# SUPPLEMENT TO "MONEY AS A UNIT OF ACCOUNT" <br> (Econometrica, Vol. 85, No. 5, September 2017, 1537-1574) 

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## APPENDIX B: Coalition-Proof Contracts

WHEN DEFINING OPTIMAL CONTRACTS in the main text, we have required only individual rationality. In an environment with multiple bilateral meetings, it is also attractive to rule out joint deviations by a pair of agents within a meeting. We now define coalitionproof contracts and point out that the optimal contract characterized by Proposition 3 is coalition proof.

Consider first morning meetings. We require that for all alternative bilateral contracts $\left(\tilde{x}_{2}, \tilde{\boldsymbol{\pi}}_{2}\right)$ that are feasible-that is, $\tilde{\boldsymbol{\pi}}_{2} \in \boldsymbol{\Pi}_{2}(\boldsymbol{\Pi})$-agents are no better off than under the economy-wide contract:

$$
\begin{align*}
E\left(U_{2}(X, \boldsymbol{\Pi} ; \theta)\right) & \geq(1+\lambda) \tilde{x}_{2}-q\left(\tilde{\boldsymbol{\pi}}_{2}\right)+E\left(q\left(\boldsymbol{\pi}_{1}(\theta)\right)-x(\theta)\right),  \tag{28}\\
U_{3}(X, \boldsymbol{\Pi}) & \geq q\left(\tilde{\boldsymbol{\pi}}_{2}\right)-\tilde{x}_{2} .
\end{align*}
$$

Proposition 3 shows that any optimal contract maximizes the scale of payment in meetings between artisans 2 and 3. As a result, there cannot be any fruitful joint deviation and the contract satisfies satisfies (28).

Consider now joint deviations by a farmer of type $\theta$ and an artisan from location 2 in a night meeting. For all alternative bilateral contracts ( $\tilde{x}_{1}, \tilde{\boldsymbol{\pi}}_{1}$ ) that allow the artisan to make his already promised payment-that is, $\boldsymbol{p}^{\prime} \tilde{\boldsymbol{\pi}}_{1} \geq \boldsymbol{p}^{\prime} \boldsymbol{\pi}_{2}$ for all $\boldsymbol{p} \in \boldsymbol{P}$-we require

$$
\begin{aligned}
E\left(U_{2}(X, \boldsymbol{\Pi} ; \theta)\right) & \geq(1+\lambda) x_{2}-q\left(\boldsymbol{\pi}_{2}\right)+E\left(q\left(\tilde{\boldsymbol{\pi}}_{1}\right)\right)-\tilde{x}_{1}, \\
U_{1}(X, \boldsymbol{\Pi} ; \theta) & \geq(1+\lambda) \tilde{x}_{1}-q\left(\tilde{\boldsymbol{\pi}}_{1}\right)+E\left(\boldsymbol{p}^{\prime} \boldsymbol{y} \mid \theta\right) .
\end{aligned}
$$

Again an optimal contract does not allow such a deviation since Proposition 3 says the optimal contract maximizes scale within meetings between farmers and artisans from location 2.

## APPENDIX C: Endowment Income for Artisans

Proposition 3 assumes that artisans in locations 2 and 3 do not receive their own endowment of tradable goods. Instead, they rely for payment on the endowment of the farmer at the head of their chain. Here we consider an extension where artisans can receive endowment income as well. Suppose that at the beginning, every agent learns an endowment type $\theta \in \Theta$. Every chain is then characterized by an endowment vector $\left(\theta_{1}, \theta_{2}, \theta_{3}\right)$, where $\theta_{i}$ indicates the type of the agent in location $i$.

[^0]It is enough to consider an economy with one endowment type in locations 2 and 3. Since types $\theta_{2}$ and $\theta_{3}$ are realized before morning meetings where the first actions in the economy are taken, the overall economy divides into many independent subeconomies according to the realization $\left(\theta_{2}, \theta_{3}\right)$. Moreover, the effect of an endowment for the artisan in location 3 affects only that agent's outside option, but does not alter any feasibility constraints.

Consider thus a modified environment in which artisans in location 2 receive an endowment with support $\boldsymbol{Y}_{2}$. The optimal payment satisfies the recursion in Proposition 3 if we define

$$
\boldsymbol{\Pi}_{1}(\boldsymbol{\Pi})=\left(\boldsymbol{\pi}_{1}(\theta)+\boldsymbol{y}_{2}: \theta \in \Theta, \boldsymbol{y}_{2} \in \boldsymbol{Y}_{2}\right)
$$

Indeed, an optimal payment in morning meetings maximizes scale given the set $\boldsymbol{\Pi}_{1}(\boldsymbol{\Pi})$; adding $\boldsymbol{Y}_{2}$ only changes the initial set of available bundles. Moreover, an optimal bundle must still maximize scale in night meetings also: the key property exploited in the proof of Proposition 3 is that $\boldsymbol{\pi}_{1}(\theta)$ maximizes the value of production for any price vector. Higher scale in night meetings thus allows higher scale in morning meetings with or without an extra endowment.

