Econometrica Supplementary Material

SUPPLEMENT TO "THE GLOBAL DIFFUSION OF IDEAS"

(*Econometrica*, Vol. 88, No. 1, January 2020, 83–114)

FRANCISCO J. BUERA
Department of Economics, Washington University in St. Louis

EZRA OBERFIELD
Department of Economics, Princeton University

APPENDIX A: STATIC EQUILIBRIUM

GIVEN A VECTOR OF STOCKS of knowledge $(\lambda_1, \dots, \lambda_n)$, a static equilibrium is given by a profile of wages $(w_1, \dots, w_n)$ such that labor market clears in all countries.

Given the isoelastic demand, if a producer had no direct competitors, it would set a price with a markup of $\frac{\varepsilon}{\varepsilon-1}$ over marginal cost. Since producers engage in Bertrand competition. the lowest cost provider of a good to a country will either use this markup or, if necessary, set a limit price to just undercut the next-lowest-cost provider of the good.

For a producer with productivity q in country j, the cost of providing one unit of the good in country i is $\frac{w_j \kappa_{ij}}{q}$. The price of good s in country i is determined as follows. Suppose that country j's best and second best producers of good s have productivities $q_{j1}(s)$ and $q_{j2}(s)$. The country that can provide good s to i at the lowest cost is given by

$$\arg\min_j \frac{w_j \kappa_{ij}}{q_{j1}(s)}.$$

If the lowest-cost-provider of good s for i is a producer from country k, the price of good s in i is

$$p_i(s) = \min\left\{\frac{\varepsilon}{\varepsilon - 1}\frac{w_k \kappa_{ik}}{q_{k1}(s)}, \frac{w_k \kappa_{ik}}{q_{k2}(s)}, \min_{j \neq k} \frac{w_j \kappa_{ij}}{q_{j1}(s)}\right\}.$$

That is, the price is either the monopolist's price or else it equals the cost of the next-lowest-cost provider of the good; the latter is either the second best producer of good s in country k or the best producer in one of the other countries.

The static equilibrium will depend on whether trade is balanced and where profit from producers is spent. For now, we take each country's expenditure as given and solve for the equilibrium as a function of these expenditures.

Labor in j is used to produce goods for all destinations. Let $S_{ij} \subseteq [0, 1]$ be the set of goods for which a producer in j is the lowest-cost-provider for country i. To deliver one unit of good $s \in S_{ij}$ to i, the producer in j uses $\kappa_{ij}/q_{j1}(s)$ units of labor. Thus the labor market clearing constraint for country j is

$$L_j = \sum_i \int_{s \in S_{ij}} \frac{\kappa_{ij}}{q_{j1}(s)} c_i(s)\, ds.$$

Francisco J. Buera: fjbuera@wustl.edu
Ezra Oberfield: edo@princeton.edu

 https://doi.org/10.3982/ECTA14044

Similarly, the total profit earned by producers in j can be written as

$$\Pi_j = \sum_i \int_{s \in S_{ij}} \left(p_i(s) - \frac{w_j \kappa_{ij}}{q_{j1}(s)} \right) c_i(s)\, ds.$$

A.1. *Distribution of Productivities*

In the model, managers engage in Bertrand competition. In that environment, an important object is the joint distribution of the productivities of the best and second best producers of a good. We denote the CDF of this joint distribution as $F_t^{12}(q_1, q_2)$, for $q_1 \geq q_2$, and note the frontier of knowledge can be expressed in terms of this joint distribution, $F_t(q) = F_t^{12}(q, q)$.

In this section, we derive results for this joint distribution that are analogous to those derived in Section 1. As in Section 1, we define $\lambda_t \equiv \int_{-\infty}^{t} \alpha_\tau \int_0^\infty x^{\beta\theta}\, dG_\tau(x)\, d\tau$.

PROPOSITION 1: *Suppose Assumption* 1 *holds and that at each* t, $\lim_{q \to \infty} q^{\beta\theta}[1 - G_t(q)] = 0$. *Then the joint distribution of best and second best productivities satisfies*, *for* $q_1 \geq q_2$:

$$F_t^{12}(q_1, q_2) = e^{-(\lambda_t - \lambda_0) q_2^{-\theta}} \left\{ F_0^{12}(q_1, q_2) + F_0^{12}(q_2, q_2)(\lambda_t - \lambda_0)\left(q_2^{-\theta} - q_1^{-\theta}\right)\right\}. \tag{A.1}$$

