sigma} - \mathbf{B}\right)\left(E[\overline{\mathbf{q}}_0] - E\left[\mathbf{q}^i_0\right]\right)\right. \\
&\quad \left. + \mathbf{B}\left(\overline{\mathbf{q}}_0 - \mathbf{q}^i_0\right)\right).
\end{aligned} \tag{S29}$$

Taking an expectation of the ex post payoff (S29) with respect to $\{\mathbf{q}^j_0\}_j$, and using that trace satisfies $E[\mathbf{x}'\mathbf{M}\mathbf{x}] = E[\mathrm{tr}(\mathbf{x}\mathbf{x}'\mathbf{M})] = \mathrm{tr}(E[\mathbf{x}\mathbf{x}']\mathbf{M}) = \mathrm{tr}(\mathrm{Var}[\mathbf{x}]\mathbf{M}) + E[\mathbf{x}']\mathbf{M}E[\mathbf{x}]$ for any $\mathbf{x} \in \mathbb{R}^K$ and $\mathbf{M} \in \mathbb{R}^{K\times K}$, the ex ante equilibrium payoff of trader i is

$$\begin{aligned}
E\left[u^i - \mathbf{p}\cdot\mathbf{q}^i\right] = {} & \underbrace{E\left[\boldsymbol{\delta}\cdot\mathbf{q}^i_0 - \frac{1}{2}\mathbf{q}^i_0\cdot\alpha\mathbf{\Sigma}\mathbf{q}^i_0\right]}_{\text{Payoff without trade}} + \underbrace{\left(E[\overline{\mathbf{q}}_0] - E\left[\mathbf{q}^i_0\right]\right)\cdot\mathbf{Y}^+(\mathbf{\Lambda})\left(E[\overline{\mathbf{q}}_0] - E\left[\mathbf{q}^i_0\right]\right)}_{\text{Equilibrium surplus from trade}} \\
& + \underbrace{\frac{I-1}{I}\sigma_{pv}\,\mathrm{tr}\left(\alpha\mathbf{\Sigma}\mathbf{W}'\mathbf{B} - \frac{1}{2}\mathbf{B}'\alpha\mathbf{\Sigma}^+\mathbf{B}\right)}_{\text{Payoff term due to } \mathrm{Var}[\overline{\mathbf{q}}_0|\mathbf{q}^i_0] > 0},
\end{aligned} \tag{S30}$$

where $\mathbf{Y}^+(\mathbf{\Lambda}) \in \mathbb{R}^{(\sum_n K(n))\times(\sum_n K(n))}$ is a surplus matrix, which is a function of price impact:

$$\mathbf{Y}^+(\mathbf{\Lambda}) \equiv \alpha\mathbf{\Sigma}\mathbf{W}'\left(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}\right)^{-1}\mathbf{W}\alpha\mathbf{\Sigma} - \frac{1}{2}\alpha\mathbf{\Sigma}\mathbf{W}'\left(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}'\right)^{-1}\alpha\mathbf{\Sigma}^+\left(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}\right)^{-1}\mathbf{W}\alpha\mathbf{\Sigma}. \tag{S31}$$

First, applying the Woodbury Matrix Identity (Lemma S1) to matrix $(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda})^{-1}$ in Eq. (S31), the surplus matrix $\mathbf{Y}^+(\mathbf{\Lambda})$ can be written as

$$\begin{aligned}
\mathbf{Y}^+(\mathbf{\Lambda}) &= \alpha\mathbf{\Sigma}\mathbf{W}'\mathbf{\Lambda}^{-1}\mathbf{W}\left(\widehat{\mathbf{\Lambda}}^{-1} + (\alpha\mathbf{\Sigma})^{-1}\right)^{-1} \\
&\qquad - \frac{1}{2}\left(\left(\widehat{\mathbf{\Lambda}}^{-1}\right)' + (\alpha\mathbf{\Sigma})^{-1}\right)^{-1}\mathbf{W}'\left(\mathbf{\Lambda}^{-1}\right)'\alpha\mathbf{\Sigma}^+\mathbf{\Lambda}^{-1}\mathbf{W}\left(\widehat{\mathbf{\Lambda}}^{-1} + (\alpha\mathbf{\Sigma})^{-1}\right)^{-1} \\
&= \alpha\mathbf{\Sigma}\left(\alpha\mathbf{\Sigma} + \widehat{\mathbf{\Lambda}}'\right)^{-1}\left(\frac{1}{2}\alpha\mathbf{\Sigma} + \widehat{\mathbf{\Lambda}}'\right)\left(\alpha\mathbf{\Sigma} + \widehat{\mathbf{\Lambda}}\right)^{-1}\alpha\mathbf{\Sigma}.
\end{aligned} \tag{S32}$$

Therefore, the equilibrium surplus from trade in Eq. (S30),

$$\big(E[\overline{\mathbf{q}}_0]-E[\mathbf{q}_0^i]\big)\cdot\mathbf{Y}(\widehat{\boldsymbol{\Lambda}})\big(E[\overline{\mathbf{q}}_0]-E[\mathbf{q}_0^i]\big),$$

is determined as a function of the per-unit price impact $\widehat{\boldsymbol{\Lambda}}$, where

$$\mathbf{Y}(\widehat{\boldsymbol{\Lambda}})\equiv\frac{1}{2}\alpha\boldsymbol{\Sigma}\big(\alpha\boldsymbol{\Sigma}+\widehat{\boldsymbol{\Lambda}}'\big)^{-1}\big(\alpha\boldsymbol{\Sigma}+\widehat{\boldsymbol{\Lambda}}+\widehat{\boldsymbol{\Lambda}}'\big)(\alpha\boldsymbol{\Sigma}+\widehat{\boldsymbol{\Lambda}})^{-1}\alpha\boldsymbol{\Sigma}. \tag{S33}$$

Equation (S33) replaces the matrix $(\frac{1}{2}\alpha\boldsymbol{\Sigma}+\widehat{\boldsymbol{\Lambda}}')$ in Eq. (S32) with its symmetric component $\frac{1}{2}(\alpha\boldsymbol{\Sigma}+\widehat{\boldsymbol{\Lambda}}+\widehat{\boldsymbol{\Lambda}}')$. Replacing $\mathbf{Y}^{+}(\boldsymbol{\Lambda})$ (Eq. (S32)) by $\mathbf{Y}(\boldsymbol{\Lambda})$ (Eq. (S30)) is innocuous when evaluating the equilibrium surplus from trade in Eq. (S30), which is a quadratic function of $E[\overline{\mathbf{q}}_0]-E[\mathbf{q}_0^i]$.

Second, we show that matrix $\alpha\boldsymbol{\Sigma}\mathbf{W}'\mathbf{B}-\frac{1}{2}\mathbf{B}'\alpha\boldsymbol{\Sigma}^{+}\mathbf{B}$ in Eq. (S30) is determined as a function of $\widehat{\mathbf{B}}$. By the characterization of $\mathbf{B}$ in Eq. (S19) of Corollary S1 in Appendix A, we have

$$\begin{aligned}\alpha\boldsymbol{\Sigma}\mathbf{W}'\mathbf{B}-\frac{1}{2}\mathbf{B}'\alpha\boldsymbol{\Sigma}^{+}\mathbf{B}&=\alpha\boldsymbol{\Sigma}-\frac{1}{2}\big(\mathbf{B}'\mathbf{W}-\mathbf{Id}\big)\alpha\boldsymbol{\Sigma}\big(\mathbf{W}'\mathbf{B}-\mathbf{Id}\big)\\&=\alpha\boldsymbol{\Sigma}-\frac{1}{2}(\widehat{\mathbf{B}}-\mathbf{Id})'\alpha\boldsymbol{\Sigma}(\widehat{\mathbf{B}}-\mathbf{Id}),\end{aligned} \tag{S34}$$

where the second equality holds by the definition of $\widehat{\mathbf{B}}\equiv\mathbf{W}'\mathbf{B}$ (Eq. (49)). It follows that trader i's ex ante equilibrium payoff (S30) is determined as a function of $(\widehat{\boldsymbol{\Lambda}},\widehat{\mathbf{B}})$:

$$\begin{aligned}E\big[u^i-\mathbf{p}\cdot\mathbf{q}^i\big]&=E\bigg[\boldsymbol{\delta}\cdot\mathbf{q}_0^i-\frac{1}{2}\mathbf{q}_0^i\cdot\alpha\boldsymbol{\Sigma}\mathbf{q}_0^i\bigg]+\big(E[\overline{\mathbf{q}}_0]-E[\mathbf{q}_0^i]\big)\cdot\mathbf{Y}(\widehat{\boldsymbol{\Lambda}})\big(E[\overline{\mathbf{q}}_0]-E[\mathbf{q}_0^i]\big)\\&\quad+\frac{I-1}{I}\sigma_{pv}\operatorname{tr}(\alpha\boldsymbol{\Sigma})-\frac{I-1}{2I}\sigma_{pv}\operatorname{tr}\big((\widehat{\mathbf{B}}-\mathbf{Id})'\alpha\boldsymbol{\Sigma}(\widehat{\mathbf{B}}-\mathbf{Id})\big).\end{aligned} \tag{S35}$$

We now show that equilibrium payoff (S35) of each trader coincides between market structures N and N' if and only if $\widehat{\boldsymbol{\Lambda}}$ and $\widehat{\mathbf{B}}$ coincide between N and N'.

(If and only if: $0\leq|\rho_{k\ell}|<1$ for all k and $\ell\neq k$). We first assume that no assets are perfectly correlated, that is, $\boldsymbol{\Sigma}$ is invertible. Then, in Eq. (S33), the per-unit price impact $\widehat{\boldsymbol{\Lambda}}$ is one-to-one with $\mathbf{Y}(\widehat{\boldsymbol{\Lambda}})$, while in Eq. (S34), cross-asset inference $\widehat{\mathbf{B}}$ is one-to-one with $\alpha\boldsymbol{\Sigma}\mathbf{W}'\mathbf{B}-\frac{1}{2}\mathbf{B}'\alpha\boldsymbol{\Sigma}^{+}\mathbf{B}$. Consequently, the per-unit price impact $\widehat{\boldsymbol{\Lambda}}\in\mathbb{R}^{K\times K}$ is the sufficient statistic for the surplus matrix $\mathbf{Y}(\widehat{\boldsymbol{\Lambda}})$, while cross-asset inference $\widehat{\mathbf{B}}\in\mathbb{R}^{K\times K}$ is the sufficient statistic for the payoff term due to $\operatorname{Var}[\overline{\mathbf{q}}_0|\mathbf{q}_0^i]$ in Eq. (S30).

(If and only if: $|\rho_{k\ell}|=1$ for some k and $\ell\neq k$). Suppose that the payoffs of some assets are perfectly correlated, that is, $\boldsymbol{\Sigma}$ is singular. When the asset payoffs of assets k and $\ell\neq k$ are perfectly correlated, the equilibrium coincides with that in which assets k and ℓ are treated as the same asset, that is, the asset payoffs are defined by $(r_m)_{m\neq\ell}\in\mathbb{R}^{K-1}$ that is jointly Normally distributed according to $\mathcal{N}(\boldsymbol{\delta}^{-},\boldsymbol{\Sigma}^{-})$, where $\boldsymbol{\delta}^{-}\in\mathbb{R}^{K-1}$ and $\boldsymbol{\Sigma}^{-}\in\mathbb{R}^{(K-1)\times(K-1)}$. Given trader i's endowment $\mathbf{q}_0^i\in\mathbb{R}^K$, his endowment in $\mathbb{R}^{K-1}$ is $\mathbf{q}_0^{i,-}\equiv(q_{0,m}^{i,-})_m\in\mathbb{R}^{K-1}$ such that $q_{0,k}^{i,-}=q_{0,k}^i+\operatorname{sign}(\rho_{k\ell})\frac{\sigma_{kk}}{\sigma_{\ell\ell}}q_{0,\ell}^i$ and $q_{0,m}^{i,-}=q_{0,m}^i$ for all $m\neq\ell,k$.

The same argument as for the case $0\leq|\rho_{k\ell}|<1$, $\ell\neq k$, applies with endowments defined in $\mathbf{R}^{(K-1)\times(K-1)}$ rather than $\mathbf{R}^{K\times K}$: Equilibrium payoff (S29) is a function of $\boldsymbol{\Sigma}E[\mathbf{q}_0^i]=$

$\mathbf{W}^{-}\mathbf{\Sigma}^{-}E[\mathbf{q}_0^{i,-}]$ and $\mathbf{\Sigma}\mathbf{q}_0^i = \mathbf{W}^{-}\mathbf{\Sigma}^{-}\mathbf{q}_0^{i,-}$, where the ℓth row $W_\ell^- = (w_{\ell m}^-)_{m\neq\ell}$ of $\mathbf{W}^- \in \mathbb{R}^{K\times(K-1)}$ is such that $w_{\ell k}^- = \text{sign}(\rho_{k\ell})\frac{\sigma_{\ell\ell}}{\sigma_{kk}}$ and $w_{\ell m}^- = 0$ for all $m \neq k$, and the $(K-1)\times(K-1)$ submatrix of $\mathbf{W}^-$ excluding the ℓth row is the identity matrix. Then, the trade of asset k (and zero trade of asset ℓ) in the market with $K-1$ assets is the same as the total trade for assets k and ℓ, defined by $\widehat{q}_k^i = q_k^i + \text{sign}(\rho_{k\ell})\frac{\sigma_{kk}}{\sigma_{\ell\ell}} q_\ell^i$ in the market with K assets.

(Part (2)). We show that $\widehat{\mathbf{\Lambda}}$ maps one-to-one to $\widehat{\mathbf{B}}$ if and only if $\mathbf{\Lambda}$ is symmetric, that is, $\mathbf{\Lambda} = \mathbf{\Lambda}'$. Then, $\widehat{\mathbf{\Lambda}}$ is a sufficient statistic for equilibrium payoffs (S30).

