Bounded Normal Mean

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```r
library("NPRHonest")
```

Let $cv_{1-\alpha}(B)$ denote the half-length of a fixed-length $1-\alpha$ confidence interval (CI) around a normally distributed estimator with variance one and maximum bias B. This critical value can be computed as `CVb(B, alpha)`. The function `CVb` returns a dataframe of the critical values:

```r
RDHonest::CVb(B = 1, alpha = 0.5)
```

bias alpha cv TeXDescription 1 1 0.5 1.05054
$alpha = 0.5$

```r
d <- RDHonest::CVb(B = sort(c(seq(0, 1, by = 0.1), sqrt(1/(2/3) - 1), sqrt(1/(6/7) -
    1), 1.5, 2)), alpha = c(0.01, 0.05, 0.1))
t1 <- reshape2::dcast(d[, -2], bias ~ TeXDescription, value.var = "cv")
pander::pandoc.table(t1, justify = "left", split.cells = 5)
```

bias	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$
0	2.576	1.96	1.645
0.1	2.589	1.97	1.653
0.2	2.626	1.999	1.677
0.3	2.683	2.045	1.717
0.4	2.757	2.107	1.772
0.4082	2.764	2.113	1.777
0.5	2.842	2.181	1.839
0.6	2.934	2.265	1.916
0.7	3.03	2.356	2.001
0.7071	3.037	2.362	2.008
0.8	3.128	2.45	2.093
0.9	3.227	2.548	2.187
1	3.327	2.646	2.284
1.5	3.826	3.145	2.782
2	4.326	3.645	3.282

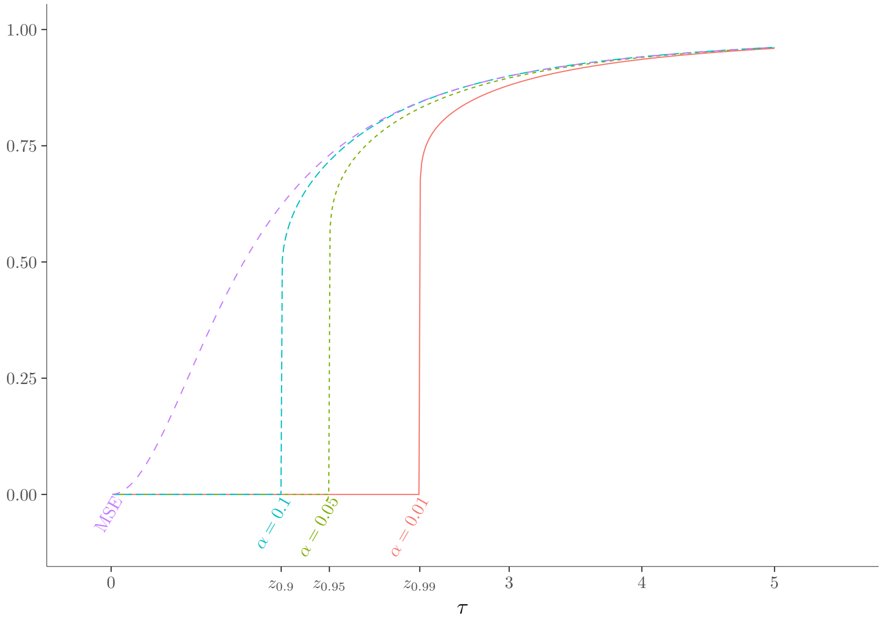
Given an observation $X \sim N(\theta, 1)$ with $\theta \in [-\tau, \tau]$, the optimal affine fixed-length confidence interval can be computed with `bnmflci`. This gives the optimal shrinkage `c` and optimal half-length `chi`, so that the resulting Ci is given by $cX \pm \chi$:

```r
bnmflci(tau = 2, alpha = 0.05)
#> $c
#> [1] 0.771448
#>
#> $chi
#> [1] 1.74259
```

Plots of critical values:

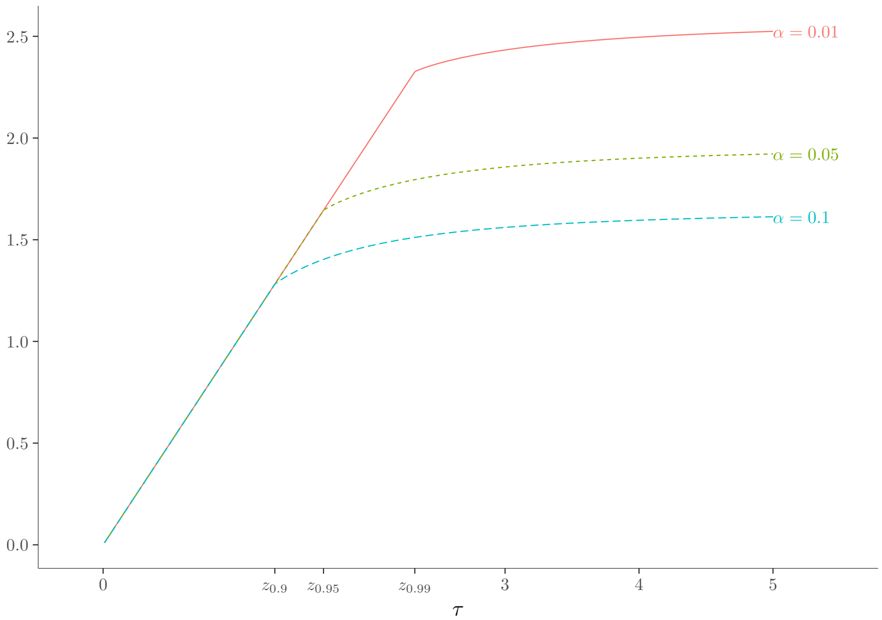
```r
library("ggplot2")  # otherwise directlabels freaks out
p <- plot_bnm()
```

```
p$p1
```



1.00
0.75
0.50
0.25
0.00
MSE
$\alpha = 0.1$
$\alpha = 0.05$
$\alpha = 0.01$
0
$z_{0.9}$
$z_{0.95}$
$z_{0.99}$
3
4
5
τ


```
p$p2
```



2.5
2.0
1.5
1.0
0.5
0.0
$\alpha = 0.01$
$\alpha = 0.05$
$\alpha = 0.1$
0
$z_{0.9}$
$z_{0.95}$
$z_{0.99}$
3
4
5
τ