If $\lim_{t \to \infty} \lambda_t = \infty$, *then*

$$\lim_{t \to \infty} F_t^{12}\left(\lambda_t^{1/\theta} q_1, \lambda_t^{1/\theta} q_2\right) = \left(1 + q_2^{-\theta} - q_1^{-\theta}\right) e^{-q_2^{-\theta}}. \tag{A.2}$$

PROOF: We first derive the law of motion for F^{12}. Note first that under Assumption 1 and the restriction on the tail of the source distribution, the arrival rate of techniques that deliver efficiency better than q at t is $\int A_t(q/x^\beta)\, dG_t(x) = \alpha_t \int x^{\beta\theta}\, dG_t(x) = \dot{\lambda}_t q^{-\theta}$. The number of new ideas that deliver efficiency in the range $(q_2, q_1]$ between t_0 and t_1 follows a Poisson distribution with mean $(\lambda_{t_1} - \lambda_{t_0})(q_2^{-\theta} - q_1^{-\theta})$.

We next claim that the joint distribution $F_t^{12}(q_1, q_2)$ can be expressed as

$$F_t^{12}(q_1, q_2) = F_0^{12}(q_2, q_2) \underbrace{e^{-(\lambda_t - \lambda_0) q_1^{-\theta}}}_{\text{no new ideas} > q_1} \underbrace{\left[e^{-(\lambda_t - \lambda_0)(q_2^{-\theta} - q_1^{-\theta})}\left(1 + (\lambda_t - \lambda_0)\left(q_2^{-\theta} - q_1^{-\theta}\right)\right)\right]}_{\text{0 or 1 new ideas} \in (q_2, q_1]}$$

$$+ \left[F_0^{12}(q_1, q_2) - F_0^{12}(q_2, q_2)\right] \underbrace{e^{-(\lambda_t - \lambda_0) q_2^{-\theta}}}_{\text{no new ideas} > q_2}.$$

Consider first a good for which there are initially no ideas with productivity exceeding q_2; there are $F_0^{12}(q_2, q_2)$ such ideas. For it to be the case that, at time t, the best idea for that good has productivity no greater than q_1 and the second best idea has productivity no greater than q_2, it must be that both no ideas arrived with productivity exceeding q_1 and at most one idea arrived with productivity in the range $(q_2, q_1]$. To find these probabilities, note that the arrival between time 0 and t of ideas with productivity exceeding q_1 follows a Poisson distribution with mean $(\lambda_t - \lambda_0) q_1^{-\theta}$, so the probability of no such events is $e^{-(\lambda_t - \lambda_0) q_1^{-\theta}}$. Similarly, the arrival between time 0 and t of ideas with productivity in the range $(q_2, q_1]$ follows a Poisson distribution with mean $(\lambda_t - \lambda_0)(q_2^{-\theta} - q_1^{-\theta})$, so the probability of at most one such event is $e^{-(\lambda_t - \lambda_0)(q_2^{-\theta} - q_1^{-\theta})} + e^{-(\lambda_t - \lambda_0)(q_2^{-\theta} - q_1^{-\theta})}(\lambda_t - \lambda_0)(q_2^{-\theta} - q_1^{-\theta})$.

Consider next a good for which initially there is exactly one idea with productivity in the range $(q_2, q_1]$ and no other ideas exceeding q_2; there are $F_0^{12}(q_1, q_2) - F_0^{12}(q_2, q_2)$ such ideas. For it to be the case that, at time t, the best idea for that good has productivity no greater than q_1 and the second best idea has productivity no greater than q_2, it must be that both no ideas arrived with productivity exceeding q_2. Such events follow a Poisson distribution with mean $(\lambda_t - \lambda_0)q_2^{-\theta}$, so the probability of no such events is $e^{-(\lambda_t-\lambda_0)q_2^{-\theta}}$.

Rearranging this equation yields (A.1). To find (A.2), we evaluate (A.1) at $\lambda_t^{1/\theta}q_1$, $\lambda_t^{1/\theta}q_2$ and note that $\lim_{t\to\infty} F_0^{12}(\lambda_t^{1/\theta}q_1, \lambda_t^{1/\theta}q_2) = \lim_{t\to\infty} F_0^{12}(\lambda_t^{1/\theta}q_2, \lambda_t^{1/\theta}q_2) = \lim_{t\to\infty} \frac{\lambda_t-\lambda_0}{\lambda_t} = 1$. *Q.E.D.*

This proposition nests Proposition 1 as a special case when $q_1 = q_2$. Next, we refine Assumption 2.

ASSUMPTION A.1: *The initial joint distribution of best and second best productivities satisfies* $F_0^{12}(q_1, q_2) = (1 + \lambda_0 q_2^{-\theta} - \lambda_0 q_1^{-\theta})e^{-\lambda_0 q_2^{-\theta}}$.