By Eq. (49), the per-unit cross-asset inference $\widehat{\mathbf{B}}$ is characterized as follows:

$$\begin{aligned}\widehat{\mathbf{B}} &= \mathbf{W}'\big((1-\sigma_0)\big(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}\big) + \sigma_0(I-1)\mathbf{\Lambda}'\big)^{-1}\mathbf{W}\alpha\mathbf{\Sigma} \\ &= \Big((1-\sigma_0)\alpha\mathbf{\Sigma} + \Big(\mathbf{W}'\big((1-\sigma_0)\mathbf{\Lambda} + \sigma_0(I-1)\mathbf{\Lambda}'\big)^{-1}\mathbf{W}\Big)^{-1}\Big)^{-1}\alpha\mathbf{\Sigma}. \end{aligned} \tag{S36}$$

The second equality holds by applying the Woodbury Matrix Identity (Lemma S1) to $((1-\sigma_0)\alpha\mathbf{\Sigma}^+ + (1-\sigma_0)\mathbf{\Lambda} + \sigma_0(I-1)\mathbf{\Lambda}')^{-1}$. Given the invertibility of $\mathbf{\Sigma}$, Eq. (S36) shows that $\widehat{\mathbf{B}}$ maps one-to-one to $\mathbf{W}'((1-\sigma_0)\mathbf{\Lambda} + \sigma_0(I-1)\mathbf{\Lambda}')^{-1}\mathbf{W}$, which is a function of $\widehat{\mathbf{\Lambda}} = (\mathbf{W}'\mathbf{\Lambda}\mathbf{W})^{-1}$ if and only if $\mathbf{\Lambda} = \mathbf{\Lambda}'$:

$$\mathbf{W}'\big((1-\sigma_0)\mathbf{\Lambda} + \sigma_0(I-1)\mathbf{\Lambda}'\big)^{-1}\mathbf{W} = \frac{1}{1+(I-2)\sigma_0}\widehat{\mathbf{\Lambda}}^{-1} \quad \text{if and only if} \quad \mathbf{\Lambda} = \mathbf{\Lambda}'.$$

Hence, the sufficient statistic $(\widehat{\mathbf{\Lambda}}, \widehat{\mathbf{B}})$ of the equilibrium payoffs reduces to a single variable $\widehat{\mathbf{\Lambda}}$ or $\widehat{\mathbf{B}}$ if and only if $\mathbf{\Lambda} = \mathbf{\Lambda}'$. *Q.E.D.*

PROOF OF THEOREM 4: (NONREDUNDANCY OF CHANGES IN MARKET STRUCTURE: CONDITIONS). Suppose that $K > 1$ and $|\rho_{k\ell}| < 1$ for all k and $\ell \neq k$. By the same argument as in the proof of Proposition 4, it is without loss of generality to treat the perfectly correlated assets as the same asset. Given a market structure $N = \{K(n)\}_n$, let $\mathbf{\Lambda}^N$ be the equilibrium price impact. Suppose that an exchange n' is introduced such that $K(n') \subset K(n)$ for some $n \in N$ and define $N' \equiv N \cup \{n'\}$. Indicator matrices $\mathbf{W}^N$ and $\mathbf{W}^{N'}$ represent market structures N and N' (Definition 6), respectively.

(Part "If"). We show that, when one of conditions (i)–(iii) holds, the equilibrium payoffs in market N and N' coincide. By Proposition 4, it suffices to show that the equilibrium price impact $\mathbf{\Lambda}^{N'}$ in market $N' \equiv N \cup \{n'\}$ satisfies $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^N$ and $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$. Given the equilibrium price impact $\mathbf{\Lambda}^N$ in market N, we will first construct a block-diagonal matrix $\mathbf{\Lambda}^{N'} \in \mathbb{R}^{(\sum_n K(n)+K(n'))\times(\sum_n K(n)+K(n'))}$ that equalizes the per-unit price impact $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^N$ and cross-asset inference $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$. Then, we will show that such matrix $\mathbf{\Lambda}^{N'}$ is an equilibrium price impact in N'.

(Construction of matrix $\mathbf{\Lambda}^{N'}$ such that $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^N$ and $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$). Define a block-diagonal matrix $\mathbf{\Lambda}^{N'} = \text{diag}(\mathbf{\Lambda}_{K(n'')}^{N'})_{n''\in N'}$ such that

$$\big(\big(\mathbf{\Lambda}_{K(n)}^{N'}\big)^{-1}\big)_{\ell,m} = \big(\big(\mathbf{\Lambda}_{K(n)}^{N}\big)^{-1}\big)_{\ell,m} \quad \forall \ell, m \in K(n) \text{ and } \{\ell, m\} \not\subset K\big(n'\big), \tag{S37}$$

$$\begin{aligned}\big(\big(\mathbf{\Lambda}_{K(n')}^{N'}\big)^{-1}\big)_{\ell,m} &= \xi\big(\big(\mathbf{\Lambda}_{K(n)}^{N}\big)^{-1}\big)_{\ell,m}, \big(\big(\mathbf{\Lambda}_{K(n)}^{N'}\big)^{-1}\big)_{\ell,m} \\ &= (1-\xi)\big(\big(\mathbf{\Lambda}_{K(n)}^{N}\big)^{-1}\big)_{\ell,m} \quad \forall \ell, m \in K\big(n'\big), \end{aligned} \tag{S38}$$

$$\mathbf{\Lambda}_{K(n'')}^{N'} = \mathbf{\Lambda}_{K(n'')}^{N} \quad \forall n'' \neq n, n', \tag{S39}$$

for some $\xi \in (0,1)$ subject to $\mathbf{\Lambda}^{N'}_{K(n')} > 0$ and $\mathbf{\Lambda}^{N'}_{K(n)} > 0$. This implies that each trader's demand coefficient $\mathbf{C}^N_{K(n)} = \frac{1}{I-1}((\mathbf{\Lambda}^N_{K(n)})^{-1})'$ in exchange n of market N is a linear function of $\mathbf{C}^{N'}_{K(n)}$ and $\mathbf{C}^{N'}_{K(n')}$ in exchanges n and n' of market N'. Moreover, demands in other exchanges $n'' \neq n, n'$ coincide between markets N and N'.

First, $\mathbf{\Lambda}^{N'}$ defined in Eqs. (S37)–(S39) satisfies $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^N$ and $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$. By construction, $\widehat{\mathbf{\Lambda}}^{N'} = ((\mathbf{W}^{N'})'(\mathbf{\Lambda}^{N'})^{-1}\mathbf{W}^{N'})^{-1}$ is the same as the per-unit price impact $\widehat{\mathbf{\Lambda}}^N = ((\mathbf{W}^N)'(\mathbf{\Lambda}^N)^{-1}\mathbf{W}^N)^{-1}$ in N when the indicator matrix in N' is

$$\mathbf{W}^{N'} = \begin{bmatrix} \mathbf{W}^N \\ \mathbf{W}_{K(n')} \end{bmatrix}.$$

In addition, when one of conditions (i)–(iii) holds, the cross-asset inference coincides, that is, $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$. By Eq. (49), $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^N$ if and only if $\mathbf{\Lambda}^{N'}$ and $\mathbf{\Lambda}^N$ satisfy

$$\begin{aligned} &(\mathbf{W}^{N'})'\big((1-\sigma_0)\mathbf{\Lambda}^{N'} + \sigma_0(I-1)(\mathbf{\Lambda}^{N'})'\big)^{-1}\mathbf{W}^{N'} \\ &\quad = (\mathbf{W}^N)'\big((1-\sigma_0)\mathbf{\Lambda}^N + \sigma_0(I-1)(\mathbf{\Lambda}^N)'\big)^{-1}\mathbf{W}^N. \end{aligned} \tag{S40}$$

Because the trader's demands in exchanges $n'' \neq n, n'$ coincide between markets N and N' (Eq. (S39)), Eq. (S40) simplifies to an equation for price impacts in exchanges n and n' alone:

$$\sum_{n'' \in \{n,n'\}} (\mathbf{W}^{N'}_{K(n'')})'\big(\mathbf{\Lambda}^{N'}_{K(n'')} + \kappa(\mathbf{\Lambda}^{N'}_{K(n'')})'\big)^{-1}\mathbf{W}^{N'}_{K(n'')} = (\mathbf{W}^N_{K(n)})'\big(\mathbf{\Lambda}^N_{K(n)} + \kappa(\mathbf{\Lambda}^N_{K(n)})'\big)^{-1}\mathbf{W}^N_{K(n)}, \tag{S41}$$

where $\kappa \equiv \frac{\sigma_0(I-1)}{1-\sigma_0} \in \mathbb{R}_+$. When $K(n') = K(n'')$ (condition (i)), Eq. (S41) holds because both $(\mathbf{\Lambda}^{N'}_{K(n)} + \kappa(\mathbf{\Lambda}^{N'}_{K(n)})')^{-1}$ and $(\mathbf{\Lambda}^{N'}_{K(n')} + \kappa(\mathbf{\Lambda}^{N'}_{K(n')})')^{-1}$ are proportional to $(\mathbf{\Lambda}^N_{K(n)} + \kappa(\mathbf{\Lambda}^N_{K(n)})')^{-1}$. When the payoff of assets $K(n')$ is independent of other assets in exchange n, $K(n) \setminus K(n')$ (condition (iii)), the demand coefficient $\mathbf{C}^N_{K(n)} = \operatorname{diag}(\mathbf{C}^N_{K(n)\setminus K(n')}, \mathbf{C}^N_{K(n')})$ is a block-diagonal matrix, and so are $(\mathbf{\Lambda}^{N'}_{K(n)} + \kappa(\mathbf{\Lambda}^{N'}_{K(n)})')^{-1}$ and $(\mathbf{\Lambda}^N_{K(n)} + \kappa(\mathbf{\Lambda}^N_{K(n)})')^{-1}$ in Eq. (S41). Applying the same argument as in condition (i) to all block-diagonal submatrices that correspond to $K(n')$ and $K(n) \setminus K(n')$ shows that Eq. (S41) holds. Last, when $\mathbf{\Lambda}^N_{K(n)}$ is symmetric (condition (ii)), both $\mathbf{\Lambda}^{N'}_{K(n)}$ and $\mathbf{\Lambda}^{N'}_{K(n')}$ are symmetric by construction (Eqs. (S37)–(S38)), and Eq. (S41) holds:

$$\sum_{n'' \in \{n,n'\}} \frac{1}{1+\kappa}(\mathbf{W}^{N'}_{K(n'')})'(\mathbf{\Lambda}^{N'}_{K(n'')})^{-1}\mathbf{W}^{N'}_{K(n'')} = \frac{1}{1+\kappa}(\mathbf{W}^N_{K(n)})'(\mathbf{\Lambda}^N_{K(n)})^{-1}\mathbf{W}^N_{K(n)}.$$

(Simplifying the equilibrium fixed point with $\widehat{\mathbf{\Lambda}}$ and $\widehat{\mathbf{B}}$). We now show that $\mathbf{\Lambda}^{N'}$ defined in Eqs. (S37)–(S39) is an equilibrium price impact in $N' = N \cup \{n'\}$. We first simplify equilibrium fixed point (S19)–(S21) by decomposing the terms that coincide between market N and N' (Eq. (S44) below). Applying the Woodbury Matrix Identity to $\mathbf{B}^{N'}$ gives

$$\mathbf{B}^{N'} = \frac{1}{1-\sigma_0}\big((1-\sigma_0)\mathbf{\Lambda}^{N'} + \sigma_0(I-1)(\mathbf{\Lambda}^{N'})'\big)^{-1}\mathbf{W}^{N'}\big(\widehat{\mathbf{\Phi}} + ((1-\sigma_0)\alpha\mathbf{\Sigma})^{-1}\big)^{-1}, \tag{S42}$$

where $\widehat{\mathbf{\Phi}} \equiv (\mathbf{W}^{N'})'((1-\sigma_0)\mathbf{\Lambda}^{N'} + \sigma_0(I-1)(\mathbf{\Lambda}^{N'})')^{-1}\mathbf{W}^{N'} \in \mathbb{R}^{K\times K}$ coincides between N and N' (Eq. (S40)). Substituting $\mathbf{B}^{N'}$ into the LHS of the equilibrium fixed point equation (Eq. (S20)):

$$\left[\left(\alpha\mathbf{\Sigma}^{+} + \mathbf{\Lambda}^{N'} - (I-1)\left(\mathbf{\Lambda}^{N'}\right)'\right)\left((1-\sigma_0)\mathbf{\Lambda}^{N'} + \sigma_0(I-1)\left(\mathbf{\Lambda}^{N'}\right)'\right)^{-1}\mathbf{W}^{N'}\widehat{\mathbf{V}}\left(\mathbf{W}^{N'}\right)'\right]_{N'}, \quad \text{(S43)}$$

where $\widehat{\mathbf{V}} \equiv (\widehat{\mathbf{\Phi}} + ((1-\sigma_0)\alpha\mathbf{\Sigma})^{-1})^{-1}\mathbf{\Omega}(\widehat{\mathbf{\Phi}}' + ((1-\sigma_0)\alpha\mathbf{\Sigma})^{-1})^{-1} \in \mathbb{R}^{K\times K}$ represents the covariance of K linearly independent random variables that determines the residual supply intercepts (cf. Eq. (43)). The term $\widehat{\mathbf{V}}$ in Eq. (S43) coincides in N and N'. Equation (S43) further simplifies using $\mathbf{\Lambda}^{N'} - (I-1)(\mathbf{\Lambda}^{N'})' = -\frac{1}{\sigma_0}((1-\sigma_0)\mathbf{\Lambda}^{N'} + \sigma_0(I-1)(\mathbf{\Lambda}^{N'})') + \frac{1}{\sigma_0}\mathbf{\Lambda}^{N'}$:

$$\left[\mathbf{W}^{N'}\left(\alpha\mathbf{\Sigma}\widehat{\mathbf{\Phi}}\widehat{\mathbf{V}} - \frac{1}{\sigma_0}\widehat{\mathbf{V}}\right)\left(\mathbf{W}^{N'}\right)'\right]_{N'}$$
$$+ \frac{1}{\sigma_0}\left((1-\sigma_0)\mathbf{Id} + \sigma_0(I-1)\left(\mathbf{\Lambda}^{N'}\right)'\left(\mathbf{\Lambda}^{N'}\right)^{-1}\right)^{-1}\left[\mathbf{W}^{N'}\widehat{\mathbf{V}}\left(\mathbf{W}^{N'}\right)'\right]_{N'}. \quad \text{(S44)}$$

Equation (S44) is a function of a block-diagonal matrix $((1-\sigma_0)\mathbf{Id} + \sigma_0(I-1)(\mathbf{\Lambda}^{N'})' \times (\mathbf{\Lambda}^{N'})^{-1})^{-1}$ and terms that coincide in markets N and N'. $\mathbf{\Lambda}^{N'}$ is the equilibrium price impact in N' if and only if Eq. (S44) equals $\mathbf{0}$.

