

SUPPLEMENT TO “PARALYZED BY FEAR: RIGID AND DISCRETE PRICING UNDER DEMAND UNCERTAINTY”
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APPENDIX A

A.1. *Updating With More Observed Prices*

WE CAN READILY EXPAND THE UPDATING FORMULAS that we have developed in Section 2.2 for one observed price to the case of multiple observed past price points. Let the firm’s information set ε^{t-1} contain T unique price points collected in the vector $\mathbf{p}_T = [p_1, \dots, p_T]'$, where $T > 0$ is arbitrarily large but finite. We label the average realized quantity sold at each of these unique price points \bar{y}_i , and similarly collect them in the vector $\mathbf{y}_T = [y_1, \dots, y_T]'$. Last, let N_i be the number of times the firm has seen price point p_i in the past, and thus this is the number of signals at p_i the firm has. The vector $\mathbf{N}_T = [N_1, \dots, N_T]'$ collects these values.

The joint distribution between demand at any price p and the vector of signals \mathbf{y} is similarly joint Normal:

$$\begin{bmatrix} x(p) \\ \mathbf{y}_T \end{bmatrix} \sim N \left(\begin{bmatrix} m(p) \\ m(\mathbf{p}_T) \end{bmatrix}, \Sigma(p, \mathbf{p}_T) \right),$$

where the variance–covariance matrix is given by

$$\Sigma(p, \mathbf{p}_T) = \begin{bmatrix} \sigma_x^2 & K(p, \mathbf{p}_T) \\ K(\mathbf{p}_T, p) & K(\mathbf{p}_T, \mathbf{p}_T) + \text{diag}(\mathbf{N}_T)^{-1} \sigma_z^2 \end{bmatrix}.$$

The conditional expectation of $x(p)$, given a prior mean function $m(p)$ and the vector of signals \mathbf{y}_T , follows from applying the standard formula for conditional Gaussian expectations:

$$E(x(p)|\mathbf{y}_T, m(p)) = m(p) + K(p, \mathbf{p}_T)(K(\mathbf{p}_T, \mathbf{p}_T) + \text{diag}(\mathbf{N}_T)^{-1} \sigma_z^2)^{-1}(\mathbf{y}_T - m(\mathbf{p}_T)). \quad (34)$$

Expanding the above expression, we can show that the conditional expectation is again linear in the prior and a weighted sum of the demeaned signals, leading to

$$E(x(p)|\mathbf{y}_T, m(p)) = m(p) + \alpha_1(p)(y_1 - m(p_1)) + \dots + \alpha_T(p)(y_T - m(p_T)),$$

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where $\alpha_i \in (0, 1)$ is the i th element of the $1 \times T$ vector $K(p, \mathbf{p}_T)(K(\mathbf{p}_T, \mathbf{p}_T) + \text{diag}(\mathbf{N}_T)^{-1}\sigma_z^2)^{-1}$.

Without loss of generality, assume the prices in \mathbf{p} are sorted in ascending order, with the last element being the largest price value. In building the worst-case expectation, one can work from back to front and first characterize the worst-case prior $m^*(p; p_t)$ for entertained price values $p_t > p_T$. The firm wants the prior level of demand at the entertained price p_t , $m^*(p; p_t)$, to be the lowest possible, so it sets it equal to the lower bound of Y_0 so that

$$m^*(p; p_t) = -\gamma - bp_t.$$

Again similarly to the case of only one previously observed price, the firm is worried that demand decreases a lot as it increases its price away from its previous observations. Now, however, this worry does not apply only to the closest signal at the price value of p_T , but to all previous signals. Since all previous signals were observed at prices below p_t , the worst case $m^*(p; p_t)$ for any $p < p_t$ is given by

$$m^*(p; p_t) = \min[\gamma - bp, -\gamma - bp_t + (b + \delta)(p_t - p)].$$

Next consider, $p_t \in (p_{T-1}, p_T]$. The worst case $m^*(p; p_t)$ is again at the lower bound of the admissible set Y_0 . And the basic intuition for the rest of the worst-case prior is similar to before—the firm worries that setting the price p_t away from its previous observations \mathbf{p}_T makes demand change for the worse. Thus, the firm is worried that $m^*(\mathbf{p}_T; p_t)$ is the highest possible level, given constraints on the admissible set Y_0 and the fact that $m^*(p; p_t) = -\gamma - bp_t$. This concern yields

$$m^*(p; p_t) = \begin{cases} \min[\gamma - bp, -\gamma - bp_t + (b + \delta)(p_t - p)] & \text{for } p < p_t, \\ \min[\gamma - bp, -\gamma - bp_t + (b - \delta)(p_t - p)] & \text{for } p \geq p_t. \end{cases}$$

Hence, for all price points below the currently entertained price p_t , the worst-case prior is restricted by the maximum admissible derivative $b + \delta$, while for prices above p_t , it is restricted by the lowest admissible derivative $b - \delta$.

Substituting this worst-case prior in (34), it is easy to evaluate the worst-case expectation $\widehat{x}^*(p_t | y_T, m^*(p; p_t))$. Given the piecewise nature of $m^*(p; p_t)$, it follows that there is a kink in the worst-case expected demand $\widehat{x}^*(p_t | y_T, m^*(p; p_t))$ around any $p \in \mathbf{p}_T$.

A.2. Proofs for Section 2

PROPOSITION 1: Define $\delta^* = \delta \text{sgn}(p_t - p_0)$. For a given realization of c_t , the difference in worst-case expected profits at p_t and p_0 , up to a first-order approximation around p_0 , is

$$\ln v^*(\varepsilon^{t-1}, c_t, p_t) - \ln v_0^*(\varepsilon^{t-1}, c_t, p_0) \approx \left[\frac{e^{p_0}}{e^{p_0} - e^{c_t}} - (b + \alpha_{t-1}(p_0)\delta^*) \right] (p_t - p_0).$$

PROOF: Consider $\ln v^*(\varepsilon^{t-1}, c_t, p_t)$ at some $p_t \in [p_0 - \frac{2\gamma}{\delta}, p_0 + \frac{2\gamma}{\delta}]$. When $p_t > p_0$, we have

$$\ln(e^{p_t} - e^{c_t}) + \{-\gamma - bp_t + \alpha_{t-1}(p_t)\widehat{z}_0 - \alpha_{t-1}(p_t)\delta(p_t - p_0) + 0.5\widehat{\sigma}_{t-1}^2(p_t) + 0.5\sigma_z^2\},$$

while at $p_t < p_0$, this equals

$$\ln(e^{p_t} - e^{c_t}) + \{-\gamma - bp_t + \alpha_{t-1}(p_t)\widehat{z}_0 + \alpha_{t-1}(p_t)\delta(p_t - p_0) + 0.5\widehat{\sigma}_{t-1}^2(p_t) + 0.5\sigma_z^2\},$$

where for convenience we have defined $\widehat{z}_0 \equiv -\gamma - bp_0$. In turn, $\ln v^*(\varepsilon^{t-1}, c_t, p_0)$ equals

$$\ln(e^{p_t} - e^{c_t}) + \{-\gamma - bp_0 + \alpha_{t-1}(p_t)\widehat{z}_0 + 0.5\widehat{\sigma}_{t-1}^2(p_0) + 0.5\sigma_z^2\}.$$

Fix some c_t and take a first-order approximation of $\ln v^*(\varepsilon^{t-1}, c_t, p_t)$ with respect to p_t , evaluated at p_0 . Since this function is not differentiable at p_0 , we analyze its right and left derivative separately. The former derivative equals

$$\frac{e^{p_0}}{e^{p_0} - e^{c_t}} - b - \alpha_{t-1}(p_0)\delta + \frac{\partial \alpha_{t-1}(p_t)}{\partial p_t} [\widehat{z}_0 - \delta(p_t - p_0)] + 0.5 \frac{\partial \widehat{\sigma}_{t-1}^2(p_t)}{\partial p_t},$$

where the partial derivatives $\frac{\partial \alpha_{t-1}(p_t)}{\partial p_t}$ and $\frac{\partial \widehat{\sigma}_{t-1}^2(p_t)}{\partial p_t}$ are evaluated locally at p_0 . In particular, given that

$$\alpha_{t-1}(p_t) = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2/N_0} e^{-\psi(p_t - p_0)^2}; \quad \widehat{\sigma}_{t-1}^2(p_t) = \sigma_x^2(1 - \alpha_{t-1}(p_t)),$$

then these two functions are differentiable p_0 , with marginal effects equal to zero at p_0 . Therefore, the local approximation to the right of p_0 simplifies to

$$\frac{e^{p_0}}{e^{p_0} - e^{c_t}} - [b + \alpha_{t-1}(p_0)\delta].$$

The first term in the brackets reflects the effect of changing the price on profits, while the second captures the movement of demand along a curve with elasticity $-b$. The third term arises from the effect of demand of moving along a steeper demand curve, which is a characteristic of the worst-case belief about the demand elasticity.

Therefore, we obtain the local approximation to the right of p_0

$$\ln v^*(\varepsilon^{t-1}, c_t, p_t) - \ln v_0^*(\varepsilon^{t-1}, c_t, p_0) \approx \left[\frac{e^{p_0}}{e^{p_0} - e^{c_t}} - (b + \alpha_{t-1}(p_0)\delta) \right] (p_t - p_0). \quad (35)$$

A similar derivation follows for the derivative to the left of p_0 , where we obtain

$$\frac{e^{p_0}}{e^{p_0} - e^{c_t}} - [b - \alpha_{t-1}(p_0)\delta],$$

and therefore the local approximation to the left of p_0 is simply

$$\ln v^*(\varepsilon^{t-1}, c_t, p_t) - \ln v_0^*(\varepsilon^{t-1}, c_t, p_0) \approx \left[\frac{e^{p_0}}{e^{p_0} - e^{c_t}} - (b - \alpha_{t-1}(p_0)\delta) \right] (p_t - p_0). \quad (36)$$

We obtain the result in Proposition 1 by putting together equations (35) and (36) and using the signum function to define $\delta^* = \delta \operatorname{sgn}(p_t - p_0)$. Q.E.D.

