Econometrica Supplementary Material

SUPPLEMENT TO "ORGANIZING THE GLOBAL VALUE CHAIN"

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This supplement documents several detailed proofs from Sections 2 and 3 of the main text of the paper, that were omitted due to space constraints. It also contains Tables S.I–S.X and Figures S.2 and S.3, which were mentioned in the main text of the paper.

A. OPTIMAL DIVISION OF SURPLUS: SUFFICIENT CONDITION

IN THIS SECTION, we show that the function $\beta^*(j)$ in equation (15) of the main text, characterizing the optimal division of surplus at stage j, indeed satisfies a sufficient condition for the associated profit-maximization problem of the firm. Our approach builds on recasting this as a dynamic programming problem.

Remember that in the main text, we reduced the problem to that of finding a function v that maximizes

$$\pi_F(v) = \kappa \int_0^1 \big(1 - v'(j)^{(1-\alpha)/\alpha}\big) v'(j) v(j)^{(\rho-\alpha)/(\alpha(1-\rho))}\, dj. \tag{A.1}$$

As a reminder, $\kappa \equiv A\frac{\rho}{\alpha}(\frac{1-\rho}{1-\alpha})^{(\rho-\alpha)/(\alpha(1-\rho))}(\frac{\rho}{c})^{\rho/(1-\rho)}$ is a positive constant.

Define the *value function* $V(j, v)$ associated with this problem as

$$V(j, v) = \kappa \sup_{v'_{[j,1]}} \int_j^1 \big(1 - v'(k)^{(1-\alpha)/\alpha}\big) v'(k) v(k)^{(\rho-\alpha)/(\alpha(1-\rho))}\, dk,$$

where v in the argument of the value function satisfies $v = v(j)$. The Hamilton–Jacobi–Bellman equation associated with this problem is

$$-V_j(j, v) = \sup_{v'} \big\{\kappa\big(1 - (v')^{(1-\alpha)/\alpha}\big) v' v^{(\rho-\alpha)/(\alpha(1-\rho))} + V_v(j, v) v'\big\}, \tag{A.2}$$

with boundary condition $V(1, v) = 0$. The right-hand-side problem is strictly concave and delivers a unique solution:

$$\alpha\left(1 + \frac{V_v(j, v)}{\kappa v^{(\rho-\alpha)/(\alpha(1-\rho))}}\right) = (v')^{(1-\alpha)/\alpha},$$

which we can plug back into (A.2). After some simplifications, we have

$$-V_j(j, v) = (1-\alpha)\left(\frac{\alpha}{\kappa}\right)^{\alpha/(1-\alpha)} \times \big(V_v(j, v) + \kappa v^{(\rho-\alpha)/(\alpha(1-\rho))}\big)^{1/(1-\alpha)} v^{(\alpha-\rho)/((1-\rho)(1-\alpha))}. \tag{A.3}$$

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From well-known results (see, for instance, Bertsekas (2005, Proposition 3.2.1), or Liberzon (2011, Section 5.1.4)), if the value function associated with the solution v' that satisfies the necessary conditions for optimality also satisfies the HJB equation (A.3), then that would be sufficient to conclude that v' (and thus β^*) delivers a maximum.

To prove so, let us begin by defining

$$J\big(j, v, v'_{[j,1]}\big) = \kappa \int_j^1 \big(1 - \big(v'(k)\big)^{(1-\alpha)/\alpha}\big) v'(k) \big(v(k)\big)^{(\rho-\alpha)/(\alpha(1-\rho))} \, dk,$$

which is the functional in the main text, but with the lower limit of the integral starting at $j \in [0, 1]$. The optimization problem for this functional is analogous to the problem in our Benchmark Model except for the lower limit of the integral. As shown in the Appendix of the main text, we must have

$$v(k) = \left(\frac{(1-\alpha)C_1}{1-\rho}(k - C_2)\right)^{(1-\rho)/(1-\alpha)}, \quad \text{and}$$
$$v'(k) = \left(\frac{(1-\alpha)C_1}{1-\rho}(k - C_2)\right)^{(\alpha-\rho)/(1-\alpha)} C_1.$$

Here, C_1 and C_2 are the associated constants of integration. The key difference here from our Benchmark Model is in the initial condition, which is now $v(j) = v$, while the transversality condition continues to be given by $v'(1)^{(1-\alpha)/\alpha} = \alpha$. Using these two conditions, we find that C_1 and C_2 are implicitly defined by

$$\left(\frac{(1-\alpha)C_1}{1-\rho}(j - C_2)\right)^{(1-\rho)/(1-\alpha)} = v, \quad \text{and} \tag{A.4}$$

$$\frac{1-\rho}{1-\alpha}(1 - C_2)^{(\alpha-\rho)/(1-\alpha)} \frac{v}{(j - C_2)^{(1-\rho)/(1-\alpha)}} = \alpha^{\alpha/(1-\alpha)}. \tag{A.5}$$

Note that C_1 and C_2 are therefore functions of j and v. Moreover, since we must have $C_2 < j < 1$ in order for v to be greater than 0, the constants of integration C_1 and C_2 are continuously differentiable in j and v.

The value function $V(j, v)$ is then

$$V(j, v) = \kappa \sup_{v'_{[j,1]}} \int_j^1 \big(1 - \big(v'(k)\big)^{(1-\alpha)/\alpha}\big) v'(k) \big(v(k)\big)^{(\rho-\alpha)/(\alpha(1-\rho))} \, dk$$
$$= \kappa \int_j^1 \left(1 - \left(\left(\frac{(1-\alpha)C_1}{1-\rho}(k - C_2)\right)^{(\alpha-\rho)/(1-\alpha)} C_1\right)^{(1-\alpha)/\alpha}\right)$$

$$\times \left(\frac{(1-\alpha)C_1}{1-\rho}(k-C_2)\right)^{(\alpha-\rho)/(1-\alpha)}$$
$$\times C_1\left(\frac{(1-\alpha)C_1}{1-\rho}(k-C_2)\right)^{(\rho-\alpha)/(\alpha(1-\alpha))} dk,$$

which can be simplified to

$$V(j,v) = \kappa \int_j^1 \left(\frac{1-\rho}{1-\alpha}\left(\frac{(1-\alpha)C_1}{1-\rho}\right)^{\rho/\alpha}(k-C_2)^{(\rho-\alpha)/\alpha} - C_1^{1/\alpha}\right) dk.$$

Evaluating the integral, we have

$$V(j,v) = \kappa\left\{\frac{1-\rho}{1-\alpha}\frac{\alpha}{\rho}\left(\frac{(1-\alpha)C_1}{1-\rho}\right)^{\rho/\alpha}\left[(1-C_2)^{\rho/\alpha} - (j-C_2)^{\rho/\alpha}\right] - C_1^{1/\alpha}(1-j)\right\},$$

which, using equations (A.4) and (A.5), can be reduced to

$$V(j,v) = \kappa\left\{\frac{1-\rho}{1-\alpha}\frac{\alpha}{\rho}\left[C_1^{\rho(1-\alpha)/(\alpha(\rho-\alpha))}\alpha^{\rho/(\alpha-\rho)} - v^{\rho(1-\alpha)/(\alpha(1-\rho))}\right] - C_1^{1/\alpha}(1-j)\right\}. \quad \text{(A.6)}$$

By eliminating C_2 from equations (A.4) and (A.5), one can see that C_1 itself is given implicitly by

$$\frac{(1-\alpha)C_1}{1-\rho}(1-j) + v^{(1-\alpha)/(1-\rho)} = \alpha^{\alpha/(\alpha-\rho)}(C_1)^{(1-\alpha)/(\rho-\alpha)}. \quad \text{(A.7)}$$

Our final step is to show that the value function defined by equations (A.6) and (A.7) indeed satisfies the Hamilton–Jacobi–Bellman equation in (A.3). Implicit differentiation of (A.7) produces

$$\frac{dC_1}{dj} = -\frac{C_1}{\alpha^{\alpha/(\alpha-\rho)}\dfrac{1-\rho}{\rho-\alpha}C_1^{(1-\rho)/(\rho-\alpha)} - (1-j)}, \quad \text{and}$$

$$\frac{dC_1}{dv} = \frac{v^{(\rho-\alpha)/(1-\rho)}}{\alpha^{\alpha/(\alpha-\rho)}\dfrac{1-\rho}{\rho-\alpha}C_1^{(1-\rho)/(\rho-\alpha)} - (1-j)}.$$

Using these expressions to (totally) differentiate $V(j, v)$ in (A.6) with respect to j and v, and simplifying, we obtain

$$V_j(j, v) = -\kappa \frac{1-\alpha}{\alpha} C_1^{1/\alpha}, \quad \text{and}$$
$$V_v(j, v) = \kappa \left(\frac{1}{\alpha} C_1^{(1-\alpha)/\alpha} v^{(\rho-\alpha)/(1-\rho)} - v^{(\rho-\alpha)/(\alpha(1-\rho))} \right).$$

Plugging the above expressions for $V_j(j, v)$ and $V_v(j, v)$ into (A.3), it is straightforward to verify that the Hamilton–Jacobi–Bellman equation in (A.3) is indeed satisfied. This confirms that the function $\beta^*(j)$ satisfies the sufficient condition for a maximum. Note, finally, that because we have only one candidate solution for a maximum that satisfies the Euler–Lagrange equation, we can comfortably state that $\beta^*(j)$ is the global maximizer within the set of piecewise continuously differentiable real-valued functions.

B. OPTIMAL DIVISION OF SURPLUS: CONSTRAINED PROBLEM

In solving for the optimal division of surplus at each stage in the main paper, we have not constrained the optimal bargaining share $\beta^*(m)$ to be nonnegative or no larger than 1. The latter assumption is without loss of generality, since the solution to the problem satisfies $\beta^*(m) = 1 - \alpha m^{(\alpha-\rho)/\alpha} \leq 1$. Note, however, that in the sequential complements case ($\rho > \alpha$), when m is sufficiently small we necessarily have $\beta^*(m) < 0$. As argued in the main text, a negative $\beta^*(m)$ can be justified by appealing to the fact that the firm might find it optimal to compensate certain suppliers with a payoff that exceeds their marginal contribution. Still, it is worth exploring how the optimal division of surplus is affected by imposing the constraint $\beta^*(m) \geq 0$. It might seem natural that the modified solution in the complements case would be given by $\beta^*(m) = \max\{1 - \alpha m^{(\alpha-\rho)/\alpha}, 0\}$, but we will show below that this would be an incorrect guess.

The problem that we seek to solve can be written as

$$\max_{\{v'(j)\}_{j\in[0,1]}} \int_0^1 \left(1 - v'(j)^{(1-\alpha)/\alpha}\right) v'(j) v(j)^{(\rho-\alpha)/(\alpha(1-\rho))} \, dj$$
$$\text{s.t.} \quad 0 \leq v'(j) \leq 1,$$

with initial condition $v(0) = 0$. Remember that this formulation follows from defining

$$v(j) \equiv \int_0^j \left(1 - \beta(k)\right)^{\alpha/(1-\alpha)} \, dk,$$

which in turn implies $\beta(j) = 1 - v'(j)^{(1-\alpha)/\alpha}$.

Observe first that in the sequential substitutes case ($\rho < \alpha$), the solution to the unconstrained problem does not violate the constraint $0 \leq v'(j) \leq 1$, since $0 \leq \alpha j^{(\alpha-\rho)/\alpha} < 1$. Thus, the solution obtained from solving the unconstrained problem is necessarily also that which yields the maximum for the constrained problem.

We therefore concentrate below on the sequential complements case ($\rho > \alpha$). As mentioned above for the unconstrained problem, we necessarily have that $v'(j) > 0$ (or $\beta(j) < 1$) for all $j > 0$. As we will show below, the same will be true for the solution to the constrained problem (i.e., when imposing $v'(j) \leq 1$), and thus, for the time being, we ignore the constraint $v'(j) \geq 0$.