Plugging this initial distribution into (A.1) gives

$$F_t^{12}(q_1, q_2) = \left(1 + \lambda_t q_2^{-\theta} - \lambda_t q_1^{-\theta}\right)e^{-\lambda_t q_2^{-\theta}}, \quad q_1 \geq q_2.$$

A.2. *Equilibrium*

This section gives expressions for price indices, trade shares, and market clearing conditions that determine equilibrium wages. Throughout this section, we maintain that $F_i^{12}(q_1, q_2) = [1 + \lambda_i q_2^{-\theta} - \lambda_i q_1^{-\theta}]e^{-\lambda_i q_2^{-\theta}}$.

For a variety $s \in S_{ij}$ (produced in j and exported to i) that is produced with productivity q, the producer's cost of providing the good to country i is $\frac{w_j \kappa_{ij}}{q}$. If the total expenditure in i is X_i, then the expenditure on consumption in i of that variety is $(\frac{p_i(s)}{P_i})^{1-\varepsilon} X_i$, consumption is $\frac{1}{p_i(s)}(\frac{p_i(s)}{P_i})^{1-\varepsilon} X_i$, and the labor used in j to produce variety s for i is $\frac{\kappa_{ij}/q_{j1}(s)}{p_i(s)}(\frac{p_i(s)}{P_i})^{1-\varepsilon} X_i$.

Define $\pi_{ij} \equiv \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}}$. We will eventually show this is the share of i's total expenditure that is spent on goods from j. We begin with a lemma which will be useful in deriving a number of results.

LEMMA 2: *Suppose* τ_1 *and* τ_2 *satisfy* $\tau_1 + \tau_2 < 1$. *Then*

$$\int_{s \in S_{ij}} q_{j1}(s)^{\tau_1 \theta} p_i(s)^{-\tau_2 \theta}\, ds = B(\tau_1, \tau_2)\left[\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}\right]^{\tau_2} \pi_{ij}\left(\frac{\lambda_j}{\pi_{ij}}\right)^{\tau_1},$$

where $B(\tau_1, \tau_2) \equiv \{1 - \frac{\tau_2}{1-\tau_1} + \frac{\tau_2}{1-\tau_1}(\frac{\varepsilon}{\varepsilon-1})^{-\theta(1-\tau_1)}\}\Gamma(1 - \tau_1 - \tau_2)$.

We relegate the proof to the Online Appendix C.1. We first use this lemma to provide expressions for each country's price index, expenditure shares, expenditure on labor, and profit.

CLAIM 3: *The price index for* i *satisfies* $P_i = B(0, \frac{\varepsilon-1}{\theta})^{\frac{1}{1-\varepsilon}}[\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}]^{-\frac{1}{\theta}}$. $\pi_{ij} = \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}}$ *is the share of* i*'s expenditure on goods from* j. *Country* j*'s expenditure on labor is* $w_j L_j = \frac{\theta}{\theta+1}\sum_i \pi_{ij} X_i$ *and the profit earned by firms based in* j *is* $\Pi_j = \frac{1}{\theta+1}\sum_i \pi_{ij} X_i$.

PROOF: The price aggregate of goods provided to i by j is $\int_{s\in S_{ij}} p_i(s)^{1-\varepsilon}\,ds$. Using Lemma 2, this equals $\int_{s\in S_{ij}} p_i(s)^{1-\varepsilon}\,ds = B(0, \frac{\varepsilon-1}{\theta})[\sum_k \lambda_k(w_k\kappa_{ik})^{-\theta}]^{\frac{\varepsilon-1}{\theta}}\pi_{ij}$. The price index for i therefore satisfies $P_i^{1-\varepsilon} = \sum_j \int_{s\in S_{ij}} p_i(s)^{1-\varepsilon}\,ds = B(0, \frac{\varepsilon-1}{\theta})[\sum_k \lambda_k(w_k\kappa_{ik})^{-\theta}]^{\frac{\varepsilon-1}{\theta}}$ and i's expenditure share on goods from j is $\frac{\int_{s\in S_{ij}} p_i(s)^{1-\varepsilon}\,ds}{P_i^{1-\varepsilon}} = \pi_{ij}$.