PROPOSITION 2: Let $\delta_i^* \equiv \delta \operatorname{sgn}(p_t - p_i)$ for all $p_i \in \varepsilon^{t-1}$. For a given realization of c_t , up to a first-order approximation around each such $p_i \in \varepsilon^{t-1}$:

$$\ln v^*(\varepsilon^{t-1}, c_t, p_t) - \ln v_0^*(\varepsilon^{t-1}, c_t, p_i) \approx \left[\frac{e^{p_i}}{e^{p_i} - e^{c_t}} - (b + \alpha_{t-1,i}(p_i)\delta^* + A_i) \right] (p_t - p_i).$$

PROOF: The structure of the proof is very similar to the previous one. Consider $\ln v^*(\varepsilon^{t-1}, c_t, p_t)$ at some $p_t \in [p_i - \frac{2\gamma}{\delta}, p_i + \frac{2\gamma}{\delta}]$. Using $\delta_i^* \equiv \delta \operatorname{sgn}(p_t - p_i)$, we can write $\ln v^*(\varepsilon^{t-1}, c_t, p_t)$ as

$$\begin{aligned} & \ln(e^{p_t} - e^{c_t}) + \left\{ -\gamma - bp_t + \sum_{p_k \in \varepsilon^{t-1}} \alpha_{t-1,k}(p_t) (\widehat{z}_k - \delta_k^*(p_t - p_k) \mathbb{1}(p_t \in (\underline{p}_k, \overline{p}_k))) \right. \\ & \left. + 0.5 \widehat{\sigma}_{t-1}^2(p_t) + 0.5 \sigma_z^2 \right\}. \end{aligned}$$

Fixing some c_t , take a first-order approximation of $\ln v^*(\varepsilon^{t-1}, c_t, p_t)$ with respect to p_t , evaluated at p_i . Since this function is not differentiable at p_0 , we analyze its right and left derivative separately as before. Using the notation $\delta_i^* \equiv \delta \operatorname{sgn}(p_t - p_i)$, we can express both the right and left derivatives around one of the $p_i \in \varepsilon^{t-1}$ as

$$\begin{aligned} & \frac{e^{p_i}}{e^{p_i} - e^{c_t}} - b - \alpha_{t-1,i}(p_i) \delta_i^* + \frac{\partial \alpha_{t-1,i}(p_i)}{\partial p} \widehat{z}_i + 0.5 \frac{\partial \widehat{\sigma}_{t-1}^2(p_i)}{\partial p} \\ & + \sum_{p_k \in \varepsilon^{t-1}/p_i} \frac{\partial \alpha_{t-1,k}(p_i)}{\partial p} (\widehat{z}_k - \delta_k^*(p_i - p_k) \mathbb{1}(p_i \in (\underline{p}_k, \overline{p}_k))) \\ & - \sum_{p_k \in \varepsilon^{t-1}/p_i} \alpha_{t-1,k}(p_i) (-\delta_k^* \mathbb{1}(p_i \in (\underline{p}_k, \overline{p}_k))). \end{aligned}$$

The partial derivatives of the signal-to-noise ratios and the posterior variance are no longer zero; however, they are not a function of the sign of $(p_t - p_i)$; hence, when considering a local approximation around p_i , all of the additional terms (as compared to Proposition 1) can be treated as a constant. We call that constant A_i :

$$\begin{aligned} A_i &= \frac{\partial \alpha_{t-1,i}(p_i)}{\partial p} \widehat{z}_i + 0.5 \frac{\partial \widehat{\sigma}_{t-1}^2(p_i)}{\partial p} \\ & + \sum_{p_k \in \varepsilon^{t-1}/p_i} \frac{\partial \alpha_{t-1,k}(p_i)}{\partial p} (\widehat{z}_k - \delta_k^*(p_i - p_k) \mathbb{1}(p_i \in (\underline{p}_k, \overline{p}_k))) \\ & - \sum_{p_k \in \varepsilon^{t-1}/p_i} \alpha_{t-1,k}(p_i) (-\delta_k^* \mathbb{1}(p_i \in (\underline{p}_k, \overline{p}_k))). \end{aligned}$$

Using the fact that the A_i term is not a function of p_t , it just updates the coefficients in the first-order approximation of $\ln v^*(\varepsilon^{t-1}, c_t, p_t)$, but does not change the basic observation that there is a kink in the profit function at p_i , so that

$$\begin{aligned} & \ln v^*(\varepsilon^{t-1}, c_t, p_t) - \ln v_0^*(\varepsilon^{t-1}, c_t, p_i) \\ & \approx \left[\frac{e^{p_i}}{e^{p_i} - e^{c_t}} - (b + \alpha_{t-1,i}(p_i) \delta_i^* + A_i) \right] (p_t - p_i). \end{aligned} \quad Q.E.D.$$

A.3. Forward-Looking Behavior

We solve the recursive optimization problem in two steps. First, we compute the value function at time $t + 1$. The key insight is that from this point onward, the firm solves a

series of static maximization problems because the endogenous state variable, the information set ε^t , remains the same from period to period. Still, the firm faces a dynamic, recursive problem because of the law of motion of the exogenous state variable, the cost shock c_t , which evolves according to its law of motion $g(c_{t+1}|c_t)$. Hence, the value function at $t + 1$, which we label with $\tilde{V}(\cdot)$ to differentiate from the time- t value function $V(\cdot)$, is given by

$$\tilde{V}(\varepsilon^t, c_{t+1}) = \max_{p_{t+1}} \min_{m(p) \in Y_0} E \left[\nu(\varepsilon_{t+1}, c_{t+1}) + \beta \int \tilde{V}(\varepsilon^t, c_{t+2}) g(c_{t+2}|c_{t+1}) dc_{t+2} \middle| \varepsilon^t \right].$$

Since the information set is not growing over time, the state space for this problem is finite and tractable. As a result, we can solve for $\tilde{V}(\varepsilon^t, c_{t+1})$ through standard techniques and use it as the continuation value perceived by the firm at time t :

$$V(\varepsilon^{t-1}, c_t) = \max_{p_t} \min_{m(p) \in Y_0} E \left[\nu(\varepsilon_t, c_t) + \beta \int \tilde{V}(\varepsilon^t, c_{t+1}) g(c_{t+1}|c_t) dc_{t+1} \middle| \varepsilon^{t-1} \right]$$

s.t.

$$\varepsilon^t = \{ \varepsilon^{t-1}, p_t, y_t \}.$$

Thus, at time t , the firm fully takes into account that p_t , and the resulting new demand signal y_t , will serve as informative signals for future profit-maximization decisions. Importantly, this information is useful not only in the very next period, but propagates through the infinite future according to the law of motion of c_t .

For the following analytical results, we work with the case where $\psi = \infty$ and the firm has perfect foresight on future costs, s.t. $c_{t+k} = c$ for all $k \geq 1$, for some constant c . In this case, the time- $t + 1$ value function is just the present discounted value of worst-case expected profits when the cost shock equals c :

$$\tilde{V}(\varepsilon^t, c) = \frac{\max_p \min_{m(p) \in Y_0} E[\nu(\varepsilon_{t+1}, c) | \varepsilon^t]}{1 - \beta}.$$

Hence, the only remaining uncertainty in $\tilde{V}(\cdot)$ from the perspective of time t is the uncertainty about the realization of the time- t signal y_t . Next, we turn to characterizing the expectation of \tilde{V} , given the time- t information set ε^{t-1} .

For all analytical results below, we assume that (i) $\psi \rightarrow \infty$ and (ii) there is perfect foresight on future costs so that $c_{t+k} = c$ for some c .

Exploration makes prices more flexible when ε^{t-1} contains demand observations at only one previous price p_0

We start with the case where the time- t information set, ε^{t-1} , contains only one price point, p_0 , observed N_0 times with an average signal y_0 . To be specific, call that information set ε^0 . We will assume that the realization of the signal y_0 is good enough, so that when $c = c_0^* = p_0 - \ln(\frac{b}{b-1})$, p_0 is not just locally optimal (recall Corollary 1), but that it is the global maximizer conditional on ε^{t-1} . The relevant condition is

$$\widehat{z}_0 = y_0 - (-\gamma - bp_0) > \frac{\sigma_x^2}{2},$$

in which case

$$p_0 = \arg \max_p \min_{m(p) \in Y_0} E[\nu(\varepsilon_{t+1}, c_0^*) | \varepsilon^0, m(p)].$$

Hence, in the absence of any new information, in future periods the firm will optimally set p_0 , since it essentially faces a static problem with marginal cost equal to c_0^* . The signal pair $\{p_t, y_t\}$ provides such new information and could lead to a different optimal action p_{t+k} .

Our first result is a characterization of the current price p_t that maximizes the expected continuation value when $c = c_0^*$. It turns out that when the firm has collected prior information about demand only at p_0 , then even at that value of the cost, the optimal exploration strategy is to deviate from p_0 .

PROPOSITION 3: *The expected continuation value $E[\tilde{V}(\{\varepsilon^0, p_t, y_t\}, c_0^*) | \varepsilon^0, p_t]$ achieves its maximum at*

$$p_t^* = \arg \min_p (p - p_0)^2 \quad s.t. \quad p \neq p_0.$$

PROOF: In order to simplify notation, throughout the proofs we will use the standard expectation notation $E(\cdot)$ to define the worst-case expectation of the firm.