($\mathbf{\Lambda}^{N'}$ in Eqs. (S37)–(S39) is an equilibrium price impact in N'). Each block submatrix $\mathbf{\Lambda}^{N'}_{K(n'')}$ of $\mathbf{\Lambda}^{N'}$ satisfies Eq. (S44): For any exchange $n'' \neq n, n'$, $\mathbf{W}^{N'}_{K(n'')} = \mathbf{W}^{N}_{K(n'')}$ and $\mathbf{\Lambda}^{N'}_{K(n'')} = \mathbf{\Lambda}^{N}_{K(n'')}$, and hence, the block submatrix of Eq. (S44) that corresponds to exchange n'' is the same as the corresponding submatrix in N. Therefore,

$$\mathbf{W}^{N'}_{K(n'')}\left(\alpha\mathbf{\Sigma}\widehat{\mathbf{\Phi}}\widehat{\mathbf{V}} - \frac{1}{\sigma_0}\widehat{\mathbf{V}}\right)\left(\mathbf{W}^{N'}_{K(n'')}\right)'$$
$$+ \frac{1}{\sigma_0}\left((1-\sigma_0)\mathbf{Id} + \sigma_0(I-1)\left(\mathbf{\Lambda}^{N'}_{K(n'')}\right)'\left(\mathbf{\Lambda}^{N'}_{K(n'')}\right)^{-1}\right)^{-1}\mathbf{W}^{N'}_{K(n'')}\widehat{\mathbf{V}}\left(\mathbf{W}^{N'}_{K(n'')}\right)' = \mathbf{0}.$$

In exchange n, when the price impact matrix is a symmetric matrix (condition (ii)), we have $((1-\sigma_0)\mathbf{Id} + \sigma_0(I-1)(\mathbf{\Lambda}^{N'}_{K(n)})'(\mathbf{\Lambda}^{N'}_{K(n)})^{-1})^{-1} = ((1-\sigma_0)\mathbf{Id} + \sigma_0(I-1)(\mathbf{\Lambda}^{N}_{K(n)})'(\mathbf{\Lambda}^{N}_{K(n)})^{-1})^{-1} = \frac{1}{1+(I-2)\sigma_0}\mathbf{Id}$. This shows that the block submatrix in Eq. (S44) corresponding to $K(n)$ equals to $\mathbf{0}$ in N', given that $\mathbf{\Lambda}^{N}_{K(n)}$ is the equilibrium price impact in exchange n in market N. If condition (iii) holds, $\mathbf{\Lambda}^{N'}_{K(n)}$ is a block-diagonal matrix, whose each block submatrix corresponds to assets $K(n')$ and $K(n)\setminus K(n')$. By construction, the block submatrix of $\mathbf{\Lambda}^{N'}_{K(n)}$ is proportional to the corresponding submatrix of $\mathbf{\Lambda}^{N}_{K(n)}$, and hence, the block submatrix in Eq. (S44) for exchange n equals to $\mathbf{0}$. The same argument applies to condition (i). Finally, in the new exchange n', Eq. (S44) is equivalent to the $K(n') \times K(n')$ submatrix of $(\alpha\mathbf{\Sigma}\widehat{\mathbf{\Phi}}\widehat{\mathbf{V}} - \frac{1}{\sigma_0}\widehat{\mathbf{V}})_{K(n),K(n)} + \frac{1}{\sigma_0}\frac{1}{1+(I-2)\sigma_0}\widehat{\mathbf{V}}_{K(n),K(n)} = \mathbf{0}$. It follows that $\mathbf{\Lambda}^{N'}$ defined in Eqs. (S37)–(S39) is the equilibrium price impact in N'.

(Part "Only if"). We prove the contrapositive: Suppose that $K(n') \neq K(n'')$ for all $n'' \in N$ (condition (i)) and $0 < |\rho_{k\ell}| < 1$ for some assets $k \in K(n')$ and $\ell \in K(n)\setminus K(n')$ (condition (iii)). We show that if a block-diagonal matrix $\mathbf{\Lambda}^{N'} \in \mathbb{R}^{(\sum_n K(n)+K(n'))\times(\sum_n K(n)+K(n'))}$ satisfies $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^{N}$, then $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^{N}$ generally does not hold unless $\mathbf{\Lambda}^{N}_{K(n)} = (\mathbf{\Lambda}^{N}_{K(n)})'$ for an exchange

n such that $K(n') \subset K(n)$ (condition (ii)). Then, by Proposition 4, introducing new exchange n' in market N is nonredundant.

(Construction of $\mathbf{\Lambda}^{N'}$ *that equalizes per-unit price impact* $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^{N}$*).* We first assume a block-diagonal matrix $\mathbf{\Lambda}^{N'} = \text{diag}(\mathbf{\Lambda}^{N'}_{K(n'')})_{n'' \in N'}$ such that $\widehat{\mathbf{\Lambda}}^{N'} = \widehat{\mathbf{\Lambda}}^{N}$:

$$(\mathbf{W}^{N'})'(\mathbf{\Lambda}^{N'})^{-1}\mathbf{W}^{N'} = (\mathbf{W}^{N})'(\mathbf{\Lambda}^{N})^{-1}\mathbf{W}^{N}. \tag{S45}$$

Given that $K(n') \subset K(n)$ for an existing exchange $n \in N$, the indicator matrix (Definition 6) $\mathbf{W}^{N'}$ in market $N' = N \cup \{n'\}$ can be represented as a function of $\mathbf{W}^{N}$:

$$\mathbf{W}^{N'} = \begin{bmatrix} \mathbf{W}^{N} \\ \mathbf{W}^{N'}_{K(n')} \end{bmatrix} = \begin{bmatrix} \mathbf{W}^{N} \\ [\mathbf{0} \quad \mathbf{Id}]\mathbf{W}^{N} \end{bmatrix}. \tag{S46}$$

This is because $\mathbf{W}^{N'}_{K(n')} \in \mathbb{R}^{K(n') \times K}$ for the new exchange n' is a submatrix of the matrix $\mathbf{W}^{N}$. Replacing $\mathbf{W}^{N'}$ by Eq. (S46) simplifies Eq. (S45) in terms of $\mathbf{W}^{N}$ itself rather than $\mathbf{W}^{N}$ and $\mathbf{W}^{N'}$:

$$(\mathbf{W}^{N})'\big((\mathbf{\Lambda}^{N})^{-1} - (\mathbf{\Lambda}^{N'}_{-n'})^{-1}\big)\mathbf{W}^{N} = (\mathbf{W}^{N})'[\mathbf{0} \quad \mathbf{Id}]'(\mathbf{\Lambda}^{N'}_{K(n')})^{-1}[\mathbf{0} \quad \mathbf{Id}]\mathbf{W}^{N}. \tag{S47}$$

The subscript "$-n'$" denotes the existing exchanges $n'' \neq n'$ in market $N' = N \cup \{n'\}$: that is, $\mathbf{\Lambda}^{N'}_{-n'} = \text{diag}(\mathbf{\Lambda}^{N'}_{K(n'')})_{n'' \neq n'}$.

($\mathbf{\Lambda}^{N'}$ *does not satisfy* $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^{N}$*).* We now show that $\mathbf{\Lambda}^{N'}$ that satisfies Eq. (S47) does not satisfy $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^{N}$ unless one of conditions (i)–(iii) holds. The same argument near Eq. (S41) shows that $\widehat{\mathbf{B}}^{N'} = \widehat{\mathbf{B}}^{N}$ holds if and only if the following equation holds:

$$\begin{aligned}(\mathbf{W}^{N})'\big((\mathbf{\Lambda}^{N} + \kappa(\mathbf{\Lambda}^{N})')^{-1} - (\mathbf{\Lambda}^{N'}_{-n'} + \kappa(\mathbf{\Lambda}^{N'}_{-n'})')^{-1}\big)\mathbf{W}^{N} \\ = (\mathbf{W}^{N})'[\mathbf{0} \quad \mathbf{Id}]'\big(\mathbf{\Lambda}^{N'}_{K(n')} + \kappa(\mathbf{\Lambda}^{N'}_{K(n')})'\big)^{-1}[\mathbf{0} \quad \mathbf{Id}]\mathbf{W}^{N}.\end{aligned} \tag{S48}$$

Equalization of the per-unit price impact (Eq. (S47)) gives a *linear* relation between the demand coefficients $\mathbf{C} = \frac{1}{I-1}(\mathbf{\Lambda}^{-1})'$ in N and N': $(\mathbf{W}^{N})'(\mathbf{C}^{N'}_{-n'})'\mathbf{W}^{N} = \frac{1}{I-1}(\mathbf{W}^{N})'(\mathbf{\Lambda}^{N'}_{-n'})^{-1}\mathbf{W}^{N}$ is a linear function of $(\mathbf{W}^{N})'\mathbf{C}^{N}\mathbf{W}^{N}$ and $(\mathbf{W}^{N})'[\mathbf{0} \quad \mathbf{Id}]'\mathbf{C}^{N'}_{K(n')}[\mathbf{0} \quad \mathbf{Id}](\mathbf{W}^{N})$ (Eq. (S47)). However, given that $\mathbf{C} = \frac{1}{I-1}(\mathbf{\Lambda}^{-1})'$, Eq. (S48) equalizes the *harmonic* means of $\mathbf{C}^{N'}_{-n'}$ and $(\mathbf{C}^{N'}_{-n'})'$ with the sum of the *harmonic* means of $\{\mathbf{C}^{N}, (\mathbf{C}^{N})'\}$ and $\{\mathbf{C}^{N'}_{K(n')}, (\mathbf{C}^{N'}_{K(n')})'\}$.

Using the different relations—linear and harmonic mean—between the inverses of price impacts $(\mathbf{\Lambda}^{N})^{-1}$ and $(\mathbf{\Lambda}^{N'})^{-1}$ in Eqs. (S47)–(S48), we show that if Eq. (S47) holds, then Eq. (S48) generally does not hold. The RHS of Eq. (S48) has zero elements for all $k, \ell \in K$, unless assets k and ℓ are both traded in the new exchange n', that is, $\{k, \ell\} \subset K(n')$. If Eq. (S48) holds, then the LHS of Eq. (S48) must have zero elements for all k and ℓ such that $\{k, \ell\} \not\subset K(n')$. By applying the Woodbury Matrix Identity to $(\mathbf{\Lambda}^{N} + \kappa(\mathbf{\Lambda}^{N})')^{-1}$ and $(\mathbf{\Lambda}^{N'}_{-n'} + \kappa(\mathbf{\Lambda}^{N'}_{-n'})')^{-1}$, the LHS of Eq. (S48) can be represented as

$$\begin{aligned}(\mathbf{W}^{N})'(\mathbf{C}^{N} - \mathbf{C}^{N'}_{-n})\mathbf{W}^{N} - \kappa(\mathbf{W}^{N})'\big(\mathbf{C}^{N}\big((\mathbf{C}^{N})^{-1}(\mathbf{C}^{N})' + \kappa\mathbf{Id}\big)^{-1} \\ - \mathbf{C}^{N'}_{-n'}\big((\mathbf{C}^{N'}_{-n'})^{-1}(\mathbf{C}^{N'}_{-n'})' + \kappa\mathbf{Id}\big)^{-1}\big)\mathbf{W}^{N}.\end{aligned} \tag{S49}$$

The first term in Eq. (S49) has zero elements for all k, ℓ such that $\{k, \ell\} \not\subset K(n')$, while the second term in Eq. (S49) has a zero (k, ℓ)th element if and only if

$$\sum_{\{n''\in N|\{k,\ell\}\subset K(n'')\}} \left(\mathbf{C}^N_{K(n'')}\left(\left(\mathbf{C}^N_{K(n'')}\right)^{-1}\left(\mathbf{C}^N_{K(n'')}\right)' + \kappa\mathbf{Id}\right)^{-1} - \mathbf{C}^{N'}_{K(n'')}\left(\left(\mathbf{C}^{N'}_{K(n'')}\right)^{-1}\left(\mathbf{C}^{N'}_{K(n'')}\right)' + \kappa\mathbf{Id}\right)^{-1}\right)_{k\ell}$$
$$= 0. \tag{S50}$$

By Eq. (S47), however, the $\mathbf{C}^{N'}$ that matches the per-unit price impact satisfies

$$\sum_{\{n''\in N|\{k,\ell\}\subset K(n'')\}} \left(c^N_{k\ell,n''} - c^{N'}_{k\ell,n''}\right) = 0.$$

The demand Jacobian $\mathbf{C}^{N'}_{K(n'')}$ in exchange n'' has a nonzero off-diagonal element $c^{N'}_{k\ell,n''}$ except when $[\mathbf{BB}']_{k,K(n'')}$ is proportional to $[\alpha\mathbf{\Sigma}\mathbf{BB}']_{k,K(n'')}$ (see Eq. (S20) in Corollary S1), that is, $c^{N'}_{k\ell,n''} \neq 0$ unless $\sigma_{k\ell} = 0$ (condition (iii)). When condition (iii) does not hold, for Eq. (S50) to hold, for each $n'' \in N$, the demand coefficient $\mathbf{C}^N_{K(n'')}$ must either be the same in market structures N and N', that is, $\mathbf{C}^N_{K(n'')} = \mathbf{C}^{N'}_{K(n'')}$, or must be symmetric, that is, $\mathbf{C}^N_{K(n'')} = (\mathbf{C}^N_{K(n'')})'$ so that $((\mathbf{C}^N_{K(n'')})^{-1}(\mathbf{C}^N_{K(n'')})' + \kappa\mathbf{Id})^{-1} = \frac{1}{1+\kappa}\mathbf{Id}$. The former condition cannot hold for all exchanges n'' such that $K(n') \subsetneq K(n'')$ unless condition (i) holds: When an equilibrium exists, the demand coefficient in the new exchange n' is positive semi-definite $\mathbf{C}^{N'}_{K(n')} > \mathbf{0}$ (i.e., demands are downward-sloping); using Eq. (S47), for each $\{k, \ell\} \subset K(n')$, there exists an exchange n'' such that $\{k, \ell\} \subset K(n') \cap K(n'')$ and $\mathbf{C}^N_{K(n'')} \neq \mathbf{C}^{N'}_{K(n'')}$. Hence, for such an exchange n'', $\mathbf{C}^N_{K(n'')}$ must be symmetric, that is, condition (ii) must hold. *Q.E.D.*

LEMMA S4—Price Equalization Across Exchanges: *Given a market structure* $N = \{K(n)\}_n$, *the equilibrium prices of asset k are the same in the exchanges where k is traded*,

$$p_{k,n} = p_{k,n'} \quad \forall n, \forall n' \neq n \; s.t. \; k \in K(n) \cap K(n') \; \forall k \; \forall \left(\mathbf{q}^i_0\right)_i \in \mathbb{R}^{IK},$$

if and only if price impact $\mathbf{\Lambda}$ *is a symmetric matrix*, *that is*, $\mathbf{\Lambda} = \mathbf{\Lambda}'$.

PROOF OF LEMMA S4 (PRICE EQUALIZATION ACROSS EXCHANGES): Using the indicator matrix $\mathbf{W}$ (Definition 6), we can write the equilibrium price equation (S16) as follows:

$$\mathbf{p} = \boldsymbol{\delta}^+ - \mathbf{W}\alpha\mathbf{\Sigma}E[\overline{\mathbf{q}}_0] - \mathbf{C}^{-1}\mathbf{B}\left(\overline{\mathbf{q}}_0 - E[\overline{\mathbf{q}}_0]\right) = \mathbf{W}\left(\boldsymbol{\delta} - \alpha\mathbf{\Sigma}E[\overline{\mathbf{q}}_0]\right) - \mathbf{C}^{-1}\mathbf{B}\left(\overline{\mathbf{q}}_0 - E[\overline{\mathbf{q}}_0]\right). \tag{S51}$$

Prices for each asset k are the same in all exchanges where k is traded if and only if there exists a price vector $\widehat{\mathbf{p}} \in \mathbb{R}^K$, such that $\mathbf{p} = \mathbf{W}\widehat{\mathbf{p}} = (\mathbf{W}_n\widehat{\mathbf{p}})_n$ for all realizations of endowments $(\mathbf{q}^i_0)_i \in \mathbb{R}^{IK}$. From Eq. (51), the price equalization holds if and only if $\mathbf{C}^{-1}\mathbf{B} \in \mathbb{R}^{(\sum_n K(n))\times K}$ is characterized as $\mathbf{WM}$ for a matrix $\mathbf{M} \in \mathbb{R}^{K\times K}$.