We next compute j's expenditure on labor. i's consumption of good s is $p_i(s)^{-\varepsilon}\frac{X_i}{P_i^{1-\varepsilon}}$. If j is the lowest-cost provider to i, then j's expenditure on labor per unit delivered is $w_j\frac{\kappa_{ij}}{q_{j1}(s)}$. The total expenditure on labor in j to produce goods for i is then $\int_{s\in S_{ij}} \frac{w_j\kappa_{ij}}{q_{j1}(s)} p_i(s)^{-\varepsilon}\frac{X_i}{P_i^{1-\varepsilon}}\,ds$. Using Lemma 2, this equals $B(-\frac{1}{\theta}, \frac{\varepsilon}{\theta})w_j\kappa_{ij}\frac{X_i}{P_i^{1-\varepsilon}}[\sum_k \lambda_k(w_k\kappa_{ik})^{-\theta}]^{\frac{\varepsilon}{\theta}}\pi_{ij}(\frac{\lambda_j}{\pi_{ij}})^{-\frac{1}{\theta}}$. Summing across i, the expression for the expenditure on labor follows from $B(-\frac{1}{\theta}, \frac{\varepsilon}{\theta}) = \frac{\theta}{\theta+1}B(0, \frac{\varepsilon-1}{\theta})$ and $\frac{w_j\kappa_{ij}}{P_i^{1-\varepsilon}}[\sum_k \lambda_k(w_k\kappa_{ik})^{-\theta}]^{\frac{\varepsilon}{\theta}}(\frac{\lambda_j}{\pi_{ij}})^{-\frac{1}{\theta}} = B(0, \frac{\varepsilon-1}{\theta})^{-1}$.

Profit in j is total revenue minus cost, or $\Pi_j = \sum_i \pi_{ij}X_i - w_jL_j = \frac{1}{\theta+1}\sum_i \pi_{ij}X_i$. *Q.E.D.*

Finally, we note that if trade is balanced and all profit from domestic producers is spent domestically, then $X_i = w_iL_i + \Pi_i$ and the labor market clearing conditions can be expressed as $w_jL_j = \sum_i \pi_{ij}w_iL_i$.

A.3. *Learning From Sellers*

Here, we characterize the learning process when insights are drawn uniformly from sellers. If producers are equally likely to learn from all active sellers, the source distribution is $G_i(q) = \sum_j H_{ij}(q)$. The change in i's stock of knowledge depends on $\int_0^\infty q^{\beta\theta}\,dG_i(q) = \sum_j \int_0^\infty q^{\beta\theta}\,dH_{ij}(q) = \sum_j \int_{s\in S_{ij}} q_{j1}(s)^{\beta\theta}\,ds$. Using Lemma 2, this is

$$\int_0^\infty q^{\beta\theta}\,dG_i(q) = B(\beta, 0)\sum_j \pi_{ij}\left(\frac{\lambda_j}{\pi_{ij}}\right)^\beta = \Gamma(1-\beta)\sum_j \pi_{ij}\left(\frac{\lambda_j}{\pi_{ij}}\right)^\beta. \tag{A.3}$$

A.4. *Learning From Producers*

Here, we briefly describe the learning process in which insights are equally likely to be drawn from all active domestic producers. As discussed in the text, we consider only the case in which trade costs satisfy the triangle inequality which implies that, all producers that export also sell domestically. As a consequence, the source distribution is $G_i(q) = \frac{H_{ii}(q)}{H_{ii}(\infty)}$. The change in i's stock of knowledge depends on $\int_0^\infty q^{\beta\theta}\,dG_i(q) = \frac{\int_0^\infty q^{\beta\theta}\,dH_{ii}(q)}{\int_0^\infty dH_{ii}(q)} = \frac{\int_{s\in S_{ii}} q_{i1}(s)^{\beta\theta}\,ds}{\int_{s\in S_{ii}} ds}$. Using Lemma 2, this is

$$\int_0^\infty q^{\beta\theta}\,dG_i(q) = \frac{B(\beta, 0)\pi_{ii}\left(\dfrac{\lambda_i}{\pi_{ii}}\right)^\beta}{B(0, 0)\pi_{ii}} = \Gamma(1-\beta)\left(\frac{\lambda_i}{\pi_{ii}}\right)^\beta.$$

APPENDIX B: QUANTITATIVE MODEL

This Appendix derives expressions for the price index, expenditure shares, and the law of motion of the stock of knowledge for the extended model discussed in Section 4, incorporating nontradable goods, intermediate inputs, and equipped labor. The price index satisfies

$$p_i^{1-\varepsilon} \propto (1-\mu)\left[\left(p_i^{\eta} w_i^{1-\eta}\right)^{-\theta} \lambda_i\right]^{-\frac{1-\varepsilon}{\theta}} + \mu\left[\sum_{j=1}^{n}\left(p_j^{\eta} w_j^{1-\eta} \kappa_{ij}\right)^{-\theta} \lambda_j\right]^{-\frac{1-\varepsilon}{\theta}}$$

and the share of i's spending on nontradable goods is

$$\pi_i^{\mathrm{NT}} = \frac{(1-\mu)\left[\left(p_i^{\eta} w_i^{1-\eta}\right)^{-\theta} \lambda_i\right]^{\frac{\varepsilon-1}{\theta}}}{(1-\mu)\left[\left(p_i^{\eta} w_i^{1-\eta}\right)^{-\theta} \lambda_i\right]^{\frac{\varepsilon-1}{\theta}} + \mu\left[\sum_{k}\left(p_k^{\eta} w_k^{1-\eta} \kappa_{ik}\right)^{-\theta} \lambda_k\right]^{\frac{\varepsilon-1}{\theta}}}.$$