To solve the constrained problem, it is simplest to write down the Hamiltonian associated with the problem, where, for simplicity, we drop the argument j and define $u \equiv v'$:

$$H = \left(1 - u^{(1-\alpha)/\alpha}\right) u v^{(\rho-\alpha)/(\alpha(1-\rho))} + \lambda u + \theta(1-u).$$

Here, λ is the costate variable and θ is the multiplier associated with the constraint $u \leq 1$. The first-order conditions associated with this problem are

$$\text{(B.1)} \qquad H_u = \left(1 - \frac{1}{\alpha} u^{(1-\alpha)/\alpha}\right) v^{(\rho-\alpha)/(\alpha(1-\rho))} + \lambda - \theta = 0, \quad \text{and}$$

$$\text{(B.2)} \qquad H_v = \left(1 - u^{(1-\alpha)/\alpha}\right) u \frac{\rho-\alpha}{\alpha(1-\rho)} v^{(\rho-\alpha)/(\alpha(1-\rho))-1} = -\lambda'.$$

Combining these to eliminate λ and λ', we have

$$\begin{aligned} &\frac{1-\alpha}{\alpha^2} u^{1/\alpha} \frac{\rho-\alpha}{(1-\rho)} v^{(\rho-\alpha)/(\alpha(1-\rho))-1} \\ &\quad = -\frac{1-\alpha}{\alpha^2} u^{(1-\alpha)/\alpha-1} u' v^{(\rho-\alpha)/(\alpha(1-\rho))} - \theta'. \end{aligned}$$

Note that when the constraint $u \leq 1$ (or $\beta \geq 0$) does not bind, we have $\theta' = \theta = 0$, and this reduces to

$$v^{(\rho-\alpha)/(\alpha(1-\rho))} u^{(1-\alpha)/\alpha-1} \left[\frac{\rho-\alpha}{1-\rho} \frac{u^2}{v} + u'\right] = 0,$$

which is identical to equation (14) in the main text. For reasons analogous to those in the unconstrained problem, the profit-maximizing function u must set the term in the square brackets to 0, which implies

$$u = C_1 v^{-(\rho-\alpha)/(1-\rho)},$$

where C_1 is a strictly positive constant. Note that for v sufficiently small (in particular, in the neighborhood of $j = 0$), and given $\rho > \alpha$, we necessarily have that $u > 1$, so that the constraint $u \leq 1$ will have to bind, implying $\theta > 0$. Notice then that equation (B.2) implies that $\lambda' = 0$, which, in light of equation (B.1), in turn implies that θ is a monotonically decreasing function of j as long as the constraint binds. As a result, if the constraint binds at some $\hat{j} \in (0, 1)$, then $\theta > 0$ for all $j < \hat{j}$ and so the constraint must bind as well for all $j < \hat{j}$, implying $u(j) = 1$ for all $j \leq \hat{j}$.

The solution for the optimal share for $j > \hat{j}$ thus solves the differential equation

$$v' = C_1 v^{-(\rho-\alpha)/(1-\rho)}, \tag{B.3}$$

with the boundary condition $v'(\hat{j}) = 1$ and the transversality condition $v'(1) = \alpha^{\alpha/(1-\alpha)}$. As in the unconstrained problem, equation (B.3) implies

$$v(j) = \left(\frac{(1-\alpha)C_1}{1-\rho}(j - C_2) \right)^{(1-\rho)/(1-\alpha)}, \quad \text{and} \tag{B.4}$$

$$v'(j) = C_1 \left(\frac{(1-\alpha)C_1}{1-\rho}(j - C_2) \right)^{(\alpha-\rho)/(1-\alpha)}, \tag{B.5}$$

but now C_1 and C_2 follow from solving

$$C_1 \left(\frac{(1-\alpha)C_1}{1-\rho}(\hat{j} - C_2) \right)^{(\alpha-\rho)/(1-\alpha)} = 1, \quad \text{and}$$

$$C_1 \left(\frac{(1-\alpha)C_1}{1-\rho}(1 - C_2) \right)^{(\alpha-\rho)/(1-\alpha)} = \alpha^{\alpha/(1-\alpha)}.$$

This system yields

$$C_1 = \alpha^{\alpha/(1-\rho)} \left(\frac{1-\alpha}{1-\rho} \left(\frac{1-\hat{j}}{1-\alpha^{\alpha/(\rho-\alpha)}} \right) \right)^{(\rho-\alpha)/(1-\rho)}, \quad \text{and} \tag{B.6}$$

$$C_2 = \frac{\hat{j} - \alpha^{\alpha/(\rho-\alpha)}}{1-\alpha^{\alpha/(\rho-\alpha)}}. \tag{B.7}$$

Note, however, that at $\hat{j}$, we must also have

$$v(\hat{j}) \equiv \int_0^{\hat{j}} u \, dk = \hat{j}.$$

Plugging (B.6) and (B.7) into (B.4) then yields

$$\hat{j} = \frac{(1-\alpha)\alpha^{\alpha/(\rho-\alpha)}}{(1-\rho)(1-\alpha^{\alpha/(\rho-\alpha)}) + (1-\alpha)\alpha^{\alpha/(\rho-\alpha)}}.$$

Finally, substituting this expression for $\hat{j}$, together with (B.6) and (B.7), into (B.5) produces

$$v'(j) = \alpha^{\alpha/(1-\alpha)}\left(j - (1-j)\frac{(\rho-\alpha)\alpha^{\alpha/(\rho-\alpha)}}{1-\rho}\right)^{(\alpha-\rho)/(1-\alpha)},$$

which, given $\beta(j) \equiv 1 - v'(j)^{(1-\alpha)/\alpha}$, finally implies

$$\beta^*(j) = 1 - \alpha\left(j - (1-j)\frac{(\rho-\alpha)\alpha^{\alpha/(\rho-\alpha)}}{1-\rho}\right)^{(\alpha-\rho)/\alpha} \quad \text{for} \quad j > \hat{j}.$$

Note that we can then summarize the solution to the constrained problem as

$$\beta^*(j) = \begin{cases} 1 - \alpha j^{(\alpha-\rho)/\alpha}, & \text{if } \alpha > \rho, \\ \max\left\{1 - \alpha\left(j - (1-j)\dfrac{(\rho-\alpha)\alpha^{\alpha/(\rho-\alpha)}}{1-\rho}\right)^{(\alpha-\rho)/\alpha}, 0\right\}, \\ \qquad \text{if } \rho > \alpha, \end{cases}$$

for all $j \in [0, 1]$.

In the accompanying Figure S.1, we plot the above solution (thick curve) and compare it to that which solves the unconstrained problem (thin curve). Obviously, in the sequential substitutes case, the two solutions coincide. Interestingly, for all $j > \hat{j}$, we find that the optimal bargaining share received by the firm in the unconstrained problem is higher than its bargaining share under the constrained problem. Intuitively, in the constrained problem, the firm would have preferred to incentivize upstream suppliers by offering them a payoff exceeding their marginal contribution, but when it is not able to do so, it attempts to alleviate upstream investment inefficiencies by offering their full

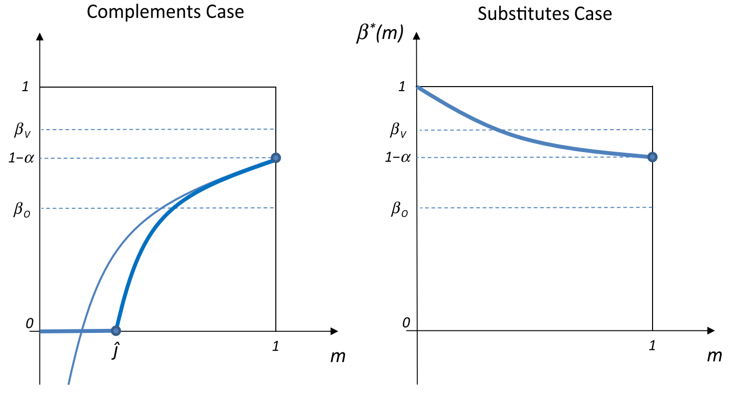

FIGURE S.1.—Profit-maximizing division of surplus for stage m.

marginal contribution to a larger measure of suppliers and by offering a higher share of their marginal contribution to the remaining suppliers.

Despite these differences, notice that the optimal bargaining share $\beta^*(j)$ continues to be (weakly) increasing in the sequential complements case and strictly decreasing in the sequential substitutes case. Hence, the statement in Proposition 1 of the paper remains valid except for the fact that $\beta^*(j)$ is now only *weakly* increasing in m when $\rho > \alpha$.

C. THE BENCHMARK MODEL WITH EX ANTE TRANSFERS

In Section 3.1.1 of the paper, we argue that introducing ex ante lump-sum transfers between the firm and the suppliers has very little impact on our main results. Because these ex ante transfers have no effect on ex post decisions made after agents are locked in by the contracts, investment levels continue to be characterized by equation (10) in our main text. The key implication of introducing ex ante transfers is that the objective function of the firm is no longer their ex post payoff (as in equation (11) of the main paper), but rather the joint surplus created along the value chain, or

$$\pi_T = A^{1-\rho}\theta^\rho\left(\int_0^1 x(j)^\alpha\, dj\right)^{\rho/\alpha} - \int_0^1 cx(j)\, dj. \tag{C.1}$$

This might reflect, as in Antràs (2003) and Antràs and Helpman (2004), the fact that the firm has full bargaining power ex ante, in the sense that it can make take-it-or-leave-it offers to suppliers that include an initial transfer to the firm. With a perfectly elastic supply of potential suppliers, each with an ex ante outside option equal to 0, these ex ante transfers would thus be set in a way that allows the firm to appropriate all the surplus created along the value chain. Alternatively, even when both the firm and suppliers have some ex ante bargaining power (perhaps because the number of potential suppliers is limited), the fact that agents have access to a means to transfer utility ex ante in a distortionary manner implies, by the Coase theorem, that the organization of production along the value chain (i.e., which stages get integrated and which get outsourced) will be decided efficiently, namely, in a joint-profit-maximizing manner.

Note from equation (6) in the main text that $cx(j) = \alpha(1-\beta(j))r'(j)$ for all $j \in [0,1]$. Plugging this into (C.1), we have

$$\pi_T = r(1) - \alpha\int_0^1 \bigl(1-\beta(j)\bigr)r'(j)\, dj = \int_0^1 \bigl(1-\alpha\bigl(1-\beta(j)\bigr)\bigr)r'(j)\, dj.$$

After substituting in the expressions from equations (8) and (9) of the main paper, we find that

$$\pi_T = \kappa\int_0^1 \bigl(1-\alpha\bigl(1-\beta(j)\bigr)\bigr)\bigl(1-\beta(j)\bigr)^{\alpha/(1-\alpha)}$$

$$\times \left[\int_0^j \big(1-\beta(k)\big)^{\alpha/(1-\alpha)}\,dk \right]^{(\rho-\alpha)/(\alpha(1-\rho))} dj,$$

where $\kappa \equiv A\frac{\rho}{\alpha}(\frac{1-\rho}{1-\alpha})^{(\rho-\alpha)/(\alpha(1-\rho))}(\frac{\rho\theta}{c})^{\rho/(1-\rho)}$ is a positive constant.

Defining

$$v(j) \equiv \int_0^j \big(1-\beta(k)\big)^{\alpha/(1-\alpha)}\,dk,$$

we can write π_T as

$$\pi_T = \kappa \int_0^1 \big(1-\alpha v'(j)^{(1-\alpha)/\alpha}\big)v'(j)\big[v(j)\big]^{(\rho-\alpha)/(\alpha(1-\rho))}\,dj. \tag{C.2}$$

The Euler–Lagrange equation associated with choosing the function $v'(j)$ from the set of piecewise continuously differentiable real-valued functions to maximize (C.2) can be derived in a manner analogous to the case without ex ante transfers. This Euler–Lagrange equation reduces to

$$v(j)^{(\rho-\alpha)/(\alpha(1-\rho))}v'(j)^{(1-\alpha)/\alpha-1}\frac{1-\alpha}{\alpha}\left[\frac{\rho-\alpha}{1-\rho}\frac{v'(j)^2}{v(j)}+v''\right]=0,$$

which, as in the case without ex ante transfers, implies

$$v(j) = \left(\frac{(1-\alpha)C_1}{1-\rho}(j-C_2)\right)^{(1-\rho)/(1-\alpha)}, \quad \text{and}$$

$$v'(j) = C_1\left(\frac{(1-\alpha)C_1}{1-\rho}(j-C_2)\right)^{(\alpha-\rho)/(1-\alpha)},$$

with initial condition $v(0)=0$. The main difference relative to the case without ex ante transfers is that the transversality condition is now $v'(1)^{(1-\alpha)/\alpha}=1$ (instead of $v'(1)^{(1-\alpha)/\alpha}=\alpha$), which implies that the optimal division of surplus is now given instead by

$$\beta_T^*(j) = 1 - v'(j)^{(1-\alpha)/\alpha} = 1 - j^{(\alpha-\rho)/\alpha}.$$

Note that the slope of $\beta_T^*(j)$ with respect to j continues to be crucially shaped by whether ρ is higher or lower than α, just as in our Benchmark Model. It follows then that Proposition 1, which we reproduce below, continues to hold in the setup with ex ante transfers.