We now show that $\mathbf{C}^{-1}\mathbf{B} = \mathbf{WM}$ if and only if $\mathbf{C}$ is a symmetric matrix (equivalently, $\mathbf{\Lambda} = \frac{1}{I-1}(\mathbf{C}^{-1})'$ is a symmetric matrix). Using demand coefficients $\mathbf{B}$ and $\mathbf{C}$ in Eqs. (S19)–(S20), the price weight matrix coefficient $\mathbf{C}^{-1}\mathbf{B} \in \mathbb{R}^{(\sum_n K(n))\times K}$ in Eq. (51) can be characterized as follows:

$$\mathbf{C}^{-1}\mathbf{B} = \mathbf{C}^{-1}\left((1-\sigma_0)\mathbf{W}\alpha\mathbf{\Sigma}\mathbf{W}' + \frac{1-\sigma_0}{I-1}\left(\mathbf{C}^{-1}\right)' + \sigma_0\mathbf{C}^{-1}\right)^{-1}\mathbf{W}\alpha\mathbf{\Sigma}$$

$$= \left(\frac{1-\sigma_0}{I-1}(\mathbf{C}^{-1})'\mathbf{C} + \sigma_0 \mathbf{Id}\right)^{-1}$$
$$\times \mathbf{W}\left(\mathbf{W}'\left(\frac{1}{I-1}(\mathbf{C}^{-1})' + \frac{\sigma_0}{1-\sigma_0}\mathbf{C}^{-1}\right)^{-1}\mathbf{W} + (\alpha\mathbf{\Sigma})^{-1}\right)^{-1}, \qquad \text{(S52)}$$

where the second equality applies the Woodbury Matrix Identity (Lemma S1) to $((1-\sigma_0)\mathbf{W}\alpha\mathbf{\Sigma}\mathbf{W}' + \frac{1-\sigma_0}{I-1}(\mathbf{C}^{-1})' + \sigma_0\mathbf{C}^{-1})^{-1}$. Equation (S52) shows that $\mathbf{C}^{-1}\mathbf{B} = \mathbf{W}\mathbf{M}$ if and only if $(\frac{1-\sigma_0}{I-1}(\mathbf{C}^{-1})'\mathbf{C} + \sigma_0\mathbf{Id})^{-1}$ is a diagonal matrix whose diagonal elements corresponding to asset k are the same for all exchanges: that is, $(\mathbf{C}^{-1})'\mathbf{C} = \text{diag}(m_{k,n})_{k,n}$, where $m_{k,n} = m_k$ for all n such that $k \in K(n)$. Given $\mathbf{C} > \mathbf{0}$, $m_{k,n} = 1$ for all k and n must hold, so $\mathbf{C} = \mathbf{C}'\,\text{diag}(m_{k,n})_{k,n} = \mathbf{C}'$. We conclude that $\mathbf{C}^{-1}\mathbf{B} = \mathbf{W}\mathbf{M}$ if and only if $\mathbf{C}$ is a symmetric matrix, that is, $\mathbf{C} = \mathbf{C}'$. *Q.E.D.*

PROOF OF COROLLARY 2: (REDUNDANCY OF CHANGES IN MARKET STRUCTURE: A CONDITION ON EXCHANGES). Suppose that $0 < |\rho_{k\ell}| < 1$ for all k and $\ell \neq k$. This assumption is without loss of generality, as shown in the proof of Proposition 4.

(Part (ii) $\Leftrightarrow \widehat{\mathbf{\Lambda}} = \mathbf{\Lambda}^c$ *and* $\widehat{\mathbf{B}} = \mathbf{B}^c$*).* We show that the equilibrium in the market with exchanges N is ex post if and only if traders' equilibrium payoffs are the same as in the contingent market: The equilibrium price and trades are characterized as a function of price impact $\mathbf{\Lambda}$:

$$\mathbf{p} = \boldsymbol{\delta}^+ - \big(\mathbf{W}\alpha\mathbf{\Sigma} - \mathbf{C}^{-1}\mathbf{B}\big)E[\overline{\mathbf{q}}_0] - \mathbf{C}^{-1}\mathbf{B}\overline{\mathbf{q}}_0,$$
$$\mathbf{q}^i = \big((\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda})^{-1}\mathbf{W}\alpha\mathbf{\Sigma} - \mathbf{B}\big)\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big) + \mathbf{B}\overline{\mathbf{q}}_0 - \mathbf{B}\mathbf{q}_0^i.$$

The equilibrium is ex post if and only if $\mathbf{B} = (\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda})^{-1}\mathbf{W}\alpha\mathbf{\Sigma}$ and $\mathbf{C}^{-1}\mathbf{B} = \mathbf{W}\alpha\mathbf{\Sigma}$ so that the equilibrium price and total trades are independent of the distribution of endowments. Applying the Woodbury Matrix Identity (Lemma S1) to $(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda})^{-1}$ and $\mathbf{B} = ((1-\sigma_0)(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda}) + \sigma_0(I-1)\mathbf{\Lambda}')^{-1}\mathbf{W}\alpha\mathbf{\Sigma}$ (Eq. (S19)), the matrix condition $\mathbf{B} = (\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda})^{-1}\mathbf{W}\alpha\mathbf{\Sigma}$ simplifies to

$$(1-\sigma_0)\alpha\mathbf{\Sigma} + \big(\mathbf{W}'\big((1-\sigma_0)\mathbf{\Lambda} + \sigma_0(I-1)\mathbf{\Lambda}'\big)^{-1}\mathbf{W}\big)^{-1} = \alpha\mathbf{\Sigma} + \widehat{\mathbf{\Lambda}}. \qquad \text{(S53)}$$

Equation (S53) holds if and only if $\widehat{\mathbf{\Lambda}} = \frac{\alpha}{I-2}\alpha\mathbf{\Sigma} = \mathbf{\Lambda}^c$ and $\mathbf{\Lambda} = \mathbf{\Lambda}'$. Then, by Eqs. (S19)–(S20), $\widehat{\mathbf{B}} = \frac{I-2}{I-1}\mathbf{Id} = \mathbf{B}^c$ and $\mathbf{C}^{-1}\mathbf{B} = \mathbf{W}\alpha\mathbf{\Sigma}$ also hold.

(Part (ii) $\Leftarrow$ *(iii)).* From (Part (ii) $\Leftrightarrow \widehat{\mathbf{\Lambda}} = \mathbf{\Lambda}^c$ and $\widehat{\mathbf{B}} = \mathbf{B}^c$), an equilibrium in a market with exchanges N is ex post if and only if it is payoff-equivalent to the equilibrium in $N' = \{K\}$. Suppose that for every pair of assets k and $\ell \neq k$ such that $0 < |\rho_{k\ell}| < 1$, these assets are traded in a same exchange in N: that is, $k, \ell \in K(n)$ for some $n \in N$.

We first show that, given market structure N, there exists a symmetric block-diagonal matrix $\mathbf{\Lambda} = \text{diag}(\mathbf{\Lambda}_{K(n)})_n \in \mathbb{R}^{(\sum_n K(n))\times(\sum_n K(n))}$ such that $(\mathbf{W}'\mathbf{\Lambda}^{-1}\mathbf{W})^{-1} = \widehat{\mathbf{\Lambda}}^c = \frac{\alpha}{I-2}\mathbf{\Sigma}$, which we then show is the equilibrium price impact in market N. Given that $\mathbf{W}$ is the indicator matrix of market N, the (k,ℓ)th element of $\mathbf{W}'\mathbf{\Lambda}^{-1}\mathbf{W} = (I-1)\mathbf{W}'\mathbf{C}'\mathbf{W}$ is the sum of demand coefficients $\sum_{\{n|k,\ell\in K(n)\}} c_{\ell k,n}$. Because condition (iii) implies that $\{n|k,\ell \in K(n)\} \neq \emptyset$, $\sum_{\{n|k,\ell\in K(n)\}} c_{\ell k,n} \neq 0$ for any k and $\ell \neq k$ except when $\rho_{k\ell} = 0$ (Proposition S2). Matching each element of $\mathbf{W}'\mathbf{C}'\mathbf{W}$ and $(\mathbf{C}^c)'$ gives the system of K^2 equations for $\sum_n (K(n))^2$ variables $\{\{c_{\ell k,n}\}_{k,\ell}\}_n$. Given $\sum_n (K(n))^2 \geq K^2$, there exist $\{c_{\ell k,n}\}_{\{n|k,\ell\in K(n)\}}$ such that $\sum_{\{n|k,\ell\in K(n)\}} c_{\ell k,n} = c^c_{\ell k}$ for all k and $\ell \neq k$. Moreover, when $\mathbf{\Lambda}$ satisfies $(\mathbf{W}'\mathbf{\Lambda}^{-1}\mathbf{W})^{-1} = \widehat{\mathbf{\Lambda}}^c$, so does its symmetric counterpart $\frac{1}{2}(\mathbf{\Lambda}+\mathbf{\Lambda}')$, because $\widehat{\mathbf{\Lambda}}^c$ is symmetric. It follows that there exists a symmetric matrix $\mathbf{\Lambda} = \mathbf{\Lambda}'$ that satisfies $(\mathbf{W}'\mathbf{\Lambda}^{-1}\mathbf{W})^{-1} = \widehat{\mathbf{\Lambda}}^c$.

We now argue that a symmetric matrix $\mathbf{\Lambda}$ such that $(\mathbf{W}'\mathbf{\Lambda}^{-1}\mathbf{W})^{-1} = \frac{\alpha}{I-2}\mathbf{\Sigma}$ is the equilibrium price impact. We show that it satisfies equilibrium fixed point Eqs. (S20)–(S21), that is,

$$\big[\big(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda} - (I-1)\mathbf{\Lambda}'\big)\mathbf{B}\mathbf{\Omega}\mathbf{B}'\big]_N = \mathbf{0}. \tag{S54}$$

Using $\mathbf{\Lambda} = \mathbf{\Lambda}'$, Eq. (S42) for $\mathbf{B}$ simplifies to

$$\mathbf{B} = \mathbf{\Lambda}^{-1}\mathbf{W}\big(\big(1+(I-2)\sigma_0\big)(\alpha\mathbf{\Sigma})^{-1} + (1-\sigma_0)\widehat{\mathbf{\Lambda}}^{-1}\big)^{-1}. \tag{S55}$$

Substituting for $\mathbf{B}$ from Eq. (S55) to Eq. (S54), we have

$$\begin{aligned}&\big[\mathbf{W}\big(\alpha\mathbf{\Sigma} - (I-2)\widehat{\mathbf{\Lambda}}\big)\big((1-\sigma_0)(\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}) + \sigma_0(I-1)\widehat{\mathbf{\Lambda}}\big)^{-1} \\ &\quad \times \alpha\mathbf{\Sigma}\mathbf{\Omega}\alpha\mathbf{\Sigma}\big((1-\sigma_0)(\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}) + \sigma_0(I-1)\widehat{\mathbf{\Lambda}}\big)^{-1}\mathbf{W}'\big]_N = \mathbf{0}.\end{aligned} \tag{S56}$$

When condition (iii) holds, for any matrix $\mathbf{M} \in \mathbb{R}^{K\times K}$, $[\mathbf{W}\mathbf{M}\mathbf{W}']_N = \mathbf{0}$ if and only if $\mathbf{M} = \mathbf{0}$. This is because $m_{k\ell} = 0$ for all $\ell \neq k$ and k if and only if $(\mathbf{W}\mathbf{M}\mathbf{W}')_{K(n)} = (m_{k\ell})_{k,\ell\in K(n)} = 0$ for all n. This establishes that matrix $\widehat{\mathbf{\Lambda}}$ satisfies Eq. (S56) given market N if and only if $\widehat{\mathbf{\Lambda}}$ satisfies

$$\begin{aligned}&\big(\alpha\mathbf{\Sigma} - (I-2)\widehat{\mathbf{\Lambda}}\big)\big((1-\sigma_0)(\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}) + \sigma_0(I-1)\widehat{\mathbf{\Lambda}}\big)^{-1} \\ &\quad \times \alpha\mathbf{\Sigma}\mathbf{\Omega}\alpha\mathbf{\Sigma}\big((1-\sigma_0)(\alpha\mathbf{\Sigma}+\widehat{\mathbf{\Lambda}}) + \sigma_0(I-1)\widehat{\mathbf{\Lambda}}\big)^{-1} = \mathbf{0}.\end{aligned} \tag{S57}$$

Given the positive definiteness of $\mathbf{\Sigma}$ and $\mathbf{\Omega}$, $\widehat{\mathbf{\Lambda}} = \frac{\alpha}{I-2}\mathbf{\Sigma} = \mathbf{\Lambda}^c$ is the unique matrix that satisfies Eq. (S57). It follows that $\mathbf{\Lambda}$ such that $\widehat{\mathbf{\Lambda}} = \mathbf{\Lambda}^c$, which hence satisfies Eq. (S57), is an equilibrium price impact in N.

(Part (ii) ⇒ (iii)). We prove by contradiction: Suppose that a pair of assets k and $\ell \neq k$ such that $0 < |\rho_{k\ell}| < 1$ is not traded in a same exchange in N: that is, $\{k,\ell\} \not\subset K(n)$ for all $n \in N$. By Proposition 4, the equilibrium payoffs in N coincide with ex post equilibrium payoffs only when $(\mathbf{W}'\mathbf{\Lambda}^{-1}\mathbf{W})^{-1} = \frac{\alpha}{I-2}\mathbf{\Sigma}$, or equivalently, $\mathbf{W}'\mathbf{C}\mathbf{W} = \frac{I-2}{I-1}(\alpha\mathbf{\Sigma})^{-1}$. Following the argument in (Part (ii) ⇐ (iii)), the (k,ℓ)th element of $\mathbf{W}'\mathbf{C}\mathbf{W}$ is zero, because $\{n|k,\ell \in K(n)\} = \emptyset$. This contradicts the equality $\mathbf{W}'\mathbf{C}\mathbf{W} = \frac{I-2}{I-1}(\alpha\mathbf{\Sigma})^{-1}$, because $((\alpha\mathbf{\Sigma})^{-1})_{k\ell} \neq 0$: $(\mathbf{W}'\mathbf{\Lambda}^{-1}\mathbf{W})^{-1} \neq \frac{\alpha}{I-2}\mathbf{\Sigma}$. Therefore, condition (iii) is necessary for ex post equilibrium.

(Part (i) ⇐ (iii)). If condition (iii) holds in a market structure N, an additional exchange n' cannot change the set of conditioning variables in traders' total demands. More precisely, condition (iii) also holds in market structure $N' = N \cup \{n'\}$: for every pair of assets k and $\ell \neq k$ such that $0 < |\rho_{k\ell}| < 1$, there is an exchange n'' in which these assets are traded, that is, $k,\ell \in K(n'')$ for some $n'' \in N'$. By (Part (ii) ⇔ (iii)), the equilibrium payoffs in both N and N' coincide with those in the contingent market, and thus, are the same.

