Econometrica Supplementary Material

SUPPLEMENT TO "CONSUMER SEARCH AND PRICE COMPETITION"

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A. DISTRIBUTIONS OF EFFECTIVE VALUES

IN THIS SUPPLEMENT, we provide three examples in which $H_i(w_i)$ can be explicitly calculated.

(1) Uniform: suppose V_i and Z_i are uniform over $[0, 1]$ (i.e., $F_i(v) = G_i(v) = v$). Provided that $s \leq 1/2$ (which guarantees $z_i^* \in [0, 1]$), $z_i^* = 1 - \sqrt{2s}$. It is then straightforward to show that $H_i(w_i)$ is given as follows:

$$H_i(w_i) = \begin{cases} \dfrac{w_i^2}{2} & \text{if } w_i \in \left[0, z_i^*\right), \\ w_i - z_i^* + \dfrac{\left(z_i^*\right)^2}{2} & \text{if } w_i \in \left[z_i^*, 1\right), \\ 2w_i - \dfrac{w_i^2}{2} - z_i^* + \dfrac{\left(z_i^*\right)^2}{2} - \dfrac{1}{2} & \text{if } w_i \in \left[1, 1 + z_i^*\right]. \end{cases}$$

Notice that, whereas H_i is continuous, the density function h_i has an upward jump at z_i^*. Therefore, H_i is not globally log-concave. Nevertheless, it is easy to show that both H_i and $1 - H_i$ are log-concave above z_i^*.

(2) Exponential: suppose V_i and Z_i are exponential distributions with parameters λ_1 and λ_2, respectively (i.e., $F_i(v_i) = 1 - e^{-\lambda_1 v_i}$ and $G_i(z_i) = 1 - e^{-\lambda_2 z_i}$). Provided that $s < 1/\lambda_2$ (which ensures that $z_i^* > 0$), then $z_i^* = -\log(\lambda_2 s)/\lambda_2$. For any $w_i \geq 0$,

$$H_i(w_i) = 1 - e^{-\lambda_2 \min\{w_i, z_i^*\}} - \frac{\lambda_2\left(e^{(\lambda_1 - \lambda_2)\min\{w_i, z_i^*\}} - 1\right)}{e^{\lambda_1 w_i}(\lambda_1 - \lambda_2)} + \left(1 - e^{-\lambda_1(\max\{w_i, z_i^*\} - z_i^*)}\right)e^{-\lambda_2 z_i^*}.$$

Similarly to the uniform example, H_i is not globally log-concave, because h_i has a upward jump at z_i^*, but both H_i and $1 - H_i$ are log-concave above z_i^*.

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(3) Gumbel: suppose that V_i and $-Z_i$ are standard Gumbel distributions (i.e., $F_i(v_i) = e^{-e^{-v_i}}$ and $G_i(z_i) = 1 - e^{-e^{z_i}}$). For any $w_i \in (-\infty, \infty)$,

$$H_i(w_i) = \frac{1 + e^{-w_i - e^{z_i^*}(1+e^{-w_i})}}{1 + e^{-w_i}}.$$

Since both f_i and g_i are log-concave, $1 - H_i$ is log-concave by Proposition 2. Given the solution for H_i above, we have

$$\frac{h_i(w_i)}{H_i(w_i)} = \frac{e^{z_i^* - w_i} - 1}{1 + e^{w_i + e^{z_i^*}(1+e^{-w_i})}} + \frac{1}{1 + e^{w_i}}.$$

The first term falls in w_i whenever $w_i \ge z_i^*$, while the second term constantly falls in w_i. Therefore, $H_i(w_i)$ is log-concave above z_i^*.

B. PROOF OF THE SECOND CLAIM IN PROPOSITION 2 (CONT'D)

Since

$$\left(\log H_i^\sigma(w_i^\sigma)\right)'' = \frac{(h_i^\sigma)'(w_i^\sigma) H_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)^2}{H_i^\sigma(w_i^\sigma)^2},$$

it suffices to show that $(h_i^\sigma)'(w_i^\sigma)H_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)^2 < 0$ for all w_i^σ, provided that σ is sufficiently large. Integrate equation (2) by parts; we have $H_i^\sigma(w_i^\sigma) = \int_{\underline{v}_i^\sigma}^{\bar{v}_i^\sigma} G_i(w_i^\sigma - v_i^\sigma)\, dF_i^\sigma(v_i^\sigma)$ for $w_i^\sigma < \underline{v}_i^\sigma + z_i^*$. In this case, H_i^σ is log-concave by Prékopa's theorem. For $w_i^\sigma \ge \underline{v}_i^\sigma + z_i^*$, we have

$$H_i^\sigma(w_i^\sigma) = \int_{w_i^\sigma - z_i^*}^{\bar{v}_i^\sigma} G_i(w_i^\sigma - v_i^\sigma)\, dF_i^\sigma(v_i^\sigma) + F_i^\sigma(w_i^\sigma - z_i^*).$$