PROPOSITION C.1: *The* (*unconstrained*) *optimal bargaining share* $\beta_T^*(j)$ *is a weakly increasing function of* j *in the complements case* ($\rho > \alpha$), *while it is a weakly decreasing function of* j *in the substitutes case* ($\rho < \alpha$).

In sum, Proposition C.1 confirms that whether the incentive for the firm to retain a larger surplus share increases or decreases along the value chain continues to crucially depend on the relative size of the parameters ρ and α, which we view as the central result of our paper.

The key difference from our Benchmark Model without ex ante transfers relates to the *level* of the share $\beta_T^*(j)$. In particular, note that in the sequential complements case ($\rho > \alpha$), we necessarily have $\beta^*(j) \leq 0$ for all $j \in [0, 1]$, and thus the firm finds it optimal to outsource all production stages, as discussed in the main text. In the sequential substitutes case ($\alpha > \rho$), we have $\beta^*(0) = 0$ and $\beta^*(1) = 1$, which necessarily implies that the most upstream stages will necessarily be integrated, while the most downstream stages will necessarily be outsourced. As a result, the cutoff stage separating the upstream integrated stages from the downstream outsourced stages necessarily lies strictly in the interior of $(0, 1)$.

D. LINKAGES ACROSS BARGAINING ROUNDS

In this section, we include the details related to the variant of our model outlined in Section 3.1.2, in which we allow suppliers to internalize the effect of their investment levels and their negotiations with the firm on the subsequent negotiations between the firm and downstream suppliers. As argued in the paper, it now becomes important to specify precisely the implications of an (off-the-equilibrium path) decision by a supplier to refuse to deliver its input to the firm. The simplest case to study is one in which, once the production process incorporates an incompatible input (say, because a supplier refused to trade with the firm), all downstream inputs are then necessarily incompatible as well, and thus their marginal product is zero and firm revenue remains at $r(m)$ if the deviation happened at stage m. (We will briefly discuss alternative assumptions below.)

For reasons that will become apparent, it is necessary to develop our results within a discrete-player version of the game between the firm and the suppliers, in which each of $M > 0$ suppliers controls a measure $1/M$ of production stages. We will later run the limit as $M \to \infty$ to compare our results with those in the Benchmark Model in our paper. Assuming that each supplier sets a common investment level for all the production stages under its control (remember that leaving aside the sequentiality of stages, the production function is symmetric in investments), revenue generated up to supplier $K < M$ is given by

$$R(K) = A^{1-\rho}\theta^{\rho}\left[\sum_{k=1}^{K}\frac{1}{M}X(k)^{\alpha}\right]^{\rho/\alpha},$$

if all the suppliers upstream of K have delivered compatible inputs before supplier K makes its own investment decision (thus respecting the natural sequencing of the stages). We use uppercase letters to denote variables in the discrete-player case, to distinguish them from the lowercase letters for the continuum case.

We solve the game by backward induction. Consider the negotiations between the firm and the most downstream supplier, M. Provided that all upstream suppliers have delivered compatible inputs, the value of production generated before supplier M's input is given by $R(M-1)$. If supplier M then provides a compatible input, the value of production will increase to $R(M)$. Following the reasoning in our paper, the ex post payoff for supplier M will then be

$$P_S(M) = \big(1-\beta(M)\big)\big(R(M) - R(M-1)\big), \tag{D.1}$$

where $\beta(M) = \beta_O$ in the case of outsourcing and $\beta(M) = \beta_V > \beta_O$ in the case of integration. The firm then obtains a payoff equal to $\beta(M)(R(M) - R(M-1))$ in that stage of production.

Moving to the supplier immediately upstream from M, that is, $M-1$, note that the value of production up to that point is $R(M-2)$ and will remain at that value if an incompatible input is produced. If that were to happen, not only would the incremental contribution $R(M-1) - R(M-2)$ be lost, but note that the firm would also lose its share of rents at stage M, which is $\beta(M)(R(M) - R(M-1))$. In sum, the *effective* incremental contribution of supplier $M-1$ to the joint payoff of the firm and supplier $M-1$ is given by

$$R(M-1) - R(M-2) + \beta(M)\big(R(M) - R(M-1)\big),$$

and thus its ex post payoff is

$$\begin{aligned} P_S(M-1) &= \big(1-\beta(M-1)\big) \\ &\qquad \big[R(M-1) - R(M-2) + \beta(M)\big(R(M) - R(M-1)\big)\big] \\ &= \big(1-\beta(M-1)\big)\big(R(M-1) - R(M-2)\big) \\ &\qquad + \beta(M)\frac{(1-\beta(M-1))}{(1-\beta(M))}P_S(M), \end{aligned}$$

where, in the second line, we have used equation (D.1).

Iterating this formula backward, we then find that, as stated in the main text, the profits of a supplier $K \in \{1, \dots, M-1, M\}$ are given by

$$\begin{aligned} \pi_S(K) = {} & \big(1-\beta(K)\big)\sum_{i=0}^{M-K}\mu(K,i)\big(R(K+i) - R(K+i-1)\big) \\ & - \frac{1}{M}cX(K), \end{aligned} \tag{D.2}$$

where

$$\text{(D.3)}\qquad \mu(K,i)=\begin{cases}1, & \text{if } i=0,\\ \prod_{l=1}^{i}\beta(K+l), & \text{if } i\geq 1.\end{cases}$$

The key difference relative to our Benchmark Model is that the payoff to a given supplier in equation (D.2) is now not only a fraction $1-\beta(K)$ of the supplier's own *direct* contribution $R(K)-R(K-1)$, but also incorporates a share $\mu(K,i)$ of the direct contribution of each supplier located i positions downstream from K, where $1\leq i\leq M-K$. Note, however, that the share of supplier $K+i$'s direct contribution captured by K quickly falls in the distance between K and $K+i$ (see equation (D.3)).

To assess the implications of this alternative setup for the choice of investment, note that a first-order Taylor approximation of the revenue function delivers

$$\text{(D.4)}\qquad R(K+i)-R(K+i-1) \approx A^{1-\rho}\theta^{\rho}\frac{\rho}{\alpha}\left[\sum_{k=1}^{K+i-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-\alpha)/\alpha}\frac{1}{M}X(K+i)^{\alpha}\quad \text{for all } i\geq 0.$$

We next consider the first-order condition associated with the choice of investment by the supplier at position K. For the time being, and to build intuition, consider the case in which upstream suppliers do not internalize the effect of their investments on the investment decision of downstream suppliers.

Despite this assumption (which we will relax below), the equilibrium investment choices of the current variant of the model would be expected to differ from those in our Benchmark Model because the payoff to supplier K is now a function of the direct contribution of all suppliers downstream from K, and these "downstream" contributions are themselves a function of supplier K's investments. To be more precise, plugging (D.4) into (D.2) and taking the derivative with respect to $X(K)$, the first-order condition is given, after some rearrangement, by

$$\frac{c}{\rho A^{1-\rho}\theta^{\rho}}=\big(1-\beta(K)\big)\left[\sum_{k=1}^{K-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-\alpha)/\alpha}X(K)^{\alpha-1}$$
$$+\big(1-\beta(K)\big)\mathbf{1}(K<M)\sum_{i=1}^{M-K}\mu(K,i)\frac{\rho-\alpha}{\alpha}$$
$$\times\left[\sum_{k=1}^{K+i-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-2\alpha)/\alpha}\frac{1}{M}X(K)^{\alpha-1}X(K+i)^{\alpha},$$

where $\mathbf{1}(K < M)$ is an indicator function equal to 1 if $K < M$, and equal to 0 otherwise. The first term reflects the effect of supplier K's investment on its own direct contribution, and is the key term highlighted in the Benchmark Model. The second term captures the effects of supplier K's investments on the direct contributions of downstream suppliers $K' > K$.

To formally study the convergence of these terms as $M \to \infty$, it is convenient to study the choice of investment by a supplier with a fraction m of suppliers upstream from him or her, that is, the supplier in position $K = mM$. The first-order condition above then becomes

$$\frac{c}{\rho A^{1-\rho}\theta^{\rho}} = \big(1-\beta(mM)\big)\left[\sum_{k=1}^{mM-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-\alpha)/\alpha} X(mM)^{\alpha-1}$$

$$+ \big(1-\beta(mM)\big)\mathbf{1}(m<1)\sum_{i=1}^{M-mM}\mu(mM,i)\frac{\rho-\alpha}{\alpha}$$

$$\times\left[\sum_{k=1}^{mM+i-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-2\alpha)/\alpha}\frac{1}{M}X(mM)^{\alpha-1}X(mM+i)^{\alpha}.$$

Note, however, that the term

$$\left[\sum_{k=1}^{mM-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-\alpha)/\alpha} \tag{D.5}$$

converges to the Riemann integral:

$$\left[\int_0^m x(j)^{\alpha}\,dj\right]^{(\rho-\alpha)/\alpha} = \left(\frac{r(m)}{A^{1-\rho}\theta^{\rho}}\right)^{(\rho-\alpha)/\rho}$$

when $M \to \infty$. Let us assume that when investing to produce a compatible input, the choice of investment by suppliers, $X(k)$, is uniformly bounded, so that $0 < \underline{C} \le X(k) \le \overline{C}$ for all k. We will confirm below that this is a feature of the equilbrium (both in the Benchmark Model as well as in this extended one), but we impose this assumption upfront to simplify the exposition. It then follows that $r(m)^{(\rho-\alpha)/\alpha}$ is bounded for given $m > 0$, and the same will be true for the term in (D.5) as $M \to \infty$.[1]

As for the terms

$$\left[\sum_{k=1}^{mM+i-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-2\alpha)/\alpha} \tag{D.6}$$

[1]For the case of the initial supplier ($m = 0$), these terms are irrelevant for that supplier's investment, since no value has been generated up to that supplier.

that appear in the second line of the first-order condition, we need to establish that there is a uniform bound for these as i runs from 1 to $M - mM$. If $\rho > 2\alpha$, this upper bound is given by $r(1)^{(\rho-2\alpha)/\alpha}$. On the other hand, if $\rho < 2\alpha$, then $r(m)^{(\rho-2\alpha)/\alpha}$ provides the necessary bound. (Recall here that m is fixed as we are considering m's first-order condition.) Thus, each of the terms in (D.6) is uniformly bounded from above as $M \to \infty$. Let C_1 denote this bound. We finally note that $\sum_{i=1}^{M-mM} \mu(mM, i)$ also remains uniformly bounded as $M \to \infty$. To see this, note that $\beta(k) \leq \beta_V$ for k, so that

$$0 \leq \lim_{M\to\infty} \sum_{i=1}^{M-mM} \mu(mM, i) \leq \lim_{M\to\infty} \sum_{i=1}^{M} (\beta_V)^i = \frac{\beta_V}{1-\beta_V} \equiv B.$$

With these results in hand, note that with some abuse of notation, the first line of the first-order condition converges, as $M \to \infty$, to

$$\big(1 - \beta(m)\big)\left(\frac{r(m)}{A^{1-\rho}\theta^{\rho}}\right)^{(\rho-\alpha)/\rho} x(m)^{\alpha-1},$$

while in absolute terms, the second line satisfies

$$\begin{aligned}
&\big(1 - \beta(mM)\big)\mathbf{1}(m < 1) \sum_{i=1}^{M-mM} \mu(mM, i)\left|\frac{\rho-\alpha}{\alpha}\right| \\
&\quad \times \left[\sum_{k=1}^{mM+i-1} \frac{1}{M} X(k)^{\alpha}\right]^{(\rho-2\alpha)/\alpha} \frac{1}{M} X(mM)^{\alpha-1} X(mM+i)^{\alpha} \\
&\leq \big(1 - \beta(mM)\big)\left|\frac{\rho-\alpha}{\alpha}\right| \sum_{i=1}^{M-mM} \mu(mM, i) C_1 \frac{1}{M} \underline{C}^{\alpha-1} \overline{C}^{\alpha} \\
&\leq \big(1 - \beta(mM)\big)\left|\frac{\rho-\alpha}{\alpha}\right| C_1 \frac{1}{M} \underline{C}^{\alpha-1} \overline{C}^{\alpha} B,
\end{aligned}$$

and the latter expression tends to 0 as $M \to \infty$. In sum, the second term in the first-order condition becomes negligible when $M \to \infty$, and thus the first-order condition collapses to

$$c = \rho\big(A^{1-\rho}\theta^{\rho}\big)^{\alpha/\rho}\big(1 - \beta(m)\big) r(m)^{(\rho-\alpha)/\rho} x(m)^{\alpha-1},$$

as in our Benchmark Model.