(Part (i) ⇒ (ii)). We prove the contrapositive: if an equilibrium is not ex post, then there exists a new exchange that is not redundant. Consider exchanges N. By the equivalence between (ii) and (iii), there exist imperfectly correlated assets k and $\ell \neq k$ that are not both traded in any exchange, that is, there is no n such that $\{k,\ell\} \subset K(n)$. The

(k,ℓ)th element of the total demand's Jacobian $\widehat{\mathbf{C}} = \mathbf{WCW}'$ is zero, that is, $\widehat{c}_{k\ell} = 0$. Suppose that a new exchange $n' \equiv \{k,\ell\}$ is introduced in the market structure N, that is, $N' = N \cup \{n'\}$. We show that exchange n' is not redundant: The Jacobian $\mathbf{C}^{N'}_{K(n')} \in \mathbb{R}^{2\times 2}$ in exchange n' has a nonzero off-diagonal element $c^{N'}_{k\ell}$ except when $[\mathbf{BB}']_{k,K(n')}$ is proportional to $[\alpha \mathbf{\Sigma} \mathbf{BB}']_{k,K(n')}$ (Eq. (S20) in Corollary S1), that is, $c^{N'}_{k\ell} \neq 0$ unless $\sigma_{k\ell} = 0$. This shows that $\widehat{c}^{N'}_{k\ell} \neq \widehat{c}_{k\ell} = 0$, and hence, the Jacobians of the total demands in N' and N differ: $\widehat{\mathbf{C}}^{N'} = \mathbf{W}^{N'}\mathbf{C}^{N'}(\mathbf{W}^{N'})' \neq \widehat{\mathbf{C}}$. Equivalently, $\widehat{\mathbf{\Lambda}}^{N'} \neq \widehat{\mathbf{\Lambda}}$. By Proposition 4(i), the equilibrium payoffs differ in N and N'. *Q.E.D.*

COROLLARY S2—Nonredundancy of Changes in Market Structure: A Condition on Primitives: *All market structures $\{K(n)\}_n$ give the same equilibrium payoff if and only if the payoffs of all assets are either perfectly correlated or independent.*

PROOF OF COROLLARY S2: (NONREDUNDANCY OF CHANGES IN MARKET STRUCTURE: A CONDITION ON PRIMITIVES). The proof is immediate from the proof of Corollary 2. *Q.E.D.*

PROOF OF PROPOSITION 3: (WELFARE WITH MULTIPLE EXCHANGES VERSUS JOINT MARKET CLEARING). Suppose that there is no inference error: that is, $\sigma_{cv} \to 0$, $\sigma_{pv} \to 0$, and $\sigma_0 \equiv \frac{\sigma_{cv} + \frac{1}{I}\sigma_{pv}}{\sigma_{cv}+\sigma_{pv}} < 1$. For a market structure N with multiple exchanges that is not payoff-equivalent to a single exchange, consider the difference in the equilibrium surplus $U^c - U^N$:

$$U^c - U^N = \sum_i \big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big) \cdot \big(\mathbf{\Upsilon}(\mathbf{\Lambda}^c) - \mathbf{\Upsilon}(\widehat{\mathbf{\Lambda}})\big)\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big), \tag{S58}$$

which, by Proposition 4, is zero if the per-unit price impacts $\mathbf{\Lambda}^c$ and $\widehat{\mathbf{\Lambda}}$ are the same.

The equilibrium surplus difference (Eq. (S58)) is a quadratic matrix function of $E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]$ with a quadratic coefficient of $\mathbf{\Upsilon}(\mathbf{\Lambda}^c) - \mathbf{\Upsilon}(\widehat{\mathbf{\Lambda}})$. If the surplus matrix difference $\mathbf{\Upsilon}(\mathbf{\Lambda}^c) - \mathbf{\Upsilon}(\widehat{\mathbf{\Lambda}})$ has a negative eigenvalue $\mu < 0$, then there exist ex ante trading needs $\{E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\}_i \in \mathbb{R}^{IK}$ such that $U^c - U^N < 0$. Pick a distribution of endowments such that $E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]$ is proportional to an eigenvector of matrix $\mathbf{\Upsilon}(\mathbf{\Lambda}^c) - \mathbf{\Upsilon}(\widehat{\mathbf{\Lambda}})$ (with a positive or a negative proportionality constant) associated with an eigenvalue μ: for all i,

$$\big(\mathbf{\Upsilon}(\mathbf{\Lambda}^c) - \mathbf{\Upsilon}(\widehat{\mathbf{\Lambda}})\big)\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big) = \mu\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big). \tag{S59}$$

Substituting the trading needs vector $\{E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\}_i$ that satisfies Eq. (S59) into Eq. (S58), we have

$$\begin{aligned}&\sum_i \big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big) \cdot \big(\mathbf{\Upsilon}(\mathbf{\Lambda}^c) - \mathbf{\Upsilon}(\widehat{\mathbf{\Lambda}})\big)\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big)\\ &\quad = \sum_i \big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big) \cdot \mu\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big) < 0,\end{aligned}$$

and hence, $U^N > U^c$. Because the difference in equilibrium surplus (S58) is a quadratic function of (and hence continuous with respect to) expected trading needs, $U^N > U^c$ holds for trading needs $\{E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\}_i$ that are sufficiently close to the eigenvector of

$\mathbf{Y}(\mathbf{\Lambda}^c) - \mathbf{Y}(\widehat{\mathbf{\Lambda}})$. Furthermore, given σ_0, the ex ante equilibrium payoff (25) is a linear function of (and hence continuous with respect to) variances $(\sigma_{cv}, \sigma_{pv})$. Therefore, $U^N > U^c$ holds for distribution $F((\mathbf{q}_0^i)_i)$ with sufficiently small $(\sigma_{cv}, \sigma_{pv})$.

Lemma S5 gives a sufficient condition for a negative eigenvalue to exist: any market structure whose exchanges are demergers (Definition 5) of a single venue for all assets. If the surplus matrix difference $\mathbf{Y}(\mathbf{\Lambda}^c) - \mathbf{Y}(\widehat{\mathbf{\Lambda}})$ does not have a negative eigenvalue, then $U^c - U^N \geq 0$ for any distribution of endowments. *Q.E.D.*

LEMMA S5—Price Impacts in Multiple Exchanges Versus Joint Market Clearing: *Let $K > 1$ and $I < \infty$. Consider a market structure $N = \{K(n)\}_n$ that consists of exchanges that partition the set of K assets: $K(n) \cap K(n') = \emptyset$ for all n and $n' \neq n$ and $\bigcup_n K(n) = K$. The equilibrium price impact $\mathbf{\Lambda}$ in N and the price impact in the contingent market $\mathbf{\Lambda}^c$ are not ranked in the positive semi-definite sense, that is, neither $\mathbf{\Lambda} \geq \mathbf{\Lambda}^c$ nor $\mathbf{\Lambda} \leq \mathbf{\Lambda}^c$ holds, except when $\mathbf{\Lambda} = \mathbf{\Lambda}^c$.*

PROOF OF LEMMA S5: (PRICE IMPACTS IN MULTIPLE EXCHANGES VERSUS JOINT MARKET CLEARING). The equilibrium fixed point equation (S20) for the equilibrium price impact $\mathbf{\Lambda} \in \mathbb{R}^{K\times K}$ can be written as follows:

$$\big[\big(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda} - (I-1)\mathbf{\Lambda}'\big)\mathbf{B\Omega B}'\big]_N = 0. \tag{S60}$$

To demonstrate that $\mathbf{\Lambda}^c - \mathbf{\Lambda}$ is neither positive semi-definite nor negative semi-definite, we argue by contradiction: Suppose that $\mathbf{\Lambda}^c - \mathbf{\Lambda} = \frac{\alpha}{I-2}\mathbf{\Sigma}^+ - \mathbf{\Lambda}$ is positive semi-definite. By the Trace Inequality for Matrix Product,[7] the trace of the matrix on the LHS of Eq. (S60) is nonnegative:

$$\begin{aligned}&\operatorname{tr}\big(\big(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda} - (I-1)\mathbf{\Lambda}'\big)\mathbf{B\Omega B}'\big)\\ &\quad\geq (I-2)\mu_K\big(\mathbf{B\Omega B}'\big)\operatorname{tr}\left(\frac{\alpha}{I-2}\mathbf{\Sigma}^+ - \frac{1}{2}\big(\mathbf{\Lambda} + \mathbf{\Lambda}'\big)\right) \geq 0,\end{aligned} \tag{S61}$$

where $\mu_K(\mathbf{M}) \in \mathbb{R}$ is the lowest eigenvalue of matrix $\mathbf{M}$. Because matrix $\mathbf{B\Omega B}'$ is symmetric and positive definite, its lowest eigenvalue is positive, and hence, (S61) holds with equality if and only if $\frac{\alpha}{I-2}\mathbf{\Sigma}^+ = \frac{1}{2}(\mathbf{\Lambda} + \mathbf{\Lambda}')$, or equivalently $\mathbf{\Lambda}^c = \mathbf{\Lambda}$.

Except when $\mathbf{\Lambda}^c = \mathbf{\Lambda}$, however, Eq. (S61) contradicts the equilibrium fixed point equation (S60). Hence, by the definition of operator $[\cdot]_N$, the matrix trace must be zero:

$$\operatorname{tr}\big(\big(\alpha\mathbf{\Sigma}^+ + \mathbf{\Lambda} - (I-1)\mathbf{\Lambda}'\big)\mathbf{B\Omega B}'\big) = 0.$$

An analogous argument shows that $\mathbf{\Lambda}^c - \mathbf{\Lambda}$ is not negative semi-definite except when $\mathbf{\Lambda}^c = \mathbf{\Lambda}$. *Q.E.D.*

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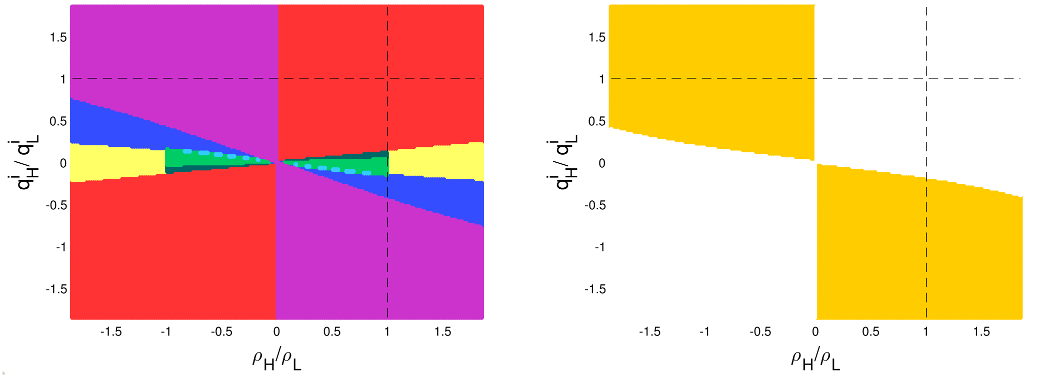
[7]For a real matrix $\mathbf{S} \in \mathbb{R}^{K\times K}$ and a positive semi-definite matrix $\mathbf{T} \in \mathbb{R}^{K\times K}$, the following inequality holds:

$$\mu_K(\mathbf{S})\operatorname{tr}(\mathbf{T}) \leq \operatorname{tr}(\mathbf{ST}) = \operatorname{tr}(\mathbf{TS}) \leq \mu_1(\mathbf{S})\operatorname{tr}(\mathbf{T}),$$

where $\mu_k(\mathbf{S})$ is the kth largest eigenvalue of the Hermitian part $\frac{1}{2}(\mathbf{S} + \mathbf{S}')$.



(A) $\rho_L = 0.2$ (B) $\rho_L = -0.2$

FIGURE S2.—Heterogeneous asset correlations and trading needs. *Notes*: Each color indicates which market structure provides the highest ex ante welfare. Red = $\{\{1\},\{2\},\{3\}\}$ (i.e., the uncontingent market); Orange = $\{\{1\},\{2,3\}\}$; Yellow = $\{\{1,2\},\{3\}\}$; Blue = $\{\{1,2\},\{1,3\}\}$; Light blue = $\{\{1,2\},\{1,3\},\{3\}\}$; Purple = $\{\{1,2\},\{1,3\},\{1\}\}$; Green = $\{\{1,2\},\{2\},\{3\}\}$; Olive = $\{\{1,2\},\{1\},\{3\}\}$; and White = $\{\{1,2,3\}\}$ (i.e., the contingent market). The welfare effect of the inference error is sufficiently small not to dominate the welfare benefit from diversification ($\sigma_{cv} = 0$, $\sigma_{pv} = 0.01$). The number of traders is $I = 10$. The trading needs for assets 2 and 3 are $|E[\overline{q}_{0,L}] - E[q^i_{0,L}]| = 1$ for all i. Panel (A) assumes the asset payoff correlation $\rho_L = 0.2$ (i.e., substitutes), and panel (B) assumes $\rho_L = -0.2$ (i.e., complements).

APPENDIX C: SYMMETRIC MARKETS

C.1. *Additional Results: Symmetric Markets*

This appendix presents results for markets that are symmetric in the following sense.

DEFINITION S1—Symmetric Market: Assume $K = MN$ for some $M \geq 1$. A market structure $N = \{K(n)\}_n$ is *symmetric* if

- asset distribution is symmetric, that is, $\sigma \equiv \text{Var}[r_k]$ for all k and $\rho \equiv \text{Corr}[r_k, r_\ell]$ for all k and $\ell \neq k$, and
- exchanges N partition the set of K assets into exchanges with the same number of assets, that is, $K(n) \cap K(n') = \emptyset$ for all n and $n' \neq n$, and $K(n) = M$ for all n.

For results in this part of the appendix, we assume that traders' endowments are independent across assets: $\mathbf{\Omega} = \mathbf{Id} \in \mathbb{R}^{K\times K}$.

In a symmetric market, the asset covariance is $\mathbf{\Sigma} = \sigma((1-\rho)\mathbf{Id} + \rho\mathbf{1}\mathbf{1}')$ and the price impact matrix $\mathbf{\Lambda} = \text{diag}(\mathbf{\Lambda}_{K(n)})_n \in \mathbb{R}^{K\times K}$ is symmetric across exchanges and assets and can be written as follows: for all n,

$$\mathbf{\Lambda}_{K(n)} = (\lambda_k - \lambda_{k\ell})\mathbf{Id} + \lambda_{k\ell}\mathbf{1}\mathbf{1}' \in \mathbb{R}^{M\times M}, \tag{S62}$$

where $\lambda_k \in \mathbb{R}_+$ is the diagonal price impact for asset k and $\lambda_{k\ell} \in \mathbb{R}$ is the off-diagonal price impact for assets $k, \ell \neq k$.

PROPOSITION S1—Equilibrium Existence and Uniqueness: Symmetric Environment: *Let $I < \infty$ and $K = MN > 1$ for some $M \geq 1$. In a symmetric market $\{K(n)\}_n$ defined in Definition S1, there exists a unique equilibrium.*

PROOF OF PROPOSITION S1: (EQUILIBRIUM EXISTENCE AND UNIQUENESS: SYMMETRIC ENVIRONMENT). *(Scalar equations for price impact)*. By Corollary S1 in Appendix A, the price impact $\mathbf{\Lambda}$ is determined by Eqs. (S19)–(S21):

$$\big[\big(\alpha\mathbf{\Sigma}-(I-2)\mathbf{\Lambda}\big)\big(\mathbf{Id}+\kappa(\alpha\mathbf{\Sigma})^{-1}\mathbf{\Lambda}\big)^{-1}\big(\mathbf{Id}+\kappa\mathbf{\Lambda}(\alpha\mathbf{\Sigma})^{-1}\big)^{-1}\big]_N=\mathbf{0}, \tag{S63}$$

where $\kappa \equiv \frac{1+(I-2)\sigma_0}{1-\sigma_0} \in \mathbb{R}_+$.