By straightforward calculus,

$$\frac{h_i^\sigma(w_i^\sigma)}{H_i^\sigma(w_i^\sigma)} = \frac{\int_{w_i^\sigma - z_i^*}^{\bar{v}_i^\sigma} g_i(w_i^\sigma - v_i^\sigma)\, dF_i^\sigma(v_i^\sigma) + \left(1 - G_i(z_i^*)\right) f_i^\sigma(w_i^\sigma - z_i^*)}{\int_{w_i^\sigma - z_i^*}^{\bar{v}_i^\sigma} G_i(w_i^\sigma - v_i^\sigma)\, dF_i^\sigma(v_i^\sigma) + F_i^\sigma(w_i^\sigma - z_i^*)}.$$

Changing the variables with $a = F_i^\sigma(v_i^\sigma)$ and $r = F_i^\sigma(w_i^\sigma - z_i^*)$, the above equation becomes

$$\frac{h_i^\sigma\left((F_i^\sigma)^{-1}(r) + z_i^*\right)}{H_i^\sigma\left((F_i^\sigma)^{-1}(r) + z_i^*\right)} = \frac{\int_r^1 g_i\left((F_i^\sigma)^{-1}(r) - (F_i^\sigma)^{-1}(a) + z_i^*\right) da + \left(1 - G_i(z_i^*)\right) f_i^\sigma\left((F_i^\sigma)^{-1}(r)\right)}{\int_r^1 G_i\left((F_i^\sigma)^{-1}(r) - (F_i^\sigma)^{-1}(a) + z_i^*\right) da + r}.$$

Since $V_i^\sigma \equiv \sigma V_i$, we have $F_i^\sigma(v_i^\sigma) = F_i(v_i^\sigma/\sigma)$, $(F_i^\sigma)^{-1}(r) = \sigma F_i^{-1}(r)$, $f_i^\sigma((F_i^\sigma)^{-1}(r)) = f_i(F_i^{-1}(r))/\sigma$, and $(f_i^\sigma)'(F_i^{-1}(r)) = f_i(F_i^{-1}(r))/\sigma^2$. Arranging the terms in the right-hand side above yields

$$\begin{aligned}&\frac{\sigma h_i^\sigma\big((F_i^\sigma)^{-1}(r)+z_i^*\big)}{H_i^\sigma\big((F_i^\sigma)^{-1}(r)+z_i^*\big)}\\ &\quad= \frac{\displaystyle\int_r^1 \sigma g_i\big(\sigma\big(F_i^{-1}(r)-F_i^{-1}(a)\big)+z_i^*\big)\,da + \big(1-G_i(z_i^*)\big)f_i\big(F_i^{-1}(r)\big)}{\displaystyle\int_r^1 G_i\big(\sigma\big(F_i^{-1}(r)-F_i^{-1}(a)\big)+z_i^*\big)\,da + r}.\end{aligned}$$

Since $F_i^{-1}(r) - F_i^{-1}(a) \le 0$, the denominator converges to r as σ explodes. Integrating $\int_r^1 \sigma g_i(\sigma(F_i^{-1}(r) - F_i^{-1}(a)) + z_i^*)\,da$ in the numerator by parts yields

$$G_i\big(z_i^*\big)f_i\big(F^{-1}(r)\big) + \int_r^1 G_i\big(\sigma\big(F_i^{-1}(r)-F_i^{-1}(a)\big)+z_i^*\big)\,df\big(F_i^{-1}(a)\big).$$

Again, since $F_i^{-1}(r) - F_i^{-1}(a) \le 0$, the second term vanishes as σ tends to infinity, and thus the numerator converges to $G_i(z_i^*)f_i(F_i^{-1}(r))$. Therefore,

$$\lim_{\sigma\to\infty} \frac{\sigma h_i^\sigma\big((F_i^\sigma)^{-1}(r)+z_i^*\big)}{H_i^\sigma\big((F_i^\sigma)^{-1}(r)+z_i^*\big)} = \frac{f_i\big(F_i^{-1}(r)\big)}{r}.$$

Following a similar procedure, we have

$$\lim_{\sigma\to\infty} \frac{\sigma\big(h_i^\sigma\big)'\big((F_i^\sigma)^{-1}(r)+z_i^*\big)}{h_i^\sigma\big((F_i^\sigma)^{-1}(r)+z_i^*\big)} = \frac{\big(1-G_i(z_i^*)\big)f_i'\big(F_i^{-1}(r)\big)}{f_i\big(F_i^{-1}(r)\big)}.$$