The limiting case $\psi \rightarrow \infty$ simplifies the construction of the worst-case expected demand because $\text{corr}(x(p), x(p')) = 0$ for all $p \neq p'$. Thus, when updating beliefs about demand at any price p , only past signals observed at that particular price p matter. For future reference, it will be convenient to define the following notation for signal-to-noise ratios that will show up repeatedly:

$$\begin{aligned} \alpha_0 &\equiv \alpha_{t-1}(p_0; p_0) = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2/N_0}, \\ \alpha_{t|0} &\equiv \alpha_t(p_0; p_0 | p_t = p_0) = \frac{\sigma_x^2}{\sigma_x^2(N_0 + 1) + \sigma_z^2}, \\ \alpha_t &\equiv \alpha_{t-1}(p_t; p_t | p_t \neq p_0) = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2}, \end{aligned}$$

where the first is the signal-to-noise ratio of the signal y_0 conditional on ε^0 information, $\alpha_{t|0}$ and α_t are the (recursive) signal-to-noise ratios applicable to the new signal y_t given the signal y_0 , in the two cases where $p_t = p_0$ and $p_t \neq p_0$, respectively. Since $p_0 = \ln(\frac{b}{b-1}) + c_0^*$, it is the optimal myopic price for $c_{t+k} = c_0^*$, which is the relevant case in the future. Thus, if its information set does not change, the firm will price $p_{t+k} = p_0$ in the future. The information set changes, of course, as a function of the current period pricing choice p_t and the resulting new signal y_t . For convenience, define the perceived innovations in the existing signal y_0 and the new signal y_t as

$$\begin{aligned} \widehat{z}_0 &\equiv y_0 - (-\gamma - bp_0), \\ \widehat{z}_t &\equiv y_t - (-\gamma - bp_t), \end{aligned}$$

and the variance adjusted innovation of y_0 as

$$\widetilde{z}_0 \equiv \widehat{z}_0 - \frac{1}{2} \sigma_x^2.$$

Observe that since $c_{t+k} = c_0^*$ with probability 1, the only uncertainty over future profits is in the innovation of the new signal \widehat{z}_t . Hence, the expected continuation value is simply the expected discounted value of a stream of worst-case static profits at $c_{t+k} = c_0^*$, after taking the expectation over the unknown \widehat{z}_t : $E[\tilde{V}(\{\varepsilon^0, p_t, y_t\}, c_0^*) | \varepsilon^0, p_t] = \frac{\beta}{1-\beta} E[E(v(p_{t+k}^*, c_0^*) | \{\varepsilon^0, p_t, y_t\}) | \varepsilon^0, p_t] = \frac{\beta}{1-\beta} E[v_{t+k}^*(p_{t+k}^*, c_0^*) | \varepsilon^0, p_t]$, where p_{t+k}^* is the resulting static optimal price, given the updated information set $\{\varepsilon^0, p_t, y_t\}$.

If $p_t = p_0$, this optimal price is still p_0 unless the information in the new signal y_t is particularly bad and sufficiently erodes the firm's beliefs about profits at p_0 , in which case the firm switches to the interior optimal price p_{t+k}^{int} —the ex ante second best option. To find this interior optimum, note that for all prices $p_{t+k} \neq p_0$, the worst-case demand is simply

$$\widehat{x}_t^*(p_{t+k}; m^*(p; p_{t+k})) = -\gamma - bp,$$

hence the interior optimal price is

$$p_{t+k}^{\text{int}} = \min\{p | (p - p_0)^2 > 0\},$$

which gets you as close as possible to the optimal markup $\frac{b}{b-1}$ while still staying on the smooth portion of the firm's demand curve (recall: there is a kink in the worst-case belief at p_0 , but is smooth everywhere else). Thus, if $p_t = p_0$, optimal p_{t+k}^* is equal to p_0 unless $\widehat{z}_t < \underline{z}_0$, where \underline{z}_0 is such that

$$\frac{E_{t-1}(v_{t+k}^*(p_0, c_0^*) | \varepsilon^0, p_t = p_0, \widehat{z}_t = \underline{z}_0)}{\lim_{p \rightarrow p_0} E(v_{t+k}^*(p, c_0^*) | \varepsilon^0, p_t = p_0, \widehat{z}_t = \underline{z}_0)} = 1.$$

Substituting in the relevant expressions and simplifying, we can derive

$$\underline{z}_0 = \frac{\sigma_x^2}{2}(1 - \alpha_0) - \frac{\alpha(p_0)}{\alpha_{t|0}} \widetilde{z}_0.$$

Hence if $p_t = p_0$, the optimal p_{t+k}^* is equal to p_0 as long as the innovation in the new signal is good enough—namely, $\widehat{z}_t \geq \underline{z}_0$.

If $p_t \neq p_0$, p_0 remains the optimal price at $t+k$ unless the new signal y_t is *good enough* to convince the firm to deviate from its ex ante optimum p_0 and move to the newly observed p_t itself. In the limiting case $\psi \rightarrow \infty$, we know that the only potential alternative is p_t , because y_t does not update beliefs anywhere else, and hence p_0 dominates all other prices. In particular, for every possible p_t , there is an upper threshold for the innovation in y_t , such that $p_{t+k}^* = p_t$ if and only if $\widehat{z}_t > \bar{z}(p_t)$. This threshold $\bar{z}(p_t)$ satisfies

$$\frac{E(v_{t+k}^*(p_t, c_0^*) | \varepsilon^0, p_t \neq p_0, \widehat{z}_t = \bar{z}(p_t))}{E(v_{t+k}^*(p_0, c_0^*) | \varepsilon^0, p_t \neq p_0, \widehat{z}_t = \bar{z}(p_t))} = 1.$$

Substituting in the respective expressions, and simplifying, we can derive

$$\bar{z}(p_t) = \frac{\alpha_0}{\alpha_t} \widetilde{z}_0 + \frac{\sigma_x^2}{2} - \frac{1}{\alpha_t} \left[\ln \left(\frac{\exp(p_t) - \exp(c_0^*)}{\exp(p_0) - \exp(c_0^*)} \right) + b(p_0 - p_t) \right].$$

With the two thresholds thusly characterized, we can conclude that the optimal pricing policy at time $t + k$ is given by

$$p_{t+k}^* = \begin{cases} p_0 & \text{if } p_t = p_0 \text{ and } \widehat{z}_t \geq \underline{z}_0 \text{ or } p_t \neq p_0 \text{ and } \widehat{z}(p_t) \leq \bar{z}(p_t), \\ p_t & \text{if } p_t \neq p_0 \text{ and } \widehat{z}_t > \bar{z}(p_t), \\ p_{t+k}^{\text{int}} & \text{if } p_t = p_0 \text{ and } \widehat{z}_t < \underline{z}_0. \end{cases}$$

We can then evaluate the expected continuation value $E[\tilde{V}(\{\varepsilon^0, p_t, y_t\}, c_0^*) | \varepsilon^0, p_t]$ —we do so separately for the cases $p_t = p_0$ and $p_t \neq p_0$, since the expected continuation value (which we will denote by the short-hand $E_{t-1}(\tilde{V})$ to save space) is potentially discontinuous at $p_t = p_0$, so that

$$\begin{aligned} & E_{t-1}(\tilde{V} | p_t = p_0) \\ &= \Phi\left(\frac{\underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right) (\exp(p_0) - \exp(c_0^*)) \exp\left(-\gamma - bp_0 + \frac{1}{2}(\sigma_x^2 + \sigma_z^2)\right) \\ &+ \left(1 - \Phi\left(\frac{\underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)\right) (\exp(p_0) - \exp(c_0^*)) \\ &\times \exp\left(-\gamma - bp_0 + \alpha_0 \widehat{z}_0 + \frac{1}{2}(\sigma_x^2(1-\alpha_0) + \sigma_z^2)\right) \frac{\Phi\left(\frac{\alpha_{t|0}(\sigma_x^2(1-\alpha_0) + \sigma_z^2) - \underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)}{1 - \Phi\left(\frac{\underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)} \\ &= (\exp(p_0) - \exp(c_0^*)) \exp\left(-\gamma - bp_0 \frac{1}{2}(\sigma_x^2 + \sigma_z^2)\right) \\ &\times \left(\Phi\left(\frac{\alpha_{t|0}(\sigma_x^2(1-\alpha_0) + \sigma_z^2) - \underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right) \exp(\alpha_0 \widehat{z}_0) + \Phi\left(\frac{\underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)\right), \end{aligned}$$

while

$$\begin{aligned} & E_{t-1}(\tilde{V} | p_t \neq p_0) \\ &= P(\widehat{z}_t < \bar{z}(p_t)) (\exp(p_0) - \exp(c_0^*)) \exp\left(-\gamma - bp_0 + \alpha_0 \widehat{z}_0 + \frac{1}{2}(\sigma_x^2(1-\alpha_0) + \sigma_z^2)\right) \\ &+ P(\widehat{z}_t \geq \bar{z}(p_t)) (\exp(p_t) - \exp(c_0^*)) \exp\left(-\gamma - bp_t + \frac{1}{2}(\sigma_x^2(1-\alpha_t) + \sigma_z^2)\right) \\ &\times E(\exp(\alpha_t \widehat{z}_t) | \widehat{z}_t > \bar{z}(p_t)) \\ &= \Phi\left(\frac{\bar{z}(p_t)}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right) (\exp(p_0) - \exp(c_0^*)) \exp\left(-\gamma - bp_0 + \alpha_0 \widehat{z}_0 + \frac{1}{2}(\sigma_x^2(1-\alpha_0) + \sigma_z^2)\right) \end{aligned}$$

$$+ \Phi\left(\frac{\alpha_t(\sigma_x^2 + \sigma_z^2) - \bar{z}(p_t)}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right) \left(\exp(p_t) - \exp(c_0^*)\right) \exp\left(-\gamma - bp_t + \frac{1}{2}(\sigma_x^2 + \sigma_z^2)\right),$$

where we use the fact that the firm perceives $\hat{z}_t \sim N(0, \hat{\sigma}_{t-1}^2(p_t) + \sigma_z^2)$, and $\Phi(\cdot)$ denotes the CDF of the standard normal distribution.