We first rewrite the matrix fixed point equation (S63) for $\mathbf{\Lambda}$ as a system of equations in $\mathbb{R}$ for λ_k and $\lambda_{k\ell}$ (Eqs. (S69)–(S70) below). Market symmetry simplifies Eq. (S63). In particular, the symmetry of the price impact $\mathbf{\Lambda}$ implies that vector $\mathbf{1} \in \mathbb{R}^K$ is an eigenvector of $\mathbf{\Lambda}$:

$$\mathbf{\Lambda}\mathbf{1}=\overline{\lambda}\mathbf{1}, \tag{S64}$$

where $\overline{\lambda} \equiv \lambda_k+(M-1)\lambda_{k\ell}$ is the sum of elements in each row of $\mathbf{\Lambda}$. Using Eq. (S64), the inverse matrix $(\mathbf{Id}+\kappa(\alpha\mathbf{\Sigma})^{-1}\mathbf{\Lambda})^{-1}$ in Eq. (S63) can be decomposed as a linear combination of a block-diagonal matrix $(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\mathbf{\Lambda})^{-1}$ and matrix $\mathbf{1}\mathbf{1}' \in \mathbb{R}^{K\times K}$:

$$\begin{aligned}
&\big(\mathbf{Id}+\kappa(\alpha\mathbf{\Sigma})^{-1}\mathbf{\Lambda}\big)^{-1}\\
&\quad=\left(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\mathbf{\Lambda}-\frac{\kappa\rho\overline{\lambda}}{\alpha\sigma(1-\rho)\big(1+(K-1)\rho\big)}\mathbf{1}\mathbf{1}'\right)^{-1}\\
&\quad=\left(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\mathbf{\Lambda}\right)^{-1}+\frac{\kappa\overline{\lambda}\rho\overline{v}^2}{\alpha\sigma(1-\rho)\big(1+(K-1)\rho\big)-K\overline{v}\kappa\overline{\lambda}\rho}\mathbf{1}\mathbf{1}',
\end{aligned} \tag{S65}$$

where the second equality applies the Woodbury Matrix Identity (Lemma S1) to $(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\mathbf{\Lambda}-\frac{\kappa\rho\overline{\lambda}}{\alpha\sigma(1-\rho)(1+(K-1)\rho)}\mathbf{1}\mathbf{1}')^{-1}$. Here, $\overline{v} \in \mathbb{R}_+$ is the eigenvalue of matrix $(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\mathbf{\Lambda})^{-1}$ associated with the eigenvector $\mathbf{1}$:

$$\begin{aligned}
\overline{v}\mathbf{1}&=\left(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\mathbf{\Lambda}\right)^{-1}\mathbf{1},\\
\overline{v}&=\frac{\alpha\sigma(1-\rho)}{\kappa\big(\lambda_k+(M-1)\lambda_{k\ell}\big)+\alpha\sigma(1-\rho)}=\frac{\alpha\sigma(1-\rho)}{\kappa\overline{\lambda}+\alpha\sigma(1-\rho)}.
\end{aligned} \tag{S66}$$

Substituting $(\mathbf{Id}+\kappa(\alpha\mathbf{\Sigma})^{-1}\mathbf{\Lambda})^{-1}$ (Eq. (S65)) into Eq. (S63), the LHS of Eq. (S63) can be decomposed as a linear combination of a block-diagonal matrix and matrix $[\mathbf{1}\mathbf{1}']_N \in \mathbb{R}^{K\times K}$:

$$\begin{aligned}
&\left[\left(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\mathbf{\Lambda}\right)^{-1}\big(\alpha\mathbf{\Sigma}-(I-2)\mathbf{\Lambda}\big)\left(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\mathbf{\Lambda}\right)^{-1}\right]_N\\
&\qquad+\big(\alpha\sigma\big(1+(K-1)\rho\big)\\
&\qquad-(I-2)\overline{\lambda}\big)K\left(\left(\frac{\kappa\overline{\lambda}\rho\overline{v}^2}{\alpha\sigma(1-\rho)\big(1+(K-1)\rho\big)-K\overline{v}\kappa\overline{\lambda}\rho}+\frac{\overline{v}}{K}\right)^2-\frac{\overline{v}^2}{K^2}\right)[\mathbf{1}\mathbf{1}']_N\\
&\quad=\mathbf{0}.
\end{aligned} \tag{S67}$$

Because $(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\boldsymbol{\Lambda})^{-1}$ is a block-diagonal matrix, the matrix equation (S67) for $\boldsymbol{\Lambda}=\operatorname{diag}(\boldsymbol{\Lambda}_{K(n)})$ simplifies to a fixed point equation for $\boldsymbol{\Lambda}_{K(n)}$ in each exchange n:

$$\begin{aligned}&\left(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\boldsymbol{\Lambda}_{K(n)}\right)^{-1}\big(\alpha\boldsymbol{\Sigma}_{K(n),K(n)}\\&\quad-(I-2)\boldsymbol{\Lambda}_{K(n)}\big)\left(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\boldsymbol{\Lambda}_{K(n)}\right)^{-1}\\&\quad+\big(\alpha\sigma\big(1+(K-1)\rho\big)-(I-2)\overline{\lambda}\big)\\&\quad\times K\left(\left(\frac{\kappa\overline{\lambda}\rho\overline{v}^2}{\alpha\sigma(1-\rho)\big(1+(K-1)\rho\big)-K\overline{v}\kappa\overline{\lambda}\rho}+\frac{\overline{v}}{K}\right)^2-\frac{\overline{v}^2}{K^2}\right)\mathbf{1}\mathbf{1}'\\&=\mathbf{0}.\end{aligned}\tag{S68}$$

We remark that the second line of the LHS of Eq. (S68) is proportional to matrix $\mathbf{1}\mathbf{1}'\in\mathbb{R}^{M\times M}$. Thus, for Eq. (S68) to hold, its first line must be proportional to matrix $\mathbf{1}\mathbf{1}'$. Equivalently, because $(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\boldsymbol{\Lambda}_{K(n)})^{-1}$ is invertible, multiplying the first line by $(\mathbf{Id}+\frac{\kappa}{\alpha\sigma(1-\rho)}\boldsymbol{\Lambda}_{K(n)})$ shows that matrix $(\alpha\boldsymbol{\Sigma}_{K(n),K(n)}-(I-2)\boldsymbol{\Lambda}_{K(n)})$ is proportional to $\mathbf{1}\mathbf{1}'$, and hence,

$$\alpha\sigma-(I-2)\lambda_k=\alpha\sigma\rho-(I-2)\lambda_{k\ell},\quad\text{i.e., }\lambda_k-\lambda_{k\ell}=\frac{\alpha}{I-2}\sigma(1-\rho).\tag{S69}$$

Furthermore, using Eq. (S69) and $\overline{v}\kappa\overline{\lambda}=\alpha\sigma(1-\rho)(1-\overline{v})$ from Eq. (S66), the matrix equation (S68) simplifies to a fixed point equation for λ_k in $\mathbb{R}$:

$$\lambda_k-\lambda_k^c=\frac{\alpha\sigma\rho}{I-2}\left(\frac{K}{M+(K-M)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2}-1\right).\tag{S70}$$

(Equilibrium existence and uniqueness). Substituting $\lambda_k=\frac{1}{M}(\overline{\lambda}+\frac{\alpha\sigma(1-\rho)}{I-2}(M-1))$ (by the definition of $\overline{\lambda}$ in Eq. (S64) and by Eq. (S69)) and $\overline{\lambda}=\frac{\alpha\sigma(1-\rho)(1-\overline{v})}{\kappa\overline{v}}$ (by the definition of $\overline{v}$ in Eq. (S66)) into Eq. (S70) gives a third-order polynomial equation for $\overline{v}\in(0,1)$:

$$(I-2)(1-\rho)(1-\overline{v})=\kappa\left[1+\rho\left(\frac{KM}{M+(K-M)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2}-1\right)\right]\overline{v}.\tag{S71}$$

When $\overline{v}=0$, the LHS of Eq. (S71) is positive while the RHS of Eq. (S71) is zero. When $\overline{v}=1$, the LHS of Eq. (S71) is zero while the RHS of Eq. (S71) is $\kappa(1+(M-1)\rho)>0$ by the positive definiteness of $\boldsymbol{\Sigma}$. Given the continuity of both sides of Eq. (S71), it follows from the Intermediate Value Theorem that Eq. (S71) has a solution $\overline{v}\in(0,1)$, and hence an equilibrium exists.

We show that the equilibrium is unique for $\rho \geq 0$ and $\rho < 0$. Suppose first that $\rho \geq 0$. Twice differentiating the RHS of Eq. (S71) with respect to $\overline{v}$, we have

$$
\begin{aligned}
&\frac{\partial^2 \mathrm{RHS}}{\partial \overline{v}^2} \\
&\quad = -\frac{2\kappa K^2(K-M)M\rho^2}{1+(K-1)\rho} \\
&\qquad \times \left(\left(M+(K-M)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2\right)\right. \\
&\qquad + \left.\frac{K\rho}{1+(K-1)\rho}\left[(1-\overline{v})\left(M+(K-M)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2\right)-2\right]\right) \\
&\qquad \Big/ \left(\left(M+(K-M)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2\right)^3\right) \leq 0.
\end{aligned}
$$

Hence, the RHS of Eq. (S71) is concave with respect to $\overline{v}$. In addition, the LHS of Eq. (S71) is linearly decreasing in $\overline{v}$. The concavity of RHS and the linearity of LHS imply that the solution of Eq. (S71) is unique in $\overline{v} \in (0, 1)$: by the Mean Value Theorem, Eq. (S71) cannot have more than two solutions $\overline{v} \in (0, 1)$. Otherwise, LHS < RHS must hold at both boundaries $\overline{v} = 0$ and $\overline{v} = 1$, which contradicts the discussion below Eq. (S71), that is, that LHS > RHS when $\overline{v} = 1$. Therefore, the solution $\overline{v}$ to Eq. (S71) and, equivalently, the equilibrium price impacts λ_k and $\lambda_{k\ell}$ in Eqs. (S69) and (S70) are unique.

Suppose now that $\rho < 0$. Dividing both sides of Eq. (S71) by $\overline{v}$ makes the LHS and the RHS convex and concave with respect to $\overline{v}$, respectively. Analogously to the argument for $\rho \geq 0$, the Mean Value Theorem implies that the solution $\overline{v}$ to Eq. (S71) and, equivalently, the equilibrium price impacts λ_k and $\lambda_{k\ell}$ in Eqs. (S69) and (S70) are unique. *Q.E.D.*

A counterpart of Theorem 3, Proposition S2 characterizes the within-exchange equilibrium price impact in symmetric markets.

PROPOSITION S2—Price Impact: Comparative Statics; Symmetric Markets, General Design: *The within-exchange price impact $\boldsymbol{\Lambda}_{K(n)}$ satisfies the following properties for each n:*

(1) (Magnitude) *With K assets, the diagonal price impact λ_k maximally increases N-fold relative to $\lambda_k^c = \frac{\alpha}{I-2}\sigma$; this is the case if and only if $|\rho| = 1$:*

$$\frac{\alpha}{I-2}\sigma \leq \lambda_k \leq \frac{\alpha}{I-2}N\sigma.$$

(2) (Comparative statics) *Relative to the contingent market*:

(i) $\frac{\partial(\lambda_k - \lambda_k^c)}{\partial I} \leq 0$ *and* $\frac{\partial(\lambda_{k\ell} - \lambda_{k\ell}^c)}{\partial I} \leq 0$ *for all $k, \ell \in K(n)$;*

(ii) $\frac{\partial(\lambda_k - \lambda_k^c)}{\partial|\rho|} \geq 0$ *and* $\frac{\partial(\lambda_{k\ell} - \lambda_{k\ell}^c)}{\partial|\rho|} \geq 0$ *for all $k, \ell \in K(n)$. Either inequality holds with equality if and only if $\rho = 0$.*

Note. With one asset per exchange (i.e., $N = \{\{k\}\}_k$ and $M = 1$), the proof of Proposition S2 specializes to that of Theorem 3.

PROOF OF PROPOSITION S2: (PRICE IMPACT: COMPARATIVE STATICS; SYMMETRIC MARKETS, GENERAL DESIGN). From the proof of Proposition S1, equilibrium price impact matrix $\mathbf{\Lambda} = \text{diag}((\lambda_k - \lambda_{k\ell})\mathbf{Id} + \lambda_{k\ell}\mathbf{11}')$ (Eq. (S62)) is characterized by scalar equations (Eqs. (S69) and (S70)) for λ_k and $\lambda_{k\ell}$:

$$\lambda_k - \lambda_{k\ell} = \frac{\alpha}{I-2}\sigma(1-\rho),$$

$$\lambda_k - \lambda_k^c = \frac{\alpha\sigma\rho}{I-2}\left(\frac{K}{M + (K-M)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2} - 1\right).$$

(Part (1)). We are now ready to show the inequality $\lambda_k \geq \frac{\alpha}{I-2}\sigma = \lambda_k^c$. Because $\overline{\lambda} > 0$ in Eq. (S64), the following inequality holds for $\overline{v}$:

$$0 < \overline{v} = \frac{\alpha\sigma(1-\rho)}{\kappa\overline{\lambda} + \alpha\sigma(1-\rho)} < 1. \tag{S72}$$

This implies that the term $\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}$ in the denominator of the RHS of (S70) satisfies $\text{sign}(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho} - 1) = -\text{sign}(\rho)$, and thus, $\text{sign}(\frac{K}{M+(K-M)(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho})^2} - 1) = \text{sign}(\rho)$. Hence, by Eq. (S70), $\lambda_k \geq \frac{\alpha}{I-2}\sigma = \lambda_k^c$; $\lambda_k = \lambda_k^c$ if and only if $\rho = 0$.

Furthermore, the proof of Theorem 2 demonstrated the existence of an upper bound $\overline{\mathbf{\Lambda}} = \frac{\alpha}{I-2}N\sigma\mathbf{Id}$ such that equilibrium price impact $\mathbf{\Lambda}$ satisfies $\mathbf{\Lambda} \leq \overline{\mathbf{\Lambda}}$. It follows that $\lambda_k \leq \frac{\alpha}{I-2}N\sigma$ for any k. The equality holds if and only if $|\rho_{k\ell}| = 1$ for all k and $\ell \neq k$ as we showed in the proof of Theorem 2.