Altogether,

$$\begin{aligned}&\lim_{\sigma\to\infty}\sigma\left[\frac{\big(h_i^\sigma\big)'\big((F_i^\sigma)^{-1}(r)+z_i^*\big)}{h_i^\sigma\big((F_i^\sigma)^{-1}(r)+z_i^*\big)} - \frac{h_i^\sigma\big((F_i^\sigma)^{-1}(r)+z_i^*\big)}{H_i^\sigma\big((F_i^\sigma)^{-1}(r)+z_i^*\big)}\right]\\ &\quad= \frac{\big(1-G_i(z_i^*)\big)f_i'\big(F_i^{-1}(r)\big)}{f_i\big(F_i^{-1}(r)\big)} - \frac{f_i\big(F_i^{-1}(r)\big)}{r}\\ &\quad= \big(1-G_i\big(z_i^*\big)\big)\left[\frac{f_i'\big(F_i^{-1}(r)\big)}{f_i\big(F_i^{-1}(r)\big)} - \frac{f_i\big(F_i^{-1}(r)\big)}{r}\right] - \frac{G_i\big(z_i^*\big)f_i\big(F_i^{-1}(r)\big)}{r} < 0. \qquad (8)\end{aligned}$$

Provided s_i is not too large, then $G_i(z_i^*)$ and $1 - G_i(z_i^*)$ are in $(0, 1)$, so the sign of the expression is determined by both terms.[1] The square bracket term is weakly negative because F is log-concave; thus the entire expression is weakly negative. The strict inequality (8) holds for each $r \in [0, 1]$ because $f_i(F_i^{-1}(r))/r > 0$ when $r \in [0, 1)$ and

[1]If s_i is large so that $G_i(z_i^*) = 0$, then $W_i = V_i + z_i^*$ and H_i has the same shape as F_i, and thus is log-concave.

$f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ when $r = 1$.[2] Altogether, for each $r \in [0,1]$ there is a $\bar{\sigma}_r < \infty$ such that if $\sigma > \bar{\sigma}_r$, then $(\log H_i^\sigma(w_i^\sigma))'' \propto (h_i^\sigma)'(w_i^\sigma)/h_i^\sigma(w_i^\sigma) - h_i^\sigma(w_i^\sigma)/H_i^\sigma(w_i^\sigma) < 0$ where $w_i^\sigma = F_i^{-1}(r) + z_i^*$. Since $[0,1]$ is a compact convex set and $(\log H_i^\sigma(w_i^\sigma))''$ is continuous in r, there exists $\bar{\sigma} = \max_{r\in[0,1]} \bar{\sigma}_r < \infty$ such that if $\sigma > \bar{\sigma}$, then $(h_i^\sigma)'/h_i^\sigma - h_i^\sigma/H_i^\sigma < 0$ for all $r \in [0,1]$, or equivalently $H_i^\sigma(w)$ is log-concave for all $w_i^\sigma \geq \underline{v}_i^\sigma + z_i^*$. Finally, if $f_i(\underline{v}_i) = 0$, then the ratio $h_i^\sigma(w_i^\sigma)/H_i^\sigma(w_i^\sigma)$ is continuous at $\underline{v}_i^\sigma + z_i^*$. Since this ratio is decreasing for $w_i < \underline{v}_i^\sigma + z_i^*$ and decreasing for $w_i \geq \underline{v}_i^\sigma + z_i^*$ when σ is large, it is globally decreasing when σ is large, or equivalently, $H_i^\sigma(w_i^\sigma)$ is globally log-concave.

C. EXAMPLE OF A MIXED-STRATEGY EQUILIBRIUM

Now we assume F_i is degenerate and characterize a symmetric mixed-strategy equilibrium. Assume there are two symmetric sellers and $u_0 = c_i = v_i = 0$. Assume Z_i is exponentially distributed with parameter λ, namely, $G_i(z) = 1 - e^{-\lambda z}$. Assume $s < 1/\lambda$ so that $z^* > 0$. Below, we characterize the distribution of prices and show that it has decreasing density.

Let $Q_i = \min\{Z_i, z^*\} - P_i$, and let Γ_i and γ_i be its distribution function and density function, repectively. Note that the equilibrium price P_i is ex ante random in a mixed-strategy equilibrium. Moreover, in a symmetric equilibrium, the distribution of P_i has no mass point, for if it has a mass point, then a seller can get an upward jump in demand by moving the location of the mass point slightly to the left. Since the density of P_i exists (its c.d.f. is atomless), the density γ_i also exists.