So far, we have ignored the fact that suppliers might internalize the effects of their investments on the investment decisions of downstream suppliers. We ignored this as well in the Benchmark Model, but that was without loss of generality since, in that model, a supplier K's payoff was only a function of the investments of upstream suppliers, which were already fixed by the time the Kth input was incorporated into production. In the current game, investments by downstream suppliers are also relevant for payoffs, so this further complicates the first-order condition. When allowing for these effects, the first-order condition for $X(K)$ now becomes

$$\begin{aligned}\frac{c}{\rho A^{1-\rho}\theta^{\rho}} = {}& \big(1-\beta(K)\big)\left[\sum_{k=1}^{K-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-\alpha)/\alpha} X(K)^{\alpha-1} \\ & + \big(1-\beta(K)\big)\mathbf{1}(K<M) \\ & \times \left\{\sum_{i=1}^{M-K}\mu(K,i)\left[\sum_{k=1}^{K+i-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-\alpha)/\alpha}\right. \\ & \times X(K+i)^{\alpha-1}\frac{\partial X(K+i)}{\partial X(K)} \\ & + \sum_{i=1}^{M-K}\mu(K,i)\frac{\rho-\alpha}{\alpha}\left[\sum_{k=1}^{K+i-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-2\alpha)/\alpha} \\ & \left. \times\left(\sum_{l=0}^{i-1}\frac{1}{M}\frac{\partial X(K+l)}{\partial X(K)}X(K+l)^{\alpha-1}\right)X(K+i)^{\alpha}\right\}.\end{aligned}$$

Using analogous arguments to those above, it is easy to show that, provided that, as $M\to\infty$, $\frac{\partial X(K+i)}{\partial X(K)}\to 0$ for any $K<M$ and any i with $0<i<M-K$, then these extra terms will again vanish and the first-order condition of this extended game will again converge to that in our Benchmark Model. Quite intuitively, this new force will only matter when upstream investments have a measurable impact on downstream investments.

It thus suffices to show that, for any K and any $i=1,\dots,K-1$, we indeed have $\frac{\partial X(K)}{\partial X(K-i)}\to 0$ as $M\to\infty$. For this, consider the objective function of supplier K in equation (D.2). We will simply show that, as $M\to\infty$, the effect of any upstream investment $X(K-i)$ on this payoff is negligible, thus implying that the choice of investment $X(K)$ obtained by maximizing $\pi_S(K)$ in (D.2) cannot possibly be measurably affected by these upstream investments. More specifically, simple differentiation of (D.2) after plugging in (D.4) delivers

$$\begin{aligned}\left|\frac{\partial\pi_S(K)}{\partial X(K-i)}\right| = {}& \big(1-\beta(K)\big)\sum_{i=0}^{M-K}\mu(K,i) \\ & \times\left(\rho A^{1-\rho}\theta^{\rho}\left|\frac{\rho-\alpha}{\alpha}\right|\left[\sum_{k=1}^{K+i-1}\frac{1}{M}X(k)^{\alpha}\right]^{(\rho-2\alpha)/\alpha}\right.\end{aligned}$$

$$\times X(K-i)^{\alpha-1}\frac{1}{M^2}X(K+i)^{\alpha}\Bigg)$$

$$\leq \big(1-\beta(K)\big)\sum_{i=0}^{M-K}\mu(K,i)$$

$$\times\left(\rho A^{1-\rho}\theta^{\rho}\left|\frac{\rho-\alpha}{\alpha}\right|C_1\frac{1}{M^2}\underline{C}^{\alpha-1}\overline{C}^{\alpha}\right)$$

$$\leq \big(1-\beta(K)\big)\rho A^{1-\rho}\theta^{\rho}\left|\frac{\rho-\alpha}{\alpha}\right|C_1\frac{1}{M^2}\underline{C}^{\alpha-1}\overline{C}^{\alpha}B,$$

which clearly goes to 0 when $M\to\infty$ at a faster rate than c/M does. Consequently, we have $\frac{\partial X(K)}{\partial X(K-i)}\to 0$ as $M\to\infty$, and this completes the proof of the following result:

PROPOSITION D.1: *The investment levels associated with this more general game that allows for linkages across bargaining stages delivers the same investment levels as our Benchmark Model when $M\to\infty$, that is, when there is a continuum of suppliers.*

As argued in the main text, since investment levels are identical to those in the Benchmark Model, the total surplus generated along the value chain will also remain unaltered. Hence, when ownership structure along the value chain is decided in a joint-profit-maximizing manner, as in the model with ex ante transfers outlined in Section C, the introduction of linkages across bargaining stages delivers the exact same predictions as the model without these linkages.

In the absence of ex ante transfers, the choice of ownership structure of this expanded model becomes significantly more complicated due to the fact that the ex post rents obtained by the firm in a given stage are now lower than in the Benchmark Model, and more so the more upstream the supplier is. This is apparent from equation (D.2) above, which implies that the profits of the firm would be

$$\pi_F=R(M)-\sum_{k=1}^{M}\beta(k)\sum_{i=0}^{M-k}\mu(k,i)\big(R(k+i)-R(k+i-1)\big).$$

Other things equal, relative to our Benchmark Model, there is an additional incentive for the firm to integrate relatively upstream suppliers, regardless of the relative size of ρ and α, because of what in our paper we have termed the rent extraction effect. Unfortunately, a simple explicit formula for π_S and π_F cannot be obtained even in the limiting case $M\to\infty$, thus precluding an analytical characterization of ownership structure decisions along the value chain. We would hypothesize, however, that Proposition 2 in our paper would survive in the sequential substitutes case (since this new force should only reinforce the incentive to integrate upstream suppliers), while our results regarding the sequential complements case might become more nuanced in the absence of lump-sum transfers.

Our derivations above have relied on the strong assumption that once the production process incorporates an incompatible input, all downstream inputs are then necessarily incompatible as well. We have also worked out a variant of the game in which, when a supplier refuses to deliver an input and that stage is completed with an incompatible input, the production process continues without implying that the marginal productivity of downstream investments is driven down to 0. Of course, such a deviation would still affect the subsequent negotiations between the firm and downstream suppliers because, by providing an incompatible input, a supplier still affects the marginal productivity of downstream investments and thus affects the amount of surplus that the firm will obtain in subsequent negotiations. Foreseeing this, a supplier contemplating a deviation might insist on obtaining a share of its effective contribution, rather than a share of its direct contribution, as in our Benchmark Model. Without delving into the details, when solving for the payoffs of this variant of the game, we find that the ex post payoff of a supplier K in the discrete-player case with M suppliers can again be represented as

$$\pi_S(K) = \big(1-\beta(K)\big)\sum_{i=0}^{M-K}\tilde{\mu}(K,i)\big(R(K+i)-R(K+i-1)\big) - \frac{1}{M}cX(K),$$

and thus is a weighted sum of shares of direct contributions of all suppliers located downstream from K, where $1 \le i \le M-K$. The key difference is that the weights are no longer simply given by the expression in (D.3), but instead are now given by

$$\tilde{\mu}(K,i) = \sum_{j=0}^{M-K-i}(-1)^j\mu(K,i+j)a(i,j),$$

where $\mu(K,i)$ is given by (D.3) and

$$a(i,j) = \begin{cases} 1, & \text{if } i=0,\\ \displaystyle\sum_{k=0}^{j} a(i-1,k), & \text{if } i>0. \end{cases}$$

An important difference between this solution and the one developed above is that even for $i=0$ (i.e., even focusing on the direct contribution of supplier

K), the share of surplus accruing to supplier K is no longer given by $1-\beta(K)$, but instead is given by

$$\begin{aligned}\tilde{\mu}(K,0) &= 1+\sum_{j=1}^{M-K}(-1)^j\prod_{l=1}^{j}\beta(K+l)\\ &= 1-\beta(K+1)+\beta(K+1)\beta(K+2)\\ &\quad -\beta(K+1)\beta(K+2)\beta(K+3)+\cdots,\end{aligned}$$

and thus depends on all ownership decisions downstream from K. As a result, even when the effect on $X(K)$ of supplier K obtaining a share of the direct contributions of downstream suppliers is negligible (as shown above in our simpler extended model), the investment levels will differ from those in the Benchmark Model. In particular, in that case, $X(K)$ would effectively solve

$$\big(1-\beta(K)\big)\left(1+\sum_{j=1}^{M-K}(-1)^j\prod_{l=1}^{j}\beta(K+l)\right)\big(R(K)-R(K-1)\big)$$
$$-\frac{1}{M}cX(K),$$

and would thus depend directly on all $\beta(K+i)$ with $0\le i\le M-K$. Again, the fact that we cannot analytically characterize the convergence of this objective function when $M\to\infty$ precludes a straightforward comparison of the implications of this model with those of our Benchmark Model.

E. HYBRID MODELS WITH BOTH SIMULTANEOUS AND SEQUENTIAL PRODUCTION

In this section, we provide more details regarding the two extensions briefly outlined in Section 3.1.3 of the main paper.

E.1. *A Spider With Snake Legs*

First, we develop a variant of our model where production resembles a "spider," following the terminology of Baldwin and Venables (2013). Specifically, the final good combines a continuum of measure 1 of modules or parts indexed by n, which are put together simultaneously according to a symmetric technology featuring a constant elasticity of substitution, $1/(1-\zeta)>1$, across the services of the different modules. Preferences for final goods are still given by equation (2) in the main text, so, denoting by $X(n)$ the services of module $n\in[0,1]$, revenues for the final-good producer are given by

$$r=A^{1-\rho}\left(\int_0^1 X(n)^{\zeta}\,dn\right)^{\rho/\zeta}. \tag{E.1}$$

Production of each module n in turn involves a continuum of measure 1 of stages, indexed by j, which must be carried out sequentially in a predetermined order, just as in our Benchmark Model. More specifically, the production technology for each module n is given by

$$X(n) = \theta\left(\int_0^1 x_n(j)^\alpha I(j)\,dj\right)^{1/\alpha}, \tag{E.2}$$

which is analogous to equation (1) in the Benchmark Model. The assembly of each module is controlled by a *module producer* who decides which of its module-specific inputs to integrate, just as the final-good producer in our Benchmark Model decided which stages to integrate. We assume that contracting and bargaining between each module producer and their module-specific suppliers is completely analogous to the setup described in our Benchmark Model involving the final-good producer and its suppliers.

The main difference with the Benchmark Model is that the revenue captured by module producer n is not determined by demand conditions, but rather by the share of final-good revenue (E.1) captured by module producer n in its negotiations with the final-good producer. We will assume that this division of surplus is not fixed by an initial contract (a natural assumption when the services $X(n)$ are not contractible), but is rather decided ex post once all modules have been produced. As in Acemoglu, Antràs, and Helpman (2007), we use the Shapley value to determine the (simultaneous) division of ex post surplus between the firm and the continuum of module producers, with each module producer's threat point being associated with the possibility of withholding their services $X(n)$. Given final-good revenue in (E.1), we can appeal to Lemma 1 in Acemoglu, Antràs, and Helpman (2007) to conclude that the payoff for each module producer would be given by

$$r_n = \left(\frac{\rho}{\rho+\zeta}\right) A^{1-\rho} X(n)^\zeta X(-n)^{\rho-\zeta}, \tag{E.3}$$

where $X(-n)$ denotes the symmetric level of module services of all modules other than n. Naturally, in equilibrium we have $X(n) = X(-n)$, and each module producer ends up with a share $\rho/(\rho+\zeta)$ of final-good revenue. But when calculating the effects of different levels of module services on a module producer's revenue, the relevant formula is (E.3), with ζ governing the elasticity of revenue with respect to each module producer's level of services (see Acemoglu, Antràs, and Helpman (2007) for more details).