The first question of interest is if and when the expected continuation value is discontinuous at $p_t = p_0$. To answer this question, we evaluate the ratio $\frac{E_{t-1}(\tilde{V}|p_t=p_0)}{\lim_{p_t \rightarrow p_0} E_{t-1}(\tilde{V}|p_t \neq p_0)}$. It is useful to first evaluate the denominator and collect terms, concluding that

$$\begin{aligned} & \lim_{p_t \rightarrow p_0} E_{t-1}(\tilde{V}|p_t \neq p_0) \\ &= (\exp(p_0) - \exp(c_0^*)) \exp\left(-\gamma - bp_0 + \frac{1}{2}(\sigma_x^2 + \sigma_z^2)\right) \\ & \quad \times \left(\Phi\left(\frac{\bar{z}(p_t)}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right) \exp(\alpha_0 \tilde{z}_0) + \Phi\left(\frac{\alpha_t(\sigma_x^2 + \sigma_z^2) - \bar{z}(p_t)}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right)\right). \end{aligned}$$

It then follows that the ratio

$$\begin{aligned} & \frac{E_{t-1}(\tilde{V}|p_t = p_0)}{\lim_{p_t \rightarrow p_0} E_{t-1}(\tilde{V}|p_t \neq p_0)} \\ &= \frac{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) + \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right) \exp(\alpha_0 \tilde{z}_0) + \Phi\left(\frac{(1 - \alpha_0)\frac{\sigma_x^2}{2} - \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right)}{\Phi\left(\frac{\frac{\alpha_0}{\alpha_t} \tilde{z}_0 + \frac{\sigma_x^2}{2}}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right) \exp(\alpha_0 \tilde{z}_0) + \Phi\left(\frac{\frac{\sigma_x^2}{2} - \frac{\alpha_0}{\alpha_t} \tilde{z}_0}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right)}, \end{aligned}$$

where we have substituted in the respective values of the thresholds \tilde{z}_0 and $\bar{z}(p_t)$. The ratio limits to 1 as $\tilde{z}_0 \rightarrow \infty$, and it is below 1 at $\tilde{z}_0 = 0$, as in this case

$$\frac{E_{t-1}(\tilde{V}|p_t = p_0)}{\lim_{p_t \rightarrow p_0} E_{t-1}(\tilde{V}|p_t \neq p_0)} = \frac{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0)}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right)}{\Phi\left(\frac{\frac{\sigma_x^2}{2}}{2\sqrt{\sigma_x^2 + \sigma_z^2}}\right)} < 1.$$

Next, we show that the derivative of the ratio in respect to \tilde{z}_0 is positive for the relevant values $\tilde{z}_0 \geq 0$, which is enough to conclude that $\frac{E_{t-1}(\tilde{V}|p_t=p_0)}{\lim_{p_t \rightarrow p_0} E_{t-1}(\tilde{V}|p_t \neq p_0)}$ converges to 1 from

below and hence is less than 1 for all finite $\tilde{z}_0 \geq 0$. The needed derivative,

$$\frac{\partial \frac{E_{t-1}(\tilde{V}|p_t = p_0)}{\lim_{p_t \rightarrow p_0} E(\tilde{V}|p_t \neq p_0)}}{\partial \tilde{z}_0},$$

is proportional to

$$\begin{aligned} & \underbrace{\left(\left(\phi \left(\frac{\frac{\sigma_x^2}{2}(1-\alpha_0) + \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \exp(\alpha_0 \tilde{z}_0) - \phi \left(\frac{(1-\alpha_0) \frac{\sigma_x^2}{2} - \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \right)}_{=0} \\ & \times \frac{\alpha_0}{\alpha_{t|0} \sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} + \Phi \left(\frac{\frac{\sigma_x^2}{2}(1-\alpha_0) + \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \exp(\alpha_0 \tilde{z}_0) \alpha_0 \\ & * \left(\Phi \left(\frac{\frac{\alpha_0}{\alpha_t} \tilde{z}_0 + \frac{\sigma_x^2}{2}}{\sqrt{(\sigma_x^2 + \sigma_z^2)}} \right) \exp(\alpha_0 \tilde{z}_0) + \Phi \left(\frac{\frac{\sigma_x^2}{2} - \frac{\alpha_0}{\alpha_t} \tilde{z}_0}{\sqrt{(\sigma_x^2 + \sigma_z^2)}} \right) \right) \\ & - \left(\Phi \left(\frac{\frac{\sigma_x^2}{2}(1-\alpha_0) + \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \exp(\alpha_0 \tilde{z}_0) + \Phi \left(\frac{(1-\alpha_0) \frac{\sigma_x^2}{2} - \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \right) \\ & * \underbrace{\left(\left(\phi \left(\frac{\frac{\sigma_x^2}{2} + \frac{\alpha_0}{\alpha_t} \tilde{z}_0}{\sqrt{\sigma_x^2 + \sigma_z^2}} \right) \exp(\alpha_0 \tilde{z}_0) - \phi \left(\frac{\frac{\sigma_x^2}{2} - \frac{\alpha_0}{\alpha_t} \tilde{z}_0}{\sqrt{\sigma_x^2 + \sigma_z^2}} \right) \right) \frac{\alpha_0}{\alpha_1 \sqrt{\sigma_x^2 + \sigma_z^2}} \right)}_{=0} \\ & + \Phi \left(\frac{\frac{\sigma_x^2}{2} + \frac{\alpha_0}{\alpha_1} \tilde{z}_0}{\sqrt{\sigma_x^2 + \sigma_z^2}} \right) \exp(\alpha_0 \tilde{z}_0) \alpha_0 \\ & = \alpha_0 \exp(\alpha_0 \tilde{z}_0) \left[\Phi \left(\frac{\frac{\sigma_x^2}{2}(1-\alpha_0) + \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \Phi \left(\frac{\frac{\sigma_x^2}{2} - \frac{\alpha_0}{\alpha_t} \tilde{z}_0}{\sqrt{\sigma_x^2 + \sigma_z^2}} \right) \right. \\ & \left. - \Phi \left(\frac{\frac{\sigma_x^2}{2}(1-\alpha_0) - \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \Phi \left(\frac{\frac{\sigma_x^2}{2} + \frac{\alpha_0}{\alpha_t} \tilde{z}_0}{\sqrt{\sigma_x^2 + \sigma_z^2}} \right) \right]. \end{aligned}$$

Thus, the derivative is positive if and only if

$$\frac{\Phi\left(\frac{\frac{\sigma_x^2}{2} - \frac{\alpha_0 \tilde{z}_0}{\alpha_t}}{\sqrt{\sigma_x^2 + \sigma_z^2}}\right)}{\Phi\left(\frac{\frac{\sigma_x^2}{2} + \frac{\alpha_0 \tilde{z}_0}{\alpha_t}}{\sqrt{\sigma_x^2 + \sigma_z^2}}\right)} > \frac{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) - \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right)}{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) + \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right)}.$$

This inequality holds since

$$\frac{\Phi\left(\frac{\frac{\sigma_x^2}{2} - \frac{\alpha_0 \tilde{z}_0}{\alpha_t}}{\sqrt{\sigma_x^2 + \sigma_z^2}}\right)}{\Phi\left(\frac{\frac{\sigma_x^2}{2} + \frac{\alpha_0 \tilde{z}_0}{\alpha_t}}{\sqrt{\sigma_x^2 + \sigma_z^2}}\right)} > \frac{\Phi\left(\frac{\frac{\sigma_x^2}{2} - \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2 + \sigma_z^2}}\right)}{\Phi\left(\frac{\frac{\sigma_x^2}{2} + \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2 + \sigma_z^2}}\right)} > \frac{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) - \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right)}{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) + \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right)},$$

where the first inequality follows from $\alpha_{t|0} < \alpha_t$, and the second from the fact that

$$\frac{\frac{\sigma_x^2}{2}(1 - \tilde{\alpha}_0) - \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \tilde{\alpha}_0) + \sigma_z^2}} \frac{\partial}{\partial \tilde{\alpha}_0} < \frac{\frac{\sigma_x^2}{2}(1 - \tilde{\alpha}_0) + \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \tilde{\alpha}_0) + \sigma_z^2}} \frac{\partial}{\partial \tilde{\alpha}_0}$$

and the fact that the term

$$\frac{\partial}{\partial \tilde{\alpha}_0} \left(\frac{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \tilde{\alpha}_0) - \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \tilde{\alpha}_0) + \sigma_z^2}}\right)}{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \tilde{\alpha}_0) + \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \tilde{\alpha}_0) + \sigma_z^2}}\right)} \right)$$

equals

$$\phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) - \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right) \phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) + \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right) \frac{\partial}{\partial \tilde{\alpha}_0} \frac{\frac{\sigma_x^2}{2}(1 - \tilde{\alpha}_0) - \frac{\alpha_0 \tilde{z}_0}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \tilde{\alpha}_0) + \sigma_z^2}}$$

$$\begin{aligned}
& - \Phi \left(\frac{\frac{\sigma_x^2}{2}(1-\alpha_0) - \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \phi \left(\frac{\frac{\sigma_x^2}{2}(1-\alpha_0) + \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}} \right) \frac{\partial \frac{\frac{\sigma_x^2}{2}(1-\tilde{\alpha}_0) + \frac{\alpha_0}{\alpha_{t|0}} \tilde{z}_0}{\sqrt{\sigma_x^2(1-\tilde{\alpha}_0) + \sigma_z^2}}}{\partial \tilde{\alpha}_0} \\
& < 0.
\end{aligned}$$

Thus, we can conclude that

$$\frac{E_{t-1}(\tilde{V}|p_t = p_0)}{\lim_{p_t \rightarrow p_0} E_{t-1}(\tilde{V}|p_t \neq p_0)} < 1$$

for all $\tilde{z}_0 \geq 0$, meaning that there is discontinuous jump down in the continuation value at $p_t = p_0$.

Last, consider what value of p_t optimizes the expected continuation value. Since the discontinuity at p_0 (the only potential corner solution) is a jump down, the maximizing p_t must be the interior maximum, which satisfies the FOC condition that $\frac{\partial E_{t-1}(\tilde{V}|p_t \neq p_0)}{\partial p_t} = 0$. Taking the derivative,

$$\begin{aligned}
& \frac{\partial E_{t-1}(\tilde{V}|p_t \neq p_0)}{\partial p_t} \\
& = \phi \left(\frac{\bar{z}(p_t)}{\sqrt{\sigma_x^2 + \sigma_z^2}} \right) (e^{p_0} - e^{c_0^*}) \exp \left(-\gamma - bp_0 + \alpha_0 \hat{z}_0 + \frac{1}{2}(\sigma_x^2(1-\alpha_0) + \sigma_z^2) \right) \frac{\frac{\partial \bar{z}(p_t)}{\partial p_t}}{\sqrt{\sigma_x^2 + \sigma_z^2}} \\
& \quad - \phi \left(\frac{\alpha_t(\sigma_x^2 + \sigma_z^2) - \bar{z}(p_t)}{\sqrt{(\sigma_x^2 + \sigma_z^2)}} \right) (e^{p_t} - e^{c_0^*}) \\
& \quad \times \exp \left(-\gamma - bp_t + \frac{1}{2}(\sigma_x^2(1-\alpha_t) + \sigma_z^2 + \alpha_t^2(\sigma_x^2 + \sigma_z^2)) \right) \frac{\frac{\partial \bar{z}(p_t)}{\partial p_t}}{\sqrt{\sigma_x^2 + \sigma_z^2}} \\
& \quad + \Phi \left(\frac{\alpha_t(\sigma_x^2 + \sigma_z^2) - \bar{z}(p_t)}{\sqrt{(\sigma_x^2 + \sigma_z^2)}} \right) \\
& \quad \times \exp \left(-\gamma - bp_t + \frac{1}{2}(\sigma_x^2(1-\alpha_t) + \sigma_z^2 + \alpha_t^2(\sigma_x^2 + \sigma_z^2)) \right) (e^{p_t} - b(e^{p_t} - e^{c_0^*})).
\end{aligned}$$