First, we derive the demand function in a mixed-strategy equilibrium. By the eventual purchase theorem, consumers buy from seller 1 if $\min\{z^*, Z_1\} - p_1 > \max\{Q_2, 0\}$. Therefore, no consumer will buy from seller 1 if $p_1 > z^*$. For all $p_1 \leq z^*$, consumers buy from seller 1 when $z^* - p_1 > Q_2$ and $Z_1 - p_1 > \max\{Q_2, 0\}$. Therefore, for all $p_1 \leq z^*$, seller 1's demand and its derivative are given by

$$D_1(p_1) = \int_{\underline{q}}^{z^*-p_1} \big(1 - G\big(p_1 + \max\{q,0\}\big)\big)\, d\Gamma_2(q) = \int_{\underline{q}}^{z^*-p_1} e^{-\lambda(p_1+\max\{q,0\})}\, d\Gamma_2(q),$$

$$D_1'(p_1) = -e^{-\lambda z^*}\gamma_2\big(z^* - p_1\big) - \lambda \int_{\underline{q}}^{z^*-p_1} e^{-\lambda(p_1+\max\{q,0\})}\, d\Gamma_2(q).$$

Therefore, the first-order necessary condition with respect to p_1 is

$$\frac{1}{p_1} = \frac{-D_1'(p_1)}{D_1(p_1)} = \frac{e^{-\lambda z^*}\gamma_2\big(z^* - p_1\big)}{D_1(p_1)} + \lambda.$$

Let π^* be the equilibrium profit for the sellers in a symmetric equilibrium. Since seller 1 is indifferent between offering any prices in the support of P_1 in equilibrium, $\pi^* = p_1 D(p_1)$ for every p_1 in the support of P_1. Using $D_1(p_1) = \pi^*/p_1$, the first-order

[2]For $r \in (0,1)$, the strict inequality (8) is true as $f_i(F_i^{-1}(r)) > 0$ within the support. Since $f_i(F_i^{-1}(r))/r$ falls in r by log-concavity of F_i, $f_i(F_i^{-1}(r))/r > 0$ at $r = 0$, and thus the strict inequality (8) also holds for $r = 0$. For $r = 1$, since f_i has unbounded upper support, $f_i(F_i^{-1}(r))$ falls in r when r is large. Therefore $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ for some $r \in (0,1)$. Since $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r))$ falls in r by the log-concavity of f_i, $f_i'(F_i^{-1}(r))/f_i(F_i^{-1}(r)) < 0$ when $r = 1$ and thus the inequality (8) holds when $r = 1$.

condition can be rewritten as

$$\gamma_2\big(z^* - p_1\big) = \frac{\pi^*}{p_1}\left(\frac{1}{p_1} - \lambda\right)e^{\lambda z^*}. \tag{9}$$

The first-order condition implies $p_1 \leq 1/\lambda$ in equilibrium. Since $p_1 \geq 0$, the support of P_1 is a subset of the interval $[0, \min\{z^*, 1/\lambda\}]$. From equation (9), it is clear that the density γ_i of Q_i is monotonically increasing (because the right-hand side falls in p_1).

Now we use the density of Q_i (i.e., γ_i) and that of Z_i to solve for the distribution of P_i, by exploiting the equation $Q_i = \min\{Z_i, z^*\} - P_i$. This is generally a hard problem because one must solve a complex differential equation. Below, we show that the problem is especially tractable when Z_i is exponentially distributed. Let $B(p)$ be the distribution function of P_i in a symmetric equilibrium. The c.d.f. and p.d.f. of Q_i can be written as

$$\Gamma_i(q) = \int_0^\infty \big[1 - B\big(\min\{z, z^*\} - q\big)\big]\lambda e^{-\lambda z}\, dz,$$

$$\gamma_i(q) \equiv \Gamma_i'(q) = \int_0^\infty b\big(\min\{z, z^*\} - q\big)\lambda e^{-\lambda z}\, dz.$$

Substitute the equation for γ_i into the first-order condition (9); then

$$\begin{aligned}\frac{\pi^*}{p}\left(\frac{1}{p} - \lambda\right)e^{\lambda z^*} &= \int_0^\infty b\big(\min\{z - z^* + p, p\}\big)\lambda e^{-\lambda z}\, dz \\ &= \int_{-z^*}^0 b(y + p)\lambda e^{-\lambda(y+z^*)}\, dy + b(p)e^{-\lambda z^*}.\end{aligned}$$

The last line uses a change of variable $y = z - z^*$. Now multiply both sides by $e^{\lambda(z^*-p)}$, and let $\tau(p) \equiv b(p)e^{-\lambda p}$ and $T(p) \equiv \int_0^p \tau(y)\, dy$. Then we can rewrite the above equation as

$$\frac{\pi^*}{p}\left(\frac{1}{p} - \lambda\right)e^{\lambda(2z^*-p)} = \lambda \int_{-z^*}^0 \tau(y + p)\, dy + \tau(p).$$