Plugging $X(n)$ in (E.2) into (E.3), we have that the revenue available for each module producer and its suppliers to bargain over is given by

$$r_n = \tilde{A}^{1-\zeta}\theta^\zeta\left(\int_0^1 x_n(j)^\alpha I(j)\,dj\right)^{\zeta/\alpha}, \tag{E.4}$$

where $\tilde{A}$ is a positive term that each module producer takes as given.[2] It should then be clear that the characterization of the negotiations between each module producer and its suppliers, as well as the optimal ownership structure along each module's value chain, will be isomorphic to those in our Benchmark Model, except for the fact that the concavity of revenue is governed by the degree of substitution across modules—as captured by ζ—rather than by the elasticity of demand for the final good (as the parameter ρ did in our Benchmark Model). In sum, if $\zeta > \alpha$, then module inputs will be sequential complements and the propensity to integrate will once again be increasing with the downstreamness of module inputs, while the converse statement applies when $\zeta < \alpha$ and module inputs are sequential substitutes.

This extension with a "spider"-like production structure has some bearings on our empirical strategy, which we briefly discussed toward the end of Section 5.3 of our main paper. If production were more aptly described by a "spider," this would call for using a proxy for the elasticity of substitution across module services in order to empirically distinguish between the sequential substitutes and sequential complements cases. Toward this end, we experimented with using the Broda–Weinstein import demand elasticities for more aggregate product categories (at the SITC Rev. 3, three-digit level), in place of the import demand elasticities for highly disaggregate products (at the HS ten-digit level) that we have been using for our baseline empirical specification. The Broda–Weinstein elasticities for SITC three-digit categories were estimated in part using the substitution seen across constituent products within each SITC three-digit category in the U.S. import data (see footnote 22, Broda and Weinstein (2006)). To the extent that this speaks to the elasticity of substitution across module services, this would provide a first-pass look at whether the data continue to support our model's predictions when allowing for a "spider"-like production structure.

The results from this exploration are reported in Tables S.IX and S.X, which can be found toward the end of this Supplement. This yields patterns qualitatively similar to our baseline results in the main paper, albeit with reduced significance levels. That said, several caveats from this exercise need to be kept in mind. First, our proxy for ζ is a crude one at best, given the loose mapping made from the SITC three-digit category elasticities to the degree of substitution across module services in typical production processes. The above discussion moreover puts aside the organizational decision faced by the final-good producer over his/her module producers, which, in principle, could also get reflected in the extent of intrafirm trade observed in the data.

E.2. *A Snake With Spider Legs*

Consider next an alternative hybrid model in which we revert back to the "snake"-like sequential production structure in our Benchmark Model. How-

[2]More specifically, $\tilde{A}^{1-\zeta} = \rho A^{1-\rho} X(-n)^{\rho-\zeta}/(\rho+\zeta)$.

ever, we now allow each stage input j to itself be composed of a unit measure of distinct components produced *simultaneously*, each by a different supplier, under a symmetric technology featuring a constant elasticity of substitution, $1/(1-\xi) > 1$, across components. More formally, firm revenue is now again given by

$$r = A^{1-\rho}\theta^{\rho}\left(\int_0^1 x(j)^{\alpha} I(j)\,dj\right)^{\rho/\alpha}, \tag{E.5}$$

as in our Benchmark Model. However, $x(j)$ is now given by

$$x(j) = \left(\int_0^1 x_j(n)^{\xi}\,dn\right)^{1/\xi},$$

where $x_j(n)$ denotes the services provided by component $n \in [0, 1]$ toward the input of the stage-j supplier. Notice that when $\xi \to 1$, this formulation captures situations in which firms might contract with multiple suppliers to provide "essentially" the same intermediate input.

The main difference with respect to our Benchmark Model is that the firm now bargains with a continuum of suppliers at each stage m rather than just with a single supplier. Nevertheless, the total surplus over which these agents negotiate continues to be given by

$$\begin{aligned} r'(m) &= \frac{\rho}{\alpha}\left(A^{1-\rho}\theta^{\rho}\right)^{\alpha/\rho} r(m)^{(\rho-\alpha)/\rho} x(m)^{\alpha} \\ &= \frac{\rho}{\alpha}\left(A^{1-\rho}\theta^{\rho}\right)^{\alpha/\rho} r(m)^{(\rho-\alpha)/\rho}\left(\int_0^1 x_m(n)^{\xi}\,dn\right)^{\alpha/\xi}. \end{aligned}$$

To determine how this surplus is distributed among the firm and the suppliers at stage m, we follow Acemoglu, Antràs, and Helpman (2007) in using the Shapley value associated with each player. An individual supplier's threat point is affected by whether that supplier is integrated by the firm or not. In the case of outsourcing, the supplier can threaten to withhold the entirety of its input services $x_m(n)$, while in the case of integration, it can only threaten to withhold a share $1-\delta$ of these input services. We can then appeal to Lemmas 1 and 3 in Acemoglu, Antràs, and Helpman (2007) to conclude that the payoff of a supplier n in stage m will be given by

$$P_{Sm}(n) = \left(1-\beta(m)\right)\frac{\rho}{\alpha}\left(A^{1-\rho}\theta^{\rho}\right)^{\alpha/\rho} r(m)^{(\rho-\alpha)/\rho} x_m(n)^{\xi} x_m(-n)^{\alpha-\xi}, \tag{E.6}$$

where $x_m(-n)$ is the (symmetric) investment level chosen by all suppliers other than n, and

$$\beta(m) = \begin{cases} \dfrac{\xi}{\alpha+\xi}, & \text{if the firm outsources stage } m, \\ \dfrac{\xi}{\alpha+\xi}\dfrac{1-\delta^{\alpha+\xi}}{1-\delta^{\xi}} > \dfrac{\xi}{\alpha+\xi}, & \text{if the firm integrates stage } m. \end{cases}$$

Note that the payoff in (E.6) is analogous to that in the Benchmark Model—see equation (6) in the main paper—except for the fact that the concavity of the payoff with respect to the supplier's investment is governed by ξ and not α.

Suppliers will then choose investments to maximize $P_{Sm}(n) - cx_m(n)$. Solving for the first-order condition of the problem and imposing symmetry (i.e., $x_m(n) = x_m(-n)$), we find that

$$\text{(E.7)} \qquad x_m(n) = \left[\big(1-\beta(m)\big)\frac{\rho(A^{1-\rho}\theta^{\rho})^{\alpha/\rho}}{c}\frac{\xi}{\alpha}\right]^{1/(1-\alpha)} r(m)^{(\rho-\alpha)/(\rho(1-\alpha))},$$

which is identical to equation (7) in the Benchmark Model except for the term ξ/α inside the square brackets. Following then analogous steps as in the main text—see the derivation of equations (8)–(11)—we finally find that firm profits are given by

$$\text{(E.8)} \qquad \pi_F = A\frac{\rho}{\alpha}\left(\frac{1-\rho}{1-\alpha}\right)^{(\rho-\alpha)/(\alpha(1-\rho))}\left(\frac{\rho\theta}{c}\frac{\xi}{\alpha}\right)^{\rho/(1-\rho)}\int_0^1 \beta(j)\big(1-\beta(j)\big)^{\alpha/(1-\alpha)} \times\left[\int_0^j \big(1-\beta(k)\big)^{\alpha/(1-\alpha)}\,dk\right]^{(\rho-\alpha)/(\alpha(1-\rho))} dj,$$

where again the only difference is the presence of the extra term ξ/α. It should be clear, however, that this term has no impact on the optimal choice of the function $\beta^*(j)$ that determines the ex post division of surplus. Thus, regardless of the particular value of ξ, it will continue to be the case that the incentive to integrate decreases or increases along the value chain depending on the relative magnitudes of ρ and α. Note, however, that because ξ shapes the bargaining share associated with integration or outsourcing, this parameter will indeed affect the actual interval of stages that the firm will find optimal to integrate. (In other words, ξ will affect where the cutoff stage between integration and outsourcing occurs in both the sequential complements and substitutes cases.)

TABLE S.I

SUMMARY STATISTICS[a]

Variable	10th	25th	Median	75th	90th	Mean	Std. Dev.
Share of intrafirm trade (year = 2000)	0.107	0.209	0.372	0.535	0.659	0.382	0.207
Share of intrafirm trade (year = 2005)	0.132	0.222	0.386	0.557	0.650	0.392	0.203
Share of intrafirm trade (year = 2010)	0.133	0.236	0.404	0.560	0.663	0.402	0.209
Of seller industries:							
DUse_TUse	0.265	0.456	0.646	0.798	0.885	0.614	0.228
DownMeasure	0.316	0.370	0.492	0.744	0.907	0.559	0.222
Final use/Output	0	0.011	0.313	0.781	0.919	0.396	0.373
Skill intensity, $\log(s/l)$	−1.723	−1.541	−1.306	−1.006	−0.766	−1.276	0.382
Physical capital intensity, $\log(k/l)$	3.875	4.244	4.747	5.263	6.091	4.835	0.825
log(equipment k/l)	3.271	3.785	4.311	4.852	5.664	4.368	0.904
log(plant k/l)	2.930	3.273	3.67	4.186	4.855	3.796	0.757
Materials intensity, log(materials/l)	4.054	4.311	4.734	5.258	5.711	4.841	0.719
R&D intensity, $\log(0.001 + R\&D/Sales)$	−6.908	−6.908	−6.239	−4.300	−2.912	−5.436	1.764
Dispersion	1.636	1.744	1.844	1.988	2.16	1.882	0.224
Value-added/Value of shipments	0.355	0.435	0.511	0.594	0.645	0.509	0.116
Input "Importance"	0.0003	0.0009	0.002	0.003	0.0066	0.0034	0.0066
Intermediation	0.28	0.31	0.339	0.498	0.61	0.401	0.127
Own contractibility	0	0	0	0.6	0.993	0.263	0.386
Of buyer industries:							
Import elasticity, ρ	3.154	4.900	7.695	10.468	18.465	10.217	11.117
Skill intensity, $\log(s/l)$	−1.693	−1.485	−1.295	−1.034	−0.766	−1.260	0.348
Physical capital intensity, $\log(k/l)$	3.999	4.392	4.755	5.131	5.574	4.767	0.629
log(equipment k/l)	3.410	3.873	4.318	4.686	5.142	4.284	0.702
log(plant k/l)	3.054	3.365	3.676	4.042	4.533	3.746	0.570
Materials intensity, log(materials/l)	4.212	4.533	4.861	5.221	5.643	4.900	0.579
R&D intensity, $\log(0.001 + R\&D/Sales)$	−6.904	−6.655	−5.675	−4.551	−3.328	−5.408	1.361
Dispersion	1.710	1.787	1.907	2.007	2.122	1.908	0.177
Buyer contractibility	0	0.003	0.067	0.297	0.653	0.207	0.283

[a]Tabulated based on the 253 IO2002 manufacturing industries in the regression sample. For details on the construction of the data variables, please see the Data Appendix.

TABLE S.II

CORRELATIONS OF INDUSTRY VARIABLES WITH DOWNSTREAMNESS[a]

	Correlation With:	
	DUse_TUse	*DownMeasure*
Of seller industries:		
Skill intensity, $\log(s/l)$	−0.081	0.072
Physical capital intensity, $\log(k/l)$	−0.400***	−0.374***
log(equipment k/l)	−0.413***	−0.418***
log(plant k/l)	−0.347***	−0.272***
Materials intensity, log(materials$/l$)	−0.209***	−0.142**
R&D intensity, $\log(0.001 + R\&D/Sales)$	−0.144**	−0.072
Dispersion	−0.225***	−0.072
Value-added/Value of shipments	0.177***	0.134**
Input "Importance"	−0.031	−0.095
Intermediation	0.317***	0.249***
Own contractibility	−0.355***	−0.348***
Of buyer industries:		
Import elasticity, ρ	0.046	0.104*
Skill intensity, $\log(s/l)$	−0.053	0.034
Physical capital intensity, $\log(k/l)$	−0.255***	−0.319***
log(equipment k/l)	−0.284***	−0.364***
log(plant k/l)	−0.174***	−0.204***
Materials intensity, log(materials$/l$)	−0.112*	−0.193***
R&D intensity, $\log(0.001 + R\&D/Sales)$	−0.132**	−0.104*
Dispersion	−0.137**	−0.136**
Buyer contractibility	−0.189***	−0.239***

[a] ***, ** and * indicate significance at the 1%, 5%, and 10% levels, respectively. Calculated from the 253 IO2002 manufacturing industries in the regression sample.