The above expression limits to zero as $p_t \rightarrow p_0$. To see that, note that $\lim_{p_t \rightarrow p_0} \frac{\partial \bar{z}(p_t)}{\partial p_t} = 0$, thus the first two terms of the FOC expression above fall out. For the last term, using $p_0 = \ln(\frac{b}{b-1}) + c_0$, it follows that

$$(e^{p_0} - b(e^{p_0} - e^{c_0^*})) = \frac{b}{b-1} e^{c_0^*} - \frac{b}{b-1} e^{c_0^*} = 0.$$

Therefore, we can conclude that $\lim_{p_t \rightarrow p_0} \frac{\partial E_{t-1}(\tilde{V} | p_t \neq p_0)}{\partial p_t} = 0$, and thus the interior maximum of the expected continuation value is $p_t \rightarrow p_0$. *Q.E.D.*

Intuitively, $p_t^* = \arg \min_p (p - p_0)^2$ s.t. $p \neq p_0$ ensures that the new signal y_t will be informative about a price as close as possible to the ex ante expected optimal p_0 , and thus achieves almost the same markup; this makes the new information highly relevant. As a result, if the realization of \hat{z}_t happens to be good enough, that is, \hat{z}_t is above a threshold $\bar{z}_t(p_t^*)$ that is characterized in the proof above, then the firm will stick with this price in the future, set $p_{t+k} = p_t^*$, and take advantage of the unexpectedly high demand at that price. On the other hand, if the signal realization happens to be bad, the firm can safely switch back to the ex ante optimal p_0 , where the belief about demand is not affected by \hat{z}_t , and still offers lower uncertainty and a good perceived markup.

The reason for not picking $p_t = p_0$ is that a bad signal realization at p_0 erodes the ex ante best available pricing option, p_0 , and at the same time the firm does not have a good fall-back alternative, as it has no observations of demand at other prices. If in that case the realization of \hat{z}_t falls below the threshold \underline{z}_0 , the news about $x(p_0)$ is bad enough to incentivize the firm to set p_{t+k} to a previously unvisited price. Due to this downside risk at p_0 , there is a first-order gain of obtaining information at a new price, which manifests in the discontinuous jump-down in the expected continuation value at p_0 .

As shown in Proposition 3, the best forward-looking strategy is therefore to experiment by posting a new price. This exploration incentive could potentially overturn the rigidity result implied by the static maximization pricing choice analyzed earlier, but as we show next, it turns out that this result is specific to the firm having seen only one price in the past. In more general situations, when the firm has seen more than one distinct price point in the past, forward-looking behavior can in fact *reinforce* the static rigidity incentives.

Exploration makes prices stickier, when ε^t contains observations at multiple prices

PROPOSITION 4: *There is a non-singleton interval of costs (\underline{c}, \bar{c}) around c_0^* , and a threshold $\chi > 0$, such that if $\hat{z} > \chi$, then for any $c \in (\underline{c}, \bar{c})$:*

$$p_0 = \arg \max_{p_t} E[\tilde{V}(\{\varepsilon^1, p_t, y_t\}, c) | \varepsilon^1, p_t].$$

Moreover, the threshold χ is decreasing in $|p_1 - p_0|$.

PROOF: The proof follows a similar logic as the previous one. First, we characterize the optimal p_{t+k} for $c = c_0^*$, but now conditional on ε^1 , and then use it to compute the expected continuation value and show that it is maximized at $p_t = p_0$. Last, we appeal to continuity to conclude that $p_t = p_0$ is optimal for an interval of cost values around c_0^* . In addition to the signal-to-noise ratio notation $\alpha_0, \alpha_{t|0}, \alpha_t$ defined in the previous proof, we define

$$\alpha_1 \equiv \alpha_{t-1}(p_1; p_1) = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_z^2/N_1},$$

$$\alpha_{t|1} \equiv \alpha_t(p_1; p_1 | p_t = p_1) = \frac{\sigma_x^2}{\sigma_x^2(N_1 + 1) + \sigma_z^2}.$$

Similarly, we define the (variance-corrected) innovation in the signal at p_1 as

$$\tilde{z}_1 \equiv \hat{z}_1 - \frac{1}{2}\sigma_x^2 = y_1 - (-\gamma - bp_1) - \frac{1}{2}\sigma_x^2.$$

The optimal policy at $t + k$ follows a similar structure to the one described in the previous proof. Conditional on just ε^1 , the optimal p_{t+k} is equal to p_0 , and the way the new information contained in y_t affects the optimal p_{t+k} depends on the position of p_t . If $p_t = p_0$, then the firm stays at p_0 unless the new signal is too bad ($\widehat{z}_t < \underline{z}_0$). If $p_t = p_1$, then the firm *moves* to p_1 if the signal is good enough ($\widehat{z}_t > \bar{z}_1$), otherwise stays at p_0 . And if $p_t \notin \{p_0, p_1\}$, then the firm again stays at p_0 unless the signal is too good, but compared to a different threshold: $\widehat{z}_t > \bar{z}(p_t)$. The key difference from the previous proof is what happens if $p_t = p_0$ and the signal is sufficiently bad to prompt a move ($\widehat{z}_t < \underline{z}_0$). There exists a $\chi_1 > 0$ such that if $\widehat{z}_1 > \chi_1$, then the firm does not move to the interior optimum p^{int} , but rather to p_1 , which, as another relatively good price at which the firm has built some information capital, is a better option than the brand new p^{int} where the firm has not accumulated any information. To see this, note that

$$\frac{E(v_{t+k}^*(p_1, c_0^*) | \varepsilon^1, p_t = p_0)}{\lim_{p \rightarrow p_0} E(v_{t+k}^*(p, c_0^*) | \varepsilon^1, p_t = p_0)} = (b \exp(p_1 - p_0) - b + 1) \exp(-b(p_1 - p_0) + \alpha_1 \widetilde{z}_1) > 1.$$

Note that the RHS is increasing in \widetilde{z}_1 , and thus in \widehat{z}_1 and limits to infinity as $\widehat{z}_1 \rightarrow \infty$, hence there exists a constant $\chi_1 > 0$ such that the above ratio is strictly greater than 1 when $\widehat{z} > \chi_1$. For the rest of the proof, we assume that $\widehat{z}_1 > \chi_1$ so that the above inequality holds. The relevant thresholds $\underline{z}_0, \bar{z}_1, \bar{z}(p_t)$ can be computed as before, by finding the value of the signal at which the firm is indifferent between p_0 and the respective alternative option:

$$\begin{aligned} \underline{z}_0 &= \frac{\sigma_x^2}{2} (1 - \alpha_0) - \frac{1}{\alpha_{t|0}} (b(p_1 - p_0) - \ln(b e^{(p_1 - p_0)} - b + 1)), \\ \bar{z}_1 &= \frac{\sigma_x^2}{2} (1 - \alpha_1) + \frac{1}{\alpha_{t|1}} (b(p_1 - p_0) - \ln(b e^{(p_1 - p_0)} - b + 1)), \\ \bar{z}(p_t) &= \frac{\alpha_0}{\alpha_t} \widetilde{z}_0 + \frac{\sigma_x^2}{2} - \frac{1}{\alpha_t} \left[\ln \left(\frac{\exp(p_t) - \exp(c_0^*)}{\exp(p_0) - \exp(c_0^*)} \right) + b(p_0 - p_t) \right]. \end{aligned}$$

So the $t + k$ optimal pricing policy is

$$p_{t+1}^* = \begin{cases} p_0 & \text{if } p_t = p_0 \text{ and } \widehat{z}_t \geq \underline{z}_0, \text{ or } p_t = p_1 \text{ and } \widehat{z}_t \leq \bar{z}_1 \text{ or } p_t \notin \{p_0, p_1\} \text{ and } \widehat{z}_t \leq \bar{z}(p_t), \\ p_1 & \text{if } p_t = p_1 \text{ and } \widehat{z}_t > \bar{z}_1 \text{ or } p_t = p_0 \text{ and } \widehat{z}_t < \underline{z}_0, \\ p_t & \text{if } p_t \notin \{p_0, p_1\} \text{ and } \widehat{z}_t > \bar{z}(p_t). \end{cases}$$

We can now use this result to characterize the expected continuation value and find its maximizer. Note that the value of p_t that maximizes $E([\tilde{V}(\{\varepsilon^1, p_t, y_t\}, c_0^*) | \varepsilon^1, p_t])$ is either one of the two corner solutions p_0 and p_1 , or the interior maximum. Moreover, we can appeal to the proof of Proposition 3 for the result that the expected continuation value achieves its interior maximum at the limit of $p_t \rightarrow p_0$. This follows because, under $\psi \rightarrow \infty$, the additional signal y_1 only matters when updating beliefs at p_1 itself; hence at $p \neq p_1$, the expected continuation value is equivalent to the one conditional on ε^0 , that we analyzed above. We proceed in two steps. First we show that the two corner solutions are in fact equivalent to each other, and then we conclude by showing that p_0 also dominates the interior solution p^{int} . The expected value $E([\tilde{V}(\{\varepsilon^1, p_t, y_t\}, c_0^*) | \varepsilon^1, p_t = p_0])$ is slightly different than before, because the fall-back option (in case of a bad new signal y_t)

is now p_1 . Now,

$$\begin{aligned}
& E_{t-1}(\tilde{V}|p_t = p_0) \\
&= \Phi\left(\frac{\underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)(\exp(p_1) - \exp(c_0^*)) \\
&\quad \times \exp\left(-\gamma - bp_1 + \alpha_1\hat{z}_1 + \frac{1}{2}(\sigma_x^2(1-\alpha_1) + \sigma_z^2)\right) \\
&\quad + \left(1 - \Phi\left(\frac{\underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)\right)(\exp(p_0) - \exp(c_0^*)) \\
&\quad \times \exp\left(-\gamma - bp_0 + \alpha_0\hat{z}_0 + \frac{1}{2}(\sigma_x^2(1-\alpha_0) + \sigma_z^2)\right) \frac{\Phi\left(\frac{\alpha_{t|0}(\sigma_x^2(1-\alpha_0) + \sigma_z^2) - \underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)}{1 - \Phi\left(\frac{\underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)} \\
&= \frac{1}{b-1} \exp\left(c_0^* - \gamma - bp_0 + \alpha_0\tilde{z}_0 + \frac{1}{2}(\sigma_x^2 + \sigma_z^2)\right) \left(\Phi\left(\frac{\alpha_{t|0}(\sigma_x^2(1-\alpha_0) + \sigma_z^2) - \underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right)\right) \\
&\quad + \Phi\left(\frac{\underline{z}_0}{\sqrt{\sigma_x^2(1-\alpha_0) + \sigma_z^2}}\right) (be^{p_1-p_0} - b + 1)e^{-b(p_1-p_0)}.
\end{aligned}$$