Notice that, since $p \geq 0$ in equilibrium, the density $b(q) = \tau(q) = 0$ for all $q < 0$. Together with $p \leq z^*$, we have $\tau(y + p) = 0$ for all $y \in (-z^*, -p)$. In light of this, the lower support of the integral term can be replaced by $-p$. Therefore, the equation above becomes

$$\frac{\pi^*}{p}\left(\frac{1}{p} - \lambda\right)e^{\lambda(2z^*-p)} = \lambda \int_{-p}^0 \tau(y + p)\, dy + \tau(p) = \lambda T(p) + \tau(p). \tag{10}$$

This equation is a first-order differential equation. The general solution is

$$T(p) = Ce^{-\lambda p} - \pi^* e^{\lambda(2z^*-p)}\left(\lambda \log(p) + \frac{1}{p}\right),$$

where C is a constant. By $b(p) = \tau(p)e^{\lambda p}$ and equation (10), the density $b(p)$ is

$$b(p) = \frac{\pi^*}{p}\left(\frac{1}{p} - \lambda\right)e^{2\lambda z^*} - \lambda T(p)e^{\lambda p} = \pi^* e^{2\lambda z^*}\left(\frac{1}{p^2} + \lambda^2 \log(p)\right) - \lambda C.$$

The constant C is chosen so that $\int_0^{\min\{z^*,1/\lambda\}} b(p)\,dp = 1$. The value of π^* can be solved by substituting the solution of $b(p)$ into the seller's profit function. One can easily show that the density $b(p)$ falls in p by the equation above and $p \le 1/\lambda$.

D. UNOBSERVABLE PRICES AND SEARCH COSTS

Anderson and Renault (1999) studied a stationary search model with unobservable prices, and showed that $\partial p^*/\partial s > 0$ provided that $1 - G(z)$ is log-concave. We argue that this insight may not hold when search is non-stationary, due to the presence of a prior value V. Assume there is no outside option and sellers are symmetric. Below, we show $\partial p^*/\partial s < 0$ is possible if the density of V is log-concave and increasing, even when $1 - G(z)$ is log-concave.

CLAIM 1: *The equilibrium price p^* falls in s when* (i) *s is sufficiently small and* (ii) $f'(\bar{v})/f(\bar{v}) > \lim_{z\uparrow\bar{z}} g(z)/[1 - G(z)]$.

Since we have assumed $f(v)$ is log-concave, it is single-peaked in v. Therefore, the second condition requires $f'(v) > 0$ for all $[\underline{v}, \bar{v}]$, and the upper support $\bar{v}$ must be finite.

PROOF: Let $\tilde{W}_i \equiv \max_{j\neq i} W_j$; then the demand for seller i is given by (5). When prices are unobservable, seller i controls p_i but not p_i^e, so the measure of marginal consumers is

$$-\frac{dD_i(p_i, p_i^e, p^*)}{dp_i}\bigg|_{p_i=p_i^e=p^*} = E\left[\int_{\tilde{W}_i - z^*}^{\bar{v}} g(\tilde{W} - v_i)\,dF(v_i)\right]$$
$$= \int_{\underline{w}}^{\bar{v}+z^*}\left[\int_{w-z^*}^{\bar{v}} g(w - v_i)\,dF(v_i)\right] dH(w)^{n-1}.$$

In a symmetric equilibrium, p^* solves

$$p^* - c = -\left(n\frac{dD_i(p_i, p_i^e, p^*)}{dp_i}\bigg|_{p_i=p_i^e=p^*}\right)^{-1}.$$

Since the right-hand side does not depend on p^*, to show $\partial p^*/\partial s < 0$, it suffices to show the right-hand side falls in s, or equivalently the following derivative is positive:

$$\frac{d}{ds}\int_{\underline{w}}^{\bar{v}+z^*}\left[\int_{w-z^*}^{\bar{v}} g(w - v_i)\,dF(v_i)\right] dH(w)^{n-1}$$
$$= \frac{dz^*}{ds}\int_{\underline{w}}^{\bar{v}+z^*}\left[g(z^*)f(w - z^*)\right] dH(w)^{n-1}$$
$$+ \int_{\underline{w}}^{\bar{v}+z^*}\left[\int_{w-z^*}^{\bar{v}} g(w - v_i)\,dF(v_i)\right]\left[\frac{f'(w - z^*)}{h(w)} + \frac{(n-2)f(w - z^*)}{H(w)}\right] dH(w)^{n-1}.$$