TABLE S.III

DOWNSTREAMNESS AND THE INTRAFIRM IMPORT SHARE: DIRECT PLUS FINAL USE SHARE[a]

	Dependent Variable: Intrafirm Import Share							
	(1)	(2)	(3)	(4) *Elas* < *Median*	(5) *Elas* ≥ *Median*	(6) Weighted	(7)	(8) Weighted
$\log(s/l)$	−0.001	0.030	0.046	0.122*	0.025	−0.170*	0.005	−0.119
	[0.045]	[0.044]	[0.043]	[0.068]	[0.054]	[0.088]	[0.021]	[0.081]
$\log(k/l)$	0.052*	0.050*						
	[0.028]	[0.027]						
log(equipment k/l)			0.101***	0.033	0.175***	0.161**	0.028*	0.116**
			[0.036]	[0.049]	[0.045]	[0.066]	[0.017]	[0.052]
log(plant k/l)			−0.079*	−0.005	−0.168**	−0.095	−0.054***	−0.104**
			[0.048]	[0.061]	[0.068]	[0.071]	[0.020]	[0.048]
log(materials/l)	0.054	0.046	0.051	0.020	0.063	0.037	0.019	0.067
	[0.034]	[0.034]	[0.033]	[0.051]	[0.046]	[0.059]	[0.014]	[0.049]
$\log(0.001 + R\&D/Sales)$	0.056***	0.052***	0.051***	0.049***	0.049***	0.088***	0.032***	0.071***
	[0.009]	[0.009]	[0.009]	[0.014]	[0.014]	[0.019]	[0.004]	[0.016]
Dispersion	0.088	0.086	0.137*	0.051	0.249**	0.234	0.108***	0.135
	[0.072]	[0.074]	[0.079]	[0.110]	[0.105]	[0.149]	[0.041]	[0.121]
DFShare	0.034			−0.149*	0.236***			
	[0.051]			[0.076]	[0.065]			
DFShare × $\mathbf{1}(Elas < Median)$, β_1		−0.109	−0.079			−0.145	−0.057	−0.068
		[0.068]	[0.069]			[0.103]	[0.037]	[0.092]
DFShare × $\mathbf{1}(Elas > Median)$, β_2		0.165***	0.194***			0.407***	−0.044	0.290***
		[0.062]	[0.063]			[0.118]	[0.027]	[0.106]
$\mathbf{1}(Elas > Median)$		−0.165**	−0.160**			−0.422***	−0.008	−0.296***
		[0.069]	[0.068]			[0.096]	[0.034]	[0.084]

(*Continues*)

TABLE S.III—*Continued*

	Dependent Variable: Intrafirm Import Share							
	(1)	(2)	(3)	(4) *Elas < Median*	(5) *Elas ≥ Median*	(6) Weighted	(7)	(8) Weighted
p-value: Joint significance of β_1 and β_2		[0.0059]	[0.0028]			[0.0001]	[0.1244]	[0.0028]
p-value: Test of $\beta_2 - \beta_1 = 0$		[0.0020]	[0.0018]			[0.0000]	[0.7547]	[0.0009]
Industry controls for:	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer
Year fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes	No	No
Country-year fixed effects?	No	No	No	No	No	No	Yes	Yes
Observations	2783	2783	2783	1375	1408	2783	207,991	207,991
R-squared	0.27	0.31	0.32	0.35	0.30	0.59	0.18	0.58

[a] ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Standard errors are clustered by industry. Columns 1–6 use industry-year observations controlling for year fixed effects, while columns 7–8 use country-industry-year observations controlling for country-year fixed effects. Estimation is by OLS. In all columns, the industry factor intensity and dispersion variables are a weighted average of the characteristics of buyer industries (the industries that buy the input in question), constructed as described in Section 4.3 of the main text. Columns 4 and 5 restrict the sample to observations where the buyer industry elasticity is smaller (respectively, larger) than the industry median value. "Weighted" columns use the value of total imports for the industry-year or country-industry-year, respectively, as regression weights.

TABLE S.IV

DOWNSTREAMNESS AND THE INTRAFIRM IMPORT SHARE: FINAL USE SHARE[a]

	Dependent Variable: Intrafirm Import Share							
	(1)	(2)	(3)	(4) *Elas* < *Median*	(5) *Elas* ≥ *Median*	(6) Weighted	(7)	(8) Weighted
$\log(s/l)$	−0.018	0.005	0.022	0.071	0.017	−0.206***	−0.003	−0.142**
	[0.045]	[0.043]	[0.042]	[0.067]	[0.053]	[0.071]	[0.020]	[0.060]
$\log(k/l)$	0.063**	0.059**						
	[0.027]	[0.026]						
$\log(\text{equipment } k/l)$			0.123***	0.079*	0.171***	0.136**	0.047***	0.117***
			[0.034]	[0.044]	[0.048]	[0.066]	[0.015]	[0.045]
$\log(\text{plant } k/l)$			−0.099**	−0.041	−0.175**	−0.077	−0.068***	−0.100**
			[0.048]	[0.060]	[0.076]	[0.080]	[0.020]	[0.049]
$\log(\text{materials}/l)$	0.050	0.041	0.048	0.021	0.064	0.034	0.017	0.052
	[0.033]	[0.033]	[0.033]	[0.050]	[0.045]	[0.059]	[0.013]	[0.045]
$\log(0.001 + R\&D/Sales)$	0.058***	0.055***	0.055***	0.056***	0.048***	0.095***	0.034***	0.075***
	[0.009]	[0.009]	[0.009]	[0.014]	[0.014]	[0.018]	[0.004]	[0.014]
Dispersion	0.092	0.093	0.152*	0.087	0.252**	0.272*	0.118***	0.179
	[0.073]	[0.076]	[0.080]	[0.114]	[0.111]	[0.161]	[0.044]	[0.121]
FShare	0.069**			0.017	0.167***			
	[0.032]			[0.040]	[0.048]			
FShare × $\mathbf{1}(Elas < Median)$, β_1		0.020	0.045			−0.063	0.021	0.003
		[0.040]	[0.040]			[0.071]	[0.021]	[0.059]
FShare × $\mathbf{1}(Elas > Median)$, β_2		0.124***	0.149***			0.288***	−0.019	0.243***
		[0.048]	[0.046]			[0.071]	[0.019]	[0.059]
$\mathbf{1}(Elas > Median)$		−0.006	−0.002			−0.168***	0.019	−0.133***
		[0.034]	[0.034]			[0.052]	[0.018]	[0.047]

(*Continues*)

TABLE S.IV—*Continued*

	Dependent Variable: Intrafirm Import Share							
	(1)	(2)	(3)	(4) *Elas < Median*	(5) *Elas ≥ Median*	(6) Weighted	(7)	(8) Weighted
p-value: Joint significance of β_1 and β_2		[0.0335]	[0.0046]			[0.0001]	[0.3173]	[0.0002]
p-value: Test of $\beta_2 - \beta_1 = 0$		[0.0840]	[0.0752]			[0.0001]	[0.1311]	[0.0010]
Industry controls for:	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer
Year fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes	No	No
Country-year fixed effects?	No	No	No	No	No	No	Yes	Yes
Observations	2783	2783	2783	1375	1408	2783	207,991	207,991
R-squared	0.29	0.30	0.32	0.33	0.30	0.61	0.18	0.60

[a] ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Standard errors are clustered by industry. Columns 1–6 use industry-year observations controlling for year fixed effects, while columns 7–8 use country-industry-year observations controlling for country-year fixed effects. Estimation is by OLS. In all columns, the industry factor intensity and dispersion variables are a weighted average of the characteristics of buyer industries (the industries that buy the input in question), constructed as described in Section 4.3 of the main text. Columns 4 and 5 restrict the sample to observations where the buyer industry elasticity is smaller (respectively, larger) than the industry median value. "Weighted" columns use the value of total imports for the industry-year or country-industry-year, respectively, as regression weights.

TABLE S.V

DOWNSTREAMNESS AND THE INTRAFIRM IMPORT SHARE: *DUse_TUse* (YEAR-BY-YEAR)[a]

	Dependent Variable: Intrafirm Import Share										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Year:	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Unweighted regressions:											
$DUse\_TUse \times \mathbf{1}(Elas < Median)$, β_1	−0.162**	−0.160*	−0.181**	−0.143*	−0.149**	−0.169**	−0.178**	−0.172**	−0.189**	−0.197**	−0.212***
	[0.079]	[0.082]	[0.081]	[0.081]	[0.075]	[0.068]	[0.070]	[0.069]	[0.081]	[0.084]	[0.081]
$DUse\_TUse \times \mathbf{1}(Elas > Median)$, β_2	0.230***	0.221***	0.224***	0.173**	0.179**	0.199***	0.209***	0.211***	0.193**	0.173**	0.163**
	[0.071]	[0.074]	[0.073]	[0.073]	[0.071]	[0.069]	[0.074]	[0.073]	[0.075]	[0.078]	[0.077]
$\mathbf{1}(Elas > Median)$	−0.216***	−0.197***	−0.210***	−0.164**	−0.169***	−0.182***	−0.185***	−0.180***	−0.196***	−0.199***	−0.199***
	[0.066]	[0.070]	[0.068]	[0.068]	[0.064]	[0.059]	[0.062]	[0.062]	[0.069]	[0.072]	[0.071]
p-value: Joint significance of β_1 and β_2	[0.0005]	[0.0014]	[0.0006]	[0.0101]	[0.0042]	[0.0004]	[0.0004]	[0.0005]	[0.0019]	[0.0040]	[0.0026]
p-value: Test of $\beta_2 - \beta_1 = 0$	[0.0002]	[0.0004]	[0.0002]	[0.0029]	[0.0011]	[0.0001]	[0.0001]	[0.0001]	[0.0004]	[0.0009]	[0.0006]
Observations	253	253	253	253	253	253	253	253	253	253	253
R-squared	0.33	0.31	0.32	0.31	0.32	0.34	0.34	0.37	0.34	0.32	0.34
Weighted regressions:											
$DUse\_TUse \times \mathbf{1}(Elas < Median)$, β_1	−0.177	−0.151	−0.201**	−0.214**	−0.210**	−0.153*	−0.134	−0.111	−0.152	−0.161	−0.206*
	[0.114]	[0.101]	[0.091]	[0.097]	[0.098]	[0.088]	[0.098]	[0.084]	[0.113]	[0.130]	[0.116]
$DUse\_TUse \times \mathbf{1}(Elas > Median)$, β_2	0.560***	0.526***	0.533***	0.515***	0.486***	0.489***	0.482***	0.457***	0.452***	0.455***	0.472***
	[0.163]	[0.165]	[0.153]	[0.141]	[0.128]	[0.130]	[0.114]	[0.114]	[0.121]	[0.131]	[0.133]
$\mathbf{1}(Elas > Median)$	−0.458***	−0.418***	−0.449***	−0.449***	−0.423***	−0.385***	−0.368***	−0.346***	−0.403***	−0.439***	−0.461***
	[0.101]	[0.099]	[0.093]	[0.093]	[0.088]	[0.084]	[0.088]	[0.081]	[0.102]	[0.125]	[0.114]
p-value: Joint significance of β_1 and β_2	[0.0004]	[0.0007]	[0.0000]	[0.0000]	[0.0000]	[0.0001]	[0.0000]	[0.0000]	[0.0003]	[0.0009]	[0.0002]
p-value: Test of $\beta_2 - \beta_1 = 0$	[0.0001]	[0.0001]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0001]	[0.0006]	[0.0001]
Observations	253	253	253	253	253	253	253	253	253	253	253
R-squared	0.60	0.59	0.63	0.63	0.63	0.63	0.62	0.64	0.60	0.59	0.59
	Additional buyer industry controls included:										
Industry controls for:	$\mathbf{1}(Elas > Median)$, $\log(s/l)$, log(equipment k/l), log(plant k/l), log(materials$/l$), $\log(0.001 + R\&D/Sales)$, Dispersion										

[a] ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Year-by-year regressions are estimated by OLS, with robust standard errors. The upper panel reports unweighted regressions, while the lower panel reports results using the value of total imports in the industry-year as regression weights. All regressions include additional buyer industry control variables for factor intensity and dispersion (constructed as described in Section 4.3 of the main text), whose coefficients are not reported.