Similarly, $E([\tilde{V}(\{\varepsilon^1, p_t, y_t\}, c_0^*|\varepsilon^1, p_t = p_1)])$ can be computed as

$$\begin{aligned}
& E_{t-1}(\tilde{V}|p_t = p_1) \\
&= P(\hat{z}_t \leq \bar{z}_1)(\exp(p_0) - \exp(c_0^*)) \exp\left(-\gamma - bp_0 + \alpha_0\hat{z}_0 + \frac{1}{2}(\sigma_x^2(1-\alpha_0) + \sigma_z^2)\right) \\
&\quad + P(\hat{z}_t > \bar{z}_1)(\exp(p_1) - \exp(c_0^*)) \\
&\quad \times \exp\left(-\gamma - bp_1 + \alpha_1\hat{z}_1 + \frac{1}{2}(\sigma_x^2(1-\alpha_1)(1-\alpha_{t|1}) + \sigma_z^2)\right) E(\exp(\alpha_{t|1}\hat{z}_t)|\hat{z}_t > \bar{z}_1) \\
&= \frac{1}{b-1} \exp\left(c_0^* - \gamma - bp_0 + \alpha_0\tilde{z}_0 + \frac{1}{2}(\sigma_x^2 + \sigma_z^2)\right) \left[\Phi\left(\frac{\bar{z}_1}{\sqrt{(\sigma_x^2(1-\alpha_1) + \sigma_z^2)}}\right)\right] \\
&\quad + \Phi\left(\frac{\alpha_{t|1}(\sigma_x^2(1-\alpha_1) + \sigma_z^2) - \bar{z}_1}{\sqrt{(\sigma_x^2(1-\alpha_1) + \sigma_z^2)}}\right) (be^{p_1-p_0} - b + 1)e^{-b(p_1-p_0)}.
\end{aligned}$$

Substituting in the expressions for \underline{z}_0 and \bar{z}_1 we obtain

$$E_{t-1}(\tilde{V}|p_t = p_0) = E_{t-1}(\tilde{V}|p_t = p_1).$$

Last, note that for $p_t \notin \{p_0, p_1\}$, $E([\tilde{V}(c_0, \{\varepsilon^{t-1}, p_t, y_t\} | \varepsilon^1, p_t)])$ is the same as computed in the proof of Proposition 3 above. As a result, the interior maximum is achieved at $\lim p_t \rightarrow p_0$; hence, to conclude our argument, we need to compare $E_{t-1}(\tilde{V} | p_t = p_0)$ against $\lim_{p_t \rightarrow p_0} E_{t-1}(\tilde{V} | p_t \notin \{p_0, p_1\})$, which in turn equals

$$\begin{aligned} & (\exp(p_0) - \exp(c_0^*)) \exp\left(-\gamma - bp_0 + \frac{1}{2}(\sigma_x^2 + \sigma_z^2)\right) \\ & \times \left(\Phi\left(\frac{\bar{z}(p_t)}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right) \exp(\alpha_0 \tilde{z}_0) + \Phi\left(\frac{\alpha_t(\sigma_x^2 + \sigma_z^2) - \bar{z}(p_t)}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right) \right). \end{aligned}$$

Let $\hat{\theta} = (b(p_1 - p_0) - \ln(be^{(p_1 - p_0)} - b + 1)) > 0$; then, after substituting the expressions for \underline{z}_0 and $\bar{z}(p_t)$ and simplifying, the ratio of the two expected continuation values simplifies to

$$\begin{aligned} & \frac{E_{t-1}(\tilde{V} | p_t = p_0)}{\lim_{p_t \rightarrow p_0} E_{t-1}(\tilde{V} | p_t \notin \{p_0, p_1\})} \\ & = \frac{\Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) + \frac{\hat{\theta}}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right) + \Phi\left(\frac{\frac{\sigma_x^2}{2}(1 - \alpha_0) - \frac{\hat{\theta}}{\alpha_{t|0}}}{\sqrt{\sigma_x^2(1 - \alpha_0) + \sigma_z^2}}\right) \exp(-\hat{\theta})}{\Phi\left(\frac{\frac{\alpha_0 \tilde{z}_0 + \frac{\sigma_x^2}{2}}{\alpha_t}}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right) + \Phi\left(\frac{\frac{\sigma_x^2}{2} - \frac{\alpha_0 \tilde{z}_0}{\alpha_t}}{\sqrt{(\sigma_x^2 + \sigma_z^2)}}\right) \exp(-\alpha_0 \tilde{z}_0)}. \end{aligned} \quad (37)$$

The denominator is decreasing in \tilde{z}_0 and thus also in \hat{z}_0 ; hence, for every $\hat{\theta}$ there is a \hat{z}_0 big enough such that the above ratio is strictly greater than 0. As a result, there exists a finite constant $\chi_0 > 0$ such that when $\hat{z}_0 > \chi_0$, it follows that $p_t = p_0$ maximizes the expected continuation value. Finally, let $\chi = \max\{\chi_0, \chi_1\}$; then if $\hat{z}_1 = \hat{z}_0 > \chi$,

$$p_0 = \arg \max_{p_t} E[\tilde{V}(\{\varepsilon^1, p_t, y_t\}, c_0^*) | \varepsilon^1, p_t].$$

Since \tilde{V} is continuous in the cost shock c , it follows that there exists a non-singleton interval (\underline{c}, \bar{c}) around c_0^* , such that if $c \in (\underline{c}, \bar{c})$, then

$$p_0 = \arg \max_{p_t} E[\tilde{V}(\{\varepsilon^1, p_t, y_t\}, c) | \varepsilon^1, p_t].$$

Last, we want to show that $\frac{\partial \chi}{\partial |p_0 - p_1|} < 0$. This follows directly from the facts that (i) the numerator of (37) is decreasing in $\hat{\theta}$, and that (ii) $\hat{\theta}$ is increasing in $(p_1 - p_0)$. Hence, as we decrease the distance between p_0 and p_1 , we increase the RHS of (37), and thus we require a smaller $\hat{z} = \hat{z}_0 = \hat{z}$ to make the ratio bigger than 1. *Q.E.D.*

A.4. Household Problem

The representative household consumes and works according to

$$\max_{c_{t+k}, L_{i,t+k}} \sum_{k=0}^{\infty} E_t \left(\beta^{t+k} \left[c_{t+k} - \int L_{i,t+k} di \right] \right),$$

where c_t denotes log consumption of the aggregate good, subject to the budget constraint

$$\int e^{p_{j,t} + c_{j,t}} dj + E_t Q_{t+1} D_{t+1} = D_t + e^{p_t + w_t} \int L_{i,t} di + \int v_{i,t} di,$$

where Q_{t+1} is the stochastic discount factor, D_t are state-contingent claims on the aggregate shocks, $v_{i,t}$ is the profit from the monopolistic intermediaries, and w_t is the log real wage. The optimal labor supply condition is simply $w_t = c_t$, while the market clearing states that $c_t = y_t$. Substituting the wage into the firm's profit, we obtain equation (21).

A.5. Proofs on Learning and Nominal Rigidity

PROPOSITION A1: *The nominal price $p_{i,1} = \tilde{p}_{j,1} + \tilde{r}_{i,0}$ is a local maximizer of the worst-case expected profits for any aggregate price $p_1 \in (\underline{p}_1 + \ln(\frac{b}{b-1} \frac{b-\alpha\delta-1}{b-\alpha\delta}), \bar{p}_1 + \ln(\frac{b}{b-1} \frac{b+\alpha\delta-1}{b+\alpha\delta}))$.*

PROOF: Let $v^*(\varepsilon^0, s_1, p_{i,1})$ denote the worst-case expected profit, conditional on the history ε^0 and the current state $s_1 = \{\omega_{i,1}, p_1, y_1, \tilde{p}_{j,1}\}$, evaluated at some nominal price $p_{i,1}$. Conditional on $p_{i,1} - \tilde{p}_{j,1}$, the worst-case beliefs are given by equations (25) and (26). Take a first-order approximation of the change in profits, $v^*(\varepsilon^0, s_1, p_{i,1}) - v^*(\varepsilon^0, s_1, \tilde{p}_{j,1} + \tilde{r}_{i,0})$, evaluated around $p_{i,1} = \tilde{p}_{j,1} + \tilde{r}_{i,0}$. This equals

$$\left[\frac{e^{\tilde{p}_{j,1} + \tilde{r}_{i,0} - p_1}}{e^{\tilde{p}_{j,1} + \tilde{r}_{i,0} - p_1} - e^{y_1 - \omega_{i,1}}} - (b + \alpha\delta^*) \right] (p_{i,1} - \tilde{p}_{j,1} - \tilde{r}_{i,0}),$$

where $\delta^* = \delta \operatorname{sgn}(p_{i,1} - \tilde{p}_{j,1} - \tilde{r}_{i,0})$.