The last line uses $dH(w)/ds = f(w - z^*)$ and $dh(w)/ds = f'(w - z^*)$. Next, substitute $dz^*/ds = -1/[1 - G(z^*)]$ (by equation (1)) into the derivative and divide the entire expression by $\int_{\underline{w}}^{\bar{v}+z^*} f(w-z^*)\,dH(w)^{n-1}$; then the expression above has the same sign as

$$\frac{-g(z^*)}{1-G(z^*)}+\frac{\displaystyle\int_{\underline{w}}^{\bar{v}+z^*}\left[\int_{w-z^*}^{\bar{v}} g(w-v_i)\,dF(v_i)\right]\left[\frac{f'(w-z^*)}{h(w)}+\frac{(n-2)f(w-z^*)}{H(w)}\right]dH(w)^{n-1}}{\displaystyle\int_{\underline{w}}^{\bar{v}+z^*} f(w-z^*)\,dH(w)^{n-1}}$$

$$\geq \frac{-g(z^*)}{1-G(z^*)}+\frac{\displaystyle\int_{\underline{w}}^{\bar{v}+z^*}\left[\frac{\int_{w-z^*}^{\bar{v}} g(w-v_i)\,dF(v_i)}{h(w)}\right]\left[\frac{f'(w-z^*)}{f(w-z^*)}\right]f(w-z^*)\,dH(w)^{n-1}}{\displaystyle\int_{\underline{w}}^{\bar{v}+z^*} f(w-z^*)\,dH(w)^{n-1}}.$$

Now take $s\to 0$ and therefore $z^*\to\bar{z}$. Since (i) $h(w)\to\int_{w-z^*}^{\bar{v}} g(w-v_i)\,dF(v_i)$ as $z^*\to\bar{z}$,[3] and (ii) $f'(\bar{v})/f(\bar{v})\leq f'(v)/f(v)$ for all $v<\bar{v}$ by the log-concavity of f, the limit of the above expression is at least

$$\lim_{z^*\uparrow\bar{z}}\frac{-g(z^*)}{1-G(z^*)}+\frac{f'(\bar{v})}{f(\bar{v})}.$$

Finally, if $f'(\bar{v})/f(\bar{v})>\lim_{z^*\uparrow\bar{z}} g(z^*)/[1-G(z^*)]$, then the last line is clearly positive and thus $\partial p^*/\partial s<0$ when s is small.[4] *Q.E.D.*

To put this result in context, note that Haan, Moraga-González, and Petrikaite (2017) showed that in a symmetric duopoly model with unobservable prices, if F has full support and $1-G$ is log-concave, then $\partial p^*/\partial s>0$. Since Claim 1 allows $n=2$ and log-concave $1-G$, the sign of $\partial p^*/\partial s$ is reversed in Claim 1 precisely because F has a bounded upper support and rising density. Indeed, when $\bar{v}<\infty$ and $f'>0$, as s rises, the upper support of $H(w)$, namely, $\bar{v}+z^*$, falls while the density $h(w)$ rises at all $w<\bar{v}+z^*$. As a result, the measure of marginal consumers rises as the other sellers' search costs rise. By this logic, as the other sellers' search costs rise, seller i is willing to lower p_i to attract more marginal consumers. On the other hand, as s_i rises, seller i has an incentive to raise p_i to extract more surplus from the visiting consumers. The overall effect depends on the relative strength of the two effects. We focus on small s because the first effect is relatively stronger when s is small—indeed, the magnitude of the change in the upper support $\partial(\bar{v}+z^*)/\partial s=-1/(1-G(z^*)$ is the largest when $s\approx 0$. When $s\approx 0$, the relative strength of these two effects depends on the ratio f'/f and the hazard rate $g/(1-G)$, respectively. Finally, since $f'(v)/f(v)$ falls in v and $g(z)/(1-G(z))$ rises in z, our second sufficient condition ensures $f'/f>g/(1-G)$ at all v and z.

E. CONSUMER SURPLUS AND SEARCH COSTS

We present an example where consumer surplus rises with search costs. Consider a symmetric duopoly environment with no outside option. Assume the prior and match val-

[3]Integrate equation (2) by parts and differentiate with respect to w; then $h(w)=\int_{w-z^*}^{\bar{v}} g(w-v_i)\,dF(v_i)+(1-G(z^*))f(w-z^*)$. The second term vanishes as $z^*\to\bar{z}$.