While the recursion remains unchanged with an artisan endowment, the evolution of the optimal unit of account is different. In particular, the optimal unit of account will be tailored more closely to the endowment of artisan 2 . Like farmer endowments, artisan endowments are thus a force that can make the unit of account different across transactions. Of course, if the artisan endowment risk is similar in nature to the risk from meeting farmers, say, because it arises from a second meeting with another farmer, then it will have no effect on the unit of account and only increase the scale.

## APPENDIX D: Setup With Small Default Costs

In the main text, our results were derived for an economy in which default costs are large, so that only non-contingent contracts can be used. In this appendix, we outline how the framework can be extended to allow for small default costs, in the sense that the cost of breaking promises is sufficiently small for breaking promises to be optimal in some cases. Even so, the unit of account still matters, because an appropriately chosen unit of account is necessary to minimize the cost of settling contracts in the economy.

Most of the results for the large-cost case carry over unchanged to the small-cost setting. The key differences arise when we consider what the optimal unit of account should look like. In the small-default-cost setting, the objective is to minimize the probability of default, but not to avoid default entirely. This implies that the probability of meeting different types of agents becomes an important determinant of the optimal unit of account. Parallel to our analysis in Section 6, this feature can be developed into a model of optimal currency areas, even without relying on the ability (as assumed in Section 6) to switch trading partners ex post at a cost.

We develop the small-default-cost setting more fully in the working paper version of this paper, Doepke and Schneider (2013). Here, we describe the basic setup in the context of the partial-equilibrium setup described in Section 3. To allow for a small default cost, we introduce a distinction between a a promised payment and an actual payment. The future payment from customer to supplier is specified in two parts. The first component is a noncontingent promised payment, namely a vector of farm-good quantities $\boldsymbol{\pi}=\left(\pi^{A}, \pi^{B}\right)^{\prime}$ (as before). By promising $\pi$ to the supplier, the customer commits to delivering goods $\pi$ at date 2 . Unlike in the large-cost case, we now allow for the possibility that a customer
may not always be able to deliver on a payment promise $\pi$. To deal with this possibility, the second component of the contracted payment consists of a fully contingent payment $h(\boldsymbol{y}, \boldsymbol{p})$, where $h(\boldsymbol{y}, \boldsymbol{p}) \leq \boldsymbol{p}^{\prime} \boldsymbol{\pi}$, that is, the value of the alternative payment is no greater than the original promise. The actual payment that the customer has to make in state $(\boldsymbol{y}, \boldsymbol{p})$ is the smaller of the promised and the alternative payment:

$$
\min \left\{\boldsymbol{p}^{\prime} \boldsymbol{\pi}, h(\boldsymbol{y}, \boldsymbol{p})\right\}
$$

Given that $h(\boldsymbol{y}, \boldsymbol{p}) \leq \boldsymbol{p}^{\prime} \boldsymbol{\pi}$, the actual payment is in fact always equal to $h(\boldsymbol{y}, \boldsymbol{p})$. This actual payment is fully enforced. The full contract between customer and supplier specifies the customized artisanal good $x$ to be produced by the supplier at date 1 and delivered to the customer at date 2 , the payment promise $\boldsymbol{\pi}$, and actual payment $h(\boldsymbol{y}, \boldsymbol{p})$.

Given that the actual payment is fully contingent, the two-part payment specification as such does not constitute a deviation from complete markets. However, we assume that making a payment that is different from the initial promise is costly. If the promise is met, the customer's cost for settling the contract is zero, $s=0$. In contrast, whenever we have $h(\boldsymbol{y}, \boldsymbol{p})<\boldsymbol{p}^{\prime} \boldsymbol{\pi}$, the customer faces a fixed cost $s=\kappa \geq 0$ in terms of time at date 2 . The interpretation is that enforcing the contract and executing the alternative payment in the case of a broken promise involves a legal cost. For different values of $\kappa$, this setup captures the usual complete-market setting $(\kappa=0)$, fully non-contingent contracts $(\kappa=\infty)$ as in the main analysis above, and settings where the contracting friction affects outcomes, but is not sufficiently strong to reduce to the non-contingent case.

## REFERENCE

Doepke, M., AND M. Schneider (2013): "Money as a Unit of Account," CEPR Discussion Paper 9700. [2]


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