It then follows that for any $p_1 \in (\underline{p}, \bar{p})$, where we define

$$\underline{p} = \bar{p}_1 + \ln\left(\frac{b}{b-1} \frac{b-\alpha\delta-1}{b-\alpha\delta}\right); \quad \bar{p} = \bar{p}_1 + \ln\left(\frac{b}{b-1} \frac{b+\alpha\delta-1}{b+\alpha\delta}\right),$$

we have

$$\frac{e^{\tilde{p}_{j,1} + \tilde{r}_{i,0} - p_1}}{e^{\tilde{p}_{j,1} + \tilde{r}_{i,0} - p_1} - e^{y_1 - \omega_{i,1}}} \in (b - \alpha\delta, b + \alpha\delta),$$

which makes the first-order derivative of the change in profits negative to the right of $\tilde{p}_{j,1} + \tilde{r}_{i,0}$ and positive to its left. This gives the necessary and sufficient conditions for $\tilde{p}_{j,1} + \tilde{r}_{i,0}$ to be a local maximizer. *Q.E.D.*

PROPOSITION A2: *Let $\delta^{\text{index}} = \delta \operatorname{sgn}(p_1 - \tilde{p}_{j,1})$. Up to a first-order approximation around $p_1 = \tilde{p}_{j,1}$, the difference $\ln v^*(\varepsilon^0, s_1, \tilde{r}_{i,0} + p_1) - \ln v^*(\varepsilon^0, s_1, \tilde{r}_{i,0} + \tilde{p}_{j,1})$ equals*

$$\left[\frac{e^{\tilde{r}_{i,0}}}{e^{\tilde{r}_{i,0}} - e^{y_1 - \omega_{i,1}}} - b - \alpha\delta^{\text{index}} \right] (p_1 - \tilde{p}_{j,1}) < 0.$$

PROOF: First, analyze the worst-case expected profit under a policy rule that implements indexation, that is, $p_{i,1}^{\text{index}} = \tilde{r}_{i,0} + p_1$, given by

$$v^*(\varepsilon^0, s_1, p_{i,1}^{\text{index}}) = (e^{\tilde{r}_{i,0}} - e^{y_1 - \omega_{i,1}}) e^{\hat{x}_0^*(p_{i,1}^{\text{index}}, y_1, p_1, \tilde{p}_{j,1})},$$

where $\hat{x}_0^*(p_{i,1}^{\text{index}}, y_1, p_1, \tilde{p}_{j,1})$ equals $0.5(\hat{\sigma}_0^2 + \sigma_z^2) + c_t - b\tilde{r}_{i,0} - \gamma + \alpha[y_0 - (-\gamma - b\tilde{r}_{i,0})]$ plus

$$\min_{\delta' \in [-\delta, \delta]} \min_{\phi(p_t - \tilde{p}_{j,t}) \in [-\gamma_p, \gamma_p]} -\alpha\delta'(p_1 - \tilde{p}_{j,1}) + \alpha\delta'[\phi(p_1 - \tilde{p}_{j,1}) - \phi(p_0 - \tilde{p}_{j,0})].$$

The joint worst-case demand shape and co-integrating relationship are given by

$$\delta^{\text{index}} = \delta \operatorname{sgn}(p_1 - \tilde{p}_{j,1}); \quad \phi^{\text{index}}(p_1 - \tilde{p}_{j,1}) - \phi^{\text{index}}(p_0 - \tilde{p}_{j,0}) = -2\gamma_p \operatorname{sgn}(p_1 - \tilde{p}_{j,1}).$$

Given the presence of the kink, we compute a log-linear approximation of $v^*(\varepsilon^0, s_1, p_{i,1}^{\text{index}})$ around $p_1 = \tilde{p}_{j,1}$. At its right, we have

$$\frac{d \ln v^*(\varepsilon^0, s_1, p_{i,1}^{\text{index}})}{dp_1} = -\alpha\delta,$$

while at its left, the derivative is

$$\frac{d \ln v^*(\varepsilon^0, s_1, p_{i,1}^{\text{index}})}{dp_1} = \alpha\delta.$$

The constant term in the approximation is given by evaluating $\ln v^*(\varepsilon^0, s_1, p_{i,1}^{\text{index}})$ at $p_1 = \tilde{p}_{j,1}$:

$$\ln(e^{\tilde{r}_{i,1}} - e^{y_1 - \omega_{i,1}}) + c_t - b\tilde{r}_{i,0} - \gamma + \alpha[y_0 - (-\gamma - b\tilde{r}_{i,0})] - 2\alpha\delta\gamma_p.$$

Second, let us analyze the worst-case expected profit under the original policy, $p_{i,1}^* = \tilde{r}_{i,0} + \tilde{p}_{j,1}$, which targets the same $\tilde{r}_{i,0}$ but by adjusting the nominal price to the review signal $\tilde{p}_{j,1}$. We have

$$v^*(\varepsilon^0, s_1, p_{i,1}^*) = (e^{\tilde{r}_{i,0} + \tilde{p}_{j,1} - p_1} - e^{y_1 - \omega_{i,1}}) e^{\hat{x}_0^*(p_{i,1}^*, y_1, p_1, \tilde{p}_{j,1})},$$

where $\hat{x}_0^*(p_{i,1}^*, y_1, p_1, \tilde{p}_{j,1})$ equals $0.5(\hat{\sigma}_0^2 + \sigma_z^2) + c_t - b(\tilde{r}_{i,0} + \tilde{p}_{j,1} - p_1) - \gamma + \alpha[y_0 - (-\gamma - b\tilde{r}_{i,0})]$ plus

$$\min_{\delta' \in [-\delta, \delta]} \min_{\phi(p_t - \tilde{p}_{j,t}) \in [-\gamma_p, \gamma_p]} \alpha\delta'[\phi(p_1 - \tilde{p}_{j,1}) - \phi(p_0 - \tilde{p}_{j,0})] = -2\alpha\delta\gamma_p.$$

Note that $v^*(\varepsilon^0, s_1, p_{i,1}^*)$ does not have a kink in the p_1 space. Approximate around $p_1 = \tilde{p}_{j,1}$ to obtain a derivative:

$$\frac{d \ln v^*(\varepsilon^0, s_1, p_{i,1}^*)}{dp_1} = -\frac{e^{\tilde{r}_{i,0}}}{e^{\tilde{r}_{i,0}} - e^{my_1}} + b.$$

The constant term is given by evaluating $\ln v^*(\varepsilon^0, s_1, p_{i,1}^*)$ at $p_1 = \tilde{p}_{j,1}$, as:

$$\ln(e^{\tilde{r}_{i,0}} - e^{y_1 - \omega_{i,1}}) + c_t - b\tilde{r}_{i,0} - \gamma + \alpha[y_0 - (-\gamma - b\tilde{r}_{i,0})] - 2\alpha\delta\gamma_p.$$

We now compute the difference $\ln v^*(\varepsilon^0, s_1, p_{i,1}^{\text{index}}) - \ln v^*(\varepsilon^0, s_1, p_{i,1}^*)$, up to their first-order approximation:

$$\left(\frac{e^{\tilde{r}_{i,0}}}{e^{\tilde{r}_{i,0}} - e^{\gamma_1 - \omega_{i,1}}} - b - \alpha \delta^{\text{index}} \right) (p_1 - \tilde{p}_{j,1}) < 0$$

using the worst-case demand shape $\delta^{\text{index}} = \delta \text{sgn}(p_1 - \tilde{p}_{j,1})$ and Proposition A2. The latter shows that the condition for having the optimal price $\tilde{r}_{i,1}$ be at the kink $\tilde{r}_{i,0}$ is that the derivatives at the right, based on demand elasticity $-b - \delta$, and at the left, using the elasticity $-b + \delta$, are negative and, respectively, positive. *Q.E.D.*

A.6. Dispersion of Forecasts

Here, we detail how we use empirical evidence from Gaur, Kesavan, Raman, and Fisher (2007) on survey data to evaluate the size of our calibrated ambiguity parameter γ . Gaur, Kesavan, Raman, and Fisher (2007) used item-level forecasts of demand data from a ski-wear manufacturer, called the Sport Obermeyer data set. The data set contains style-color level forecasts for 248 short life-cycle items for a selling season of about three months. The forecasts are done by members of a committee specifically constituted to forecast demand, consisting of: the president, a vice president, two designers, and the managers of marketing, production, and customer service. Raman, Fisher, and McClelland (2001) provided details on the forecasting procedures and on the data set.

Our model connects to the data in Gaur, Kesavan, Raman, and Fisher (2007) as follows. They observed forecasts made prior to the product being introduced. Their statistic for the dispersion of these forecasts was reported as a coefficient of variation. Our model relates to this measure through the set of multiple priors. Indeed, in our model, prior to observing any realized demand signals, the firm entertains a set of forecasts about quantity sold. We connect this set to the dispersion of forecasts made by the committee described above. In particular, in our model, the firm entertains the following time-zero set of forecasts on the level of demand:

$$[\exp(-\gamma - bp + 0.5\sigma_z^2), \exp(\gamma - bp + 0.5\sigma_z^2)].$$

While in the data the set consists of only seven forecasters, we have a continuum. But we can compute the coefficient of variation (CV) of these forecasts and compare it against the reported statistic. In particular, using a uniform distribution over the forecasts in the set above, the CV, normalized by the average forecast, equals

$$\text{CV} = \frac{1}{\sqrt{3}} \frac{e^\gamma - e^{-\gamma}}{(e^\gamma + e^{-\gamma})}.$$

Gaur, Kesavan, Raman, and Fisher (2007) reported in their Table 4 that the average level of coefficient of variation, scaled by the average forecast, across the products in the data set equals 37.6%. Plugging in the calibrated value of our ambiguity parameter $\gamma = 0.614$, we obtain a CV equal to 31.58%.

A.7. Empirical Link Between Aggregate and Industry Prices

In this section, we use U.S. CPI data to show that the relationship between aggregate and industry prices is time-varying and unstable over short horizons. In particular, an

econometrician would generally have very little confidence that short-run aggregate inflation is related to industry-level inflation, even though he can be confident that the two are co-integrated in the long run. Thus, our assumption on the uncertainty over $\phi(\cdot)$ puts the firm on an equal footing with an econometrician outside of the model.