Let $Z_{ij} \equiv \frac{(p_j^{\eta} w_j^{1-\eta} \kappa_{ij})^{-\theta} \lambda_j}{\sum_k (p_k^{\eta} w_k^{1-\eta} \kappa_{ik})^{-\theta} \lambda_k}$ denote the share of i's tradable spending spent on tradable goods from j. The fraction country i's total expenditure on goods from country $j \neq i$ is $\pi_{ij} = (1-\pi_i^{\mathrm{NT}}) Z_{ij}$. The fraction of country i's total expenditure spent on its own goods is given by the sum of the nontradable and tradable shares $\pi_{ii} = \pi_i^{\mathrm{NT}} + (1-\pi_i^{\mathrm{NT}}) Z_i$, where we have denoted $Z_i = Z_{ii}$. The evolution of i's stock of knowledge when learning is from sellers is

$$\dot{\lambda}_i \propto (1-\mu)\lambda_i^{\beta} + \mu \sum_j Z_{ij}\left(\frac{\lambda_j}{Z_{ij}}\right)^{\beta}.$$

The evolution of the stock of knowledge when learning is uniformly from domestic producers is

$$\dot{\lambda}_i \propto \frac{(1-\mu)\lambda_i^{\beta} + \mu Z_i\left(\frac{\lambda_i}{Z_i}\right)^{\beta}}{(1-\mu) + \mu Z_{ii}}.$$

The market clearing conditions are the same as in the baseline model once labor is reinterpreted as equipped labor.

To obtain an expression for bilateral trade costs in terms of observables, we use the equation of relative trade shares

$$\begin{aligned}
\frac{\pi_{ij}}{1-\pi_{ii}} &= \frac{Z_{ij}}{1-Z_i} \\
&= \frac{\left(p_j^{\eta} w_j^{1-\eta} \kappa_{ij}\right)^{-\theta} \lambda_j}{\left(p_i^{\eta} w_i^{1-\eta}\right)^{-\theta} \lambda_i} \frac{Z_i}{1-Z_i} \\
&= \kappa_{ij}^{-\theta}\left(\frac{p_j}{p_i}\right)^{-\theta} \frac{\left(\frac{w_j}{p_j}\right)^{-\theta(1-\eta)} \lambda_j}{\left(\frac{w_i}{p_i}\right)^{-\theta(1-\eta)} \lambda_i} \frac{Z_i}{1-Z_i}.
\end{aligned}$$

Using that the definition of the price index implies $\lambda_i \propto [(1-\mu)+\mu Z_i^{-\frac{\varepsilon-1}{\theta}}]^{-\frac{\theta}{\varepsilon-1}}(\frac{w_i}{p_i})^{(1-\eta)\theta}$

$$\frac{\pi_{ij}}{1-\pi_{ii}} = \kappa_{ij}^{-\theta}\left(\frac{p_j}{p_i}\right)^{-\theta}\frac{\left[(1-\mu)+\mu Z_j^{-\frac{\varepsilon-1}{\theta}}\right]^{-\frac{\theta}{\varepsilon-1}}}{\left[(1-\mu)+\mu Z_i^{-\frac{\varepsilon-1}{\theta}}\right]^{-\frac{\theta}{\varepsilon-1}}}\frac{Z_i}{1-Z_i}.$$

Solving for κ_{ij}

$$\kappa_{ij} = \frac{p_i}{p_j}\left(\frac{1-\pi_{ii}}{\pi_{ij}}\frac{Z_i}{1-Z_i}\right)^{\frac{1}{\theta}}\left[\frac{(1-\mu)+\mu Z_i^{-\frac{\varepsilon-1}{\theta}}}{(1-\mu)+\mu Z_j^{-\frac{\varepsilon-1}{\theta}}}\right]^{\frac{1}{\varepsilon-1}}.$$

Co-editor Fabrizio Zilibotti handled this manuscript.

Manuscript received 22 December, 2015; final version accepted 22 July, 2019; available online 18 September, 2019.