TABLE S.VI

DOWNSTREAMNESS AND THE INTRAFIRM IMPORT SHARE: *DownMeasure* (YEAR-BY-YEAR)[a]

	Dependent Variable: Intrafirm Import Share										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Year:	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
Unweighted regressions:											
DownMeasure $\times$ $\mathbf{1}$(*Elas* < *Median*), β_1	0.078	0.081	0.065	0.068	0.038	0.014	0.015	0.004	−0.017	−0.033	−0.041
	[0.076]	[0.075]	[0.074]	[0.075]	[0.070]	[0.066]	[0.065]	[0.066]	[0.072]	[0.074]	[0.075]
DownMeasure $\times$ $\mathbf{1}$(*Elas* > *Median*), β_2	0.286***	0.306***	0.300***	0.277***	0.291***	0.306***	0.314***	0.322***	0.287***	0.289***	0.297***
	[0.082]	[0.084]	[0.082]	[0.083]	[0.084]	[0.084]	[0.085]	[0.086]	[0.087]	[0.094]	[0.094]
$\mathbf{1}$(*Elas* > *Median*)	−0.088	−0.084	−0.088	−0.082	−0.104	−0.114*	−0.110*	−0.117*	−0.127*	−0.146**	−0.152**
	[0.067]	[0.069]	[0.067]	[0.068]	[0.066]	[0.063]	[0.063]	[0.062]	[0.066]	[0.068]	[0.068]
p-value: Joint significance of β_1 and β_2	[0.0020]	[0.0011]	[0.0013]	[0.0032]	[0.0028]	[0.0014]	[0.0012]	[0.0011]	[0.0045]	[0.0072]	[0.0051]
p-value: Test of $\beta_2 - \beta_1 = 0$	[0.0564]	[0.0422]	[0.0302]	[0.0587]	[0.0184]	[0.0046]	[0.0036]	[0.0023]	[0.0055]	[0.0046]	[0.0031]
Observations	253	253	253	253	253	253	253	253	253	253	253
R-squared	0.32	0.31	0.32	0.32	0.33	0.35	0.35	0.37	0.34	0.32	0.34
Weighted regressions:											
DownMeasure $\times$ $\mathbf{1}$(*Elas* < *Median*), β_1	−0.069	−0.046	−0.081	−0.113	−0.132	−0.102	−0.136	−0.094	−0.128	−0.155	−0.182
	[0.134]	[0.125]	[0.118]	[0.120]	[0.121]	[0.111]	[0.118]	[0.102]	[0.119]	[0.127]	[0.121]
DownMeasure $\times$ $\mathbf{1}$(*Elas* > *Median*), β_2	0.565***	0.572***	0.571***	0.562***	0.532***	0.550***	0.526***	0.514***	0.490***	0.477***	0.505***
	[0.130]	[0.126]	[0.114]	[0.105]	[0.095]	[0.093]	[0.089]	[0.090]	[0.105]	[0.126]	[0.115]
$\mathbf{1}$(*Elas* > *Median*)	−0.372***	−0.356***	−0.370***	−0.389***	−0.382***	−0.373***	−0.381***	−0.351***	−0.390***	−0.425***	−0.439***
	[0.102]	[0.094]	[0.088]	[0.086]	[0.085]	[0.081]	[0.089]	[0.080]	[0.093]	[0.109]	[0.101]
p-value: Joint significance of β_1 and β_2	[0.0001]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0002]	[0.0000]
p-value: Test of $\beta_2 - \beta_1 = 0$	[0.0003]	[0.0002]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0001]	[0.0002]	[0.0000]
Observations	253	253	253	253	253	253	253	253	253	253	253
R-squared	0.62	0.63	0.66	0.67	0.66	0.67	0.66	0.68	0.63	0.60	0.61
	Additional buyer industry controls included:										
Industry controls for:	$\mathbf{1}$(*Elas* > *Median*), $\log(s/l)$, log(equipment k/l), log(plant k/l), log(materials$/l$), $\log(0.001 + R\&D/Sales)$, Dispersion										

[a] ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Year-by-year regressions are estimated by OLS, with robust standard errors. The upper panel reports unweighted regressions, while the lower panel reports results using the value of total imports in the industry-year as regression weights. All regressions include additional buyer industry control variables for factor intensity and dispersion (constructed as described in Section 4.3 of the main text), whose coefficients are not reported.

TABLE S.VII

ROBUSTNESS CHECKS WITH THE COUNTRY-INDUSTRY-YEAR SPECIFICATIONS: *DUse_TUse*[a]

	Dependent Variable: Intrafirm Import Share					
	(1) Weighted	(2) Weighted	(3) Weighted	(4) Weighted	(5) Weighted	(6) Weighted
$DUse\_TUse \times \mathbf{1}(Elas < Median)$, β_1	−0.067	−0.098	−0.047	−0.107	−0.109	−0.104
	[0.074]	[0.078]	[0.072]	[0.073]	[0.075]	[0.074]
$DUse\_TUse \times \mathbf{1}(Elas > Median)$, β_2	0.368***	0.375***	0.340***	0.292***	0.304***	0.325***
	[0.134]	[0.102]	[0.118]	[0.095]	[0.092]	[0.073]
Value-added/Value shipments	−0.092					−0.026
	[0.264]					[0.163]
Input "Importance"		−4.275***				−4.610***
		[0.995]				[0.744]
Intermediation			−0.425***			−0.386***
			[0.137]			[0.115]
Own contractibility				0.193***	−0.002	−0.002
				[0.067]	[0.133]	[0.125]
Own contractibility × Country Rule of Law					0.276*	0.306*
					[0.165]	[0.162]
Buyer contractibility				−0.514***	−0.612***	−0.594***
				[0.100]	[0.167]	[0.168]
Buyer contractibility × Country Rule of Law					0.150	0.082
					[0.200]	[0.199]

(*Continues*)

TABLE S.VII—*Continued*

	Dependent Variable: Intrafirm Import Share					
	(1) Weighted	(2) Weighted	(3) Weighted	(4) Weighted	(5) Weighted	(6) Weighted
p-value: Joint significance of β_1 and β_2	[0.0017]	[0.0002]	[0.0038]	[0.0006]	[0.0003]	[0.0000]
p-value: Test of $\beta_2 - \beta_1 = 0$	[0.0004]	[0.0000]	[0.0009]	[0.0001]	[0.0001]	[0.0000]
	Additional buyer industry controls included: $\mathbf{1}(Elas > Median)$, $\log(s/l)$, log(equipment k/l), log(plant k $/l$), log(materials$/l$), $\log(0.001 + R\&D/Sales)$, Dispersion					
Country-year fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes
Observations	207,991	207,991	207,991	207,991	174,274	174,274
R-squared	0.59	0.61	0.60	0.62	0.62	0.65

[a] ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Standard errors are clustered by industry. All columns use country-industry-year observations controlling for country-year fixed effects. Estimation is by OLS. The Value-added/Value shipments, intermediation, input "Importance," and own contractibility variables refer to characteristics of the seller industry (namely, the industry that sells the input in question), while the buyer contractibility variable is a weighted average of the contractibility of buyer industries (the industries that buy the input in question). The contractibility variables are further interacted with a country rule of law index in columns 5–6. All columns include additional control variables whose coefficients are not reported, namely: (i) the level effect of the buyer industry elasticity dummy, and (ii) buyer industry factor intensity and dispersion variables, constructed as described in Section 4.3 of the main text. "Weighted" columns use the value of total imports for the country-industry-year as regression weights.

TABLE S.VIII

ROBUSTNESS CHECKS WITH THE COUNTRY-INDUSTRY-YEAR SPECIFICATIONS: *DownMeasure*[a]

	Dependent Variable: Intrafirm Import Share					
	(1) Weighted	(2) Weighted	(3) Weighted	(4) Weighted	(5) Weighted	(6) Weighted
$DownMeasure \times \mathbf{1}(Elas < Median)$, β_1	−0.010 [0.093]	−0.052 [0.091]	0.042 [0.091]	−0.083 [0.090]	−0.098 [0.090]	−0.144 [0.090]
$DownMeasure \times \mathbf{1}(Elas > Median)$, β_2	0.439*** [0.089]	0.397*** [0.085]	0.429*** [0.088]	0.394*** [0.074]	0.398*** [0.075]	0.317*** [0.054]
Value-added/Value shipments	0.130 [0.207]					0.173 [0.131]
Input “Importance”		−2.660*** [0.600]				−3.511*** [0.544]
Intermediation			−0.411*** [0.124]			−0.353*** [0.113]
Own contractibility				0.220*** [0.065]	−0.001 [0.130]	−0.018 [0.116]
Own contractibility × Country Rule of Law					0.313* [0.163]	0.337** [0.158]
Buyer contractibility				−0.506*** [0.095]	−0.586*** [0.169]	−0.578*** [0.166]
Buyer contractibility × Country Rule of Law					0.120 [0.200]	0.064 [0.202]

(*Continues*)

TABLE S.VIII—*Continued*

	Dependent Variable: Intrafirm Import Share					
	(1) Weighted	(2) Weighted	(3) Weighted	(4) Weighted	(5) Weighted	(6) Weighted
p-value: Joint significance of β_1 and β_2	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	[0.0000]
p-value: Test of $\beta_2 - \beta_1 = 0$	[0.0001]	[0.0000]	[0.0005]	[0.0000]	[0.0000]	[0.0000]
	Additional buyer industry controls included: $\mathbf{1}(\mathit{Elas} > \mathit{Median})$, $\log(s/l)$, log(equipment k/l), log(plant k/l), log(materials$/l$), $\log(0.001 + \mathit{R\&D}/\mathit{Sales})$, Dispersion					
Country-year fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes
Observations	207,991	207,991	207,991	207,991	174,274	174,274
R-squared	0.61	0.62	0.62	0.64	0.64	0.66

[a] ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Standard errors are clustered by industry. All columns use country-industry-year observations controlling for country-year fixed effects. Estimation is by OLS. The Value-added/Value shipments, intermediation, input "Importance," and own contractibility variables refer to characteristics of the seller industry (namely, the industry that sells the input in question), while the buyer contractibility variable is a weighted average of the contractibility of buyer industries (the industries that buy the input in question). The contractibility variables are further interacted with a country rule of law index in columns 5–6. All columns include additional control variables whose coefficients are not reported, namely: (i) the level effect of the buyer industry elasticity dummy, and (ii) buyer industry factor intensity and dispersion variables, constructed as described in Section 4.3 of the main text. "Weighted" columns use the value of total imports for the country-industry-year as regression weights.

TABLE S.IX

ALTERNATIVE ELASTICITY MEASURE CAPTURING CROSS-PRODUCT SUBSTITUTABILITY: *DUse_TUse*[a]

	Dependent Variable: Intrafirm Import Share							
	(1)	(2)	(3)	(4) *Elas < Median*	(5) *Elas ≥ Median*	(6) Weighted	(7)	(8) Weighted
$\log(s/l)$	0.005	0.003	0.016	0.077	−0.028	−0.132*	−0.001	−0.085
	[0.044]	[0.044]	[0.043]	[0.062]	[0.061]	[0.079]	[0.021]	[0.076]
$\log(k/l)$	0.044	0.048						
	[0.029]	[0.029]						
log(equipment k/l)			0.087**	0.015	0.197***	0.169***	0.026	0.118**
			[0.036]	[0.045]	[0.054]	[0.056]	[0.016]	[0.050]
log(plant k/l)			−0.063	−0.018	−0.156**	−0.098	−0.052***	−0.110**
			[0.047]	[0.058]	[0.077]	[0.063]	[0.020]	[0.048]
log(materials/l)	0.058*	0.057*	0.063*	0.104**	0.019	0.033	0.023*	0.065
	[0.035]	[0.034]	[0.034]	[0.044]	[0.052]	[0.060]	[0.014]	[0.051]
$\log(0.001 + R\&D/Sales)$	0.055***	0.054***	0.053***	0.056***	0.034**	0.094***	0.031***	0.072***
	[0.009]	[0.009]	[0.009]	[0.011]	[0.016]	[0.017]	[0.004]	[0.015]
Dispersion	0.081	0.088	0.129*	0.034	0.291**	0.179	0.113***	0.091
	[0.070]	[0.070]	[0.077]	[0.089]	[0.134]	[0.136]	[0.040]	[0.113]
DUse_TUse	−0.018			−0.083	0.055			
	[0.054]			[0.081]	[0.074]			
DUse_TUse $\times \mathbf{1}(Elas < Median)$, β_1		−0.072	−0.046			−0.005	−0.099***	0.066
		[0.078]	[0.080]			[0.103]	[0.032]	[0.077]
DUse_TUse $\times \mathbf{1}(Elas > Median)$, β_2		0.025	0.039			0.449***	−0.052	0.299**
		[0.074]	[0.074]			[0.155]	[0.034]	[0.150]
$\mathbf{1}(Elas > Median)$		−0.075	−0.073			−0.285***	−0.036	−0.149*
		[0.069]	[0.068]			[0.104]	[0.029]	[0.087]

(*Continues*)

TABLE S.IX—*Continued*

	Dependent Variable: Intrafirm Import Share							
	(1)	(2)	(3)	(4) *Elas < Median*	(5) *Elas ≥ Median*	(6) Weighted	(7)	(8) Weighted
p-value: Joint significance of β_1 and β_2		[0.6020]	[0.7064]			[0.0138]	[0.0052]	[0.1356]
p-value: Test of $\beta_2 - \beta_1 = 0$		[0.3514]	[0.4049]			[0.0081]	[0.2776]	[0.1167]
Industry controls for:	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer
Year fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes	No	No
Country-year fixed effects?	No	No	No	No	No	No	Yes	Yes
Observations	2783	2783	2783	1419	1364	2783	207,991	207,991
R-squared	0.273	0.277	0.286	0.355	0.276	0.568	0.180	0.576

[a] ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Standard errors are clustered by industry. The buyer industry elasticity used in this table is constructed from the SITC Rev. 3 three-digit U.S. import elasticities from Broda and Weinstein (2006), which are partially estimated off the substitution seen across HS10 constituent product codes for each SITC three-digit category. Columns 1–6 use industry-year observations controlling for year fixed effects, while columns 7–8 use country-industry-year observations controlling for country-year fixed effects. Estimation is by OLS. In all columns, the industry factor intensity and dispersion variables are a weighted average of the characteristics of buyer industries (the industries that buy the input in question), constructed as described in Section 4.3 of the main text. Columns 4 and 5 restrict the sample to observations where the buyer industry elasticity is smaller (respectively, larger) than the industry median value. "Weighted" columns use the value of total imports for the industry-year or country-industry-year, respectively, as regression weights.