Our analysis uses the Bureau of Labor Statistics' most disaggregated 130 CPI indices as well as aggregate CPI inflation. The empirical exercise consists of the following regression method. For a specific industry j , we define its inflation rate between $t - k$ and t as $\pi_{j,t,k}$ and similarly $\pi_{t,k}^a$ for aggregate CPI inflation. For each industry j , we run the rolling regressions

$$\pi_{j,t,k} = \beta_{j,k,t} \pi_{t,k}^a + u_t$$

over 3-year windows starting in 1995 and ending in 2010, and note that results are very similar if we use windows of 2 or 5 years instead. We repeat this exercise for k equal to 1, 3, 6, 12, and 24 months. Finally, for each of these horizons, we compute the fraction of regression coefficients $\beta_{j,k,t}$ (across industries and 3-year regression windows) that are statistically different from zero at the 95% level.

We find that for 1-month inflation rates, only 11.4% of the relationships between sectoral and aggregate inflation are statistically significant. For longer horizons k , these fractions generally remain weak but do rise over time: 26.4%, 40.6%, 58.5%, and 69.1% for the 3-, 6-, 12-, and 24-month horizons, respectively. This supports our assumption that while disaggregate and aggregate price indices might be co-integrated in the long run, their short-run relationship is weak.

In fact, not only is the relationship statistically weak in general, but it is highly unstable. This can be seen in Figure A.1, which shows the evolution of the coefficient $\beta_{j,k,t}$ for $k = 3$ for 3-year-window regressions starting in each month between 1995 and 2010,

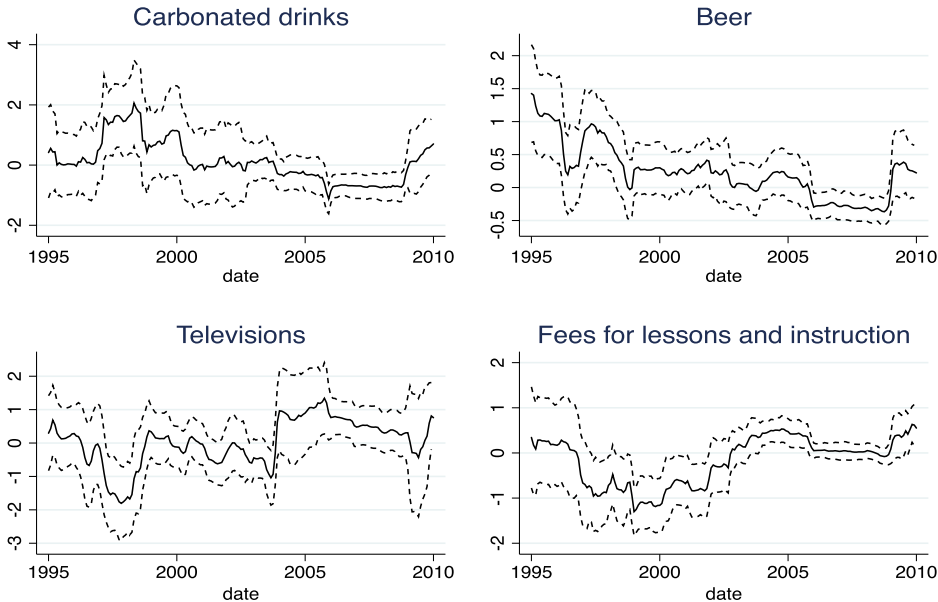


FIGURE A.1.—Three-year rolling regressions of 3-month industry inflation on 3-month aggregate inflation for four categories. The solid line plots the point estimate of regression coefficient on aggregate inflation. The dotted lines plot the 95% confidence intervals.

for four industries. Not only are there large fluctuations in the value of this coefficient over our sample, but sign reversals are common. In general, at any given date, there is little confidence that the near-future short-horizon industry-level inflation would be highly correlated with aggregate inflation, even though the data are quite clear that the two are tightly linked over the long run.

A.8. *The Typical Information Set at the Stochastic Steady State*

In this section, we analyze in more depth the typical information set at the stochastic steady state. In the model, the price histories and demand realizations differ across firms. One reason is the idiosyncratic noise in demand realizations, but more importantly, the position of the demand signals is endogenous, because it depends on the past pricing decisions of the firm. With idiosyncratic productivity shocks, firms take different pricing decisions, and thus their information sets evolve differently. Let

$$\mathcal{I}_{it} = [\tilde{\mathbf{r}}_{it}^{\text{uniq}}, \mathbf{N}_{it}, \hat{\mathbf{y}}_{it}]$$

be the 3-column matrix that characterizes the information set of firm i at time t , where $\tilde{\mathbf{r}}_{it}^{\text{uniq}}$ is the vector of *unique* unambiguously estimated relative price points in the history of past price decisions, \tilde{r}_{it}^t , of firm i ; \mathbf{N}_{it} is the associated vector of the number of times each of those unique price points has been chosen in the past; and $\hat{\mathbf{y}}_{it}$ is the average, demeaned demand realization that the firm has seen at those unique price points. So each row of $\tilde{\mathbf{r}}_{it}^{\text{uniq}}$ is one of the unique price levels the firm has posted in the past, the corresponding row of \mathbf{N}_{it} is the number of times this price has been seen in the past, and the corresponding row of $\hat{\mathbf{y}}_{it}$ is the average demeaned demand realizations the firm has experienced when choosing that price. The matrix \mathcal{I}_{it} fully described the information set of the firm, and is the sufficient statistic needed to compute the worst-case expected demand $\hat{x}_{it}(\tilde{r})$.

The most striking characteristic of \mathcal{I}_{it} is that the average cardinality of $\tilde{\mathbf{r}}_{it}^{\text{uniq}}$ is just six. Thus even though the average life span of firms in our model is 133 periods, the histories contain only six unique estimated relative prices on average. Another interesting characteristic is that the average firm has not seen each of those six price points equally often, but in fact the most often posted price accounts for 74% of all observations, on average. Moreover, the second most often chosen price accounts for another 19% of all observations.

These features of the typical information set can be helpful in understanding the pricing moments the model generates. First, the sparse nature of the information set implies that the typical firm faces substantial amount of residual demand uncertainty even in the long run. The reason behind this substantial residual demand uncertainty is that the history of observations is *endogenously sparse*. In particular, the optimal policy leads the firm to often repeat estimated relative prices, resulting in a history of observations that provides a lot of information about the average level of demand at those select prices, but leaves the firm uncertain about the *shape* of its demand in between the observed price points. Hence our mechanism, which operates specifically through the uncertainty about the local shape of demand, has a strong bite even at the steady state of the model, when firms have seen long histories of demand observations. In fact, because of the local nature of learning and the endogenous location of demand signals, learning proceeds so slowly that the mechanism survives even if firms live for thousands of periods. We explore this implication further in Supplemental Material Appendix B.6 by setting $\lambda_\phi = 0$. In the same appendix, we also show that the accumulation of new information could in fact change the optimal position of some of the reference prices.

Second, the typical information set still contains *multiple* distinct price points at which the firm has reduced demand uncertainty. Thus, the optimal pricing action does not only lead to price stickiness (i.e., reluctance to leave one of the price points with low residual demand uncertainty), but also to significant memory in prices since, conditional on a move, the firm is likely to move to one of the other price points it has learned about in the past, rather than to a brand new price.

Last, the typical information set also tends to feature one “dominant” price at which the firm has accumulated most of its past signals. This gives rise to pricing patterns where the firm has a clear “modal” or “reference” price point that it tends to stay at for prolonged periods of time and return to often.

A.9. Cell-Based Evidence on Hazard Functions

In Figure A.2, we plot the distributions of cell-based slopes obtained using the approach of Campbell and Eden (2014). A cell is a specific product sold in a given store, while the slope is computed as the difference between the price change frequencies of older and younger prices. An “old” price is one that has survived at least Γ weeks. In order to obtain a more complete comparison between the data and the model simulations than just the average slope, we plot both the empirical (left column) and simulated (right) distributions of the cell-based hazard slopes, for $\Gamma = 4, 5, 6$.

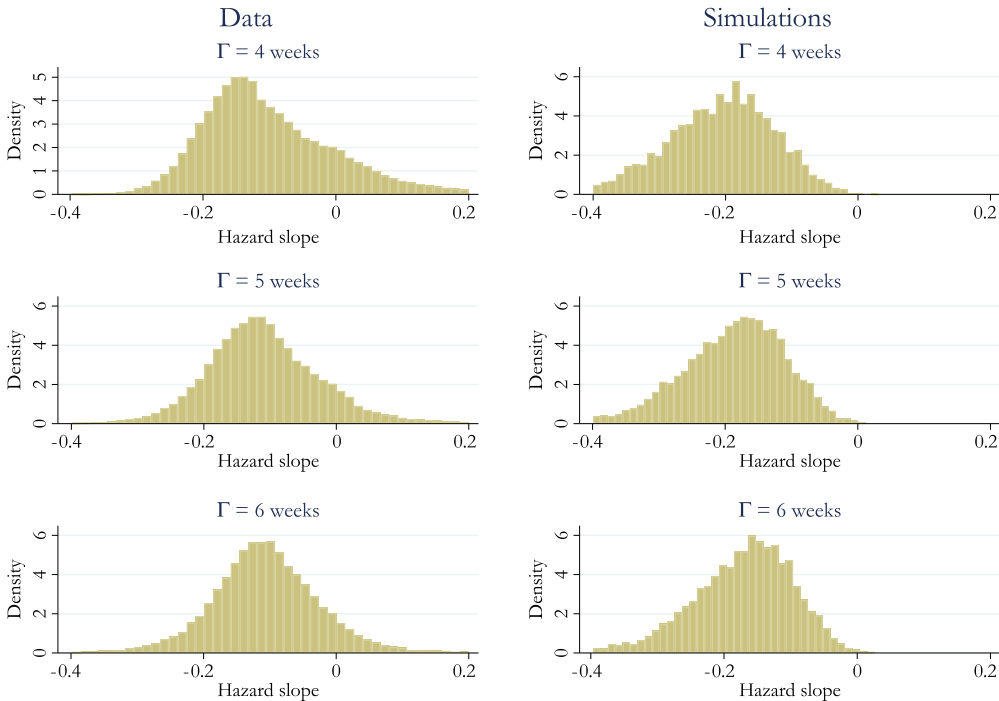


FIGURE A.2.—Distributions of the cell-based hazard slopes. A slope is defined as the difference between the price change frequencies of old ($\tau \geq \Gamma$) and young ($\tau < \Gamma$) prices. Empirical (left) and simulated (right) distributions.

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