[4]If $\bar{z}=\infty$, then $\int_{\underline{w}}^{\bar{v}+z^*} f(w-z^*)\,dH(w)^{n-1}$ vanishes as $s\to 0$, and thus $\lim_{s\to 0}\partial p^*/\partial s=0$. But by continuity, the inequality $\partial p^*/\partial s<0$ remains valid for small but strictly positive s.

ues are uniform random variables with $V \sim U[0,3/4]$ and $Z \sim U[0,1]$. Since there is no outside option and $p_1 = p_2 = p^*$ in a symmetric equilibrium, every consumer purchases the product that offers the highest effective value. By Corollary 1, a (representative) consumer's expected payoff is equal to

$$CS = E\big[\max\{W_1, W_2\}\big] - p^*.$$

First, consider the effects of s on p^*. The equilibrium price is $p^* = 6/(9+32s)$ by direct calculation.[5] This implies

$$\frac{dp^*}{ds} = \frac{-192}{(9+32s)^2}.$$

The expected value of the first-order statistic $\max\{W_1, W_2\}$ can be written as

$$E\big[\max\{W_1, W_2\}\big] = 2\int_0^1 \int_0^{\frac{3}{4}} \big(v + \min\{z, z^*\}\big) H\big(v + \min\{z, z^*\}\big)\, dv\, dz.$$

Next, we consider the effect of s on $E[\max\{W_1, W_2\}]$. By equation (1), $dz^*/ds = -1/(1 - z^*)$. This result and the equation above imply

$$\begin{aligned}\frac{dE\big[\max\{W_1, W_2\}\big]}{ds} &= -2\int_0^{\frac{3}{4}} \big[H\big(v+z^*\big) + \big(v+z^*\big)h\big(v+z^*\big)\big]\, dv \\ &\quad - \frac{2}{1-z^*}\int_0^1 \left[\int_0^{\frac{3}{4}} \big(v + \min\{z, z^*\}\big) H_{z^*}\big(v + \min\{z, z^*\}\big)\, dv\right] dz, \end{aligned} \tag{11}$$

where $H_{z^*}(w)$ is defined as

$$H_{z^*}(w) \equiv \frac{dH(w)}{dz^*} = -f\big(w - z^*\big)\big(1 - G\big(z^*\big)\big) = -\frac{4}{3}\big(1 - z^*\big) \quad \text{for } w \in \big[z^*, z^* + 4/3\big],$$

and otherwise 0.

Now we evaluate the effect of an increase in s on CS at $s = 0$. When $s = 0$, $z^* = 1$ by equation (1). By direct calculation, the density and distribution function of W are

$$h(w) = \begin{cases} 4w/3 & \text{if } w \le 3/4, \\ 1 & \text{if } 3/4 < w < 1, \\ 7/3 - 4w/3 & \text{if } 7/4 \ge w > 1, \end{cases}$$

$$H(w) = \begin{cases} 2w^2/3 & \text{if } w \le 3/4, \\ w - 3/8 & \text{if } 3/4 < w < 1, \\ 7w/3 - 2w^2/3 - 25/24 & \text{if } 7/4 \ge w > 1. \end{cases}$$

[5]This pricing formula is also provided by Haan, Moraga-González, and Petrikaite (2017). They showed that $p^* = 3\bar{z}^2\bar{v}/(3\bar{z}\bar{v} + 3s\bar{v} - \bar{v}^2)$, assuming the return to search is sufficiently high so that the consumers who visit seller 1 first will always visit seller 2 with a strictly positive probability. They showed that this assumption is satisfied when s is sufficiently small and $\bar{z} > \bar{v}$. Both conditions are satisfied in our example.

Substitute the expressions for h, H, and H_{z^*} into equation (11); then

$$\begin{aligned}\frac{dE\big[\max\{W_1, W_2\}\big]}{ds}\bigg|_{s=0} &= -2\bigg[\int_0^{\frac{3}{4}} \big[H(v+1)+(v+1)h(v+1)\big]\,dv\bigg] \\ &\quad + \frac{8}{3}\int_0^1\int_0^{\frac{4}{3}} (v+z)\mathbb{1}_{\{v+z>1\}}\,dv\,dz \\ &= -2\bigg[\int_1^{\frac{7}{4}} -2w^2+\frac{14}{3}w-\frac{25}{24}\,dw\bigg]+\frac{8}{3}\bigg(\frac{45}{128}\bigg) \\ &= -\frac{21}{16}.\end{aligned}$$

Altogether, a consumer's expected surplus rises in s when $s=0$ because

$$\frac{dCS}{ds}\bigg|_{s=0} = \frac{dE\big[\max\{W_1, W_2\}\big]}{ds}\bigg|_{s=0} - \frac{dp^*}{ds}\bigg|_{s=0} = -\frac{21}{16}+\frac{192}{81} = \frac{457}{432} > 0.$$

Intuitively, as s rises, each consumer pays a larger utility cost to visit sellers. On the other hand, they are better off because the equilibrium price p^* falls in s. This example shows that the latter effect can dominate the former when s is small.

