SUPPLEMENT TO "THE DYNAMIC PIVOT MECHANISM"

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We show that the socially efficient solution to the scheduling problem in Section 3 of the paper can be realized through a bidding mechanism, specifically a dynamic version of the ascending price auction, rather than a direct revelation mechanism. We also give a slight modification of the example where the bidding mechanism is inefficient.

IN THE SCHEDULING PROBLEM in Section 3 of the main paper, a number of bidders compete for a scare resource, namely early access to a central facility. We show here that the efficient allocation can be realized through a bidding mechanism rather than a direct revelation mechanism. We find a dynamic version of the ascending price auction where the contemporaneous use of the facility is auctioned. As a given task is completed, the number of effective bidders decreases by one. We can then use a backward induction algorithm to determine the values for the bidders starting from a final period in which only a single bidder is left without effective competition.

Consider then an ascending auction in which all tasks except that of bidder I have been completed. Along the efficient path, the final ascending auction will occur at time $t = I - 1$. Since all other bidders have vanished along the efficient path at this point, bidder I wins the final auction at a price equal to zero. By backward induction, we consider the penultimate auction in which the only bidders left are $I - 1$ and I. As agent I can anticipate to win the auction tomorrow even if she were to loose it today, she is willing to bid at most

$$(S1) \qquad b_I(v_I) = v_I - \delta(v_I - 0),$$

namely the net value gained by winning the auction today rather than tomorrow. Naturally, a similar argument applies to bidder $I - 1$: by dropping out of the competition today, bidder $I - 1$ would get a net present discounted value of $\delta\omega_{I-1}$ and hence her maximal willingness to pay is given by

$$b_{I-1}(v_{I-1}) = v_{I-1} - \delta(v_{I-1} - 0).$$

Since $b_{I-1}(v_{I-1}) \geq b_I(v_I)$, given $v_{I-1} \geq v_I$, it follows that bidder $I - 1$ wins the ascending price auction in $t = I - 2$ and receives a net payoff

$$v_{I-1} - (1 - \delta)v_I.$$

We proceed inductively and find that the maximal bid of bidder $I - k$ in period $t = I - k - 1$ is given by

$$(S2) \qquad b_{I-k}(v_{I-k}) = v_{I-k} - \delta\big(v_{I-k} - b_{I-(k-1)}\big(v_{I-(k-1)}\big)\big).$$

In other words, bidder $I - k$ is willing to bid as much as to be indifferent between being selected today and being selected tomorrow, when she would be able to realize a net valuation of $v_{I-k} - b_{I-(k-1)}$, but only tomorrow, and so the net gain from being selected today rather than tomorrow is

$$v_{I-k} - \delta\big(v_{I-k} - b_{I-(k-1)}\big).$$

The maximal bid of bidder $I - (k - 1)$ generates the transfer price of bidder $I - k$ and by solving (S2) recursively with the initial condition given by (S1), we find that the price in the ascending auction equals the externality cost in the direct mechanism. In this class of scheduling problems, the efficient allocation can therefore be implemented by a bidding mechanism.[1]

We end this section with a minor modification of the scheduling model to allow for multiple tasks. For this purpose, it is sufficient to consider an example with two bidders. The first bidder has an infinite series of single-period tasks, each delivering a value of v_1. The second bidder has only a single task with a value v_2. The utility function of bidder 1 is thus given by

$$v_1(a_t, \theta_{1,t}) = \begin{cases} v_1, & \text{if } a_t = 1 \text{ for all } t, \\ 0, & \text{if otherwise,} \end{cases}$$

whereas the utility function of bidder 2 is as described earlier.

The socially efficient allocation in this setting either has $a_t = 1$ for all t if $v_1 \geq v_2$ or $a_0 = 2$, $a_t = 1$ for all $t \geq 1$ if $v_1 < v_2$. For the remainder of this example, we assume that $v_1 > v_2$. Under this assumption, the efficient policy never completes the task of bidder 2. The marginal contributions of each bidder are

$$M_1(\theta_0) = (v_1 - v_2) + \frac{\delta}{1 - \delta} v_1$$

and

$$M_2(\theta_0) = 0.$$

Along any efficient allocation path, we have $M_i(\theta_0) = M_i(\theta_t)$ for all i and the social externality cost of agent 1, $p_1^*(\theta_t)$ for all t, is $p_1^*(\theta_t) = (1 - \delta)v_2$. The externality cost is again the cost of delay imposed on the competing bidder, namely $(1 - \delta)$ times the valuation of the competing bidder. This accurately represents the social externality cost of agent 1 in every period even though agent 2 never receives access to the facility.

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[1]The nature of the recursive bidding strategies bears some similarity to the construction of the bidding strategies for multiple advertising slots in the keyword auction of Edelman, Ostrovsky, and Schwartz (2007). In the auction for search keywords, the multiple slots are differentiated by their probability of receiving a hit and hence generating a value. In the scheduling model here, the multiple slots are differentiated by the time discount associated with different access times.

We contrast the efficient allocation and transfer with the allocation resulting in the dynamic ascending price auction. For this purpose, suppose that the equilibrium path generated by the dynamic bidding mechanism is efficient. In this case, bidder 2 is never chosen and hence receives a net payoff of 0 along the equilibrium path. But this means that bidder 2 would be willing to bid up to v_2 in every period. In consequence, the first bidder receives a net payoff of $v_1 - v_2$ in every period and her discounted sum of payoff is then

$$\text{(S3)} \qquad \frac{1}{1-\delta}(v_1 - v_2) < M_1(\theta_0).$$

But more important than the failure of the marginal contribution is the fact that the equilibrium does not support the efficient assignment policy. To see this, notice that if bidder 1 loses to bidder 2 in any single period, then the task of bidder 2 is completed and bidder 2 drops out of the auction in all future stages. Hence the continuation payoff for bidder 1 from dropping out in a given period and allowing bidder 2 to complete his task is given by

$$\text{(S4)} \qquad \frac{\delta}{1-\delta}v_1.$$

If we compare the continuation payoffs (S3) and (S4), respectively, then we see that it is beneficial for bidder 1 to win the auction in all periods if and only if

$$v_1 \geq \frac{v_2}{1-\delta},$$

but the efficiency condition is simply $v_1 \geq v_2$. It follows that for a large range of valuations, the outcome in the ascending auction is inefficient and assigns the object to bidder 2 despite the inefficiency of this assignment. The reason for the inefficiency is easy to detect in this simple setting. The forward-looking bidders consider only their individual net payoffs in future periods. The planner, on the other hand, is interested in the level of gross payoffs in future periods. As a result, bidder 1 is strategically willing and able to depress the future value of bidder 2 by letting bidder 2 win today to increase the future difference in the valuations between the two bidders. But from the point of view of the planner, the differential gains for bidder 1 are immaterial and the assignment to bidder 2 represents an inefficiency. The rule of the ascending price auction, namely that the highest bidder wins, only internalizes the individual *equilibrium payoffs* but not the social payoffs.

This small extension to multiple tasks shows that the logic of the marginal contribution mechanism can account for subtle intertemporal changes in the payoffs. On the other hand, common bidding mechanisms may not resolve the dynamic allocation problem in an efficient manner. Indirectly, it suggests that suitable indirect mechanisms have yet to be devised for scheduling and other sequential allocation problems.

REFERENCE

EDELMAN, B., M. OSTROVSKY, AND M. SCHWARTZ (2007): "Internet Advertising and the Generalized Second Price Auction: Selling Billions of Dollars Worth of Keywords," *American Economic Review*, 97, 242–259.

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