TABLE S.X

ALTERNATIVE ELASTICITY MEASURE CAPTURING CROSS-PRODUCT SUBSTITUTABILITY: *DownMeasure*[a]

	Dependent Variable: Intrafirm Import Share							
	(1)	(2)	(3)	(4) *Elas* < *Median*	(5) *Elas* ≥ *Median*	(6) Weighted	(7)	(8) Weighted
$\log(s/l)$	−0.011	−0.010	0.004	0.058	−0.019	−0.170***	0.000	−0.108**
	[0.045]	[0.044]	[0.042]	[0.063]	[0.057]	[0.061]	[0.021]	[0.054]
$\log(k/l)$	0.062**	0.063**						
	[0.027]	[0.028]						
log(equipment k/l)			0.123***	0.042	0.246***	0.228***	0.038**	0.174***
			[0.035]	[0.046]	[0.050]	[0.059]	[0.017]	[0.047]
log(plant k/l)			−0.093*	−0.038	−0.206***	−0.144**	−0.061***	−0.144***
			[0.047]	[0.058]	[0.075]	[0.064]	[0.020]	[0.048]
log(materials/l)	0.050	0.051	0.059*	0.099**	0.023	0.018	0.019	0.037
	[0.033]	[0.033]	[0.032]	[0.044]	[0.049]	[0.058]	[0.013]	[0.044]
$\log(0.001 + R\&D/Sales)$	0.058***	0.058***	0.057***	0.060***	0.038**	0.105***	0.032***	0.082***
	[0.010]	[0.009]	[0.010]	[0.012]	[0.016]	[0.017]	[0.004]	[0.014]
Dispersion	0.087	0.089	0.148*	0.051	0.302**	0.239*	0.120***	0.143
	[0.072]	[0.071]	[0.078]	[0.092]	[0.133]	[0.135]	[0.043]	[0.106]
DownMeasure	0.101*			0.052	0.208***			
	[0.055]			[0.075]	[0.078]			
DownMeasure × **1**(*Elas* < *Median*), β_1		0.060	0.113			0.008	−0.035	0.098
		[0.071]	[0.073]			[0.111]	[0.034]	[0.086]
DownMeasure × **1**(*Elas* > *Median*), β_2		0.139*	0.172**			0.505***	−0.006	0.428***
		[0.083]	[0.082]			[0.104]	[0.033]	[0.096]
1(*Elas* > *Median*)		−0.050	−0.044			−0.312***	−0.020	−0.200***
		[0.065]	[0.064]			[0.092]	[0.030]	[0.076]

(*Continues*)

TABLE S.X—*Continued*

	Dependent Variable: Intrafirm Import Share							
	(1)	(2)	(3)	(4) *Elas < Median*	(5) *Elas ≥ Median*	(6) Weighted	(7)	(8) Weighted
p-value: Joint significance of β_1 and β_2		[0.1867]	[0.0460]			[0.0000]	[0.5887]	[0.0001]
p-value: Test of $\beta_2 - \beta_1 = 0$		[0.4600]	[0.5665]			[0.0004]	[0.5223]	[0.0036]
Industry controls for:	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer	Buyer
Year fixed effects?	Yes	Yes	Yes	Yes	Yes	Yes	No	No
Country-year fixed effects?	No	No	No	No	No	No	Yes	Yes
Observations	2783	2783	2783	1419	1364	2783	207,991	207,991
R-squared	0.283	0.284	0.303	0.350	0.312	0.611	0.177	0.603

[a] ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively. Standard errors are clustered by industry. The buyer industry elasticity used in this table is constructed from the SITC Rev. 3 three-digit U.S. import elasticities from Broda and Weinstein (2006), which are partially estimated off the substitution seen across HS10 constituent product codes for each SITC three-digit category. Columns 1–6 use industry-year observations controlling for year fixed effects, while columns 7–8 use country-industry-year observations controlling for country-year fixed effects. Estimation is by OLS. In all columns, the industry factor intensity and dispersion variables are a weighted average of the characteristics of buyer industries (the industries that buy the input in question), constructed as described in Section 4.3 of the main text. Columns 4 and 5 restrict the sample to observations where the buyer industry elasticity is smaller (respectively, larger) than the industry median value. "Weighted" columns use the value of total imports for the industry-year or country-industry-year, respectively, as regression weights.

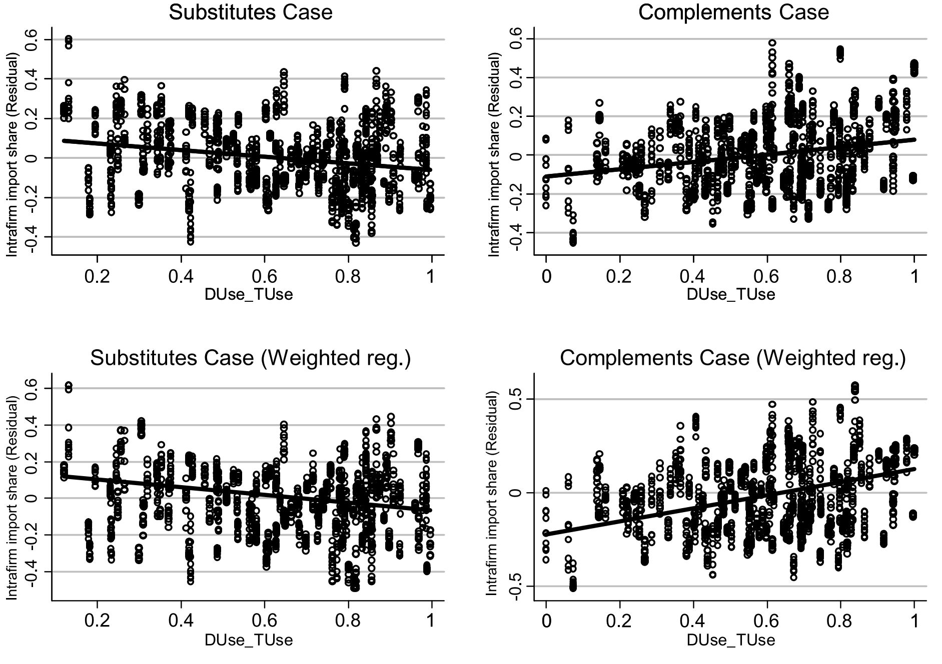

FIGURE S.2.—Partial scatterplots of the relationship between downstreamness and the intrafirm import share: *DUse_TUse*. *Notes*: The residuals plotted on the vertical axis are predicted from a regression of the intrafirm trade share on: (i) the buyer industry control variables, namely: $\mathbf{1}(Elas > Median)$, $\log(s/l)$, $\log(\text{equipment } k/l)$, $\log(\text{plant } k/l)$, $\log(\text{materials}/l)$, $\log(0.001 + R\&D/Sales)$, Dispersion, and (ii) year fixed effects. The upper panel uses residuals from an unweighted OLS regression, while the lower panel is obtained from a weighted OLS regression using the value of total imports in an industry-year as weights. These are plotted against *DUse_TUse* on the horizontal axis, for industry-year observations corresponding to the substitutes case ($Elas < Median$) on the left column, and for observations from the complements case ($Elas > Median$) on the right column. In the upper panel, an OLS prediction line is included; in the bottom panel, a weighted OLS prediction line is included (using total imports in the industry-year as weights). The slope of the prediction lines in all four graphs is significant at the 1% level based on robust standard errors.

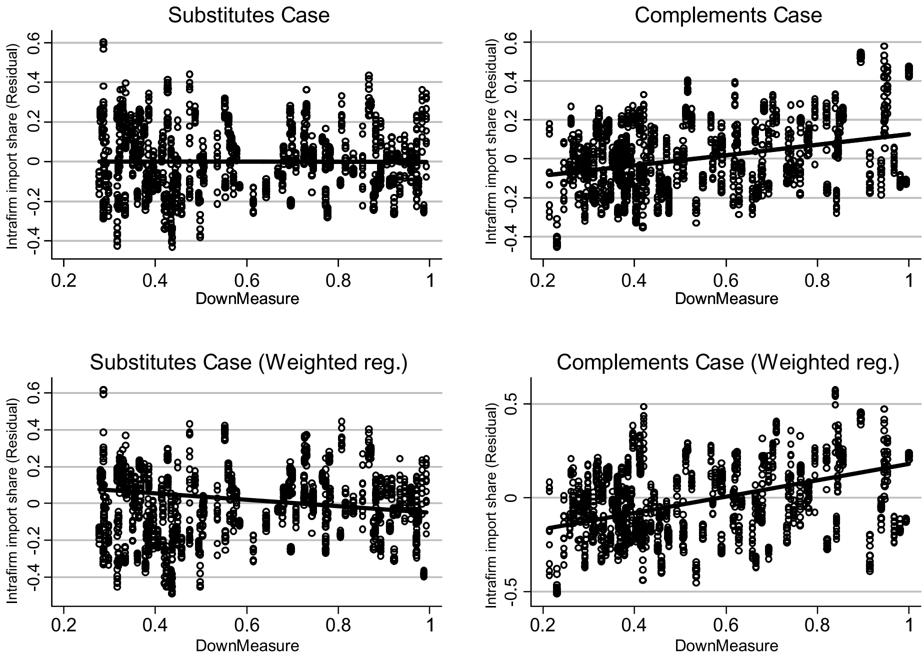

FIGURE S.3.—Partial scatterplots of the relationship between downstreamness and the intrafirm import share: *DownMeasure*. *Notes*: The residuals plotted on the vertical axis are predicted from a regression of the intrafirm trade share on: (i) the buyer industry control variables, namely: $\mathbf{1}(\textit{Elas} > \textit{Median})$, $\log(s/l)$, $\log(\text{equipment } k/l)$, $\log(\text{plant } k/l)$, $\log(\text{materials}/l)$, $\log(0.001 + \textit{R\&D}/\textit{Sales})$, Dispersion, and (ii) year fixed effects. The upper panel uses residuals from an unweighted OLS regression, while the lower panel is obtained from a weighted OLS regression using the value of total imports in an industry-year as weights. These are plotted against *DownMeasure* on the horizontal axis, for industry-year observations corresponding to the substitutes case ($\textit{Elas} < \textit{Median}$) on the left column, and for observations from the complements case ($\textit{Elas} > \textit{Median}$) on the right column. In the upper panel, an OLS prediction line is included; in the bottom panel, a weighted OLS prediction line is included (using total imports in the industry-year as weights). The slope of the prediction lines in each graph is significant at the 1% level based on robust standard errors, except in the top-left graph (Substitutes case, unweighted regression) where the slope has a negative point estimate but is not statistically significant.

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