(Part (2i)). We prove the monotonicity of the inference effect with respect to the number of traders I. By Eq. (S72), $\overline{v} \sim o(I^{-1+\varepsilon})$ for some $\varepsilon \in (0,1)$, given $\kappa = \frac{1+(I-2)\sigma_0}{1-\sigma_0}$. Then, Eq. (S70) implies that $\frac{\partial(\lambda_k - \lambda_k^c)}{\partial I} < 0$ because $\frac{\alpha\sigma\rho}{I-2} \sim o(I^{-1})$ and $\frac{K}{M+(K-M)(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho})^2} \sim o(I^{1-\varepsilon})$.

From Eq. (S69), the difference between the off-diagonal and diagonal price impacts is the same:

$$\lambda_{k\ell} - \lambda_{k\ell}^c = \left(\lambda_k - \frac{\alpha}{I-2}\sigma(1-\rho)\right) - \frac{\alpha}{I-2}\sigma\rho = \lambda_k - \lambda_k^c. \tag{S73}$$

Hence, $\frac{\partial(\lambda_{k\ell} - \lambda_{k\ell}^c)}{\partial I} < 0$.

(Part (2ii)). For any $|\rho| > 0$, $\frac{\partial}{\partial\rho}\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho} < 0$, because $K\overline{v} - 1 < K - 1$ by Eq. (S72) and $1 + (K-1)\rho > 0$ by the positive definiteness of $\mathbf{\Sigma}$. This implies

$$\frac{K}{M + (K-M)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2} - 1 > 0,$$

$$\frac{\partial}{\partial\rho}\left(\frac{K}{M + (K-M)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2} - 1\right) > 0.$$

Hence, in Eq. (S70), $\text{sign}(\frac{\partial(\lambda_k-\lambda_k^c)}{\partial\rho}) = \text{sign}(\rho)$, that is, $\frac{\partial(\lambda_k-\lambda_k^c)}{\partial|\rho|} > 0$ when $|\rho| > 0$. When $\rho = 0$, $\frac{\partial(\lambda_k-\lambda_k^c)}{\partial|\rho|} = 0$. From Eq. (S73), $\frac{\partial(\lambda_{k\ell}-\lambda_{k\ell}^c)}{\partial|\rho|} > 0$ when $|\rho| > 0$ and $\frac{\partial(\lambda_{k\ell}-\lambda_{k\ell}^c)}{\partial|\rho|} = 0$ when $\rho = 0$ for all $k, \ell \in K(n)$. *Q.E.D.*

PROOF OF COROLLARY 3: (PRICE IMPACT AND MARKET STRUCTURE). *(Part (i))*. Suppose that $K = 2$ and consider market structures $N = \{K\} = \{\{1, 2\}\}$ and $N' = \{\{k\}\}_k = \{\{1\}, \{2\}\}$. We want to show that $\lambda_k^{N'} \geq \lambda_k^N$ for all k. For simplicity, we dispense with the superscript N' for the uncontingent market N' and use the superscript c for the contingent market N.

By the equilibrium fixed point equation (15) in the uncontingent market N', the demand slope $c_k = \frac{1}{I-1}\lambda_k^{-1}$ for asset k can be decomposed into the direct effect and the (indirect) inference effect:

$$c_k \equiv -\frac{\partial q_k^j(\cdot)}{\partial p_k} = -\Bigg(\underbrace{-\frac{I-2}{I-1}(\alpha\sigma_{kk})^{-1}}_{\text{Direct effect}} + \underbrace{\frac{I-2}{I-1}(\alpha\sigma_{kk})^{-1}\alpha\sigma_{k\ell}c_\ell(\mathbf{VV}')_{\ell k}\big((\mathbf{VV}')_{kk}\big)^{-1}}_{\text{Inference effect}}\Bigg), \quad \text{(S74)}$$

where $\mathbf{V} \equiv (1-\sigma_0)\mathbf{C}^{-1}\mathbf{B} = (\mathbf{C} + \kappa(\alpha\mathbf{\Sigma})^{-1})^{-1}$ and $\kappa \equiv \frac{1+(I-2)\sigma_0}{(I-1)(1-\sigma_0)} \in \mathbb{R}_+$. We will show that $\text{sign}(\sigma_{k\ell}) = \text{sign}((\mathbf{VV}')_{\ell k})$.[8] Given the decomposition in Eq. (S74), the inference effect in Eq. (S74) is nonnegative, and hence $c_k \leq \frac{I-2}{I-1}(\alpha\sigma_{kk})^{-1}$, and $\lambda_k = \frac{c_k^{-1}}{I-1} \geq \lambda_k^c = \frac{\alpha}{I-2}\sigma_{kk}$ for all k.

We now characterize matrix $\mathbf{VV}'$. By the definition of $\mathbf{V} = (\mathbf{C} + \kappa(\alpha\mathbf{\Sigma})^{-1})^{-1}$, we have

$$\mathbf{V} = \frac{\alpha\overline{\sigma}}{\det(\mathbf{V}^{-1})}\begin{bmatrix} \alpha\overline{\sigma}c_2 + \kappa\sigma_{11} & \kappa\sigma_{12} \\ \kappa\sigma_{12} & \alpha\overline{\sigma}c_1 + \kappa\sigma_{22} \end{bmatrix}, \quad \text{(S75)}$$

where $\overline{\sigma} \equiv \det(\mathbf{\Sigma}) = \sigma_{11}\sigma_{22} - \sigma_{12}^2 > 0$ and $\det(\mathbf{V}^{-1}) = (\alpha\overline{\sigma}c_1 + \kappa\sigma_{22})(\alpha\overline{\sigma}c_2 + \kappa\sigma_{22}) - \kappa^2\sigma_{12}^2 > 0$. Using Eq. (S75), we compute $\mathbf{VV}'$, whose off-diagonal element is

$$(\mathbf{VV}')_{12} = \frac{\alpha^2\overline{\sigma}^2}{\det(\mathbf{V}^{-1})^2}\kappa\alpha\sigma_{12}\big(\alpha\overline{\sigma}(c_1+c_2) + \kappa(\sigma_{11}+\sigma_{22})\big). \quad \text{(S76)}$$

Because $\kappa > 0$, Eq. (S76) implies that $\text{sign}((\mathbf{VV}')_{\ell k}) = \text{sign}(\sigma_{k\ell})$. Hence, $\lambda_k \geq \lambda_k^c$ for all k; the equality holds if and only if $\sigma_{12} = 0$, because $(\mathbf{VV}')_{\ell k} = 0$ if and only if $\sigma_{k\ell} = 0$ in Eq. (S76).

(Part (ii)). See Figure 1(B) in Section 3.2.3 for an example of $\widehat{\lambda}_k^N > \widehat{\lambda}_k^{N'}$. *Q.E.D.*

PROPOSITION S3—Efficient Market Structure in Symmetric Markets: *Consider the class of symmetric markets* (*Definition* S1). *Assume that traders' ex ante trading needs are symmetric for all assets* $E[\overline{q}_{0,k}] - E[q_{0,k}^i] = E[\overline{q}_{0,\ell}] - E[q_{0,\ell}^i]$ *for all* k *and* ℓ *for each* i, *and there is no inference error*: $(\sigma_{cv}, \sigma_{pv}) \to 0$ *and* $\sigma_0 < 1$. *When* $\rho > 0$, *the uncontingent market maximizes total ex ante welfare*; *when* $\rho < 0$, *the contingent market does*.

[8]We note that Eq. (S74) is the counterpart of Eq. (20) for the demand coefficient $c_k = \frac{1}{I-1}\lambda_k^{-1}$ (rather than price impact λ_k). In Eq. (S74), $(\mathbf{VV}')_{\ell k} = (1-\sigma_0)^2\,\text{Cov}[p_\ell, p_k|\mathbf{q}_0^i]$ determines the sign of price correlation for all k and $\ell \neq k$ (see Section 3.2.3).

PROOF OF PROPOSITION S3: (EFFICIENT MARKET STRUCTURE IN SYMMETRIC MARKETS). We first derive the ex ante equilibrium surplus (Eq. (S30)) in a symmetric market. Given the symmetry of the ex ante trading needs across assets (i.e., $E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i] = e^i\mathbf{1}$ for some $e^i \in \mathbb{R}$), each trader's ex ante equilibrium surplus (Eq. (S30)) is

$$\begin{aligned}&\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big) \cdot \mathbf{Y}(\mathbf{\Lambda})\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big)\\ &\quad = K\alpha\sigma\big(1+(K-1)\rho\big)\left[1 - \frac{\overline{\lambda}^2}{\big(\alpha\sigma\big(1+(K-1)\rho\big)+\overline{\lambda}\big)^2}\right]\big(e^i\big)^2,\end{aligned} \tag{S77}$$

where $\overline{\lambda} \equiv \lambda_k + (M-1)\lambda_{k\ell} \in \mathbb{R}_+$ is the eigenvalue of a symmetric matrix $\mathbf{\Lambda}$ that corresponds to eigenvector $\mathbf{1}$. The ex ante equilibrium surplus (S77) is decreasing in $\overline{\lambda}$, because $\overline{\lambda}$ is nonnegative (i.e., $\mathbf{\Lambda}$ is positive semi-definite). Therefore, it suffices to show that $\operatorname{sign}(\frac{\partial\overline{\lambda}}{\partial M}) = \operatorname{sign}(\rho)$.

In the proof of Proposition S2, Eq. (S71) characterizes the price impact $\mathbf{\Lambda} = \operatorname{diag}(\mathbf{\Lambda}_{K(n)})_n = (\lambda_k - \lambda_{k\ell})\mathbf{Id} + \lambda_{k\ell}[\mathbf{11}']_N$ (and its eigenvalue $\overline{\lambda}$) by an equivalent fixed point equation for $\overline{v} \equiv \frac{\alpha\sigma(1-\rho)}{\kappa\overline{\lambda}+\alpha\sigma(1-\rho)}$: Dividing both sides of Eq. (S71) by $\overline{v}$ gives

$$(I-2)(1-\rho)\left(\frac{1}{\overline{v}} - 1\right) = \kappa\left[1 + \rho\left(\frac{K}{1 + \left(\frac{K}{M} - 1\right)\left(\frac{1+(K\overline{v}-1)\rho}{1+(K-1)\rho}\right)^2} - 1\right)\right]. \tag{S78}$$

By the definition of $\overline{v}$ in Eq. (S66), the equality $\operatorname{sign}(\frac{\partial\overline{\lambda}}{\partial M}) = \operatorname{sign}(\rho)$ holds if and only if $\operatorname{sign}(\frac{\partial\overline{v}}{\partial M}) = -\operatorname{sign}(\rho)$ does and thus, it suffices to show the latter.

From Eq. (S78), the LHS of Eq. (S78) is decreasing with respect to $\overline{v}$ (at an order of $o(\overline{v}^{-1})$); the RHS of Eq. (S78) is increasing and concave when $\rho < 0$ and decreasing and convex when $\rho > 0$ (at an order of $o(\rho\overline{v}^{-2})$). Suppose $\rho > 0$. The RHS of Eq. (S78) increases as M increases, for all $\overline{v} \in (0,1)$. Hence, the solution to Eq. (S78) decreases because the LHS of Eq. (S78) is independent of M and is decreasing with respect to $\overline{v}$. Therefore, $\frac{\partial\overline{v}}{\partial M} < 0$ holds. Similarly, when $\rho < 0$, the RHS of Eq. (S78) decreases as M increases, for all $\overline{v} \in (0,1)$; hence, $\frac{\partial\overline{v}}{\partial M} > 0$ holds. It follows that $\operatorname{sign}(\frac{\partial\overline{v}}{\partial M}) = -\operatorname{sign}(\rho)$, and hence, $\operatorname{sign}(\frac{\partial\overline{\lambda}}{\partial M}) = \operatorname{sign}(\rho)$. *Q.E.D.*

PROOF OF COROLLARY 4: (WELFARE WITH MULTIPLE EXCHANGES VERSUS JOINT MARKET CLEARING ($K = 2$)). Suppose the ex ante trading needs across assets $E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]$ are proportional to $(\xi, 1)' \in \mathbb{R}^2$ for all i for some nonzero constant: that is, $\xi \equiv \frac{E[\overline{q}_{0,1}] - E[q_{0,1}^i]}{E[\overline{q}_{0,2}] - E[q_{0,2}^i]}$. Given the symmetry of asset payoffs, the price impact in $\{\{1\},\{2\}\}$ is symmetric across assets: that is, $\mathbf{\Lambda} = \lambda\mathbf{Id}$.

We characterize the difference between the ex ante equilibrium surplus (Eq. (S30)) in the uncontingent market $\{\{1\},\{2\}\}$ and the contingent market $\{\{1,2\}\}$:

$$\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big) \cdot \big(\mathbf{Y}(\mathbf{\Lambda}) - \mathbf{Y}(\mathbf{\Lambda}^c)\big)\big(E[\overline{\mathbf{q}}_0] - E[\mathbf{q}_0^i]\big), \tag{S79}$$

where $\mathbf{\Lambda}^c = \frac{I-2}{I-1}\mathbf{\Sigma}$. Substituting $\mathbf{\Lambda} = \lambda\mathbf{Id}$ and $\mathbf{\Sigma} = \alpha\sigma(1-\rho)\mathbf{Id} + \alpha\sigma\rho\mathbf{1}\mathbf{1}'$ into Eq. (S33), we characterize the difference between surplus matrices $\mathbf{Y}(\mathbf{\Lambda}) - \mathbf{Y}(\mathbf{\Lambda}^c)$:

$$\mathbf{Y}(\mathbf{\Lambda}) - \mathbf{Y}(\mathbf{\Lambda}^c) = \frac{\alpha\sigma}{(I-1)^2}\begin{bmatrix} 1-x & \rho(1-y) \\ \rho(1-y) & 1-x \end{bmatrix}, \tag{S80}$$

where $x \equiv \frac{(I-1)^2\lambda^2}{((\alpha\sigma+\lambda)^2-(\alpha\sigma\rho)^2)^2}((\alpha\sigma+\lambda)^2 - \alpha\sigma\rho^2(\alpha\sigma+2\lambda)) \geq 1$ and $y \equiv \frac{(I-1)^2\lambda^2}{((\alpha\sigma+\lambda)^2-(\alpha\sigma\rho)^2)^2}(\lambda^2 - (\alpha\sigma)^2(1-\rho^2)) \leq 1$. Substituting $\mathbf{Y}(\mathbf{\Lambda}) - \mathbf{Y}(\mathbf{\Lambda}^c)$ (Eq. (S80)) and $E[\overline{q}_{0,1}] - E[q^i_{0,1}] = \xi(E[\overline{q}_{0,2}] - E[q^i_{0,2}])$ into Eq. (S79) shows that the ex ante equilibrium payoff in the uncontingent market is higher than in the contingent market if and only if

$$x\xi^2 + 2\xi\rho y + x < \xi^2 + 2\xi\rho + 1. \tag{S81}$$

The necessary and sufficient condition (S81) has a solution $\xi \in (\underline{\xi}(\rho, I), \overline{\xi}(\rho, I))$ with $\overline{\xi}(\rho, I) > \underline{\xi}(\rho, I)$ for any asset correlation $\rho \neq \{0, \pm 1\}$ and any finite number of traders $I < \infty$:

$$\begin{aligned} &\frac{(1-y)\rho - \sqrt{(1-y)^2\rho^2 - (x-1)^2}}{2(x-1)} \\ &\quad \leq \xi \equiv \frac{E[\overline{q}_{0,1}] - E[q^i_{0,1}]}{E[\overline{q}_{0,2}] - E[q^i_{0,2}]} \\ &\quad \leq \frac{(1-y)\rho + \sqrt{(1-y)^2\rho^2 - (x-1)^2}}{2(x-1)}. \end{aligned} \tag{S82}$$