F. PRE-SEARCH INFORMATION: PROOF OF LEMMA 1

It suffices to show there exists $a' \in (0,1)$ such that $\partial h(H^{-1}(a))/\partial\alpha < 0$ if and only if $a > a'$. Let Φ denote the standard normal distribution function and ϕ denote its density function. Since $V \sim \mathcal{N}(0,\alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$, $F(v) = \Phi(v/\alpha)$ and $G(z) = \Phi(z/\sqrt{1-\alpha^2})$. Inserting these into equation (2) and differentiating $H(w)$ with respect to α yield

$$H_\alpha(w) \equiv \frac{\partial H(w)}{\partial\alpha} = -\bigg[1-\Phi\bigg(\frac{z^*}{\sqrt{1-\alpha^2}}\bigg)\bigg]\bigg(\frac{w-z^*}{\alpha^2}\bigg)\phi\bigg(\frac{w-z^*}{\alpha}\bigg),$$

where $\partial z^*/\partial\alpha$ can be obtained from equation (1) by applying the implicit function theorem. Differentiating again with respect to w gives

$$h_\alpha(w) \equiv \frac{\partial h(w)}{\partial\alpha} = -\bigg[1-\Phi\bigg(\frac{z^*}{\sqrt{1-\alpha^2}}\bigg)\bigg]\bigg[1-\bigg(\frac{w-z^*}{\alpha}\bigg)^2\bigg]\frac{1}{\alpha^2}\phi\bigg(\frac{w-z^*}{\alpha}\bigg).$$

Now observe that

$$\frac{\partial h\big(H^{-1}(a)\big)}{\partial\alpha} = h_\alpha\big(H^{-1}(a)\big) - H_\alpha\big(H^{-1}(a)\big)\frac{h'\big(H^{-1}(a)\big)}{h\big(H^{-1}(a)\big)}.$$

Let $w = H^{-1}(a)$ and apply $H_\alpha(w)$ and $h_\alpha(w)$ to the equation. Then,

$$\frac{\partial h\big(H^{-1}(a)\big)}{\partial\alpha} = \frac{-1}{\alpha^2}\bigg[1-\Phi\bigg(\frac{z^*}{\sqrt{1-\alpha^2}}\bigg)\bigg]\phi\bigg(\frac{w-z^*}{\alpha}\bigg)\bigg[1-\frac{(w-z^*)^2}{\alpha^2}-(w-z^*)\frac{h'(w)}{h(w)}\bigg].$$

Since $V \sim \mathcal{N}(0, \alpha^2)$ and $Z \sim \mathcal{N}(0, 1-\alpha^2)$, the density of $W = V + \min\{Z, z^*\}$ is

$$h(w) = \frac{1}{\sqrt{1-\alpha^2}} \int_{-\infty}^{\infty} \phi\left(\frac{w - \min\{z, z^*\}}{\alpha}\right) \phi\left(\frac{z}{\sqrt{1-\alpha^2}}\right) dz$$
$$= \frac{1}{\sqrt{1-\alpha^2}} \int_{-\infty}^{\infty} \phi\left(\frac{w - z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^* - \alpha r}{\sqrt{1-\alpha^2}}\right) dr,$$

where the second line changes variable $r = (z^* - z)/\alpha$. Since $\partial\phi(x)/\partial x = -x\phi(x)$,

$$\frac{h'(w)}{h(w)} = -\frac{w - z^*}{\alpha^2} - \frac{\displaystyle\int_{-\infty}^{\infty} \max\{r, 0\} \phi\left(\frac{w - z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^* - \alpha r}{\sqrt{1-\alpha^2}}\right) dr}{\displaystyle\alpha \int_{-\infty}^{\infty} \phi\left(\frac{w - z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^* - \alpha r}{\sqrt{1-\alpha^2}}\right) dr}.$$

Applying this to the above equation leads to

$$\frac{\partial h\big(H^{-1}(a)\big)}{\partial \alpha} \propto -1 + \left(\frac{w - z^*}{\alpha}\right)^2 + \big(w - z^*\big)\frac{h'(w)}{h(w)}$$
$$= -1 + \frac{(z^* - w)}{\alpha} \frac{\displaystyle\int_{-\infty}^{\infty} \mathbb{1}_{\{r \geq 0\}} r \phi\left(\frac{w - z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^* - \alpha r}{\sqrt{1-\alpha^2}}\right) dr}{\displaystyle\int_{-\infty}^{\infty} \phi\left(\frac{w - z^*}{\alpha} + \max\{r, 0\}\right) \phi\left(\frac{z^* - \alpha r}{\sqrt{1-\alpha^2}}\right) dr}.$$

The last expression is clearly negative if $w > z^*$. In addition, it converges to ∞ as w tends to $-\infty$. For $w \leq z^*$, it decreases in w because $(z^* - w)$ falls in w and the density $\phi((w - z^*)/\alpha + \max\{r, 0\})$ is log-submodular in (w, r). Therefore, there exists w' less than z^* such that the expression is positive if and only if $w < w'$. The desired result follows from the fact that $w = H^{-1}(a)$ is strictly increasing in a.

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