Given that $|(1-y)\rho| > \sqrt{(1-y)^2\rho^2 - (x-1)^2}$, the bounds in the necessary and sufficient condition (S81) are both positive when $\rho > 0$ and are both negative when $\rho < 0$. It follows that inequality (S82) holds if and only if conditions (i) and (ii) hold with $\underline{\xi}(\rho, I) \equiv \frac{(1-y)\rho - \sqrt{(1-y)^2\rho^2-(x-1)^2}}{2(x-1)}$ and $\overline{\xi}(\rho, I) \equiv \frac{(1-y)\rho + \sqrt{(1-y)^2\rho^2-(x-1)^2}}{2(x-1)}$ when $\rho > 0$, and $\underline{\xi}(\rho, I) \equiv |\frac{(1-y)\rho + \sqrt{(1-y)^2\rho^2-(x-1)^2}}{2(x-1)}|$ and $\overline{\xi}(\rho, I) \equiv |\frac{(1-y)\rho - \sqrt{(1-y)^2\rho^2-(x-1)^2}}{2(x-1)}|$ when $\rho < 0$. *Q.E.D.*

C.2. *Symmetric Equilibrium Characterization in Markets With Two Assets: $K = 2$*

Suppose that $\alpha^i = \alpha$ for all i, and $\mathbf{\Sigma} = (\sigma_{k\ell})_{k,\ell}$ is characterized by $\sigma_{11} = \sigma_{22} = 1$ and $\sigma_{12} = \sigma_{21} = \rho$. Then, the price impact is symmetric across traders and assets: $\lambda^i_k = \lambda$ for all k and i and $\lambda^i_{k\ell} = 0$ for all k, $\ell \neq k$, and i. Traders' demand coefficients in (12) are symmetric: $\mathbf{b}^i_k = \mathbf{b}_k$ and $c^i_k = c$ for all i and k. Observe that vector $\mathbf{b}_k$ is symmetric across k up to a permutation: that is, if $\mathbf{b}_1 = (x, y)$, then $\mathbf{b}_2 = (y, x)$. We will continue to use the superscript i and subscript k when they are useful.

The equilibrium with uncontingent trading is characterized in two steps (Proposition 2). Step 1 characterizes the fixed point among trader i's demand coefficients for assets 1 and 2, taking as given his price impact λ and residual supply intercepts $F(\mathbf{s}^{-i}|\mathbf{q}^i_0)$. Step 2 endogenizes λ and $F(\mathbf{s}^{-i}|\mathbf{q}^i_0)$.

Step 1 (Optimization, given residual supply λ and $F(\mathbf{s}^{-i}|\mathbf{q}^i_0)$). Taking the derivative of the expected payoff (9) with respect to q^i_k gives the first-order conditions of trader i for each k:

$$E\big[\delta_1 - \alpha^i\big(\sigma_{11}\big(q_1^i + q_{0,1}^i\big) + \sigma_{12}\big(q_2^i + q_{0,2}^i\big)\big)|p_1, \mathbf{q}_0^i\big] = p_1 + \lambda_1^i q_1^i \quad \forall p_1 \in \mathbb{R}, \tag{S83}$$

$$E\big[\delta_2 - \alpha^i\big(\sigma_{22}\big(q_2^i + q_{0,2}^i\big) + \sigma_{21}\big(q_1^i + q_{0,1}^i\big)\big)|p_2, \mathbf{q}_0^i\big] = p_2 + \lambda_2^i q_2^i \quad \forall p_2 \in \mathbb{R}. \tag{S84}$$

Trader i's expected marginal utility for asset k depends on the demand coefficients of his schedule $q_\ell^i(\cdot)$ for asset $\ell \neq k$. The characterization of a trader's best-response demand $q_k^i(\cdot)$ requires solving a fixed point problem for trader i's own demand schedules $\{q_k^i(\cdot)\}_k$ across assets.

Step 1.1 (Parameterization of demands for asset $\ell \neq k$). To characterize the best-response demand of trader i for asset 1, assume that his demand for asset 2 is a linear function:

$$q_2^i(p_2) = a_2^i - \mathbf{b}_2^i \mathbf{q}_0^i - c_2^i p_2 \quad \forall p_2 \in \mathbb{R}, \tag{S85}$$

where $a_2^i \in \mathbb{R}$, $\mathbf{b}_2^i \in \mathbb{R}^{1\times 2}$, and $c_2^i \in \mathbb{R}_+$.

Step 1.2 (Price distribution and expected trades, given $F(\mathbf{s}^{-i}|\mathbf{q}_0^i)$). Market clearing for asset 2 characterizes the distribution of price p_2. Equalization of demand $q_2^i(\cdot)$ in Eq. (S85) and residual supply $S_2^{-i}(\cdot) = s_2^{-i} + (\lambda_2^i)^{-1} p_2$ gives

$$a_2^i - \mathbf{b}_2^i \mathbf{q}_0^i - c_2^i p_2 = s_2^{-i} + \big(\lambda_2^i\big)^{-1} p_2 \quad \forall s_2^{-i} \in \mathbb{R}.$$

Price p_2 maps one-to-one to s_2^{-i}:

$$p_2 = \frac{1}{c_2^i + \big(\lambda_2^i\big)^{-1}} \big(a_2^i - \mathbf{b}_2^i \mathbf{q}_0^i - s_2^{-i}\big) \quad \forall s_2^{-i} \in \mathbb{R}. \tag{S86}$$

Equation (S86) characterizes price distribution $F(p_2|\mathbf{q}_0^i)$ as a function of the intercept distribution $F(s_2^{-i}|\mathbf{q}_0^i)$ and the coefficients $\{a_2^i, \mathbf{b}_2^i, c_2^i\}$ of trader i's own demand function $q_2^i(\cdot)$ for asset 2.

The one-to-one mapping between p_2 and s_2^{-i} (Eq. (S86)) allows the expected trade $E[q_2^i|p_1, \mathbf{q}_0^i]$ in the first-order condition for asset 1 (Eq. (S83)) to be characterized conditionally on s_1^{-i}:

$$E\big[q_2^i|p_1, \mathbf{q}_0^i\big] = E\big[q_2^i|s_1^{-i}, \mathbf{q}_0^i\big].$$

From the parameterization of $q_2^i(\cdot)$ in Eq. (S85) and price distribution p_2 in Eq. (S86),

$$\begin{aligned} E\big[q_2^i|s_1^{-i}, \mathbf{q}_0^i\big] &= E\big[a_2^i - \mathbf{b}_2^i \mathbf{q}_0^i - c_2^i p_2|s_1^{-i}, \mathbf{q}_0^i\big] \\ &= a_2^i - \mathbf{b}_2^i \mathbf{q}_0^i - \frac{c_2^i}{c_2^i + \big(\lambda_2^i\big)^{-1}} \big(a_2^i - \mathbf{b}_2^i \mathbf{q}_0^i - E\big[s_2^{-i}|s_1^{-i}, \mathbf{q}_0^i\big]\big). \end{aligned}$$

The conditional expectation $E[s_2^{-i}|s_1^{-i}, \mathbf{q}_0^i]$ is characterized by the intercept distribution $F(\mathbf{s}^{-i}|\mathbf{q}_0^i)$, which trader i takes as given.

Step 1.3 (Best response for asset k, given demands for $\ell \neq k$). Substituting the expected trade into the first-order condition (S83) gives the following equation:

$$\delta_1 - \alpha^i\left(\sigma_{11}\left(q_1^i + q_{0,1}^i\right) + \sigma_{12}\left(a_2^i - \mathbf{b}_2^i\mathbf{q}_0^i - \frac{c_2^i}{c_2^i + \left(\lambda_2^i\right)^{-1}}\left(a_2^i - \mathbf{b}_2^i\mathbf{q}_0^i - E\left[s_2^{-i}|s_1^{-i}, \mathbf{q}_0^i\right]\right) + q_{0,2}^i\right)\right)$$
$$= p_1 + \lambda_1^i q_1^i,$$

from which the best response $q_1^i(\cdot)$ is derived as a linear function of s_1^{-i} and p_1:

$$\begin{aligned} q_1^i\left(p_1, s_1^{-i}\right) = {} & \frac{1}{\alpha^i\sigma_{11} + \lambda_1^i} \\ & \times \left(\delta_1 - \alpha^i\mathbf{\Sigma}_1\mathbf{q}_0^i - p_1 - \alpha^i\sigma_{12}\left(a_2^i - \mathbf{b}_2^i\mathbf{q}_0^i\right.\right. \\ & \left.\left. - \frac{c_2^i}{c_2^i + \left(\lambda_2^i\right)^{-1}}\left(a_2^i - \mathbf{b}_2^i\mathbf{q}_0^i - E\left[s_2^{-i}|s_1^{-i}, \mathbf{q}_0^i\right]\right)\right)\right). \end{aligned} \tag{S87}$$

The demand schedule $q_1^i(\cdot)$ in Eq. (S87) can be written as a function of both p_1 and s_1^{-i}. Using the one-to-one mapping between p_1 and s_1^{-i}:

$$q_1^i\left(p_1, s_1^{-i}\right) = s_1^{-i} + \left(\lambda_1^i\right)^{-1} p_1, \tag{S88}$$

the best response $q_1^i(\cdot)$ in Eq. (S87) is characterized as a function of only p_1 as an endogenous variable. Equations (S87)–(S88) characterize the demand coefficients in $q_1^i(p_1) = a_1^i - \mathbf{b}_1^i\mathbf{q}_0^i - c_1^i p_1$ as functions of $a_2^i, \mathbf{b}_2^i, c_2^i$, and $\{\lambda_k^i\}_k$. An analogous argument characterizes the demand coefficients $a_2^i, \mathbf{b}_2^i, c_2^i$ for asset 2 as functions of $a_1^i, \mathbf{b}_1^i, c_1^i$, and $\{\lambda_k^i\}_k$, which creates a fixed point for $\{a_k^i, \mathbf{b}_k^i, \mathbf{c}_k^i\}_k$.

Step 2 (Correct residual supply). Given other traders' demands (S85) for all k and $j \neq i$, the correct residual supply of trader i is determined by $S_k^{-i}(\cdot) = -\sum_{j\neq i} q_k^j(\cdot)$.

Step 2.1 (Correct distribution of residual supply intercepts and expectations). The residual supply intercepts $s_k^{-i} = -\sum_{j\neq i}(a_k^j - \mathbf{b}_k^j\mathbf{q}_0^j)$ are jointly Normally distributed. From the distribution of endowments $F((\mathbf{q}_0^j)_j|\mathbf{q}_0^i)$, the first and second moments of intercepts (s_1^{-i}, s_2^{-i}) are: for each k and ℓ,

$$E\left[s_k^{-i}|\mathbf{q}_0^i\right] = -\sum_{j\neq i} a_k^j + \mathbf{b}_k \sum_{j\neq i}\left(E\left[\mathbf{q}_0^j\right] + \frac{\sigma_{cv}}{\sigma_{cv} + \sigma_{pv}}\left(\mathbf{q}_0^i - E\left[\mathbf{q}_0^i\right]\right)\right),$$
$$\operatorname{Cov}\left[s_k^{-i}, s_\ell^{-i}|\mathbf{q}_0^i\right] = \mathbf{b}_k \sum_{j,h\neq i}\operatorname{Cov}\left[\mathbf{q}_0^j, \mathbf{q}_0^h|\mathbf{q}_0^i\right]\mathbf{b}_\ell' = I(I-1)\sigma_{pv}\sigma_0\mathbf{b}_k \cdot \mathbf{b}_\ell.$$

Applying the Projection Theorem to this distribution of the residual supply intercepts $F(\mathbf{s}^{-i}|\mathbf{q}_0^i)$ gives the expected intercepts $E[s_\ell^{-i}|s_k^{-i}, \mathbf{q}_0^i]$:

$$E\left[s_\ell^{-i}|s_k^{-i}, \mathbf{q}_0^i\right] = E\left[s_\ell^{-i}|\mathbf{q}_0^i\right] + \frac{\mathbf{b}_k \cdot \mathbf{b}_\ell}{\mathbf{b}_k \cdot \mathbf{b}_k}\left(s_k^{-i} - E\left[s_k^{-i}|\mathbf{q}_0^i\right]\right).$$

Substituting $E[s_\ell^{-i}|s_k^{-i},\mathbf{q}_0^i]$ into Eq. (S87) characterizes trader i's demand coefficients $\{a_k^i,\mathbf{b}_k^i,c_k^i\}_k$ as functions of $\{a_k^j,\mathbf{b}_k^j,c_k^j\}_{k,j\neq i}$ and price impacts $\{\lambda_k^i\}_k$. This defines a fixed point for $\{a_k^i,\mathbf{b}_k^i,\mathbf{c}_k^i\}_{i,k}$ as a function of $\{\lambda_k^i\}_{i,k}$.

Step 2.2 (Fixed point for best-response coefficients, given price impacts). By the symmetry across traders and assets, the fixed point for demand coefficients of trader i simplifies to

$$a_k^i = c_k\delta_k - c_k(1-\sigma_0)\alpha\Big(\big(x\big(\alpha-(I-2)\lambda\big)+y\rho\big)E[\overline{q}_{0,k}] + \big(y\big(\alpha-(I-2)\lambda\big)+x\rho\big)E[\overline{q}_{0,l}]\Big), \tag{S89}$$

$$c_1 = c_2 = \left((\alpha+\lambda)+\alpha\rho\frac{\mathbf{b}_1\cdot\mathbf{b}_2}{\mathbf{b}_1\cdot\mathbf{b}_1}\right)^{-1}, \tag{S90}$$

$$\mathbf{b}_1 = (x,y), \mathbf{b}_2 = (y,x), \tag{S91}$$

where $x\equiv(1-\sigma_0)(1-\rho^2)\alpha+(1+(I-2)\sigma_0)\lambda$ and $y\equiv\rho(1+(I-2)\sigma_0)\lambda$. The demand coefficients a_k^i, $\mathbf{b}_k$, and c_k are closed-form functions of λ (S89)–(S91).

Step 2.3 (Correct price impact). The price impact must equal the slope of the inverse residual supply, $\lambda_k = (\sum_{j\neq i}\frac{\partial q_k^j(\cdot)}{\partial p_k})^{-1} = \frac{1}{I-1}c_k^{-1}$ for all k. By Eqs. (S90)–(S91), the price impact $\lambda = \frac{1}{I-1}c_1^{-1} = \frac{1}{I-1}c_2^{-1}$ is characterized by

$$\lambda = \frac{\alpha}{I-2} + \frac{\alpha\rho}{I-2}\frac{2xy}{x^2+y^2}. \tag{S92}$$

Equation (S92) characterizes the equilibrium price impact, which in turn determines the demand coefficients in Eqs. (S89)–(S91).

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